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Department of Computer Science and Engineering

## Course Name: Data Structures & Applications
## Course Code: BCS304

## III Semester

**2022 Scheme**

# MODULE 1

# Introduction to Data Structures, Array Operations and Strings

## Introduction to Data Structures

Data may be organized in many different ways. The logical or mathematical model of a particular organization of data is called a data structure.

The choice of a particular data model depends on the two considerations

1. It must be rich enough in structure to mirror the actual relationships of the data in the real world.
2. The structure should be simple enough that one can effectively process the data whenever necessary.

### Basic Terminology: Elementary Data Organization

Data: Data are simply values or sets of values.

Data items: Data items refer to a single unit of values.

Data items that are divided into sub-items are called Group items. Ex: An Employee Name may be divided into three sub items- first name, middle name, and last name.

Data items that are not able to divide into sub-items are called Elementary items.

Ex: SSN

**Entity:** An entity is something that has certain attributes or properties which may be assigned values. The values may be either numeric or non-numeric.

| | | | | | |
|---|---|---|---|---|---|
| Ex: | Attributes- | Names, | Age, | Sex, | SSN |
| | Values- | Rohland Gail, | 34, | F, | 134-34-5533 |

Entities with similar attributes form an **entity set**. Each attribute of an entity set has a range of values, the set of all possible values that could be assigned to the particular attribute.

The term "information" is sometimes used for data with given attributes, of, in other words meaningful or processed data.

**Field** is a single elementary unit of information representing an attribute of an entity.

**Record** is the collection of field values of a given entity.

**File** is the collection of records of the entities in a given entity set.

Each record in a file may contain many field items but the value in a certain field may uniquely determine the record in the file. Such a field K is called a primary key and the values represent k1, k2, ….. in such a field are called keys or key values.

Records may also be classified according to length. A file can have fixed-length records or variable-length records.

- In fixed-length records, all the records contain the same data items with the same amount of space assigned to each data item.
- In variable-length records file records may contain different lengths.

**Example:** Student records have variable lengths, since different students take different numbers of courses. Variable-length records have a minimum and a maximum length.

The above organization of data into fields, records and files may not be complex enough to maintain and efficiently process certain collections of data. For this reason, data are also organized into more complex types of structures.

The study of complex data structures includes the following three steps:

1. Logical or mathematical description of the structure
2. Implementation of the structure on a computer
3. Quantitative analysis of the structure, which includes determining the amount of memory needed to store the structure and the time required to process the structure.

## Classification of Data Structures

Data structures are generally classified into

- Primitive data Structures
- Non-primitive data Structures

**Primitive data Structures:** Primitive data structures are the fundamental data types which are supported by a programming language. Basic data types such as integer, real, character and Boolean are known as Primitive Data Structures. These data types consist of characters that cannot be divided and hence they also called simple data types.

**Non- Primitive Data Structures:** Non-primitive data structures are those data structures which are created using primitive data structures. Examples of non-primitive data structures is the processing of complex numbers, linked lists, stacks, trees, andgraphs.

Based on the structure and arrangement of data, non-primitive data structures are further classified into

- Linear Data Structure
- Non-linear Data Structure

**Linear Data Structure:**

A data structure is said to be linear if its elements form a sequence or a linear list. There are basically two ways of representing such linear structure in memory.

1. One way is to have the linear relationships between the elements represented by means of sequential memory location. These linear structures are called arrays.
2. The other way is to have the linear relationship between the elements represented by means of pointers or links. These linear structures are called linked lists.

The common examples of linear data structure are Arrays, Queues, Stacks, Linked lists

**Non-linear Data Structure:**

A data structure is said to be non-linear if the data are not arranged in sequence or a linear. The insertion and deletion of data is not possible in linear fashion. This structure is mainly used to represent data containing a hierarchical relationship between elements. Trees and graphs are the examples of non-linear data structure.

## Arrays - Data Structure

The simplest type of data structure is a linear (or one dimensional) array. A list of a finite number *n* of similar data referenced respectively by a set of *n* consecutive numbers, usually 1, 2, 3 *n*. if **A** is chosen the name for the array, then the elements of **A** are denoted by subscript notation a1, a2, a3 an

or

by the parenthesis notation A (1), A (2), A (3) A (n)

or

by the bracket notation A [1], A [2], A [3] A [n]

**Example 1:** A linear array STUDENT consisting of the names of six students is pictured in below Fig. Here STUDENT [1] denotes John Brown, STUDENT [2] denotes Sandra Gold, and so on as shown in fig 1.1.

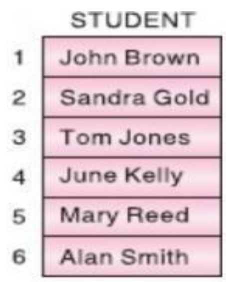



Fig 1.1 Array representation

Linear arrays are called one-dimensional arrays because each element in such an array is referenced by one subscript. Fig 1.2 illustrates two dimensional array Representation. A two-dimensional array is a collection of similar data elements where each element is referenced by two subscripts.

**Example 2:** A chain of 28 stores, each store having 4 departments, may list its weekly sales as in below fig. Such data can be stored in the computer using a two-dimensional array in which the first subscript denotes the store and the second subscript the department. If SALES is the name given to the array, then

SALES [1, 1] = 2872, SALES [1, 2] - 805, SALES [1, 3] = 3211,…., SALES [28, 4] = 982

| Store \ Dept. | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2872 | 805 | 3211 | 1560 |
| 2 | 2196 | 1223 | 2525 | 1744 |
| 3 | 3257 | 1017 | 3686 | 1951 |
| . . . | . . . | . . . | . . . | . . . |
| 28 | 2618 | 931 | 2333 | 982 |

Fig 1.2 Two dimensional array Representation

## Trees - Data Structure

Data frequently contain a hierarchical relationship between various elements. The data structure which reflects this relationship is called a rooted tree graph or a tree. Fig 1.3 shows tree Representation.

Some of the basic properties of tree are explained by means of examples

**Example 1: Record Structure**

Although a file may be maintained by means of one or more arrays a record, where one indicates both the group items and the elementary items, can best be described by means of a tree structure.

For example, an employee personnel record may contain the following data items:

**Social Security Number, Name, Address, Age, Salary, Dependents**

However, Name may be a group item with the sub-items Last, First and MI (middle initial). Also Address may be a group item with the sub-items Street address and Area address, where Area itself may be a group item having sub-items City, State and ZIP codenumber.

This hierarchical structure is pictured below

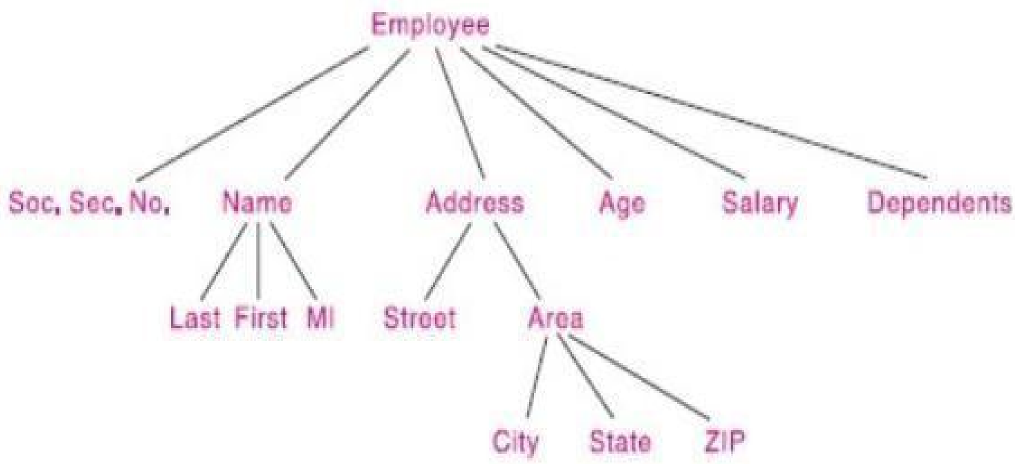



Fig 1.3 Tree Representation

## Description of data structures

### Stack

A stack, also called a fast-in first-out (LIFO) system, is a linear list in which insertions and deletions can take place only at one end, called the top. This structure is similar in its operation to a stack of dishes on a spring system as shown in fig.
Note that new 4 dishes are inserted only at the top of the stack and dishes can be deleted only from the top of the Stack as shown in fig 1.4.

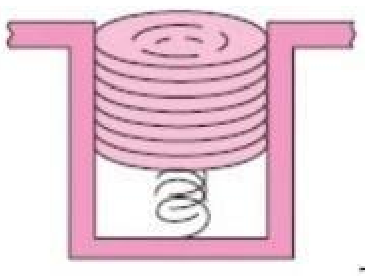


Fig 1.4 Stack of dishes

### Queue:

A queue, also called a first-in first-out (FIFO) system, is a linear list in which deletions can take place only at one end of the list, the "from" of the list, and insertions can take place only at the other end of the list, the “rear” of the list as shown in fig 1.5.
This structure operates in much the same way as a line of people waiting at a bus stop, as pictured in Fig. the first person in line is the first person to board the bus. Another analogy is with automobiles waiting to pass through an intersection the first car in line is the first car through.

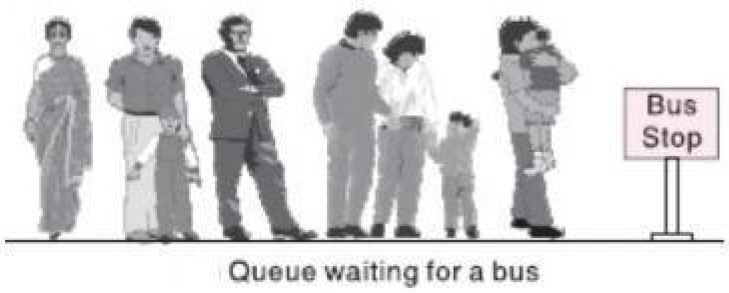



Fig 1.5 Queue representaion

**Graph:**

Data sometimes contain a relationship between pairs of elements which is not necessarily hierarchical in nature. For example, suppose an airline flies only between the cities connected by lines in Fig. The data structure which reflects this type of relationship is called a graph as shown in fig 1.6.

