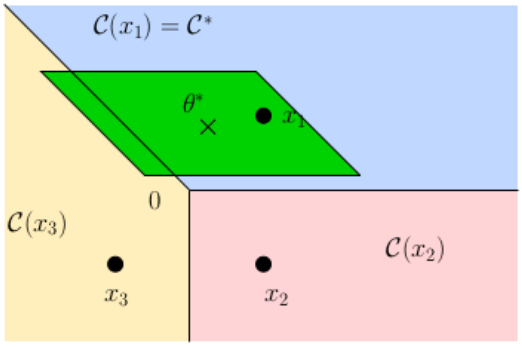
Project Report

1. **<u>Illustration of confidence set converging into the cone</u>**

   1) Compute the confidence set {a set which encapsulates the predicted theta and original theta with a given regret bound} at each phase of the algorithm .

   2) And therefore show as a result how the confidence set changes and converges into the cone of the optimal arm.



2. **<u>G/XY - static algorithm.</u>**
   Min-max optimization problem.
   Let f be,

$$x'^{\top}(A + xx^{\top})^{-1}x'$$

   then as shown in the RHS, to implement G-allocation strategy, we need to min-max f. This can be implemented as shown below

Input: decision space $\mathcal{X} \in \mathbb{R}^d$, confidence $\delta > 0$
Set: $t = 0$; $Y = \{y = (x - x'); x \neq x' \in \mathcal{X}\}$;
while Eq. 11 is not true **do**
  if G-allocation **then**
    $x_t = \arg\min_{x \in X} \max_{x' \in X} x'^{\top}(A + xx^{\top})^{-1}x'$
  else if $\mathcal{XY}$-allocation **then**
    $x_t = \arg\min_{x \in X} \max_{y \in Y} y^{\top}(A + xx^{\top})^{-1}y$
  end if
  Update $\hat{\theta}_t = A_t^{-1}b_t$, $t = t + 1$
end while
Return arm $\Pi(\hat{\theta}_t)$

Figure 2: Static allocation algorithms

Let X_1, X_2, X_3 be the arms given.

X_1	X_2 => f_12 X_3 => f_13	T1 =Max { f_12, f_13 }	Min { T1, T2, T3 }
X_2	X_1 => f_21 X_3 => f_23	T1 =Max { f_21, f_23 }	
X_3	X_1 => f_31 X_2 => f_32	T2 =Max { f_31, f_32 }	

3. <u>XY – adaptive algorithm</u>

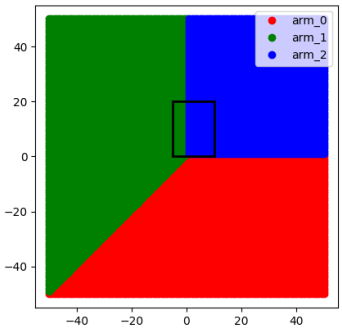
Let X_hat_j => Set of potentially optimal arms in phase j

<u>Algorithm:</u>

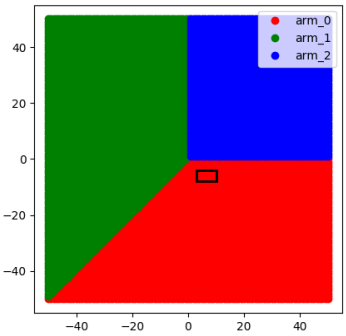
1. In each phase we implement XY iterative algo.
2. The phase length is determined by the uncertainty present in estimating the active directions between successive phases.
3. Once a phase ends then we compute theta_hat using OLS method.
4. We then use the sub-optimal condition to remove the arms from X_hat_j.
5. And loop over the above steps until | X_hat_j | = 1

Results obtained:

Visualization of confidence set



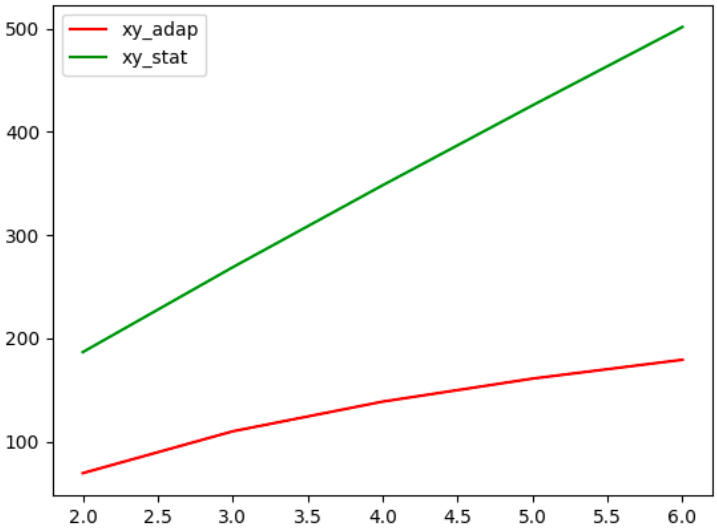
Initial phase of the algorithm



After completion of the algorithm

Plot of Samples required vs Dimensionality over an average of 15 runs

Samples required



Dimensionality

Observations: A sample run of the program, shows that XY-adaptive algorithm eliminates the dominated arms at intial phases leading it to find the best arm in lesser no.of samples when compared to XY-static which samples across all the arms evenly.

Samples/arm	*$\mathcal{XY}$-adaptive*	$\mathcal{XY}$
x_1	195	30124
x_2	48382	29517
x_3	5	31024
x_4	1	32014
x_5	1	28092
x_6	5	3
Budget	48589	118760