① solve the following instance of fractional knapsack problem? no of items = 5, m=15   profit 12 15 9 17 16
weight 3 5 3 6 7

Sol
Given, n=5, M=15, By using greedy method,

Step-①:- Calculate p_i/w_i

$p_1/w_1 = \frac{12}{3} = 4$, $p_2/w_2 = 15/3 = 3$, $p_3/w_3 = \frac{9}{3} = 3$, $p_4/w_4 = \frac{17}{6} = 2.83$,
$p_5/w_5 = 16/7 = 2.28$.

step-②:- sort the p_i/w_i ratio values in descending order $\Rightarrow \frac{p_i}{w_i} \geq \frac{p_j}{w_j}$

$(p_1, p_2, p_3, p_4, p_5) = (12, 15, 9, 17, 16)$
$(w_1, w_2, w_3, w_4, w_5) = (3, 5, 3, 6, 7)$

step ③:- $R = m$

① → add item i_1
$R = R - w_1$ $w_1 < R$
$= 15 - 3$ $3 \leq 15$ ✓
$= 12$
$\boxed{x = 1}$

② → add item i_2,
$R = R - w_2$ $3 \leq 12$
$= 12 - 3 = 9$
$\boxed{x_2 = 1}$

③ → add item i_3
$R = R - w_3$
$= 9 - 3$
$= 6$ $3 \leq 9$ ✓
$\boxed{x_3 = 1}$

⑤ → add item p_4,
$R = R - w_4$
$= 6 - 6 = 0$ $6 \leq 6$ (✓)
$\boxed{x_4 = 1}$

⑤ R=0, discard items

step ④:- profit $= \sum_{i=1}^{n} p_i x_i$
$= p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4 + p_5 x_5$
$= 12(1) + 15(1) + 9(1) + 17(1) + 16(0) = 53$

∴ optimal profit = 53

Result vector x_1 x_2 x_3 x_4 x_5
1 1 1 1 0.

② solve the following instance of fractional knapsack problem?

no of items = 5,   profit 20 15 22 09 11
m=30   weight 18 20 15 5 10.

Given n=5, m=30.
By using greedy approach,

step 1:- Calculate profit ratio p_i/w_i

$\frac{p_1}{w_1} = \frac{20}{18} = 1.11$, $\frac{p_2}{w_2} = \frac{15}{20} = 0.75$, $\frac{p_3}{w_3} = \frac{22}{15} = 1.46$, $\frac{p_4}{w_4} = \frac{9}{5} = 1.8$, $\frac{p_5}{w_5} = 1.1$

step 2:- sort $\frac{p_i}{w_i}$ values in descending order

$(p_4, p_3, p_1, p_5, p_2) = (9, 22, 20, 11, 15)$
$(w_4 = w_3, w_1, w_5, w_2) = (5, 15, 18, 10, 22)$

step 3:- $R = m$, $R = 30$.

① → add item i_4,
$R = R - w_4$
$= 30-5 \quad 5 \le 30$
$\boxed{x_4 = 1}$

② → add item i_5
$R = R - w_3 \quad 15 \le 25$ ✓
$= 10$
$\boxed{x_3 = 1}$

③ → add item i_1,
$R = R - \frac{R}{w_1}, \quad 18 \le 10 (x)$
$\boxed{x_1 = 10/18}$ - since $R=0$ discard i_5 & i_2

step 4:-

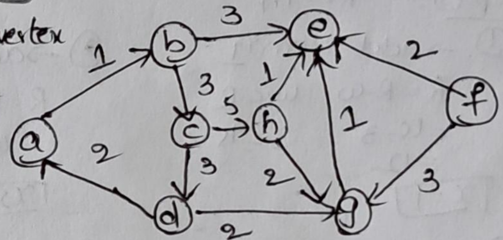
$$\text{profit} = \sum_{i=1}^{n} p_i x_i = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4 + p_5x_5$$
$$= 20\left(\frac{10}{18}\right) + 15(0) + 22(1) + 9(1)$$

$\therefore$ optimal profit $= 42.11$

Result vector

x_1	x_2	x_3	x_4	x_5
10/18	0	1	1	0.