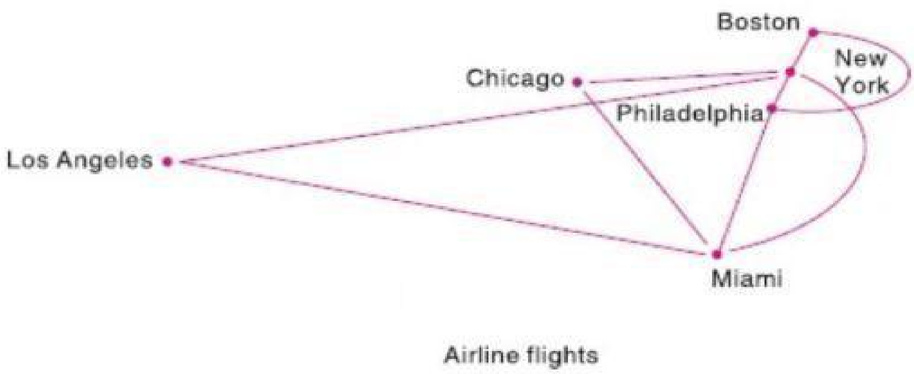



Fig 1.6 Graph representation

## Data Structures Operations

The data appearing in data structures are processed by means of certain operations. The following four operations play a major role in this text:

1. **Traversing:** accessing each record/node exactly once so that certain items in the record may be processed. (This accessing and processing is sometimes called "**visiting**" the record.)
2. **Searching:** Finding the location of the desired node with a given key value, or finding the locations of all such nodes which satisfy one or more conditions.

3. **Inserting:** Adding a new node/record to the structure.
4. **Deleting:** Removing a node/record from the structure.

The following two operations, which are used in special situations:

1. **Sorting:** Arranging the records in some logical order (e.g., alphabetically according to some NAME key, or in numerical order according to some NUMBER key, such as social security number or account number)
2. **Merging:** Combining the records in two different sorted files into a single sorted file.

## Review of Arrays and Structures

An Array is defined as, an ordered set of similar data items. All the data items of an array are stored in consecutive memory locations.

The data items of an array are of **same type** and each data items can be accessed using the same **name** but different **index** value.
An array is a set of pairs, <*index, value* >, such that each index has a value associated with it. It can be called as ***corresponding*** or a***mapping***

Ex: <index, value>

| | |
|---|---|
| < 0 , 25 > | list[0]=25 |
| < 1 , 15 > | list[1]=15 |
| < 2 , 20 > | list[2]=20 |
| < 3 , 17 > | list[3]=17 |
| < 4 , 35 > | list[4]=35 |

Here, ***list*** is the name of array. By using, list [0] to list [4] the data items in list can be accessed.

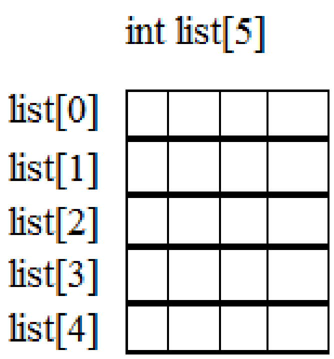
**Array in C**

**Declaration:** A one dimensional array in C is **declared** by adding brackets to the name of a variable.

Ex: **int list[5], *plist[5];**

The array **list[5],** defines 5 integers and in C array start at index 0, so list[0], list[1], list[2], list[3], list[4] are the names of five array elements which contains an integer value.

The array ***plist[5],** defines an array of 5 pointers to integers. Where, plist[0], plist[1], plist[2], plist[3], plist[4] are the five array elements which contains a pointer to an integer.



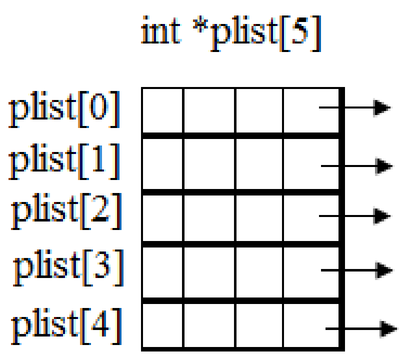



Fig 1.7 Array Representation using pointers

**Implementation:**

When the complier encounters an array declaration, **list[5],** it allocates five consecutive memory locations. Each memory is enough large to hold a single integer.

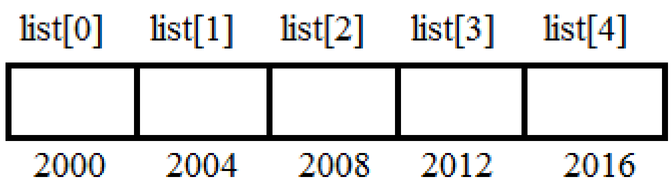
The address of first element of an array is called **Base Address.** Ex: For **list[5] t**he address of **list[0]** is called the base address.

If the memory address of **list[i]** need to compute by the compiler, then the size of the **int** would get by **sizeof (int)**, then memory address of list[i] is as follows:

**list[i] = α + i * sizeof (int)**

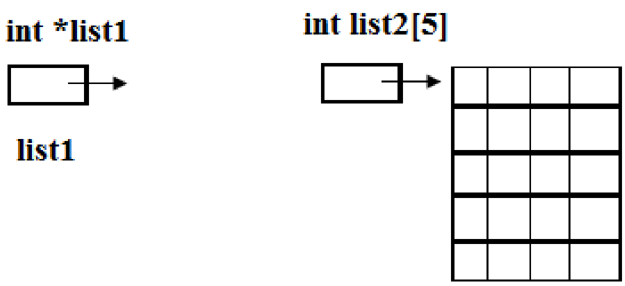
Where, α is base address.

list[3] = α+ 3 * sizeof(int)
= 2000 + 3 * 4
list[3] = 2012




**Difference between int *list1; & int list2[5];**

The variables list**1** and list**2** are both pointers to an **int,** but in **list2[5]** five memory locations are reserved for holding integers. **list2** is a pointer to **list2[0]** and **list2+i** is a pointer to **list2[i]**.




**Note:** In **C** the offset ***i*** do not multiply with the size of the type to get to the appropriate element of the array. Hence **(list2+i)** is equal **&list2[i]** and ***(list2+i)** is equal to **list2[i].**

**How C treats an array when it is parameter to a function?**

All parameters of a C functions must be declared within the function. As various parameters are passed to functions, the name of an array can be passed as parameter.

The range of a one-dimensional array is defined only in the main function since new storage for an array is not allocated within a function.

If the size of a one dimensional array is needed, it must be passed into function as a argument or accessed as a global variable.

**Example: Array Program**

```
#define    MAX_SIZE
100 float sum(float [], int);
float input[MAX_SIZE], answer; void
```

```
main(void)
{
        int i;
        for( i=0; i<MAX_SIZE; i++) input[i]=
        i;
        answer    =    sum(input,
        MAX_SIZE); printf("\n The sum is: %f
        \n",answer);
}

float sum(float list[], int n)
{
        int i;
        float tempsum = 0;
        for(i=0; i<n; i++)
                tempsum   =   tempsum   +   list[i];
        return tempsum;
}
```

When **sum** is invoked, **input=&input[0]** is copied into a temporary location and associated with the formal parameter ***list***

A function that prints out both the address of the *i*th element of the array and the value found at that address can written as shown in below program.

```
void print1 (int *ptr, int rows)
{
        int i;
        printf("    Address    contents
        \n"); for(i=0; i<rows; i++)
                printf("% 8u %5d \n", ptr+i, *(prt+i));
        printf("\n");
}
```

<u>**Output:**</u>

| **Address** | **Content** |
|---|---|
| 12244868 | 0 |
| 12344872 | 1 |
| 12344876 | 2 |
| 12344880 | 3 |
| 12344884 | 4 |

## Structures

In C, a way to group data that permits the data to vary in type. This mechanism is called the **structure**, for short **struct**.
A structure (a record) is a collection of data items, where each item is identified as to its type and name.

**Syntax: struct**

```
{       data_type member 1;
        data_type member 2;
        ………………………
        ………………………
        data_type member n;
}variable_name;
```

**Ex:** struct {
char name[10]; int age;
float salary;
} Person;

The above example creates a **structure** and variable name is **Person** and that has three fields: name
= a name that is a characterarray
age = an integer value representing the age of the person salary =
a float value representing the salary of the individual

**Assign values to fields**

To assign values to the fields, use **.** (dot) as the structure member operator. This operator is used to select a particular member of the structure

**Ex:**

```
strcpy(Person.name,"james")
; Person.age = 10;
Person.salary = 35000;
```

**Type-Defined Structure**

The structure definition associated with keyword **typedef** is called Type-Defined Structure.

**Syntax 1: typedef struct**

```
{
    data_type member 1;
    data_type member 2;
    ………………………
    ………………………
    data_type member n;
}Type_name;
```

**typedef** is the keyword used at the beginning of the definition and by using typedef user defined data type can be obtained.

**struct** is the keyword which tells structure is defined to the complier

The members are declare with their data_type

**Type_name** is not a variable, it is user defined data_type.

**Syntax 2: struct struct_name**

```
{
        data_type member 1;
        data_type member 2;
        ………………………
        ………………………
        data_type member n;
};
```

**typedef struct** struct_name Type_name;

**Ex:**

typedef struct{

c
h
a
r
n
a
m
e
[
1
0
]
;
i
n
t
a
g
e
;
f
l
o
a
t

s
a
l
a
r
y
;

}humanBeing;

In above example, **humanBeing** is the name of the type and it is a user defined data type.

Declarations of structure variables:

humanBeing person1, person2;

This statement declares the variable **person1** and **person2** are of type **humanBeing.**

**Structure Operation**

The various operations can be performed on structures and structure members.

**1. Structure Equality Check:**

Here, the equality or inequality check of two structure variable of same type or dissimilar type is not allowed

```
typedef struct{
        char
        name[10]; int age;
        float salary;
    }humanBeing;
humanBeing person1, person2;
```

**if (person1 = = person2)** is invalid.

The **valid function** is shown below

```
#define FALSE 0
#define TRUE 1
if (humansEqual(person1,person2))
        printf("The two human beings are the same\n");
else
        printf("The two human beings are not the same\n");
```

```
int humansEqual(humanBeing person1, humanBeing person2)
   { /* return TRUE if person1 and person2 are the same human being otherwise
            return FALSE */
        if (strcmp(person1.name, person2.name))
            return FALSE;
        if (person1.age != person2.age)
            return FALSE;
        if (person1.salary != person2.salary) return
            FALSE;
        return TRUE;
    }
```

Program: Function to check equality of structures

**2. Assignment operation on Structure variables:**

**person1 = person2**

The above statement means that the value of every field of the structure of person 2 is assigned as the value of the corresponding field of person 1, but this is invalid statement.

