CS & IT ENGINEERING

Discrete maths
GRAPH THEORY

Lecture No. 1

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TOPICS TO BE COVERED

01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Basics of Graph

Discrete maths:

- → G.T.
- → G.T + CS subjects

→ Graph Theory (4 – 6)

→ logic. (2 – 4)

→ Set theory. (2 – 4)

→ Combinatorics. (2 – 4)

Basics of Graph

Graph Theory :

1) A ✓

2) all bridges ×

Basics of Graph

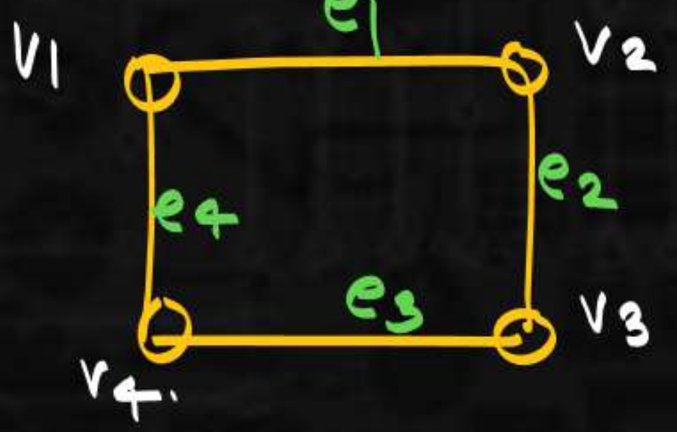
joint / point → vertex / vertices.

line / branch → edge / edges.

Graph $G = (V, E)$

V → set of vertices

E → set of edges.



$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4\}$

$G = (V, E)$

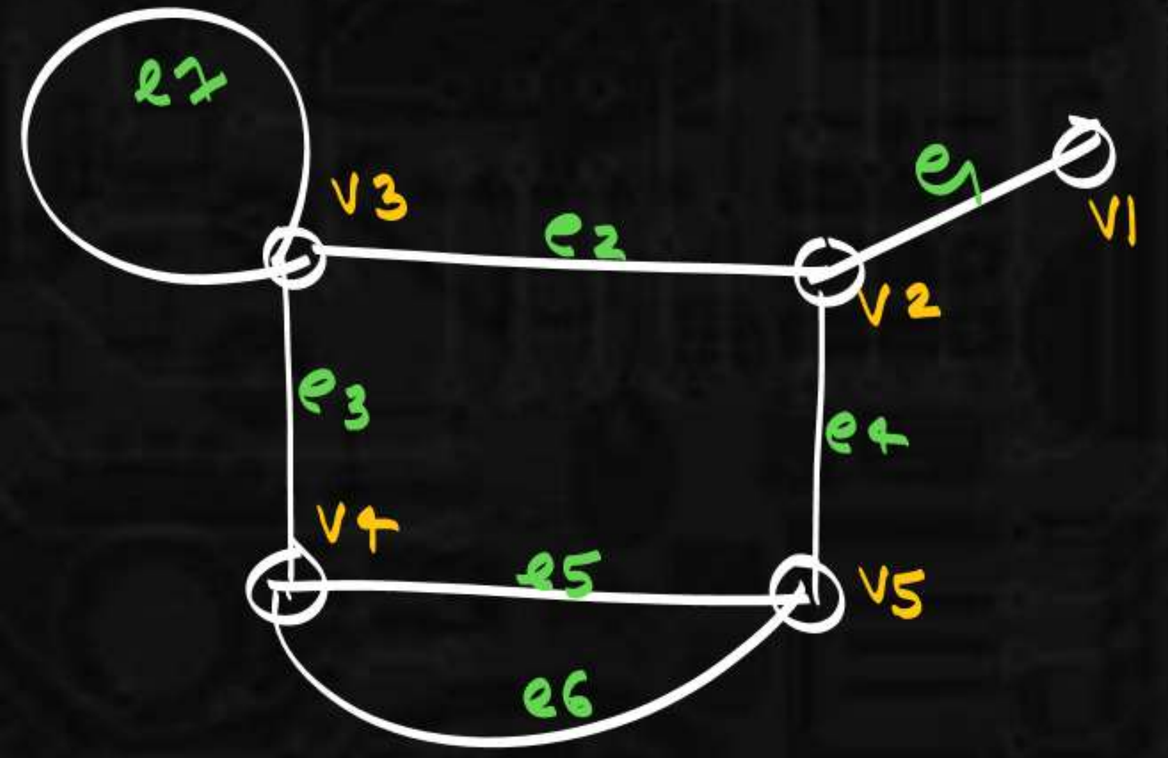
Basics of Graph

$G = (V, E)$

V = set of vertices.

E = set of edges.

→ each edge must be associated with unorder pair of vertices.



