1. Solve the following 0/1 Knapsack problem using dynamic programming $p = \{11, 21, 31, 33\}$ $w = \{2, 11, 22, 15\}$, $m = 40$, $n = 4$.

Sol given $p = \{11, 21, 31, 33\}$ $w = \{2, 11, 22, 15\}$

$S^0 = \{(0,0)\}$,

$S^1 = S^0 \cup S^0_1$

$S^0_1 =$ add p_1, w_1 correspondingly with $S^0 = \{(0+11, 0+2)\} = \{(11,2)\}$

$S^1 = \{(0,0)\} \cup \{(11,2)\} = \{(0,0), (11,2)\}$ $\quad w_i \le m$ (✓) verified mergepurgerule

$S^2 = S^1 \cup S^1_1$

$S^1_1 = \{(0+21, 0+11), (11+21, 2+11)\} = \{(21,11), (32,13)\}$

$w_i \le m$ (✓) verified merge purge rule

$S^2 = \{(0,0), (11,2)\} \cup \{(21,11), (32,13)\} = \{(0,0), (11,2), (21,11), (32,13)\}$

$S^3 = S^2 \cup S^2_1$

$S^2_1 = \{(0+31, 0+22), (11+31, 2+22), (21+31, 11+22), (32+31, 13+22)\}$

$S^2_1 = \{(31,22), (42,24), (52,33), (63,35)\}$

$S^3 = \{(0,0), (11,2) (21,11), (32,13)\} \cup \{(31,22) (42,24) (52,33) (63,35)\}$

$= \{(0,0) (11,2), (21,11), (31,\cancel{22}), (32,13), (42,24), (52,33) (63,35)\}$

$w_i \le m$

$S^3 = \{(0,0) (11,2) (21,11) (32,13) (42,24) (52,33) (63,35)\}$

$S^4 = S^3 \cup S^3_1$

$S^3_1 = \{(0+33, 0+15) (11+33, 2+15), (21+33, 11+15), (32+33, 13+15),$
$(42+33, 24+15), (52+33, 33+15), (63+33, 35+15)\}$

$S^3_1 = \{(33,15), (44,17), (54,26), (65,28), (75,39),$
$(85,48), (96,50)\}$

$S^4 = \{(0,0) (11,2), (21,11), (32,13), (42,24) (52,33), (63,35)\}$
$\cup \{(33,15) (44,17) (54,26) (65,28) (75,39), (85,48),$
$(96,50)\}$

$S^4 = \{(0,0) (11,2) (21,11) (32,13) (33,15) (42,\cancel{24}) (44,17)$
$(52,\cancel{33}) (54,26) (63,\cancel{35}) (65,28) (75,39) (85,\cancel{48})$
$(96,\cancel{50})\}$

$S^4 = \{(0,0), (11,2) (21,11) (32,13) (33,15), (44,17) (54,26)$
$(65,28) (75,39)$

optimal profit = 39.

x_1	x_2	x_3	x_4
1	0	1	1

$$\begin{array}{r} 75,\ 39 \\ -\ 33\ \ 15 \\ \hline 42,\ 24 \\ -\ 31\ \ 22 \\ \hline 11,\ \ 2 \end{array}$$

(2) Find the optimal solution to the 0/1 knapsack instance n=7 objects and the capacity of knapsack m=15. The profits and weights of the objects are $(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10, 5, 15, 7, 6, 18, 3)$ ~~object are~~ $(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (2, 3, 5, 7, 1, 4, 1)$ respectively

Sol

$S^0 = \{(0,0)\}$

$S^1 = S^0 \cup S_1^0$

$S_1^0 = \{(0+10), (0+2)\} = \{(10,2)\}$

$S^1 = \{(0,0)\} \cup \{(10,2)\} = \{(0,0); (10,2)\}$

$S^2 = S^1 \cup S_1^1$

$S_1^1 = \{ (0+5, 0+3), (10+5, 2+3)\} = \{(5,3), (15,5)\}$

$S^2 = \{(0,0), (10,2)\} \cup \{(5,3), (15,5)\} = \{(0,0) (10,2) (5,3) (15,5)\}$.
$= \{(0,0) (5,3) (10,2) (15,5)\}$

$S^3 = S^2 \cup S_1^2$

$S_1^2 = \{ (0+15, 0+5), (10+15, 2+5) (15+15, 5+5)\}$
$= \{(15,5), (25,7), (30,10)\}$

$S^3 = \{(0,0) (10,2), (15,5)\} \cup \{(15,5) (25,7) (30,10)\}$
$= \{(0,0), (10,2) (15,5), ~~(15,5)~~ (25,7) (30,10)\}$