**Valid Statements** is given below:

strcpy(person1.name, person2.name);
person1.age =
person2.age; person1.salary =
person2.salary;

**Structure within a structure:**

There is possibility to embed a structure within a structure. There are 2 ways to embed structure.

1. The structures are defined separately and a variable of structure type is declared inside the definition of another structure. The accessing of the variable of a structure type that are nested inside another structure in the same way as accessing other member of that structure

**Example:** The following example shows two structures, where both the structure are defined separately.

typedef struct {

int month; int day; int yea

}date; typedef struct

{

```
r;

char name[10]; int age; float

salary; date dob
```

```
;}
humanBeing; humanBeing Person1;
```

A person born on February 11, 1944, would have the values for the date struct set as: person1.dob.month = 2;

person1.dob.day = 11;

person1.dob.year = 1944;

2. The complete definition of a structure is placed inside the definition of another structure.

**Example:**

typedef struct {

char name[10];int age;float salar

y
;
s
t
r

u
c
t
{
int month, year;
} date;

} humanBeing;

## Self-Referential Structures

A self-referential structure is one in which one or more of its components is a pointer to itself. Self-referential structures usually require dynamic storage management routines (malloc and free) to explicitly obtain and release memory.

Consider as an example:

```
typedef struct {
    char data;
    struct list *link ;
} list;
```

Each instance of the structure **list** will have two components **data** and **link**. Fig 1.8 shows the node with data.

- **Data:** is a single character,
- **Link:** link is a pointer to a list structure. The value of link is either the address in memory of an instance of list or the null pointer.

Consider these statements, which create three structures and assign values to their respective fields:

list item1, item2, item3;
item1.data = 'a';
item2.data =
'b'; item3.data = 'c';
item1.link = item2.1ink = item3.link = NULL;

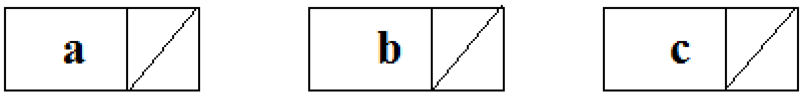



Fig 1.8 Node with data

Structures item1, item2 and item3 each contain the data item **a, b,** and **c** respectively, and the null pointer. These structures can be attached together by replacing the **null link** field in item 2 with one that points to item 3 and by replacing the null link field in item 1 with one that points to item 2.

item1.link =
&item2; item2.1ink =
&item3;

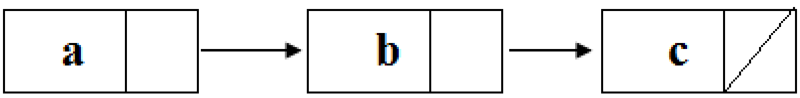



Fig 1.9 Node with two components data and link

## Unions

A union is similar to a structure, it is collection of data similar data type or dissimilar.

**Syntax:** union{
data_type member 1;
data_type member 2;
……………………..
……………………..
data_type member n;
}variable_name;

**Union Declaration:**

A union declaration is similar to a structure, but the fields of a union must share their memory space. This means that only one field of the union is "active" at any given time.

```
union{
char name;
    int age;
   float salary;
   }u;
```

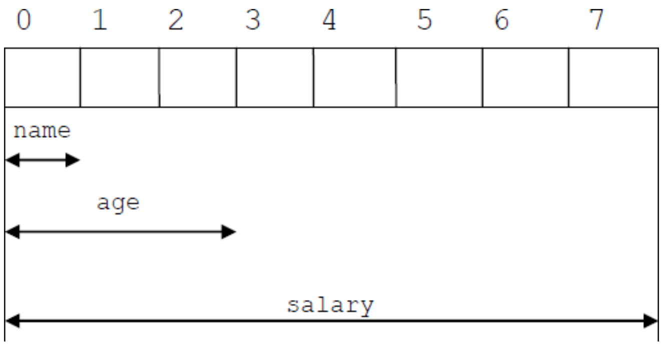



Fig 1.10 Memory representation

The major difference between a union and a structure is that unlike structure members which are stored in separate memory locations, all the members of union must share the same memory space. Fig 1.10 shows Memory representation. This means that only one field of the union is "active" at any given time.

**Example:** #include

```
<stdio.h> union job
{
            char name[32];
            float salary;
            int worker_no;
          }u;
int main( ){

printf("Enter    name:\n");    scanf("%s",    &u.name);
printf("Enter salary: \n"); scanf("%f", &u.salary);
printf("Displaying\n Name :%s\n",u.name); printf("Salary:
%.1f",u.salary); return 0;
```

**Output:**
Enter name:
Albert Enter salary:
45678.90

Displaying
Name: f%gupad (Garbage Value)
Salary: 45678.90

# Pointers

A pointer is a variable which contains the address in memory of another variable.
The two most important operator used with the pointer type are

& - The unary operator ***&*** which gives the address of a variable
* - The indirection or dereference operator * gives the content of the object pointed to by a pointer.

**Declaration**

int i, *pi;

Here, ***i*** is the integer variable and ***pi*** is a pointer to an integer

pi = &i;

Here, &i returns the address of ***i*** and assigns it as the value of ***pi***

**Null Pointer**
The null pointer points to no object or function.
The null pointer is represented by the integer 0.
The null pointer can be used in relational expression, where it is interpreted as false.

Ex: if (pi = = NULL) or if (!pi)

**Pointers can be Dangerous:**

Pointer can be very dangerous if they are misused. The pointers are dangerous in following situations:

1. Pointer can be dangerous when an attempt is made to access an area of memory that is either out of range of program or that does not contain a pointer reference to a legitimate object.

**Ex:** main ()

```
{
    int *p;
    int pa = 10;
    p = &pa;
    printf("%d", *p);       //output = 10;
```

```
        printf("%d", *(p+1)); //accessing memory which is        out of range
    }
```

2. It is dangerous when a NULL pointer is de-referenced, because on some computer it may return 0 and permitting execution to continue, or it may return the result stored in location zero, so it may produce a serious error.

3. Pointer is dangerous when use of explicit **type casts** in converting between pointer types

**Ex:** pi = malloc (sizeof (int));
pf = (float*) pi;

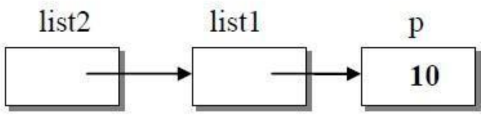
4. In some system, pointers have the same size as type **int**, since **int** is the default type specifier, some programmers omit the return type when defining a **function**. The return type defaults to **int** which can later be interpreted as a pointer. This has proven to be a dangerous practice on some computer and the programmer is made to define explicit types for functions.

**Pointers to Pointers**

A variable which contains address of a pointer variable is called pointer-to-pointer

**Example:**

```
int p;
int *list1, * *list2;
p=10;
list1=&p;
list2=&list1;
printf("%d, %d, %d", a, *list1, **list2);
```




**Output:** **10 10 10**

## Dynamic Memory Allocation Functions

**1. malloc( ):**

The function *malloc* allocates a user- specified amount of memory and a pointer to the start of the allocated memory is returned.

If there is insufficient memory to make the allocation, the returned value is NULL.

Syntax:

```
data_type *x;
x= (data_type *) malloc(size);
```

Where,

**x** is a pointer variable of data_type

**size** is the number of bytes

Ex:

```
int *ptr;
ptr = (int *) malloc(100*sizeof(int));
```

**2. calloc( ):**

The function *calloc* allocates a user- specified amount of memory and initializes the allocated memory to **0** and a pointer to the start of the allocated memory is returned.

If there is insufficient memory to make the allocation, the returned value is NULL.

**Syntax:**

data_type *x;
x= (data_type *) calloc(n, size);

> **x** is a pointer variable of type int
> **n** is the number of block to be allocated
> **size** is the number of bytes in each block

**Ex:**

```
int *x
x= calloc (10, sizeof(int));
```

The above example is used to define a one-dimensional array of integers. The capacity of this array is n=10 and x [0: n-1] (x [0, 9]) are initially 0

**Macro CALLOC**

```
#define CALLOC (p, n, s)\ if
( ! ((p) = calloc (n, s)))\
        {\
                fprintf(stderr,  "Insuffiient  memory");\
                exit(EXIT_FAILURE);\
        }\
```

**3. realloc( ):**

- Before using the realloc( ) function, the memory should have been allocated using malloc( ) or calloc( ) functions.
- The function relloc( ) resizes memory previously allocated by either ***mallor*** or ***calloc***, which means, the size of the memory changes by extending or deleting the allocated memory.
- If the existing allocated memory need to extend, the pointer value will not change.
- If the existing allocated memory cannot be extended, the function allocates a new block and copies the contents of existing memory block into new memory block and then deletes the old memory block.
- When realloc is able to do the resizing, it returns a pointer to the start of the new block and when it is unable to do the resizing, the old block is unchanged and the function returns the value NULL

**Syntax**:

data_type *x;
x= (data_type *) realloc(p, s );

The size of the memory block pointed at by p changes to S. When **s > p** the additional **s-p** memory block have been extended and when **s < p**, then **p-s** bytes of the old block are freed.

**Macro REALLOC**

```
#define REALLOC(p,S)
 if (!((p) = realloc(p,s)))
        { printf(stderr, "Insufficient memory");\ exit(EXIT_FAILURE); }

        }
```

**4. free( )**

Dynamically allocated memory with either malloc( ) or calloc ( ) does not return on its own. The programmer must use free( ) explicitly to release space.

**Syntax:**

free(ptr);

This statement cause the space in memory pointer by ptr to be deallocated

## Representation of Linear Arrays In Memory

**Linear Array**

A linear array is a list of a finite number *'n'* of homogeneous data element such that

a. The elements of the array are reference respectively by an index set consisting of n consecutive numbers.
b. The element of the array is respectively in successive memory locations.

The number *n* of elements is called the length or size of the array. The length or the numbers of elements of the array can be obtained from the index set by the formula

When LB = 0,

**Length = UB – LB + 1 Length**

When LB = 1, Where,

UB is the largest index called the Upper Bound LB
is the smallest index, called the Lower Bound

**Representation of linear arrays in memory**

Let LA be a linear array in the memory of the computer. The memory of the computer is simply a sequence of address location as shown below,

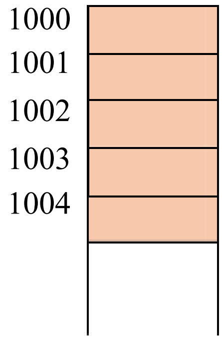



Fig 1.11 Address of the element LA [K] of the array LA

The elements of LA are stored in successive memory cells.