$G = (V, E)$

$V = \{v1, v2, v3, v4, v5\}$

$E = \{e1 \ldots e7\}$

$e1 \rightarrow (v1, v2)$

$e2 \rightarrow$

Basics of Graph

$$G = (V, E, \psi)$$

$$V = \{ \ldots\ldots \}$$

$$E = \{ \ldots\ldots \}$$

$$\psi : \underline{E} \longrightarrow \underline{V \times V}$$

$$f(x) = x^2$$

Basics of Graph

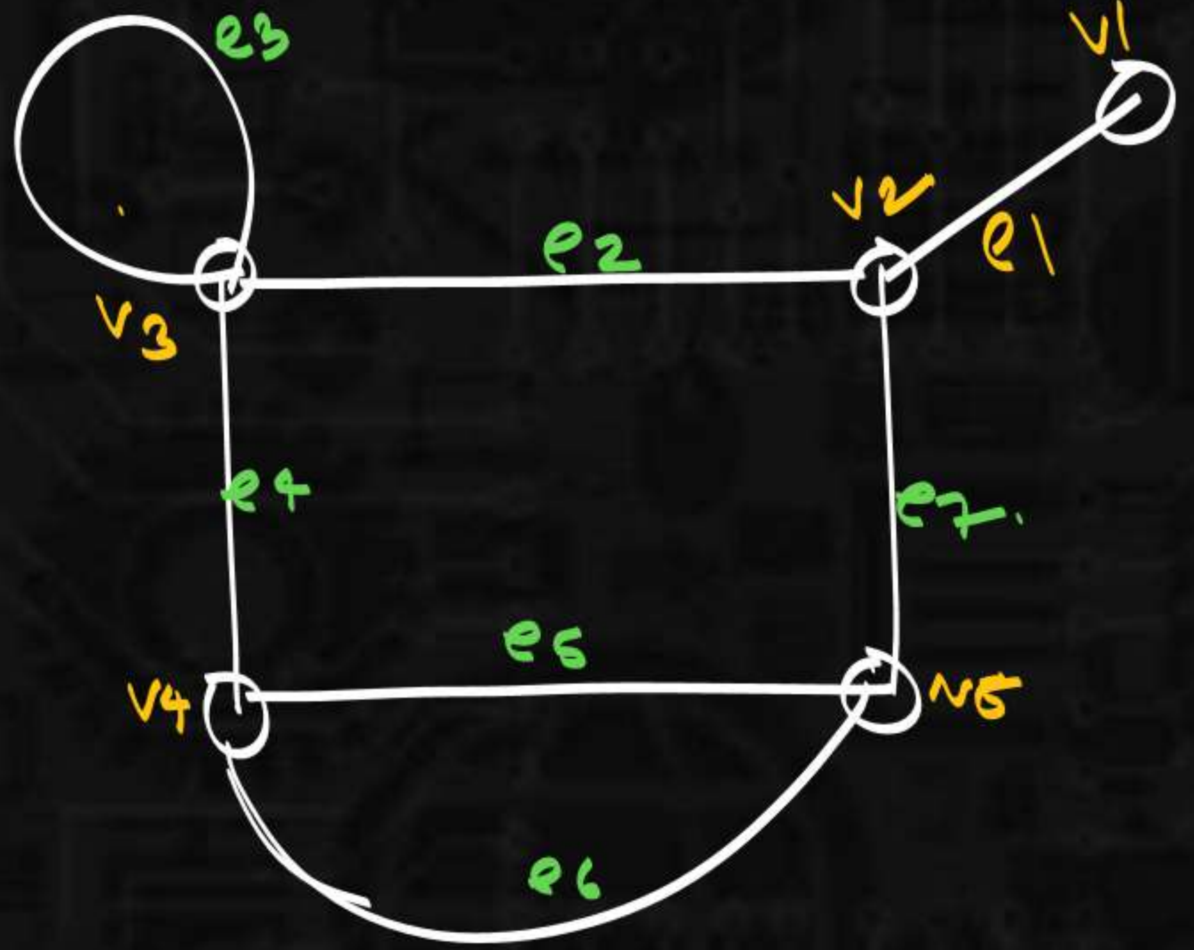
end vertices:

unordered pair of vertices are called end vertices.

loop/self-loop:

$e3 \rightarrow (v3, v3)$

if end vertices are same edge is called loop.