$S^4 = S^3 \cup S_1^3$

$S_1^3 = \{(0+7, 0+7) (10+7, 2+7) (15+7, 5+7) (25+7, 7+7), (30+7, 10+7)\}$

$S_1^3 = \{(7,7) (17,9) (22,12) (32,14) (37,17)\}$

$S^4 = \{(0,0) (10,2) (15,5) (25,7) (30,10)\} \cup \{(7,7) (17,9) (22,12) (32,14), (37,17)\}$

$S^4 = \{(0,0) (7,7) (10,2) (15,5) (17,9) (22,12) (25,7) ~~(32,14)~~, (30,10) (32,14) (37,17)\}$

$S^4 = \{(0,0)(10,2)(15,5)(17,9)(25,7)(30,10)(32,14)\}$

$S^5 = S^4 \cup S^4_1$

$S^4_1 = \{(0+6, 0+1), (10+6, 2+1)(15+6, 5+1)(17+6, 9+1), (25+6, 7+1), (30+6, 10+1)(32+6, 14+1)\}$

$S^4_1 = \{(6,1), (16,3)(21,6)(23,10)(31,8)(36,11)(38,15)\}$

$S^5 = \{(0,0)(10,2)(15,5)(17,9)(25,7)(30,10)(32,14)\} \cup \{(6,1), (16,3)(21,6)(23,10)(31,8)(36,11)(38,15)\}$

$S^5 = \{(0,0)(6,1)(10,2)\cancel{(15,5)}(16,3)\cancel{(17,9)}(21,6)\cancel{(23,10)}(25,7)\cancel{(30,10)}, (31,8)\cancel{(32,14)}(36,11)(38,15)\}.$

$\cancel{S^5 = \{(0,0)}(6$

$S^5 = \{(0,0)(6,1)(10,2)(21,6)(25,7)(31,8)(36,11)(38,15)\}$
$(16,3).$

$S^6 = S^5 \cup S^5_1$

$S^5_1 = \{(0+18, 0+4)(6+18, 1+4)(10+18, 2+4)(21+18, 6+4)(25+18, 7+4)(31+\cancel{25}\,18, 8+4)(36+\cancel{8}18, 11+4)(38+18, 15+4)(16+18, 3+4)\}$

$S^5_1 = \{(18,4)(24,5)(28,6)(39,10)(43,11)(49,12)(54,15)(56,19)(34,7)\}$

$S^6 = \{(0,0)(6,1)(10,2)(21,6)(25,7)(31,8)(36,11)(38,15)(16,3)\} \cup \{(18,4)(24,5)(28,6)(39,10)(43,11)(49,12)(54,15)(56,19)(34,7)\}$

$S^6 = \{(0,0)(6,1)(10,2)(16,3)(18,4)\cancel{(21,6)}(24,5)\cancel{(25,7)}(28,6)\cancel{(31,8)}(34,7)(36,11)\cancel{(38,15)}(39,10)(43,11)\cancel{(49,12)}(54,5)\cancel{(56,19)}\}$

$S^6 = \{(0,0)(6,1)(10,2)(16,3)(18,4)(24,5)(28,6)(34,7)(36,11)\cancel{(34,10}(39,10)(43,11)(\cancel{54}\,54,5)\}$

$S^7 = S^6 \cup S^6_1$

$S_1^6 = \{(0+3, 0+1), (6+3, 1+1)(10+3, 2+1)(16+3, 3+1)$
$(24+3, 5+1)(28+3, 6+1)(34+3, 7+1)(36+3, 11+1)$
$(39+3, 10+1)(43+3, 11+1)(54+3, 5+1)\}$

$S_1^6 = \{(3,1)(9,2),(13,3),(19,4)(27,6).(31,7)(37,8)$
$(39,12)(42,11)(46,12)(57,6)\}$

$S^7 = \{(0,0),(6,1)(10,2)(16,3)(24,5)(28,6)(34,7)$
$(36,11)(39,10)(43,11)(54,5)\} \cup \{(3,1)(9,2)(13,3)$
$(19,4)(27,6)(31,7)(37,8)(39,12)(42,11)(46,12),$
$(57,6)\}$