The computer does not keep track of the address of every element of LA, but needs to keep track only the address of the first element of LA denoted by,

*Base* (LA)

and called the base address of LA.

Using the base address of LA, the computer calculates the address of any element of LA by the formula

**LOC (LA[K]) = *Base*(LA) + w(K – lower bound)**

Where, w is the number of words per memory cell for the array LA.

**DYNAMICALLY ALLOCATED ARRAYS**

**One Dimensional Array**

While writing computer programs, if finds ourselves in a situation where we cannot determine how large an array to use, then a good solution to this problem is to defer this decision to run time and allocate the array when we have a good estimate of the required array size.

**Example:**

---

```
int i, n, *list;
printf("Enter the number of numbers to generate:");
scanf("%d", &n);
if(n<1)
```

```
{
                fprintf    (stderr,    “Improper    value    of    n
                \n”); exit(EXIT_FAILURE);
        }
```
**MALLOC (list, n*sizeof(int));**

The programs fails only when n<1 or insufficient memory to hold the list of numbers that are to be sorted.

**Two DimensionalArrays**

C uses array-of-arrays representation to represent a multidimensional array. The two dimensional arrays is represented as a one-dimensional array in which each element is itself a one-dimensional array.

Example: int x[3][5];

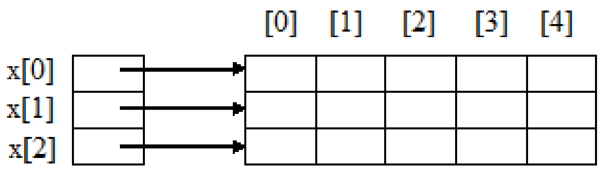



Fig 1.12 Array-of-arrays representation

C find element x[i][j] by first accessing the pointer in x[i].

Where **x[i] = α+ i* sizeof(int)**, which give the address of the zeroth element of row ***i*** of the array. Then adding **j*sizeof(int)** to this pointer ( **x[i]** ) **,** the address of the [j]th element of row ***i*** is determined**.**

**x[i] = α+ i* sizeof(int) x[j]**
**= α+ j* sizeof(int) x[i][j] =**
**x[i]+ i* sizeof(int)**

**Creation of Two-Dimensional Array Dynamically**

```
int **myArray;
myArray     =
make2dArray(5,10);
myArray[2][4]=6;

int ** make2dArray(int rows, int cols)
{ /* create a two dimensional rows X cols array */ int
        **x, i;
        MALLOC(x, rows * sizeof (*x)); /*get memory for row pointers*/ for
        (i= 0;i<rows; i++)                    /* get memory for each row */
```

```
    MALLOC(x[i], cols *
    sizeof(**x)); return x;
}
```

The second line allocates memory for a 5 by 10 two-dimensional array of integers and the third line assigns the value 6 to the [2][4] element of this array.

## Array Operations

### 1. Traversing

- Let A be a collection of data elements stored in the memory of the computer. Suppose if the contents of the each elements of array A needs to be printed or to count the numbers of elements of A with a given property can be accomplished by Traversing.
- Traversing is a accessing and processing each element in the array exactly once.

**Algorithm 1: (Traversing a Linear Array)**

Hear LA is a linear array with the lower bound LB and upper bound UB. This algorithm traverses LA applying an operation PROCESS to each element of LA using while loop.

1. [Initialize Counter] set K:= LB
2. Repeat step 3 and 4 while K ≤ UB
3. [Visit element] Apply PROCESS to LA [K]
4. [Increase counter] Set K:= K + 1

   [End of step 2 loop]
5. Exit

**Algorithm 2: (Traversing a Linear Array)**

Hear LA is a linear array with the lower bound LB and upper bound UB. This algorithm traverses LA applying an operation PROCESS to each element of LA using repeat – for loop.

1. Repeat for K = LB to UB

   Apply PROCESS to LA [K]

   [End of loop]
2. Exit.

**Example:**

Consider the array AUTO which records the number of automobiles sold each year from 1932 through 1984.

To find the number NUM of years during which more than 300 automobiles were sold, involves traversing AUTO.

1. [Initialization step.] Set NUM := 0
2. Repeat for K = 1932 to 1984:

If AUTO [K] > 300, then: Set NUM: = NUM + 1.
[End of loop.]
3. Return.

## 2. Inserting

- Let A be a collection of data elements stored in the memory of the computer. Inserting refers to the operation of adding another element to the collection A.
- Inserting an element at the "end" of the linear array can be easily done provided the memory space allocated for the array is large enough to accommodate the additional element.
- Inserting an element in the middle of the array, then on average, half of the elements must be moved downwards to new locations to accommodate the new element and keep the order of the other elements.

**Algorithm:**

INSERT (LA, N, K, ITEM)

Here LA is a linear array with N elements and K is a positive integer such that K ≤ N. This algorithm inserts an element ITEM into the $K^{th}$ position in LA.

1. [Initialize counter] set J:= N
2. Repeat step 3 and 4 while J ≥ K
3. [Move $J^{th}$ element downward] Set LA [J+1] := LA[J]
4. [Decrease counter] set J:= J – 1

   [End of step 2 loop]
5. [Insert element] set LA[K]:= ITEM
6. [Reset N] set N:= N+1
7. Exit

## 3. Deleting

- Deleting refers to the operation of removing one element to the collection A.
- Deleting an element at the "end" of the linear array can be easily done with difficulties.
- If element at the middle of the array needs to be deleted, then each subsequent elements be moved one location upward to fill up the array.

**Algorithm**

DELETE (LA, N, K, ITEM)

Here LA is a linear array with N elements and K is a positive integer such that K ≤ N. this algorithm deletes the $K^{th}$ element from LA

1. Set ITEM:= LA[K]
2. Repeat for J = K to N – 1

   [Move J + 1 element upward] set LA[J]:= LA[J+1]

   [End of loop]
3. [Reset the number N of elements in LA] set N:= N – 1

4. Exit

**Example: Inserting and Deleting**

Suppose NAME is an 8-element linear array, and suppose five names are in the array, as in Fig.(a). Observe that the names are listed alphabetically, and suppose we want to keep the array names alphabetical at all times. Suppose ***Ford*** is added to the array. Then ***Johnson, Smith*** and ***Wagner*** must each be moved downward one location, as in Fig.(b). Next suppose Taylor is added to the array; then Wagner must be moved, as in Fig.(c). Last, suppose Davis is removed from the array. Then the five names Ford, Johnson, Smith, Taylor and Wagner must each be moved upward one location, as in Fig.(d).

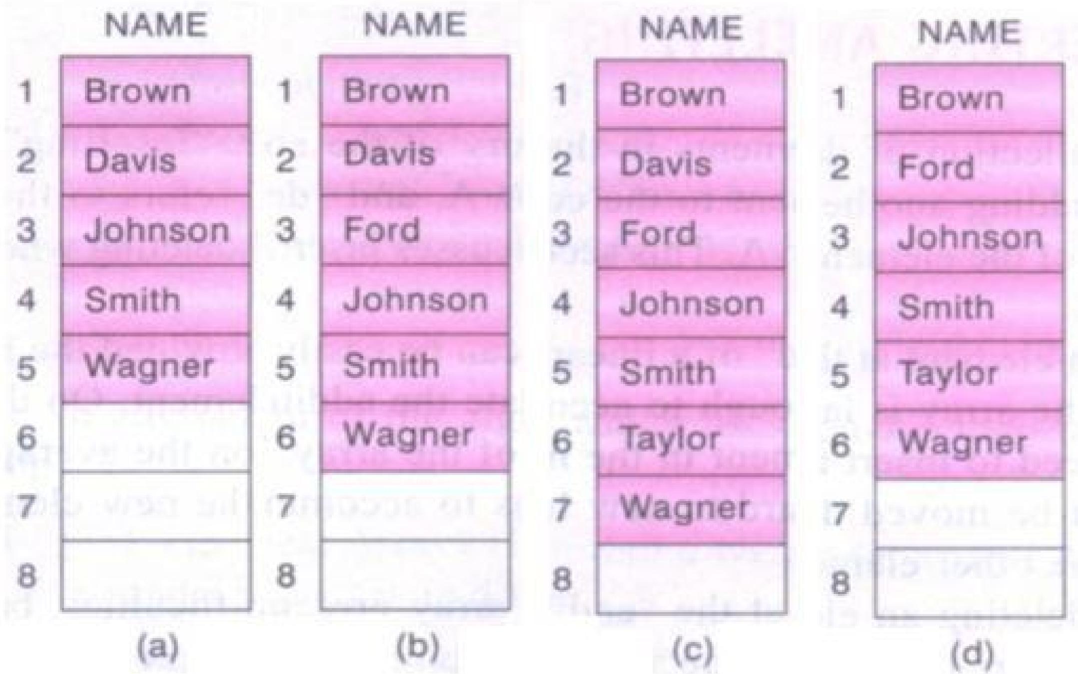



Fig 1.13 Insertion and deletion of elements in a linear array

## 4. Sorting

Sorting refers to the operation of rearranging the elements of a list. Here list be a set of n elements. The elements are arranged in increasing or decreasing order.

Ex: suppose A is the list of n numbers. Sorting A refers to the operation of rearranging the elements of A so they are in increasing order, i.e., so that,

$$A[I] < A[2] < A[3] < ... < A[N]$$

For example, suppose A originally is the list
8, 4, 19, 2, 7, 13, 5, 16
After sorting, A is the list
2, 4, 5, 7, 8, 13, 16, 19

**Bubble Sort**

Suppose the list of numbers A[l], A[2], ... , A[N] is in memory. The bubble sort algorithm works as follows:

---

Algorithm: Bubble Sort – BUBBLE (DATA, N)

Here DATA is an array with N elements. This algorithm sorts the elements in DATA.

1. Repeat Steps 2 and 3 for K = 1 to N - 1.
   2. Set PTR: = 1. [Initializes pass pointer PTR.]
   3. Repeat while PTR ≤ N - K: [Executes pass.]
      (a) If DATA[P TR] > DATA[P TR + 1], then:
          Interchange DATA [PTR] and DATA [PTR + 1]. [End of If structure.]
      (b) Set PTR: = PTR + 1.
      [End of inner loop.]
   [End of Step 1 outer loop.]
4. Exit.

---

Example:

Suppose the following numbers are stored in an array A:

32, 51, 27, 85, 66, 23, 13, 57

We apply the bubble sort to the array A. We discuss each pass separately.

Pass 1. We have the following comparisons:

(a) Compare $A_1$ and $A_2$. Since 32 < 51, the list is not altered.