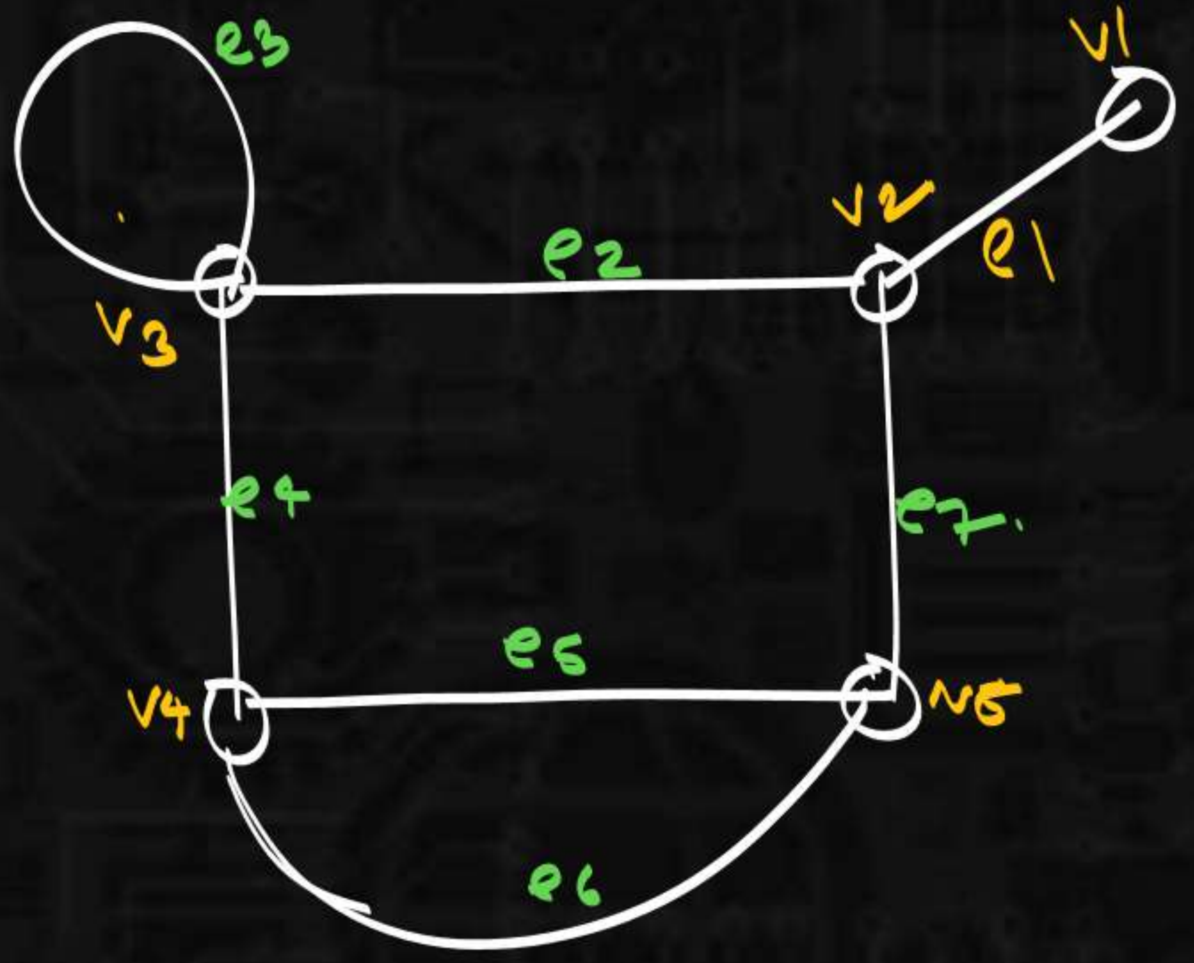
Basics of Graph

|| edges:

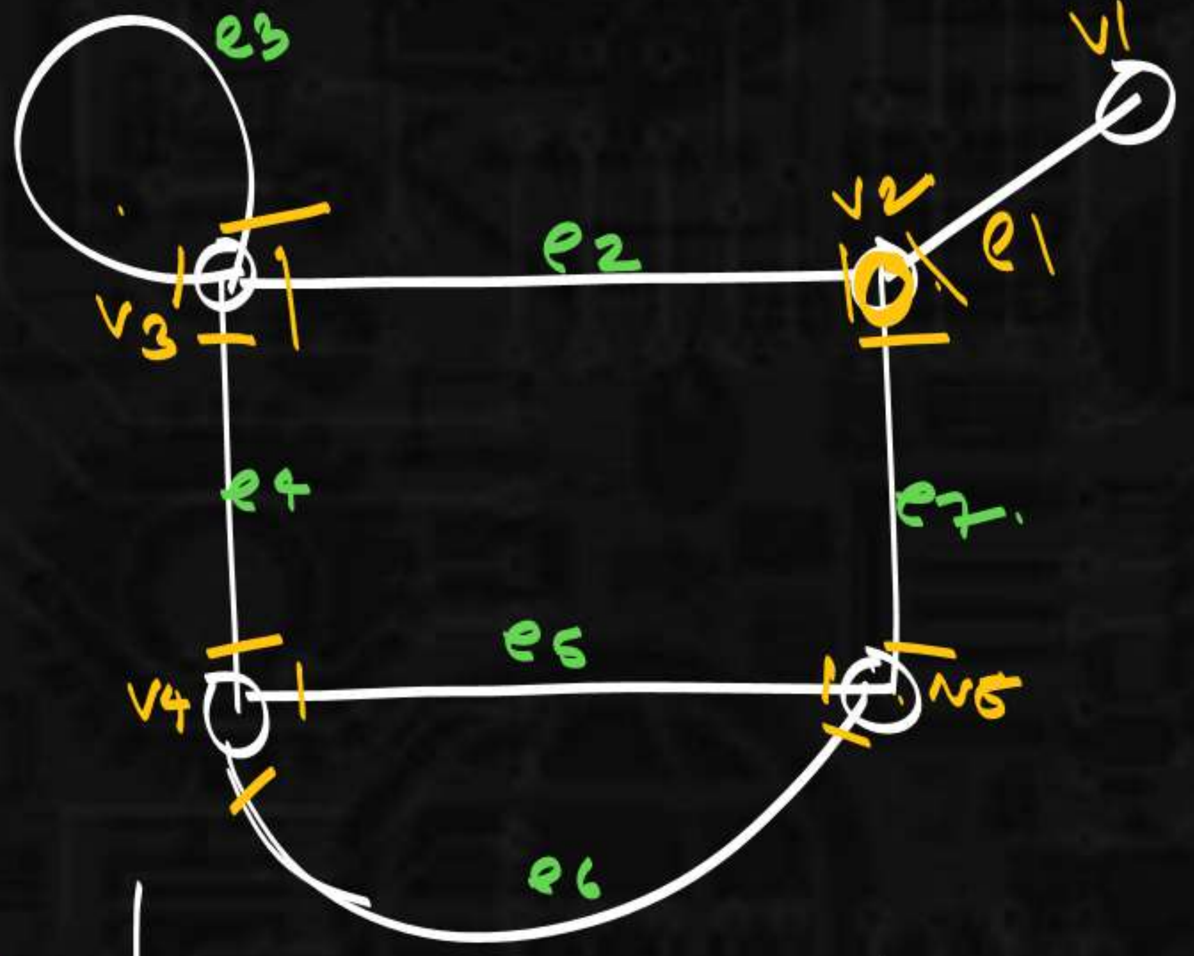
$e_5 \rightarrow (v_4, v_5)$

$e_6 \rightarrow (v_4, v_5)$

2 or more edges associated with same end vertices called as || edges.



Basics of Graph



incident point:

meeting point of vertex & edge.

Degree/valency ($d(v_i)$)

no. of edges incident on a vertex is called degree of vertex

$d(v_1) = 1.$

$d(v_2) = 3.$

isolated vertex

Degree 0

Pendant vertex

Degree 1.

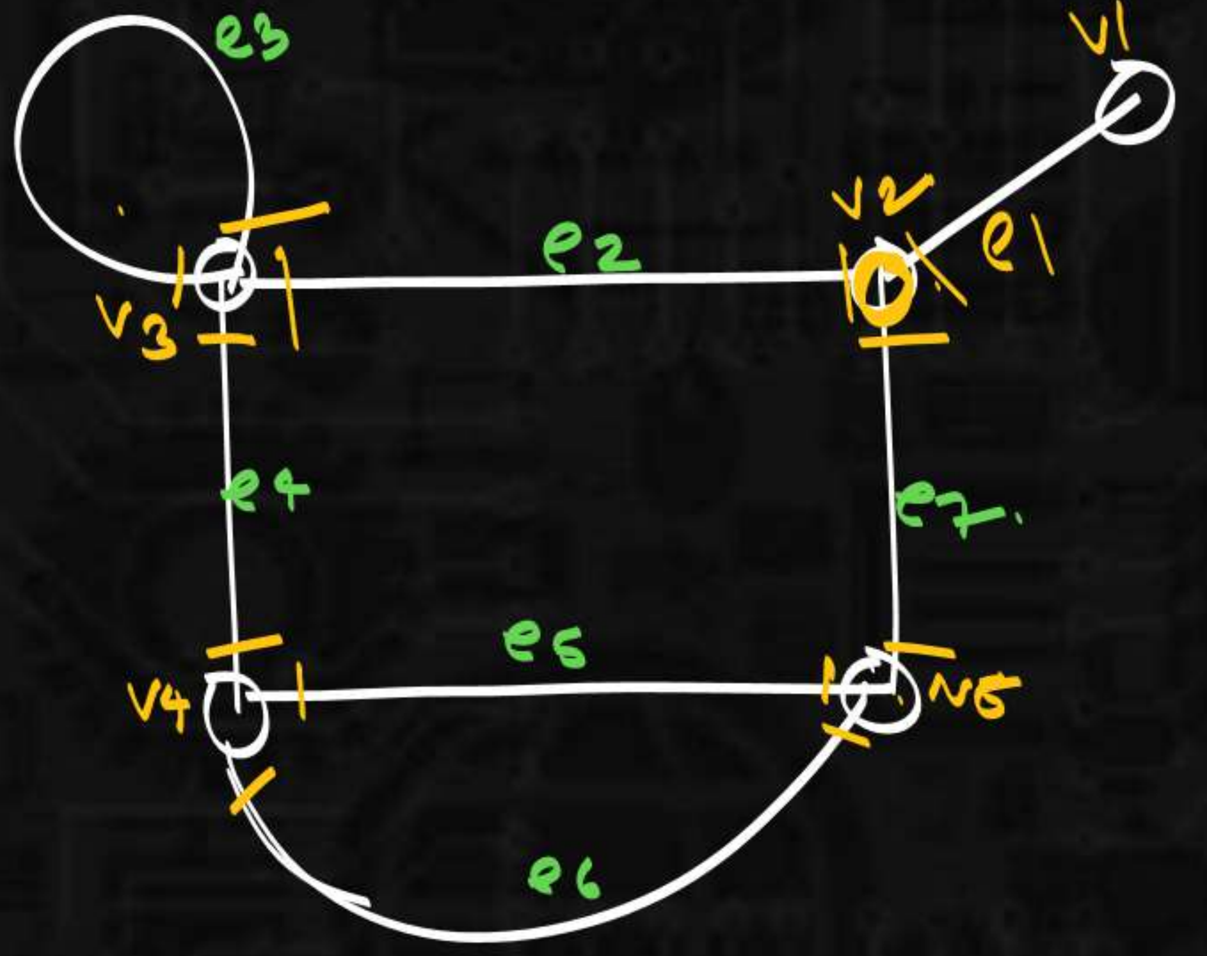
Basics of Graph

null Graph:

Set of isolated vertices.

Trivial Graph.