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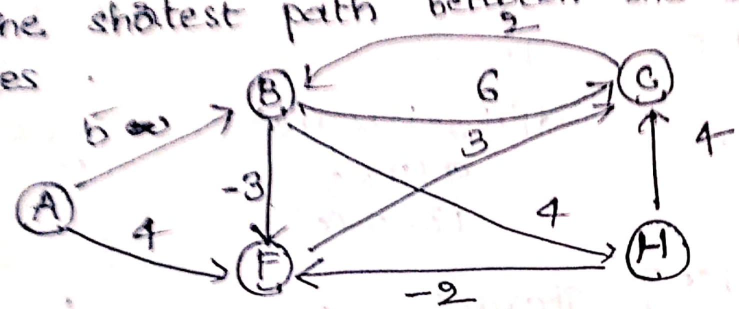
$S^7 = \{(0,0)(3,1)(6,1)(9,2)(10,2)(\cancel{16+3})(13,3)(16,3)(19,4)$
$(24,5)(27,6).(28,6)(31,7)(34,7)(36,11)$
$(37,8)(39,10)(39,12)(42,11)(43,11)(46,12)(54,5)$
$(57,6)\}$

solution vector

x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	1	1	0	1	1	0.

④ Find the shortest path between the source vertex A to every other vertices.



Sol

A → B C → B B → H

A → F F → C

B → C H → F

B → F H → C.

formula $d[u] + w < d[v]$.

	A	B	C	H	F
	0	∞	∞	∞	∞
1	0	5	~~4~~5	9	~~4~~2
2	0	5	5	9	2
3	0	5	5	9	2
4	0	5	5	9	2
5	0	5	5	9	2

iteration-1

$A \to B$

$d[A]+w < d[B]$

$0+5 < \infty$ (✓)

$d[B]=5$

$A \to F$

$d[A]+w < d[F]$

$0 + 4 < \infty$ ✓

$d[F]=4$

$B \to C$

$d[B]+w < d[C]$

$5+6 < \infty$ (✓)

$d[C]=11$

$B \to F$

$d[B]+w < d[F]$

$5+-3 < 4$

$2 < 4$ (✓)

$C \to B$

$d[C]+w < d[B]$

$11+2 < 5$ ✗

$F \to C$

$d[F]+w < d[C]$

$2+3 < 11$

$5 < 11$ (✓)

$d[C]=5$

$H \to F$

$d[H]+w < d[F]$

$9+-2 < 2$

$7 < 2$ (✗)

$H \to C$

$d[H]+w < d[C]$

$9+4 < 5$

$13 < 5$ (✗)

$B \to H$

$d[B]+w < d[H]$

$5+4 < \infty$ (✓)

$d[H]=9$

Iteration-2

$A \to B$

$d[A]+w < d[B]$

$0+5 < 5$ (✗)

$A \to F$

$d[A]+w < d[F]$

$0+4 < 2$ (✗)

$B \to C$

$d[B]+w < d[C]$

$5+6 < 5$

$11 < 5$ (✗)

$B \to F$

$d[B]+w < d[F]$

$5+-3 < 2$

$2 < 2$ (✗)

$B \to H$

$d[B]+w < d[H]$

$5+4 < 9$

$9 < 9$ (✗)

$F \to C$

$d[F]+w < d[C]$

$2+3 < 5$

$5 < 5$ (✗)

$H \to F$

$d[H]+w < d[F]$

$9+-2 < 2$

$7 < 2$ (✗)

$H \to C$

$d[H]+w < d[C]$

$9+4 < 5$

$13 < 5$ (✗)

iteration-3

$A \to B$

$d[A]+w < d[B]$

$0+5 < 5$ (✗)

$A \to F$

$d[A]+w < d[F]$

$0+4 < 2$ (✗)

$B \to C$

$5+6 < 5$

$11 < 5$ (✗)

$B \to F$

$d[B]+w < d[F]$

$2 < 2$ (✗)

$B \to H$

$d[B]+w < d[H]$

$9 < 9$ (✗)

$F \to C$

$d[F]+w < d[C]$

$5 < 5$ (✗)

$H \to F$

$d[H]+w < d[F]$

$9+-2 < 2$

$7 < 2$ (✗)

$H \to C$

$d[H]+w < d[C]$

$9+4 < 5$

$13 < 5$ (✗)

iteration-4

$A \to B$

$d[A]+w < d[B]$

$0+5 < 5$ (✗)

$A \to F$

$d[A]+w < d[F]$

$0+4 < 2$ (✗)

$B \to C$

$5+6 < 5$

$11 < 5$ (✗)

iteration-4

~~A→~~ $B \to F$

$d[B]+w < d[F]$

$2 < 2$ (✗)

$B \to H$

$9 < 9$ (✗)

$F \to C$

$d[F]+w < d[C]$

$5 < 5$ (✗)

$H \to F$

$d[H]+w < d[F]$

$7 < 2$ (✗)