③ Find the shortest path from source vertex to a every other vertex using single source shortest path algorithm.



Sol

Cost matrix:-

	a	b	c	d	e	f	g	h
a	α	1	α	α	α	α	α	α
b	α	α	2	α	3	α	α	α
c	α	α	α	3	α	α	α	α
d	2	α	α	α	α	α	2	α
e	α	α	α	α	α	2	α	5
f	α	α	α	α	α	α	α	α
g	α	α	α	α	α	α	α	α
h	α	α	α	α	α	α	α	α

Distance vector

	a	b	c	d	e	f	g	h	visited
1	0	1	α	α	α	α	α	α	{a}
2	0	1	3	α	4	α	α	8	{a,b}
3	0	1	3	6	4	α	α	8	{a,b,c}
4	0	1	3	6	4	6	~~8~~ α	8	{a,b,c,d, e}
5	0	1	3	6	4	6	8	8	{a,b,c,d, e}
6	0	1	3	6	4	6	8	8	{a,b,c,d, e,f}
7	0	1	3	6	4	6	8	8	{a,b,c,d, e,f,g,h}

① $u = b \to \overset{c}{e}$

ⓒ → dist[c] > dist[b] + cost[b,c]
$\alpha > 1+2$
$2 > 3$
$\therefore$ dist[c] = 3.

ⓔ → dist[e] > dist[b] + cost[b,e]
$\alpha > 1+3$
$\therefore$ dist[e] = 4.

③ $u = e \to f$

ⓕ → dist[f] > dist[e] + cost[e,f]
$\alpha > 4+2$
$\alpha > 6$ ✓

④ $u = d \to \frac{a \times}{g ✓}$

g → dist[g] > dist[d] + cost[d,g]
$\alpha > 6+2$
$\alpha > 8$ ✓
$\therefore$ dist[g] = 8.

② $u = c \to d$
$\quad\quad\quad\quad \to h.$

ⓓ $\to$ dist[d] > dist[c] + cost[c,d]

$\alpha > 3+3$

$\alpha > 6$ ✓

$\therefore$ dist[d] = 6

ⓗ $\to$ dist[h] > dist[c] + cost[c,h]

$\alpha > 3+5$

$\alpha > 8$ $\therefore$ dist[h] = 8

⑤ $u = f \to g$

$g \to$ dist[g] > dist[f] + cost[f,g]

$8 > 6+3$

$8 > 9$ ✗ No update

⑥ $u = g \to e$ ✗
$\quad\quad\quad\quad \to h$ ✓

ⓗ $\to$ dist[h] > dist[g] + cost[g,h]

$8 > 8+2$

$8 > 10$ ✗ No update

Finally shortest path from a to every vertex is as follows

vertex	b	c	d	e	f	g	h
shortest path from 'a'	1	3	6	4	~~8~~6	8	8

4) Find the optimal placement for 12 programs on 3 tape '7' where the programs are of lengths 8, 6, 3, 2, 4, 5, 1, 12, 7, 30 & 9.

Sol Given $n = 11$, $m = 3$

step ①:- sort all programs in ascending order as per their lengths
1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 30.

step ②:- add item into tapes

t_0: 1, 4, 7, 12

t_1: 2, 5, 8, 30

t_2: 3, 6, 9

step ③:- calculate d(I) for each

$d(I_1) = 1+5+12+24 = 42$

$d(I_2) = 2+7+15+45 = 69$

$d(I_3) = 3+9+18 = 30$

step ④:- calculate total d(I)

$d(I) = 42+69+30 = 141$

step ⑤:- calculate MRI

$MRI = \frac{d(I)}{n} = \frac{141}{4} = 12.8$.

5) Find the optimal placement for 13 programs on 3 tapes t_0, t_1, t_2 where the programs of length are 12, 5, 8, 32, 7, 5, 18, 26, 4, 3, 11, 10, 6.