Basics of Graph



$d(v_1) = 1$ $\quad d(v_2) = 3$ $\quad d(v_3) = 4$

$d(v_4) = 3$ $\quad d(v_5) = 3$

$$d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$$

$$= 1 + 3 + 4 + 3 + 3 = 14$$

$$= 2 \times 7 \rightarrow \text{(no. of edges)}$$

$$= 2e$$

Basics of Graph

Thm1: Sum of degrees of all vertices is equals to twice the no. of edges.

$$\sum d(v_i) = 2e.$$

$$\sum d(v_i) = 2 \times e$$

$$\sum d(v_i) = \text{even}.$$

7 edges (possible)

$\sum d(v_i) = 7$ ✗.

Basics of Graph

odd degree $\quad \sum d(v_i) = \text{even}.$

$\boxed{o_1, o_2, o_3} + \boxed{e_1, e_2} = \text{even}.$

$\boxed{o_1 + o_2 + o_3} \downarrow$ odd + even.

$e_1, e_2 \downarrow$ even.

$= \text{odd} \neq \text{even}.$

$d(v_1) = 1.$ odd degree vertex

$d(v_2) = 2.$ even degree vertex.

$O + O = \text{even}.$

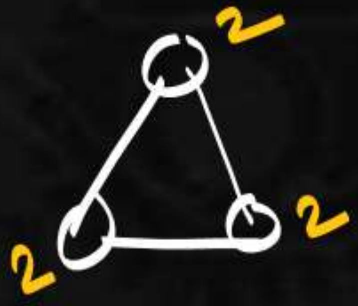
$1 + 3 = 4.$

$O + O + O = \text{odd}$

$1 + 3 + 5 = 9.$

Basics of Graph

<u>Thm 2</u> no. of odd degree vertices in a graph must be even.



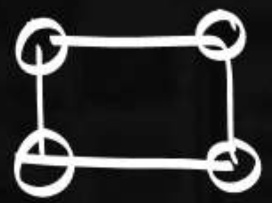
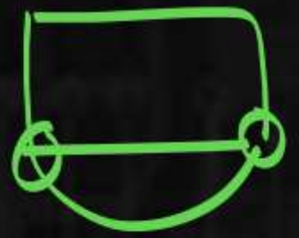
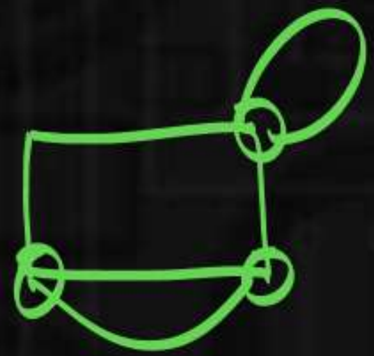
odd degrees vertices = 2

O.d.v = 2.

O.d.v = 0.

Basics of Graph

3-20

	loop	∥ edges	
Simple Graph	×	×	
multigraph	×	✓	
Pseudograph.	✓	✓	

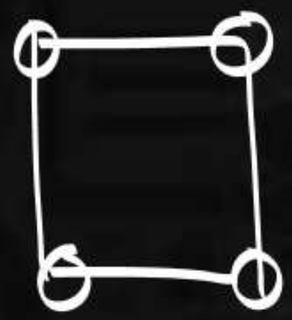
Basics of Graph

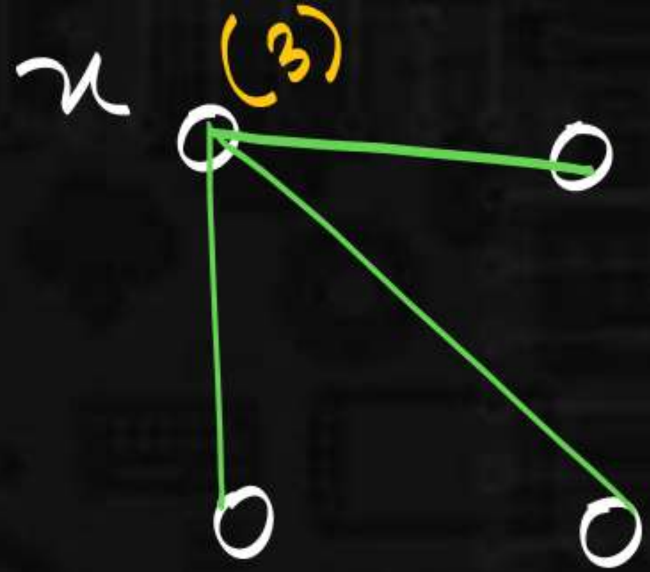
Thm 3: maximum degree in simple Graph $\leq n-1$.

$n = 4$ (vertices)

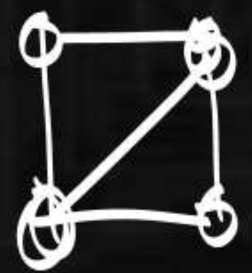
Total vertices $= n$.

$n = 4$.