(b) Compare $A_2$ and $A_3$. Since 51 > 27, interchange 51 and 27 as follows:

32, 27, 51, 85, 66, 23, 13, 57

(c) Compare $A_3$ and $A_4$. Since 51 < 85, the list is not altered.

(d) Compare $A_4$ and $A_5$. Since 85 > 66, interchange 85 and 86 as follows:

32, 27, 51, 66, 85, 23, 13, 57

(e) Compare $A_5$ and $A_6$. Since 85 > 23, interchange 85 and 23 as follows:

32, 27, 51, 66, 23, 85, 13, 57

(f) Compare $A_6$ and $A_7$. Since 85 > 13, interchange 85 and 13 to yield:

32, 27, 51, 66, 23, 13, 85, 57

(g) Compare $A_7$ and $A_8$. Since 85 > 57, interchange 85 and 51 to yield:

32, 27, 51, 66, 23, 13, 57, 85

At the end of this first pass, the largest number, 85, has moved to the last position. However, the rest of the numbers are not sorted, even though some of them have changed their positions.

For the remainder of the passes, we show only the interchanges.

Pass 2. 27, 33, 51, 66, 23, 13, 57, 85
27, 33, 51, 23, 66, 13, 57, 85
27, 33, 51, 23, 13, 66, 57, 85
27, 33, 51, 23, 13, 57, 66, 85
At the end of Pass 2, the second largest number, 66, has moved its way down to the next-to-last position.

Pass 3. 27, 33, 23, 51, 13, 57, 66, 85
27, 33, 23, 13, 51, 57, 66, 85

Pass 4. 27, 23, 33, 13, 51, 57, 66, 85
27, 23, 13, 33, 51, 57, 66, 85

Pass 5. 23, 27, 13, 33, 51, 57, 66, 85
23, 13, 27, 33, 51, 57, 66, 85

Pass 6. 13, 23, 27, 33, 51, 57, 66, 85
Pass 6 actually has two comparisons, $A_1$ with $A_2$ and $A_2$ and $A_3$. The second comparison does not involve an interchange.

Pass 7. Finally, $A_1$ is compared with $A_2$. Since 13 < 23, no interchange takes place.

Since the list has 8 elements; it is sorted after the seventh pass.

**Complexity of the Bubble Sort Algorithm**

The time for a sorting algorithm is measured in terms of the number of comparisons f(n). There are *n* – 1 comparisons during the first pass, which places the largest element in the last position; there are *n* - 2 comparisons in the second step, which places the second largest element in the next-to-last position; and so on. Thus

$$f(n) = (n - 1) + (n - 2) + ... + 2 + 1 \quad = O(n) = O(n^2)$$

**5. Searching**

- Let DATA be a collection of data elements in memory, and suppose a specific ITEM of information is given. ***Searching*** refers to the operation of finding the location LOC of ITEM in DATA, or printing some message that ITEM does not appear there.
- The search is said to be successful if ITEM does appear in DATA and unsuccessful otherwise.

## Linear Search

Suppose DATA is a linear array with n elements. Given no other information about DATA, The way to search for a given ITEM in DATA is to compare ITEM with each element of DATA one by one. That is, first test whether DATA [l] = ITEM, and then test whether DATA[2] = ITEM, and so on. This method, which traverses DATA sequentially to locate ITEM, is called ***linear search or sequential search.***

---

Algorithm: (Linear Search) LINEAR (DATA, N, ITEM, LOC)
Here DATA is a linear array with N elements, and ITEM is a given item of information. This algorithm finds the location LOC of ITEM in DATA, or sets LOC: = 0 if the search is unsuccessful.

1. [Insert ITEM at the end of DATA.] Set DATA [N + 1]: = ITEM.
2. [Initialize counter.] Set LOC: = l.
3. [Search for ITEM.]
   Repeat while DATA [LOC] ≠ ITEM: Set
   LOC:= LOC + 1.
   [End of loop.]
4. [Successful?] If LOC = N + 1, then: Set LOC:= 0
5. Exit.

---

## Complexity of the Linear Search Algorithm

Worst Case: The worst case occurs when one must search through the entire array DATA, i.e., when ITEM does not appear in DATA. In this case, the algorithm requires comparisons.

$$f(n) = n + 1$$

Thus, in the worst case, the running time is proportional to n.

Average Case: The average number of comparisons required to find the location of ITEM is approximately equal to half the number of elements in the array.

## Binary Search

Suppose DATA is an array which is sorted in increasing numerical order or, equivalently, alphabetically. Then there is an extremely efficient searching algorithm, called *binary search,* which can be used to find the location LOC of a given ITEM of information in DATA.

---

Algorithm: (Binary Search) BINARY (DATA, LB, UB, ITEM, LOC)
Here DATA is a sorted array with lower bound LB and upper bound UB, and ITEM is a given item of information. The variables BEG, END and MID denote, the beginning, end and middle locations of a segment of elements of DATA.
This algorithm finds the location LOC of ITEM in DATA or sets LOC = NULL.

1. [Initialize segment variables.]
   Set BEG: = LB, END := UB and MID = INT((BEG + END)/2).
2. Repeat Steps 3 and 4 while BEG ≤ END and DATA [MID] ≠ ITEM.
3. If ITEM < DATA [MID],
   then: Set END := MID -
   1.
   Else:
   Set BEG := MID + 1.
   [End of If structure.]
4. Set MID := INT((BEG + END)/2).
   [End of Step 2 loop.]
5. If DATA[MID] = ITEM, then:
   Set LOC := MID.
   Else:
   Set LOC := NULL.
   [End of If structure.]
6. Exit.

---

**Complexity of the Binary Search Algorithm**

The complexity is measured by the number *f(n)* of comparisons to locate ITEM in DATA where DATA contains *n* elements. Observe that each comparison reduces the sample size in half. Hence we require at most *f(n)* comparisons to locate ITEM where

$2^{f(n)} > n$ or equivalently $f(n)$ = [log2 n] + 1

That is, the running time for the worst case is approximately equal to log2 *n*. One can also show that the running time for the average case is approximately equal to the running time for the worst case.

## Multidimensional Array

- ### Two-Dimensional Arrays

A two-dimensional m x n array A is a collection of *m . n* data elements such that each element is specified by a pair of integers (such as J, K), called subscripts, with the property that

$$1 \leq J \leq m \text{ and } 1 \leq K \leq n$$

The element of A with first subscript j and second subscript k will be denoted by AJ,K or A[J, K]
Two-dimensional arrays are called ***matrices*** in mathematics and ***tables*** in business applications.

There is a standard way of drawing a two-dimensional m x n array A where the elements of A form a rectangular array with *m rows* and *n columns* and where the element A[J, K] appears in *row J and column K*.

$$\begin{array}{cc} & \text{Columns} \\ & \begin{array}{cccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\ \text{Rows} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} A[1,1] & A[1,2] & A[1,3] & A[1,4] \\ A[2,1] & A[2,2] & A[2,3] & A[2,4] \\ A[3,1] & A[3,2] & A[3,3] & A[3,4] \end{bmatrix} \end{array}$$

Fig 1.14 Two-Dimensional 3x4 Array

**Representation of Two-Dimensional Arrays in Memory**

Let A be a two-dimensional *m* x *n* array. Although A is pictured as a rectangular array of elements with *m* rows and *n* columns, the array will be represented in memory by a block of *m . n* sequential memory locations.

The programming language will store the array A either (1) column by column, is called *column-major order,* or (2) row by row, in *row-major order.*

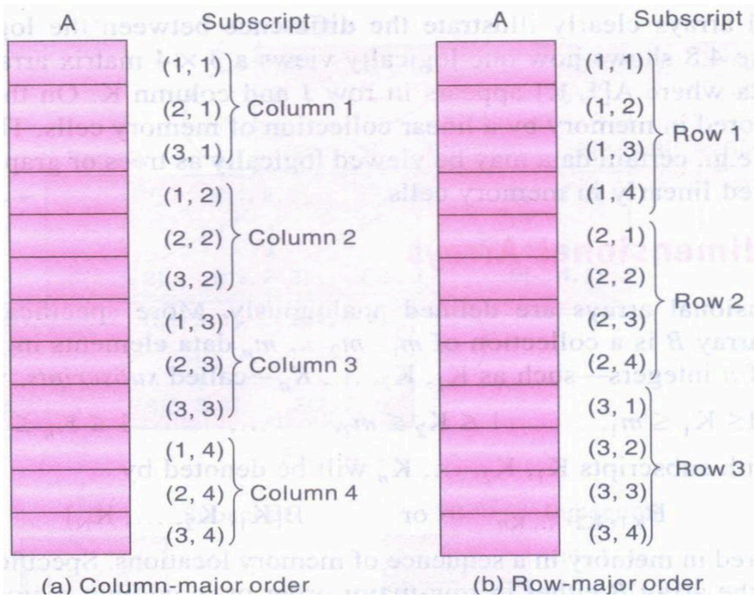



Fig 1.15 Representation of Two-Dimensional Arrays in Memory

The computer uses the formula to find the address of LA[K] in time independent of K.

LOC (LA[K]) = *Base(LA)* + w(K - 1)

The computer keeps track of *Base(A)-the* address of the first element A[1, 1] of A-and computes the address LOC(A[J, K]) of A[J, K] using the formula

(Column-major order) LOC(A[J, K]) = *Base(A)* + w[M(K - 1) + (J - 1)]

(Row-major order) LOC(A[J, K]) = *Base(A)* + w[N(J - 1) + (K - 1)]

**General Multidimensional Arrays**

An n-dimensional m1 X m2 X ... X mn array B is a collection of m1, m2 ... mn data elements in which each element is specified by a list of n integers-such as K1 K2 ... , Kn called subscripts, with the property that
$1 \le K1 \le m1$ , $1 \le K2 \le m2$ ….. $1 \le Kn \le mn$

The element of *B* with subscripts K1 K2 ... , Kn will be denoted by B[K1 K2 ... , Kn]
The programming language will store the array *B* either in row-major order or in column- major order.
Let C be such an n-dimensional array. The index set for each dimension of C consists of the consecutive integers from the lower bound to the upper bound of the dimension. The length *Li* of dimension *i* of C is the number of elements in the index set, and *Li* can be calculated, as

$$Li = \text{upper bound} - \text{lower bound} + 1$$

For a given subscript Ki, the effective index *Ei* of *Li* is the number of indices preceding Ki in the index set, and *Ei* can be calculated from

$$Ei = Ki - \text{lower bound}$$

Then the address LOC(C[K1 K2 ... , Kn] of an arbitrary element of C can be obtained from the formula

$$Base(C) + w[((( \ldots (ENLN\text{-}1\ ] + E\ N\text{-}1])LN\text{-}2) + \ldots + E3))L2 + E2)L1 + E1]$$

or from the formula

$$Base(C) + w[( \ldots ((E1L2 + E2)L3 + E3)L4 + \ldots + EN\text{-}1\ )LN + EN]$$

according to whether C is stored in column-major or row-major order.