$H \to C$

$d[H]+w < d[C]$

$13 < 5$ (✗)

iteration-5

$A \to B$

$5 < 5$ (✗)

$B \to C$

$11 < 5$ (✗)

$B \to F$

$2 < 2$ (✗)

$B \to H$

$9 < 9$ (✗)

$H \to F$

$7 < 2$ (✗)

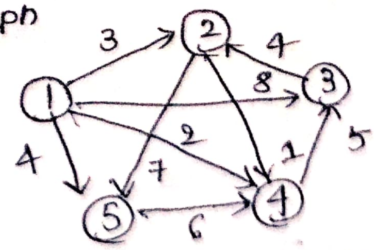
$H \to C$

$13 < 5$ (✗)

$F \to C$

$5 < 5$ (✗)

7. Find the shortest path between all pair of vertices in the following graph



Ans By using Allpairshortest path algorithm, the distance matrix updation rule is.

$D^k[i,j] = \min(D^{k-1}[i,j],\ D^{k-1}[i,k] + D^{k-1}[k,j])$

k = Intermediate node.

Cost matrix =
$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 8 & 2 & 4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \infty & 4 & 0 & \infty & \infty \\ 4 & \infty & \infty & 5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \end{array}$$

Initially Distance matrix $D^0 =$
$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 8 & 2 & 4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \infty & 4 & 0 & \infty & \infty \\ 4 & \infty & \infty & 5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \end{array}$$

$D' =$
$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 8 & 2 & 4 \\ 2 & \infty & 0 & \boxed{\infty} & \boxed{1} & \boxed{7} \\ 3 & \infty & \boxed{4} & 0 & \boxed{\infty} & \boxed{\infty} \\ 4 & \infty & \boxed{\infty} & \boxed{5} & 0 & \boxed{\infty} \\ 5 & \infty & \boxed{\infty} & \boxed{\infty} & \boxed{6} & 0 \end{array}$$

$D'[2,3] = \min(D^0[2,3],\ D^0[2,1] + D^0[1,3])$
$= \min(\infty,\ \infty+8) = \infty.$

$D'[2,4] = \min(D^0[2,4],\ D^0[2,1] + D^0[1,4])$
$= \min(1,\ \infty+2) = 1$

Illy

$D'[2,5] = \min(7,\ \infty+4) = 7$

$D'[3,2] = \min(4,\ \infty+3) = 4$

$D'[3,4] = \min(\infty,\ \infty+2) = \infty$

$D'[3,5] = \min(\infty,\ \infty+4) = \infty$

$D'[4,2] = \min(\infty,\ \infty+3) = \infty$

$D'[4,3] = \min(5,\ \infty+8) = 5$

$D'[4,5] = \min(\infty,\ \infty+4) = \infty.$

$D'[5,2] = \min(\infty,\ \infty+3) = \infty.$

$D'(5,3) = \min(\infty,\ \infty+8) = \infty$

$D'(5,4) = \min(6,\ \infty+2) = 6$

$$D^2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & \boxed{8} & \boxed{2} & \boxed{4} \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \boxed{\infty} & 4 & 0 & \boxed{5} & \boxed{11} \\ 4 & \boxed{\infty} & \infty & \boxed{5} & 0 & \boxed{\infty} \\ 5 & \boxed{\infty} & \infty & \boxed{\infty} & \boxed{6} & 0 \end{array}$$

$$D^1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 8 & 2 & 4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \infty & 4 & 0 & \infty & \infty \\ 4 & \infty & \infty & 5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \end{array}$$

$= \min(D'[1,3] , D'[1,2] + D'[2,3])$

$D^2[1,3] = \min(8, 3+\infty) = 8.$

$D^2[1,4] = \min(D'[1,4] , D'[1,2] + D'[2,4])$

$= \min(2, 3+1) = 2$

Ily $D^2[1,5] = \min(4, 3+7) = 4$

$D^2[3,1] = \min(\infty, 4+\infty) = \infty.$

$D^2[3,4] = \min(\infty, 4+1) = 5$

$D^2[3,5] = \min(\infty, 4+7) = 11$

$D^2[5,3] = \min(\infty, \infty+7) = \infty.$

$D^2[4,1] = \min(\infty, \infty+\infty) = \infty$

$D^2[4,3] = \min(5, \infty+1) = 5$

$D^2[4,5] = \min(\infty, \infty+7) = \infty$

$D^2[5,1] = \min(\infty, \infty+7) = \infty$

$D^2[5,4] = \min(6, \infty+1) = 6$

$$D^3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \boxed{3} & 8 & \boxed{2} & \boxed{4} \\ 2 & \boxed{\infty} & 0 & \infty & \boxed{1} & \boxed{7} \\ 3 & \infty & 4 & 0 & 5 & 11 \\ 4 & \boxed{\infty} & \boxed{9} & 5 & 0 & \boxed{16} \\ 5 & \boxed{\infty} & \boxed{4} & \infty & \boxed{6} & 0 \end{array}$$