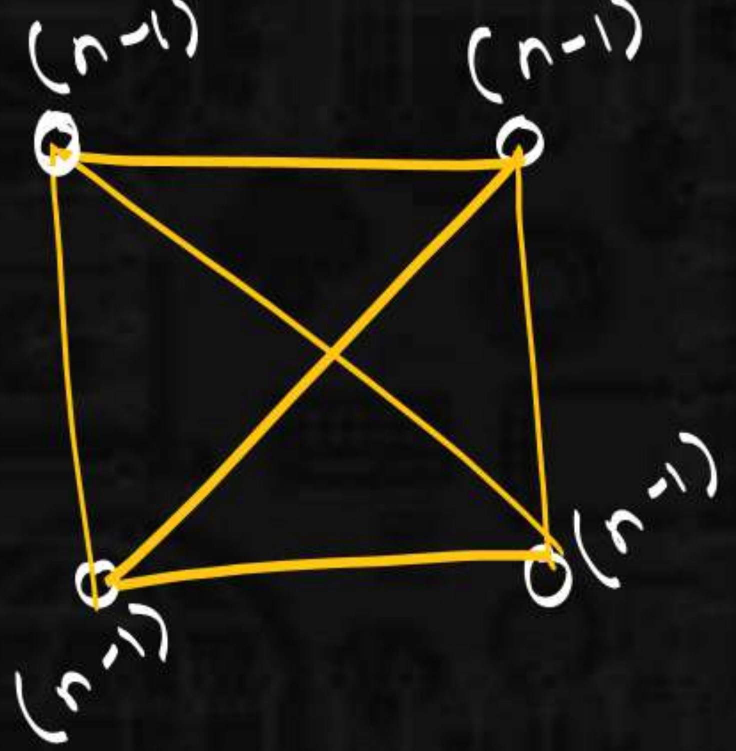
$n = 4$.



Basics of Graph

<u>Thm 4</u>: maximum no. of edges in simple Graph $\leq \frac{n(n-1)}{2}$.

Total vertices = 4.



degrees of all vertices = n-1.

Total vertices = n.

$\Sigma d(v_i) = 2e.$

$n \times (n-1) = 2e$

$$\boxed{e = \frac{n(n-1)}{2}}$$

<u>note</u>:

Basics of Graph

note: Degrees of all vertices are n-1 then it will have exactly $\frac{n(n-1)}{2}$ edges.

$e = \frac{n(n-1)}{2}$ edges $\rightarrow$ Degrees of all vertices are n-1.

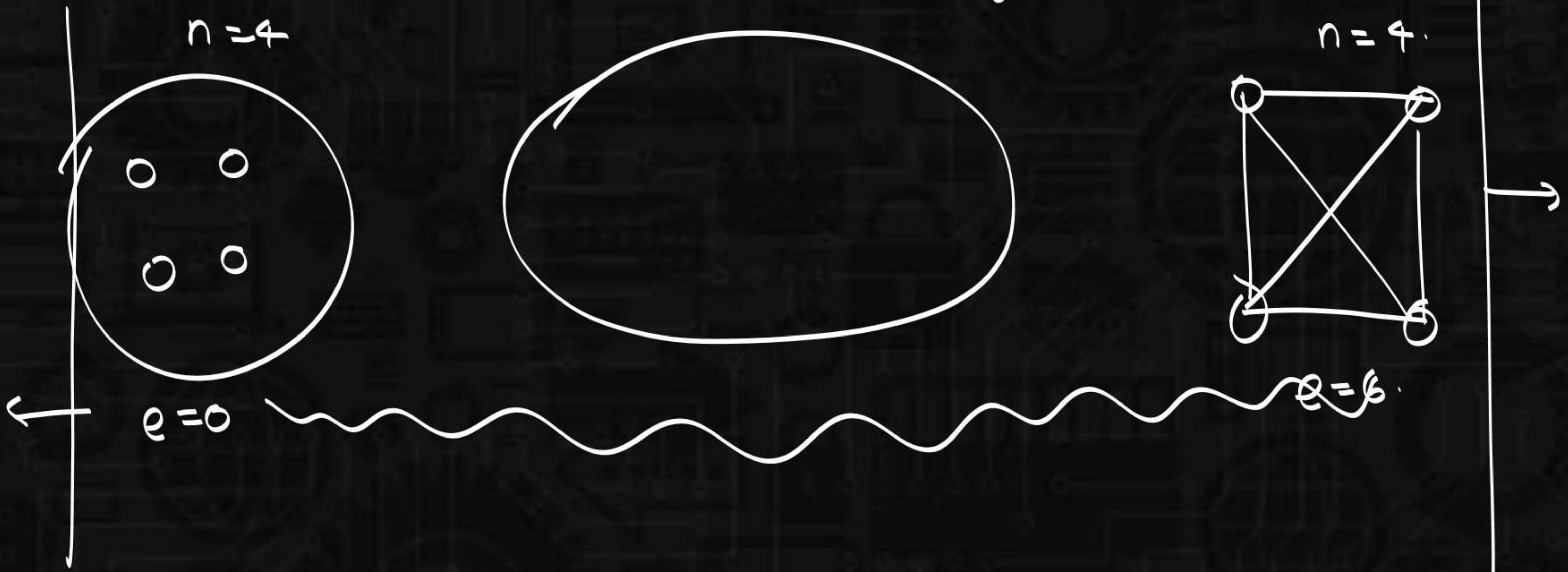
$\rightarrow$ 10 vertices, all degrees are 9. $10 \times 9/2 = 45$ edges

