ELL 714
Information Theory
Binary Symmetric Channel Coding

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Objective:

1. Design a binary symmetric channel with flip probability being 0.4.
2. Design an optimal decoder for the same channel with Joint typicality decoder.
3. Observe the trend with increasing sequence size
4. Observe the trend with epsilon

Theory:

The following are some basic results for BSC.

$$C = \sup_{p_X(x)} I(X;Y)$$

$$C = 1 + (plogp + (1-p)log(1-p))$$

$$M = \lceil 2^{nR} \rceil$$

Our value of C = 0.029.

Experimental Settings:

R = 0.01, p=0.04, theta(p(x=1)) = 0.4

H(X) = 0.97

H(Y) = 0.99

H(X,Y) = 1.94

I(X;Y) = 0.0278

Observations:

1)

n	epsilon	r	p	theta	error	Codebook(size)
100	0.01	0.01	0.4	0.4	1	2
200	0.01	0.01	0.4	0.4	0.75	4
300	0.01	0.01	0.4	0.4	0.82	8
400	0.01	0.01	0.4	0.4	0.66	16
500	0.01	0.01	0.4	0.4	0.625	32
550	0.01	0.01	0.4	0.4	0.69	46
600	0.01	0.01	0.4	0.4	0.65	64
700	0.01	0.01	0.4	0.4	0.6	128
800	0.01	0.01	0.4	0.4	0.41	256

2)

n	epsilon	r	p	theta	error	codebook
100	0.015	0.01	0.4	0.4	1	2
200	0.015	0.01	0.4	0.4	0.75	4
300	0.015	0.01	0.4	0.4	0.5	8
400	0.015	0.01	0.4	0.4	0.5	16
500	0.015	0.01	0.4	0.4	0.375	32
550	0.015	0.01	0.4	0.4	0.45	46
600	0.015	0.01	0.4	0.4	0.43	64
700	0.015	0.01	0.4	0.4	0.41	128
800	0.015	0.01	0.4	0.4	0.33	256

3)

n	epsilon	r	p	theta	error	codebook
100	0.02	0.01	0.4	0.4	1	2
200	0.02	0.01	0.4	0.4	0.75	4
300	0.02	0.01	0.4	0.4	0.5	8
400	0.02	0.01	0.4	0.4	0.56	16
500	0.02	0.01	0.4	0.4	0.46	32
550	0.02	0.01	0.4	0.4	0.5	46
600	0.02	0.01	0.4	0.4	0.42	64
700	0.02	0.01	0.4	0.4	0.46	128
800	0.02	0.01	0.4	0.4	0.5	256

4)

n	epsilon	r	p	theta	error	codebook
100	0.03	0.01	0.4	0.4	0	2
200	0.03	0.01	0.4	0.4	0.25	4
300	0.03	0.01	0.4	0.4	0.375	8
400	0.03	0.01	0.4	0.4	0.81	16
500	0.03	0.01	0.4	0.4	0.93	32
550	0.03	0.01	0.4	0.4	0.78	46
600	0.03	0.01	0.4	0.4	0.96	64
700	0.03	0.01	0.4	0.4	0.98	128
800	0.03	0.01	0.4	0.4	0.99	256