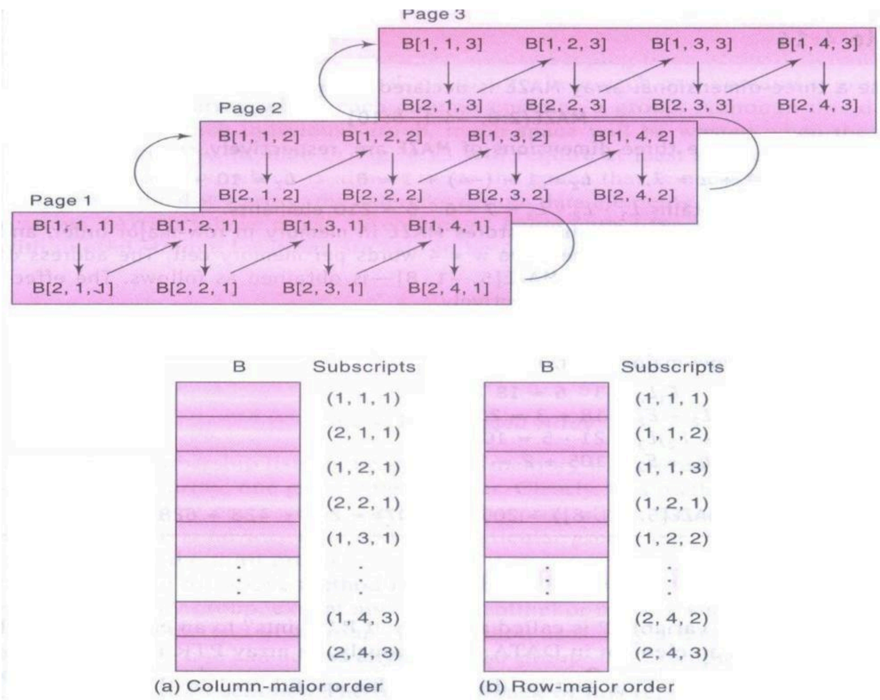



Fig 1.16 Multidimensional Arrays

# POLYNOMIALS

What is a polynomial?

"A polynomial is a sum of terms, where each term has a form **a**$\mathbf{x^e}$ , where **x** is the variable, **a** is the coefficient and **e** is the exponent."

Two example polynomials are:

$A(x) = 3x^{20} + 2x^5 + 4$ $B(x)$

$= x^4 + 10x^3 + 3x^2$

$+1$

The largest (or leading) exponent of a polynomial is called its degree. Coefficients that are zero are not displayed. The term with exponent equal to zero does not show the variable since **x** raised to a power of zero is **1**.

Assume there are two polynomials, A(x)

$= \Sigma$ ai xi and B (x) $=\Sigma$ bi xi

then:

$A(x) + B(x) = \Sigma$ (ai + bi) xi

## Polynomial Representation

One way to represent polynomials in C is to use **typedef** to create the type polynomial as below:

```
#define MAX-DEGREE 101                    /*Max degree of polynomial+1*/
typedef struct{
                    int degree;
                    float coef[MAX-DEGREE];
            } polynomial;
```

Now if **a** is a variable and is of type polynomial and n < MAX_DEGREE, the polynomial A(x) = Σai xi would be represented as:

a.degree = n

a.coef[i] = an-i , $0 \le i \le n$

In this representation, the coefficients is stored in order of decreasing exponents, such that a.coef [i] is the coefficient of $x^{n-i}$ provided a term with exponent n-i exists;

Otherwise, a.coef [i] =0. This representation leads to very simple algorithms for most of the operations, it wastes a lot of space.

To preserve space an alternate representation that uses only one global array, **terms** to store all polynomials.

The C declarations needed are:

```
MAX_TERMS 100                    /*size of terms array*/
typedef struct{
                    float   coef;
                    int expon;
             }
polynomial;                 polynomial
terms[MAX-TERMS]; int avail = 0;
```

**Consider the two polynomials**

$A(x) = 2x^{1000} + 1$

$B(x) = x^{4} + 10x^{3} + 3x^{2} + 1$

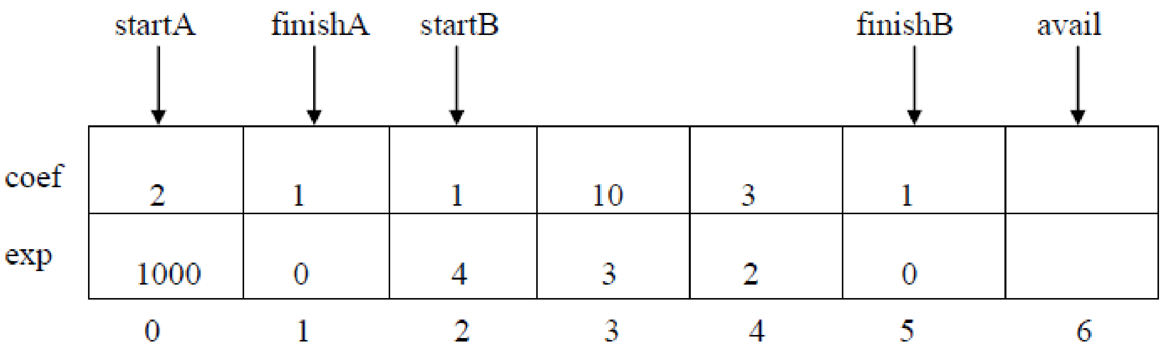



Fig 1.16 Polynomials are stored in the array terms

- The above Fig shows how these polynomials are stored in the array terms. The index of the first term of A and B is given by startA and startB, while finishA and finishB give the index of the last term of A and B.
- The index of the next free location in the array is given by avail.
- For above example, startA=0, finishA=1, startB=2, finishB=5, & avail=6.

<u>**Polynomial Addition**</u>

- C function is written that adds two polynomials, A and B to obtain D =A + B.
- To produce D (x), **padd( )** is used to add A (x) and B (x) term by term. Starting at position avail, **attach( )** which places the terms of D into the array, ***terms***.
- If there is not enough space in terms to accommodate D, an error message is printed to the standard error device & exits the program with an error condition

```
void padd(int startA, int finishA, int startB, int finishB, int *startD,int *finishD)
{ /* add A(x) and B(x) to obtain D(x) */ float
        coefficient;
        *startD = avail;
        while (startA <= finishA && startB <= finishB) switch(COMPARE(terms[startA].expon,
        terms[startB].expon))
        {
                case -1:                /* a expon < b expon */
                        attach    (terms    [startB].coef,    terms[startB].expon);
                        startB++;
                        break;

                case 0:                 /* equal exponents */
                        coefficient = terms[startA].coef + terms[startB].coef;

                        if (coefficient)
                                attach (coefficient, terms[startA].expon);
                        startA++;
                        startB++;
                        break;
                case 1:                  /* a expon > b expon */
                        attach    (terms    [startA].coef,
                        terms[startA].expon); startA++;
        }

        /* add in remaining terms of A(x) */ for(;
        startA <= finishA; startA++)
                attach (terms[startA].coef, terms[startA].expon);

        /* add in remaining terms of B(x) */ for( ;
        startB <= finishB; startB++)
                attach (terms[startB].coef, terms[startB].expon);
        *finishD = avail-i;
```

Function to add two polynomials

```
void attach(float coefficient, int exponent)
        {                       /* add a new term to the polynomial */ if
```

```
        (avail >= MAX-TERMS)
        {
                fprintf(stderr,"Too many terms in the polynomial\n");
                exit(EXIT_FAILURE);
        }
        terms[avail].coef = coefficient;
        terms[avail++].expon = exponent;
}
```

Function to add new term

**Analysis of padd( ):**

The number of non-zero terms in A and B is the most important factors in analyzing the time complexity.

Let **m** and **n** be the number of non-zero terms in A and B, If m >0 and n > 0, the **while** loop is entered. Each iteration of the loop requires O(1) time. At each iteration, the value of startA or startB or both is incremented. The iteration terminates when either startA or startB exceeds finishA or finishB. The number of iterations is bounded by **m + n -1**

The time for the remaining two **for** loops is bounded by O(n + m) because we cannot iterate the first loop more than m times and the second more than n times. So, the asymptotic computing time of this algorithm is O(n +m).

## SPARSE MATRICES

A matrix contains **m rows** and **n columns** of elements as illustrated in below Figs. In this Fig, the elements are numbers. The first matrix has five rows and three columns and the second has six rows and six columns. We write m x n (read "m by n") to designate a matrix with m rows and n columns. The total number of elements in such a matrix is **mn**. If **m** equals **n**, the matrix is square.

| | col0 | col1 | col2 |
|---|---|---|---|
| row 0 | -27 | 3 | 4 |
| row 1 | 6 | 82 | -2 |
| row 2 | 109 | -64 | 11 |
| row 3 | 12 | 8 | 9 |
| row 4 | 48 | 27 | 47 |

| | col0 | col1 | col2 | col3 | col4 | col 5 |
|---|---|---|---|---|---|---|
| row0 | 15 | 0 | 0 | 22 | 0 | -15 |
| row1 | 0 | 11 | 3 | 0 | 0 | 0 |
| row2 | 0 | 0 | 0 | -6 | 0 | 0 |
| row3 | 0 | 0 | 0 | 0 | 0 | 0 |
| row4 | 91 | 0 | 0 | 0 | 0 | 0 |
| row5 | 0 | 0 | 28 | 0 | 0 | 0 |

Fig 1.17 Sparse matrices initialization

**What is Sparse Matrix?**

A matrix which contains many zero entries or very few non-zero entries is called as Sparse matrix.

In the **Fig B** contains only 8 of 36 elements are nonzero and that is sparse.

**Important Note:**

A sparse matrix can be represented in 1-Dimension, 2- Dimension and 3- Dimensional array. When a sparse matrix is represented as a two-dimensional array as shown in **Fig B**, more space is wasted.

**Example:** consider the space requirements necessary to store a 1000 x 1000 matrix that has only 2000 non-zero elements. The corresponding two-dimensional array requires space for 1,000,000 elements. The better choice is by using a representation in which only the nonzero elements are stored.