$D^3[1,2] = \min(D^2[1,2], D^2[1,3] + D^2[3,2])$

$= \min(3, 8+4) = 3$

$D^3[1,4] = \min(D^2[1,4], D^2[1,3] + D^2[3,4])$

$= \min(2, 8+5) = 2$

Ily $D^3[1,5] = \min(4, 8+11) = 4$

$D^3[2,1] = \min(\infty, \infty+\infty) = \infty.$

$D^3[2,4] = \min(1, \infty + 5) = 1$

$D^3[2,5] = \min(7, \infty + 11) = 7$

$D^3[4,1] = \min(\infty, 5 + \infty) = \infty.$

$D^3[4,2] = \min(\infty, 5 + 4) = 9$

$D^3[4,5] = \min(\infty, 5 + 11) = 16$

$D^3[5,1] = \min(\infty, \infty + \infty) = \infty$

$D^3[5,2] = \min(\infty, \infty + 4) = 4$

$D^3[5,4] = \min(6, \infty + 5) = 6$

$$D^4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \boxed{4} & \boxed{7} & 2 & \boxed{4} \\ 2 & \boxed{\infty} & 0 & \boxed{6} & 1 & \boxed{7} \\ 3 & \boxed{\infty} & \boxed{4} & 0 & 5 & \boxed{11} \\ 4 & \infty & 9 & 5 & 0 & 16 \\ 5 & \boxed{\infty} & \boxed{4} & \boxed{11} & 6 & 0 \end{array}$$

$D^4 = [1,2] = \min(D^3[1,2], D^3[1,4] + D^3[4,2])$
$= \min(4, 5 + 9) = 4$

$D^4[1,3] = \min(D^3[1,3], D^3[1,4] + D^3[4,3])$
$= \min(8, 2 + 5) = \cancel{8}\ 7$

1ly $D^4[1,5] = \min(4, 2 + 16) = 4$

$D^4[2,1] = \min(\infty, 1 + \infty) = \infty.$

$D^4[2,3] = \min(\infty, 1 + 5) = 6.$

$D^4[2,5] = \min(7, 1 + 9) = 7$

$D^4[3,1] = \min(\infty, 5 + \infty) = \infty.$

$D^4[3,2] = \min(4, 5 + 9) = 4$

$D^4[3,5] = \min(11, 5 + 16) = 11$

$D^4[5,1] = \min(\infty, 6 + \infty) = \infty.$

$D^4[5,2] = \min(4, 6 + 9) = 4$

$D^4[5,3] = \min(\infty, 6 + 5) = 11$

$$D^5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \boxed{4} & \boxed{7} & \boxed{2} & 4 \\ 2 & \boxed{\infty} & 0 & \boxed{6} & \boxed{1} & 7 \\ 3 & \boxed{\infty} & \boxed{4} & 0 & \boxed{5} & 11 \\ 4 & \boxed{\infty} & \boxed{9} & \boxed{5} & 0 & 16 \\ 5 & \infty & 4 & 11 & 6 & 0 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 7 & 2 & 4 \\ 2 & \infty & 0 & 6 & 1 & 7 \\ 3 & \infty & 4 & 0 & 5 & 11 \\ 4 & \infty & 9 & 5 & 0 & 16 \\ 5 & \infty & 4 & 11 & 6 & 0 \end{array}$$

$D^5[1,2] = \min(D^4[1,2],\ D^4[1,5] + D^4[5,2])$

1ly $= \min(4,\ 4+4) = 4$

$D^5[1,3] = \min(7,\ 4+11) = 7$

$D^5[1,4] = \min(\infty, 2,\ 4+\infty) = 2$

$D^5[2,1] = \min(\infty,\ 7+\infty) = \infty$

$D^5[2,3] = \min(6,\ 7+11) = 6$

$D^5[2,4] = \min(1,\ 7+6) = 1$

$D^5[3,1] = \min(\infty,\ 11+\infty) = \infty$

$D^5[3,2] = \min(4,\ 11+4) = 4$

$D^5[3,4] = \min(5,\ 11+6) = 5$

$D^5[4,1] = \min(\infty,\ 16+\infty) = \infty$

$D^5[4,2] = \min(9,\ 16+4) = 9$

$D^5[4,3] = \min(5,\ 16+11) = 5$