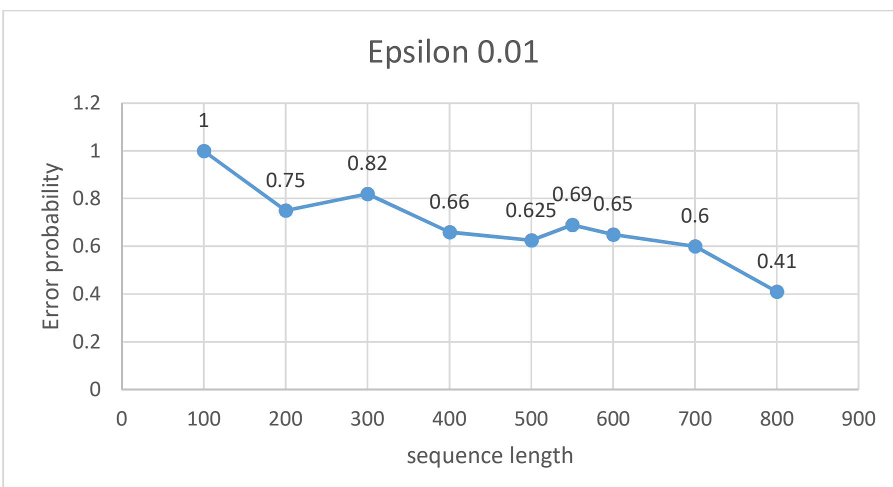

Fig1: Epsilon = 0.01

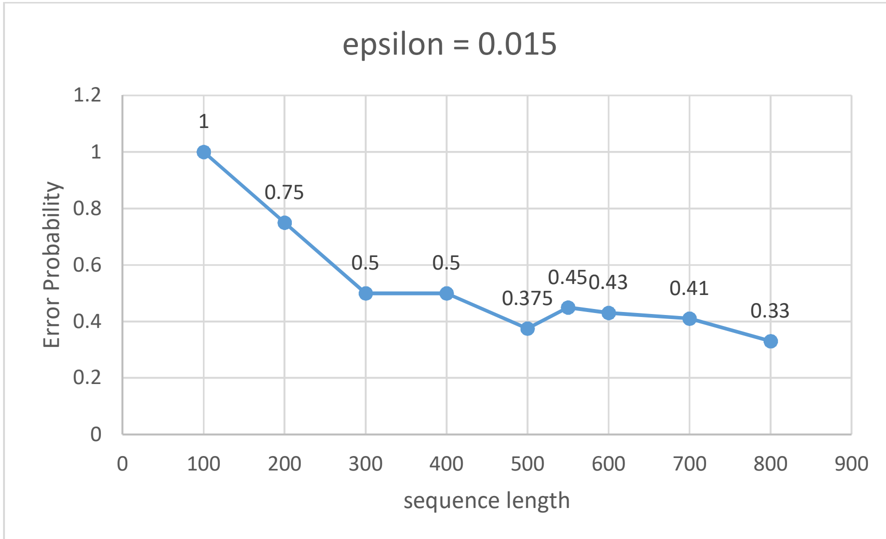

Fig2: Epsilon = 0.015

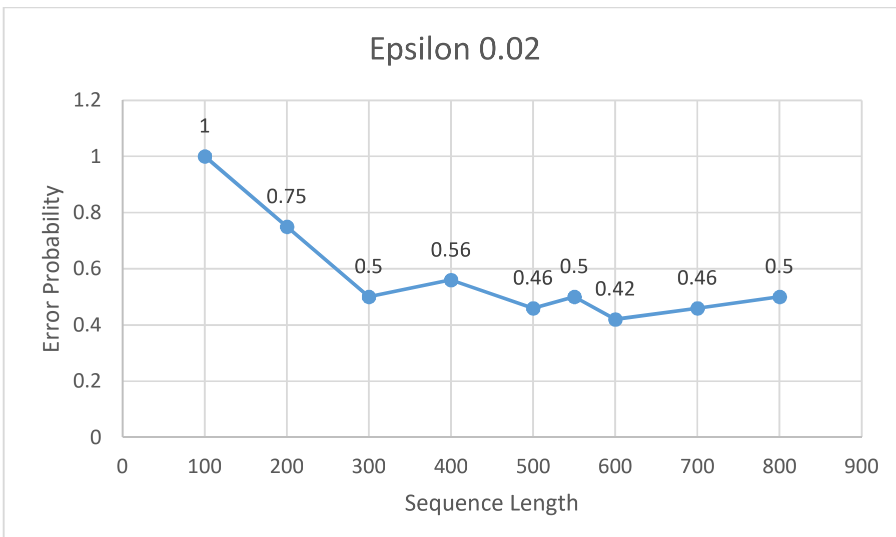

Fig3: Epsilon = 0.02

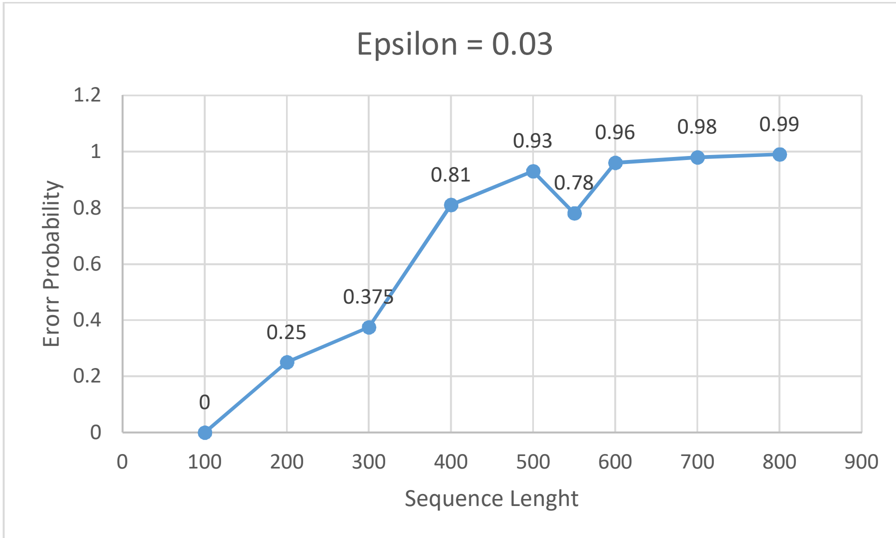

Fig4: Epsilon = 0.03

Conclusions:

1) It was observed that for only a restricted interval, we were able to see the trend where error rate was decreasing with increment in the sequence length.
2) It was our observation that with too less epsilon, there were very few elements in the typical set. Given our computing capacity, we were not able to get results for such high N.
3) With big epsilon value, we saw that many sequences were typical. Hence our decoder were not able to decode the output.
4) For epsilon 0.3, it was the case that our error rate was increasing with sequence length. We were not able to justify that. However, discussions are welcomed.
5) In general for well-behaved it was a general trend that error rate decreased as n increased.
6) This method had complexity of O(n2^nr) for decoding scheme.

Future Prospects:

1) We can try looking at the trends of changing channel capacity by changing the channel flipover probability.
2) Changing rate value and input distribution.