**Sparse Matrix Representation**

- An element within a matrix can characterize by using the **triple <row,col,value>** This means that, an array of triples is used to represent a sparse matrix.
- Organize the triples so that the row indices are in ascending order.
- The operations should terminate, so we must know the number of rows and columns, and the number of nonzero elements in the matrix.

Implementation of the **Create** operation as below:

```
SparseMatrix Create(maxRow, maxCol) ::=

#define MAX_TERMS 101              /* maximum number of terms +1*/
typedef struct {
                        int    col;
                        int
                        row; int
                        value;
                }
term; term
a[MAX_TERMS];
```

- The below Fig shows the representation of matrix in the array **"a" a[0].row** contains the number of rows, **a[0].col** contains the number of columns and **a[0].value** contains the total number of nonzero entries.
- Positions 1 through 8 store the triples representing the nonzero entries. The row index is in the field row, the column index is in the field col, and the value is in the field value. The triples are ordered by row and within rows bycolumns.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| a[0] | 6 | 6 | 8 | b[0] | 6 | 6 | 8 |
| [1] | 0 | 0 | 15 | [1] | 0 | 0 | 15 |
| [2] | 0 | 3 | 22 | [2] | 0 | 4 | 91 |
| [3] | 0 | 5 | -15 | [3] | 1 | 1 | 11 |
| [4] | 1 | 1 | 11 | [4] | 2 | 1 | 3 |
| [5] | 1 | 2 | 3 | [5] | 2 | 5 | 28 |
| [6] | 2 | 3 | -6 | [6] | 3 | 0 | 22 |
| [7] | 4 | 0 | 91 | [7] | 3 | 2 | -6 |
| [8] | 5 | 2 | 28 | [8] | 5 | 0 | -15 |

Fig 1.18 Sparse matrix stored as triple Fig 1.19 Transpose matrix stored as triple

## Transposing a Matrix

To transpose a matrix, interchange the rows and columns. This means that each element **a[i][j]** in the original matrix becomes element **a[j][i]** in the transpose matrix.

A good algorithm for transposing a matrix:

```
for each row i
        take element <i, j, value> and store it as
        element <j, i, value> of the transpose;
```

If we process the original matrix by the **row indices** it is difficult to know exactly where to place element <j, i, value> in the transpose matrix until we processed all the elements that precede it. This can be avoided by using the **column indices** to determine the placement of elements in the transpose matrix. This suggests the following algorithm:

```
for all elements in column j
        place element <i, j, value> in
        element <j, i, value>
```

The columns within each row of the transpose matrix will be arranged in ascending order.

```
void transpose (term a[], termb[])
{                   /* b is set to the transpose of a */
        int n, i, j, currentb;
        n = a[0].value;             /* total number of elements */
        b[0].row = a[0].col;        /* rows in b = columns in a */
        b[0].col = a[0].row;        /* columns in b = rows in a */
        b[0].value = n;
        if (n > 0)
```

```
        { currentb = 1;
              for (i = 0; i < a[O].col; i++) for
                    (j= 1; j<=n; j++)
                          if (a[j].col == i)
                          {
                                b[currentb].row = a[j].col; b[currentb].col
                                = a[j].row; b[currentb].value = a[j].value;
                                currentb++;
                          }
        }
}
```

Transpose of a sparse matrix

## Strings

### Basic Terminology

Each programming languages contains a character set that is used to communicate with the computer. The character set include the following:

Alphabet: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
Digits: 0 1 2 3 4 5 6 7 8 9
Special characters: + - / * ( ) , . $ = ‘ _ (Blank space)

**String:** A finite sequence S of zero or more Characters is called string.

**Length:** The number of characters in a string is called length of string.

**Empty or Null String:** The string with zero characters.

**Concatenation:** Let S1 and S2 be the strings. The string consisting of the characters of S1 followed by the character S2 is called Concatenation of S1 and S2.

Ex: ‘THE’ // ‘END’ = ‘THEEND’ ‘THE’
// ‘ ’ // ‘END’ = ‘THE END’

**Substring:** A string Y is called substring of a string S if there exist string X and Z such that S = X // Y // Z

If X is an empty string, then Y is called an Initial substring of S, and Z is an empty string then Y is called a terminal substring of S.

Ex: ‘BE OR NOT’ is a substring of ‘TO BE OR NOT TO BE’
‘THE’ is an initial substring of ‘THE END’

**STRINGS IN C**

In C, the strings are represented as character arrays terminated with the null character \0.

**Declaration 1:**

```
#define MAX_SIZE 100              /* maximum size of string */
char s[MAX_SIZE] = {“dog”};
char t[MAX_SIZE] = {“house”};
```

| s[0] | s[1] | s[2] | s[3] |
|---|---|---|---|
| d | o | g | \0 |

| t[0] | t[1] | t[2] | t[3] | t[4] | t[4] |
|---|---|---|---|---|---|
| h | O | u | s | e | \0 |

Fig 1.20 Strings representation internally in memory

**Declaration 2:**

```
char s[ ] = {“dog”};
char t[ ] = {“house”};
```

Using these declarations, the C compiler will allocate just enough space to hold each word including the null character.

**STORING STRINGS**

Strings are stored in three types of structures

1. Fixed length structures
2. Variable length structures with fixed maximum
3. Linked structures

**Record Oriented Fixed length storage:**

In fixed length structures each line of print is viewed as a record, where all have the same length i.e., where each record accommodates the same number of characters.

Example: Suppose the input consists of the program. Using a record oriented, fixed length storage medium, the input data will appear in memory as pictured below.

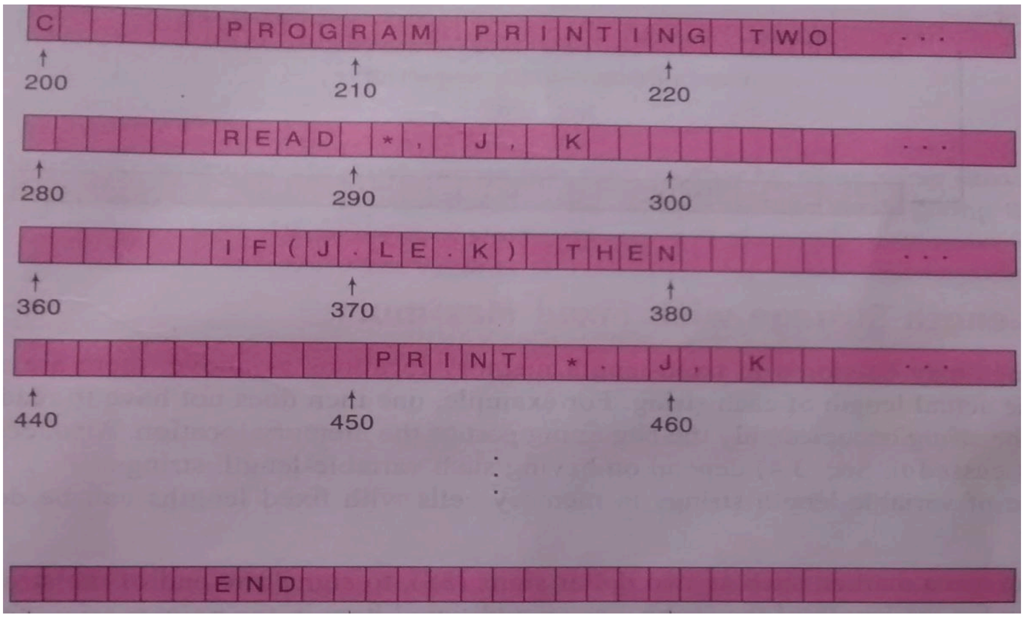



Fig 1.21 Fixed length storage

Suppose, if new record needs to be inserted, then it requires that all succeeding records be moved to new memory location. This disadvantages can be easily remedied as shown in below Fig.

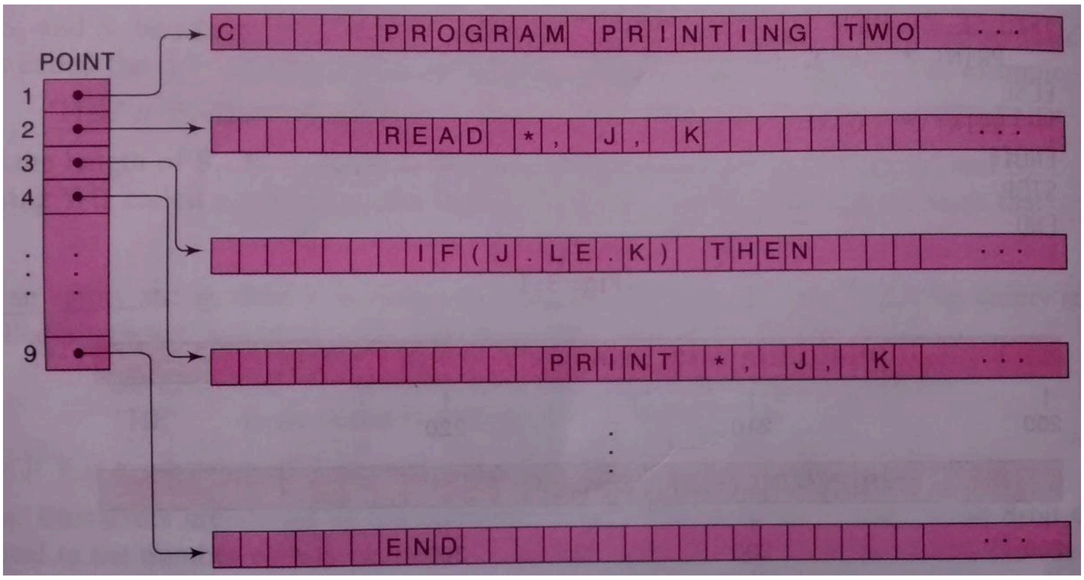



Fig 1.22 Linear array storage

That is, one can use a linear array POINT which gives the address of successive record, so that the records need not be stored in consecutive locations in memory. Inserting a new record will require only an updating of the array POINT.

The main advantages of this method are

1. The ease of accessing data from any given record
2. The ease of updating data in any given record (as long as the length of the new data does not exceed the record length)

The main disadvantages are

1. Time is wasted reading an entire record if most of the storage consists of inessential blank spaces.
2. Certain records may require more space than available
3. When the correction consists of more or fewer characters than the original text, changing a misspelled word requires record to be changed.

**Linked Storage**

Most extensive word processing applications, strings are stored by means of linked lists.