Ans Given $n = 13$, $m = 3$, i.e t_0, t_1, t_2, program lengths are:
12, ~~5~~8, 32, 7, 5, 18, 26, 4, 3, 11, 10, 6.

step ①:- sort program length in ascending order.
3, 4, 5, 6, 7, 8, 10, 11, 12, 18, 26, 32

step ②:- add to tapes

t_0: 3, 5, 8, 12, 32

t_1: 4, 6, 10, 18

t_2: 5, 7, 11, 26

step 3:- calculate d(I) for each one

$d(I_1) = 3+8+16+28+60 = 115$

$d(I_2) = 4+10+20+38 = 72$

$d(I_3) = 5+12+23+49 = 89$

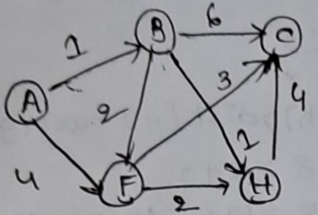
step 4:- calculate total $d(I)$

$d(I) = 115 + 72 + 89 = 276$

step 5:- calculate MRT

$MRT = \frac{d(I)}{n} = \frac{276}{13} = 21.23$

7) Find the shortest path from source vertex to every ~~visited~~ destination vertex.



	A	B	C	F	H	visited
1	0	1	α	4	α	{A}
2	0	1	7	3	2	{A, B}
3	0	1	6	3	2	{A, B, H}
4	0	1	6	3	2	{A, B, F, H}

	A	B	C	F	H
A	α	1	α	4	α
B	α	α	6	2	1
C	α	α	α	α	α
F	α	α	3	α	α
H	α	α	4	2	α

① dist [A] = 0

step - 2 - u = B = C, F, H

→ © dist[C] > dist[B] + cost [B, C]

α > 1+6

α > 7 ✓ dist [C] = 7

→ dist[F] > dist[B] + cost [B, F]

4 > 1+2

4 > 3 ∴ dist[F] = 3

→ (H) dist [H] > dist [B] + cost [B, H]

α > 1+1

α > 2

∴ dist [H] = 2

step ③

→ u = H = C, F

→ dist [C] > dist [H] + cost [H, C]

7 > 2+4

7 > 6 ∴ dist [C] = 6

(F) → dist [F] > dist [H] + cost [H, F]

3 > 2+2

3 > 4 × No update.

∴ The shortest path from source vertex path from source vertex to every destination matrix as follow

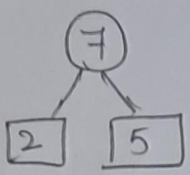
vertex	B	C	F	H
shortest path	1	6	3	2

6) Given 6 file with size of 2, 16, 5, 7, 9, 13 optimal merge sequence & cost?

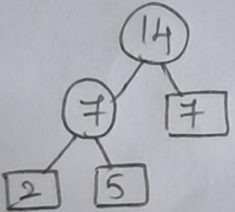
n = 6. Given list 2, 16, 5, 7, 9, 3

After sorting

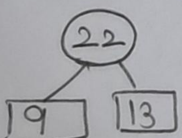
list : ~~2~~, ~~5~~, 7, 9, 13, 16.



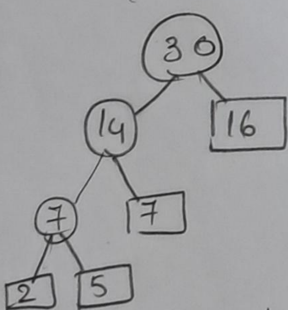
list: ~~7~~, ~~7~~, 9, 13, 16



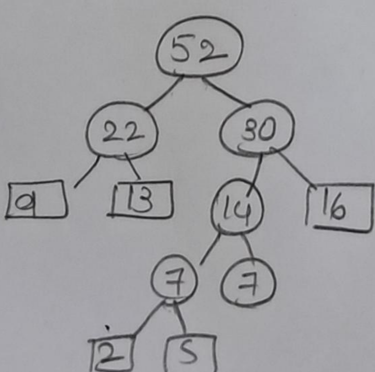
list: 14, ~~9~~, ~~13~~, 16



list: ~~14~~, 22, ~~16~~



list: ~~30~~, ~~22~~



Total cost = 52 + 22 + 30 + 14 + 7

= 125