$\rightarrow$ 10v, 45 edges $\rightarrow$ degrees of all $\rightarrow$ 9

Basics of Graph

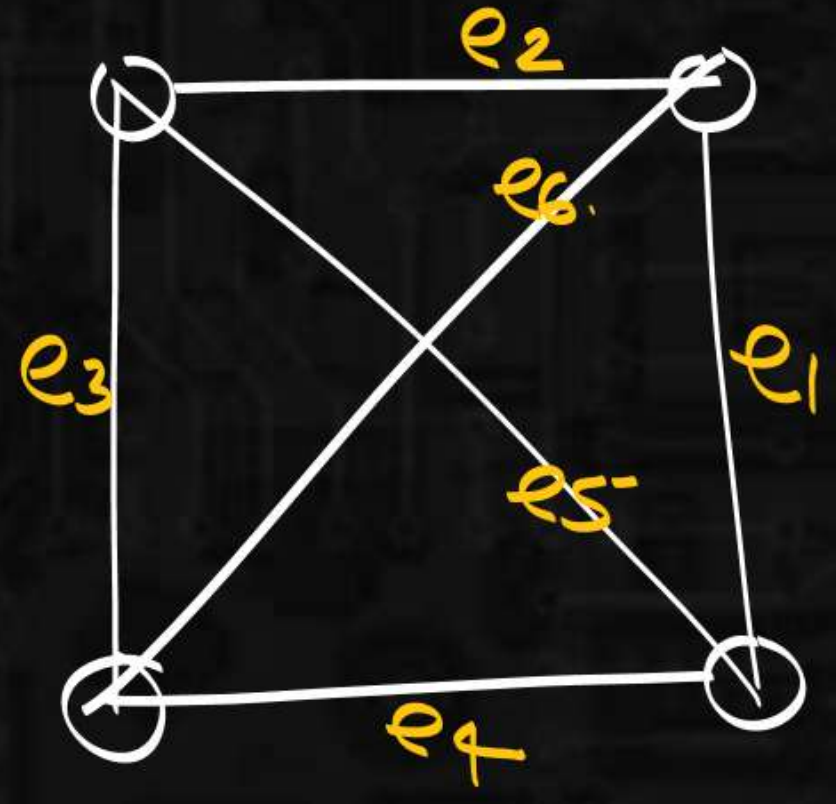
Total vertices = 4.

How many graphs are possible = ?



Basics of Graph

$\cancel{24}$



2^6

e_1	e_2	e_3	e_4	e_5	e_6
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1			
⋮					
1	1	1	1	1	1

Basics of Graph

$n = 4.$

no. of edges $= \frac{4 \times 3}{2} = 6.$

Total vertices $= 4.$

Total no. of graphs $= 2^{4 \times 3/2}.$

Total no. of vertices $= n.$

Total no. of graphs $= 2^{\frac{n(n-1)}{2}}$

Basics of Graph

Total no. of graphs:

$^6C_0 + {^6C_1} + {^6C_2} + {^6C_3} + {^6C_4} + {^6C_5} + {^6C_6} = 2^6$.

Total vertices = 4.

no. of graphs with 4 vertices & 1 edge = $^6C_1$

Total vertices = n.

Total no. of graphs n vertices & e edges. $^{\frac{n(n-1)}{2}}C_e$

Basics of Graph

$$^6C_0 + \boxed{^6C_1 + ^6C_2 + ^6C_3 + ^6C_4 + ^6C_5 + ^6C_6} = 2^6$$

How many graphs are possible with atleast 1 edges & 4 vertices.

$$= 2^6 - {}^6C_0$$

Basics of Graph

- What will be total no. of vertices if 4v will have 3 degrees, 6v will have 2 degrees & remaining vertices will have degree 3 & 27 edges?

Total no. of vertices $= 10 + 6 + 4$

$e = 27$

$\Sigma d(v_i) = 2e.$

Remaining vertices $= x$

$4v \times 3 + 6v \times 2 + xv \times 3 = 2e.$

$12 + 12 + 3x = 2 \times 27$

$3x = 54 - 24.$

$3x = 30$

$x = 10$

Basics of Graph

In Graph 15 edges & degrees of each vertex is atleast 3.
What will be maximum no. of vertices?

$e = 15 \quad \delta(G) = 3$

$n = 10$

$$\delta(G) \le \frac{2e}{n}$$

$$3 \le \frac{2 \times 15}{n}$$

$$n \le 30/3$$

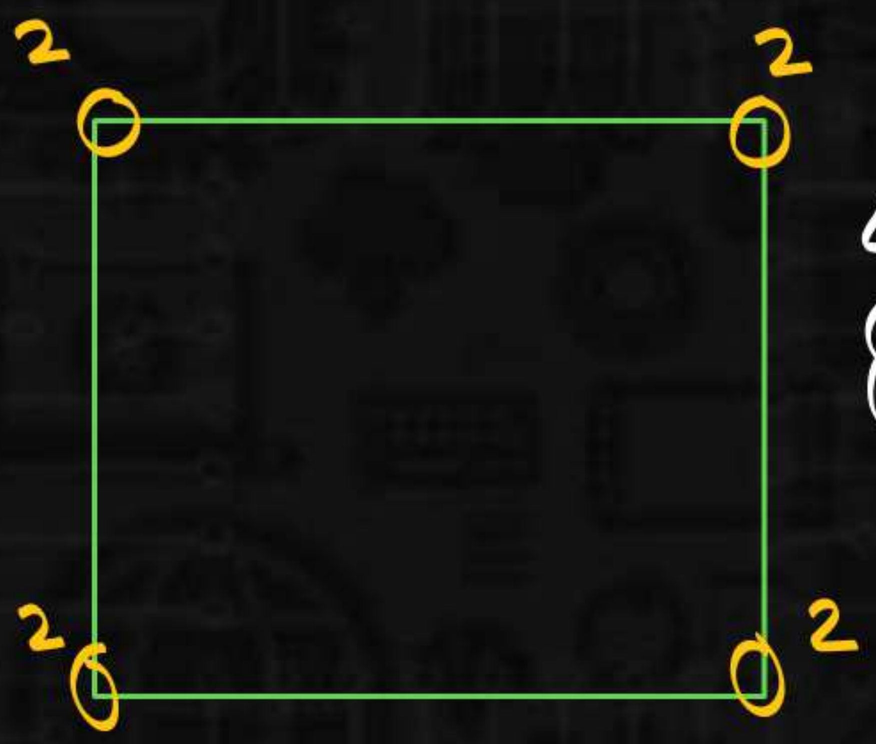
$$\boxed{n \le 10}$$

$(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10) \le 10$

Basics of Graph

minimum degree($\delta(G)$)

maximum degree ($\Delta(G)$)

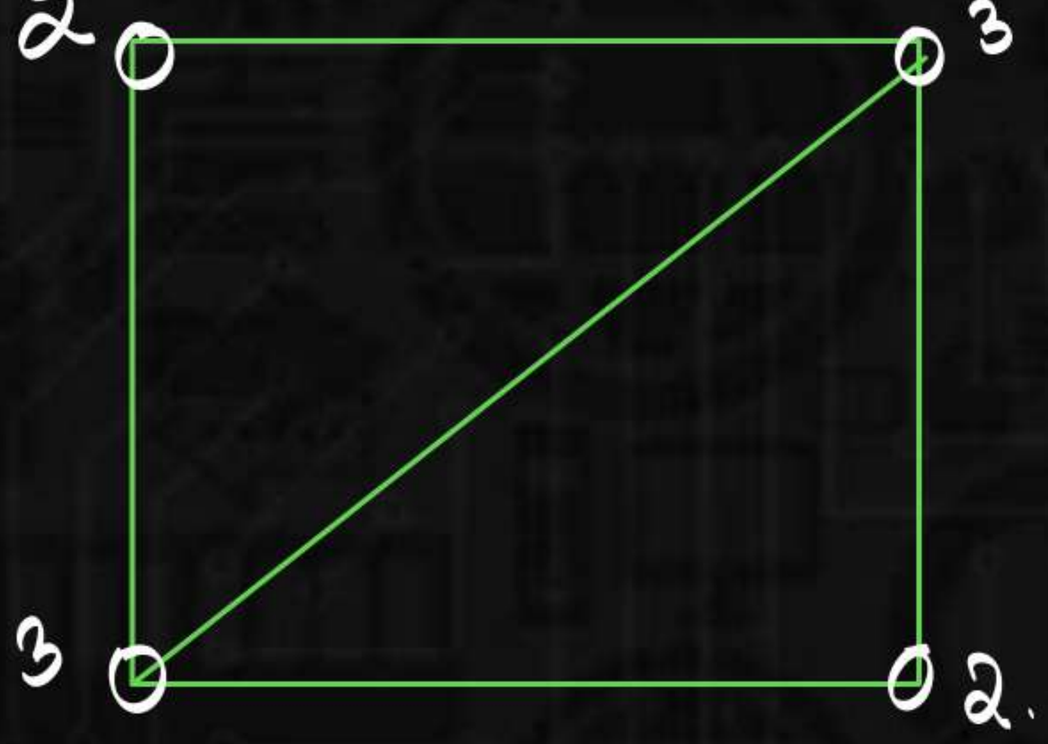


$\Delta(G)=2$

$\delta(G)=2$.

$\Delta(G)=3$

$\delta(G)=2$



Basics of Graph

$2e/n = 2.$

$$\delta(G) = \frac{2e}{n} = \Delta(G) \quad \text{— I}$$

$\delta(G) = 2$

$\Delta(G) = 3.$

avg. degree =

$$\text{Avg. degree} = \frac{d_1+d_2+d_3+d_4}{\text{Total vertices.}} = \frac{2e.}{n.}$$

$$\delta(G) < 2e/n < \Delta(G) \quad \text{— II}$$

$$= \frac{2+2+2+2.}{4}$$

$$= 2.$$

$\delta(G) = \frac{2e}{n} = \Delta(G)$ — I. $\delta(G) < 2e/n < \Delta(G)$ — II.

almost.

$$\boxed{\delta(G) \leq 2e/n} \leq \Delta(G) \leq n-1.$$

atleast

Q.1

Graph G has 14 vertices & 27 edges.

Degree of each vertex of G is 3, 4 or 5

There are six vertices of degree 4.

How many vertices of G have degree 3?

Q.2

vertices = 12

edges = 31

Degree of each vertex 4 or 6

How many vertices of degree 4?

Q.3.

$n = 25$
$e = 62$

Degree of each vertex is 3, 4, 5 or 6

2 v will have degree 4

11 v will have degree 6

How many vertices of G have degree 5?

11 vertices

Degree of each vertex is
at least 3
at most 5.

no. of edges lie between.

________ and ________.


THANK YOU GW
SOLDIERS !