In a one way linked list, a linearly ordered sequence of memory cells called nodes, where each node contains an item called a ***link*,** which points to the next node in the list, i.e., which consists the address of the nextnode.

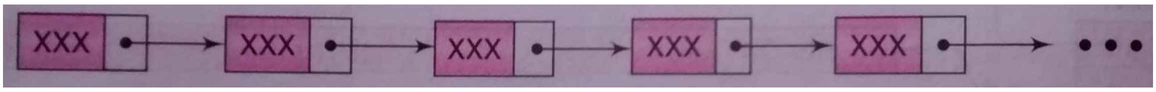



Fig 1.23 Strings stored in linked list

Each memory cell is assigned one character or a fixed number of characters and a link contained in the cell gives the address of the cell containing the next character or group of character in the string.

Ex: TO BE OR NOT TO BE

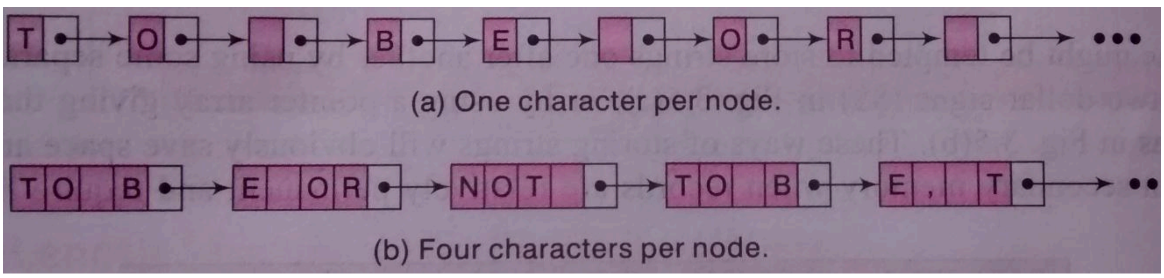



Fig 1.24 Linked list representation for strings

**CHARACTER DATA TYPE**

The various programming languages handles character data type in different ways.

**Constants**

Many programming languages denotes string constants by placing the string in either single or double quotation marks.

Ex: 'THE END'

"THE BEGINNING"

The string constants of length 7 and 13 characters respectively.

**Variables**

Each programming languages has its own rules for forming character variables. These variables fall into one of three categories

1. **Static:** In static character variable, whose length is defined before the program is executed and cannot change throughout the program
2. **Semi-static:** The length of the variable may vary during the execution of the program as long as the length does not exceed a maximum value determined by the program before the program is executed.
3. **Dynamic:** The length of the variable can change during the execution of the program.

## String Operations

**Substring**

Accessing a substring from a given string requires three pieces of information:

(1) The name of the string or the string itself
(2) The position of the first character of the substring in the given string
(3) The length of the substring or the position of the last character of the substring.

**Syntax:** SUBSTRING (string, initial, length)

The syntax denote the substring of a string S beginning in a position K and having a length L. Ex: SUBSTRING ('TO BE OR NOT TO BE', 4, 7) = 'BE OR N'

SUBSTRING ('THE END', 4, 4) = ' END'

**Indexing**

Indexing also called pattern matching, refers to finding the position where a string pattern P first appears in a given string text T. This operation is called INDEX

**Syntax:** INDEX (text, pattern)

If the pattern P does not appears in the text T, then INDEX is assigned the value 0. The arguments "text" and "pattern" can be either string constant or string variable.

## Concatenation

Let S1 and S2 be string. The concatenation of S1 and S2 which is denoted by S1 // S2, is the string consisting of the characters of S1 followed by the character of S2.

Ex:

(a) Suppose S1 = 'MARK' and S2= 'TWAIN' then S1 // S2 = 'MARKTWAIN'

Concatenation is performed in C language using ***strcat*** function as shown below

strcat (S1, S2);

Concatenates string S1 and S2 and stores the result in S1

***strcat ( )*** function is part of the ***string.h*** header file; hence it must be included at the time of pre-processing

## Length

The number of characters in a string is called its length.

**Syntax:** LENGTH (string)

Ex: LENGTH ('computer') = 8

String length is determined in C language using the ***strlen( )*** function, as shown below: X = strlen ("sunrise");

strlen function returns an integer value 7 and assigns it to the variable X

Similar to ***strcat, strlen*** is also a part of string.h, hence the header file must be included.

| Function | Description |
|---|---|
| *char *strcat(char *dest, char *src)* | concatenate *dest* and *src* strings; return result in *dest* |
| *char *strncat(char *dest, char *src, int n)* | concatenate *dest* and *n* characters from *src*; return result in *dest* |
| *char *strcmp(char *str1, char *str2)* | compare two strings; return < 0 if *str1* < *str2*; 0 if *str1* = *str2*; > 0 if *str1* > *str2* |
| *char *strncmp(char *str1, char *str2, int n)* | compare first *n* characters return < 0 if *str1* < *str2*; 0 if *str1* = *str2*; > 1 if *str1* > *str2* |
| *char *strcpy(char *dest, char *src)* | copy *src* into *dest*; return *dest* |
| *char *strncpy(char *dest, char *src, int n)* | copy *n* characters from *src* string into *dest*; return *dest*; |
| *size_t strlen(char *s)* | return the length of a *s* |
| *char *strchr(char *s, int c)* | return pointer to the first occurrence of *c* in *s*; return *NULL* if not present |
| *char *strrchr(char *s, int c)* | return pointer to last occurrence of *c* in *s*; return *NULL* if not present |
| *char *strtok(char *s, char *delimiters)* | return a token from *s*; token is surrounded by *delimiters* |
| *char *strstr(char *s, char *pat)* | return pointer to start of *pat* in *s* |
| *size_t strspn(char *s, char *spanset)* | scan *s* for characters in *spanset*; return length of span |
| *size_t strcspn(char *s, char *spanset)* | scan *s* for characters not in *spanset*; return length of span |
| *char *strpbrk(char *s, char *spanset)* | scan *s* for characters in *spanset*; return pointer to first occurrence of a character from *spanset* |

# Pattern Matching Algorithms

Pattern matching is the problem of deciding whether or not a given string pattern P appears in a string text T. The length of P does not exceed the length of T.

**First Pattern Matching Algorithm**

- The first pattern matching algorithm is one in which comparison is done by a given pattern P with each of the substrings of T, moving from left to right, until a match is found.

**$W_K$ = SUBSTRING (T, K, LENGTH (P))**

- Where, WK denote the substring of T having the same length as P and beginning with the $K^{th}$ character of T.
- First compare P, character by character, with the first substring, W1. If all the characters are the same, then P = W1 and so P appears in T and INDEX (T, P) =1.
- Suppose it is found that some character of P is not the same as the corresponding character of W1. Then P ≠ W1
- Immediately move on to the next substring, W2 That is, compare P with W2. If P ≠ W2 then compare P with W3 and so on.
- The process stops, When P is matched with some substring WK and so P appears in T and INDEX(T,P) = K or When all the WK'S with no match and hence P does not appear in T.
- The maximum value MAX of the subscript K is equal to LENGTH(T) -LENGTH(P) +1.

---

Algorithm: (Pattern Matching)
P and T are strings with lengths R and S, and are stored as arrays with one character per element. This algorithm finds the INDEX of P in T.

1. [Initialize.] Set K: = 1 and MAX: = S - R + 1
2. Repeat Steps 3 to 5 while K ≤ MAX
3. Repeat for L = 1 to R: [Tests each character of P]
   If P[L] ≠ T[K + L – l], then: Go to Step 5
   [End of inner loop.]
4. [Success.] Set INDEX = K, and Exit
5. Set K := K + 1
   [End of Step 2 outer loop]
6. [Failure.] Set INDEX = O
7. Exit

---

Observation of algorithms

- P is an r-character string and T is an s-character string
- Algorithm contains two loops, one inside the other. The outer loop runs through each successive R-character substring WK = T[K] T[K + 1] ... T[K+R-l] of T.
- The inner loop compares P with WK, character by character. If any character does not match, then control transfers to Step 5, which increases K and then leads to the next substring of T.
- If all the R characters of P do match those of some WK then P appears in T and K is the INDEX of P in T.
- If the outer loop completes all of its cycles, then P does not appear in T and so INDEX = 0.

**Complexity**

The complexity of this pattern matching algorithm is equal to $O(n^2)$

**Second Pattern Matching Algorithm**

The second pattern matching algorithm uses a table which is derived from a particular pattern P but is independent of the text T.

For definiteness, suppose

P = aaba

This algorithm contains the table that is used for the pattern P = *aaba.*

**The table is obtained as follows.**

- Let Qi denote the initial substring of *P* of length *i,* hence Q0 = A, Q1 = *a,* Q2 = *a2,* Q3 = *aab,* Q4 = *aaba* = *P* (Here Q0 = A is the empty string.)
- The rows of the table are labeled by these initial substrings of *P,* excluding *P* itself.
- The columns of the table are labeled *a, b* and *x,* where *x* represents any character that doesn't appear in the pattern *P.*
- Let f be the function determined by the table; i.e., let *f(Qi, t)* denote the entry in the table in row Qi and column *t* (where *t* is any character). This entry *f(Qi, t)* is defined to be the largest Q that appears as a terminal substring in the string (*Qi t)* the concatenation of Qi and *t.*

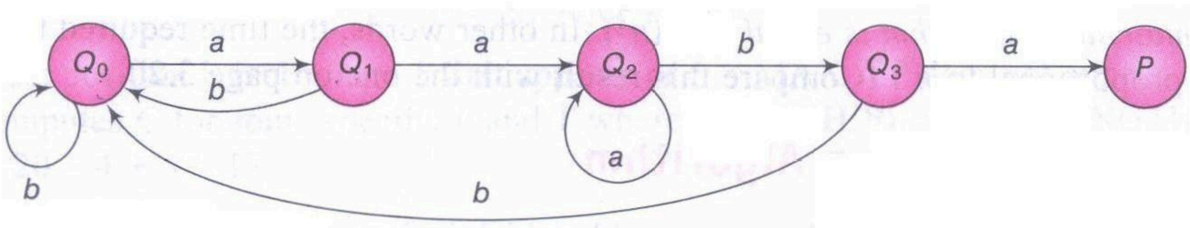
For example,

*a2* is the largest Q that is a terminal substring of *Q2a* = $a^3$, so *f(Q2, a)* = Q2 A is the largest Q that is a terminal substring of Q1*b* = *ab,* so *f(Q1, b)* = Q0 *a* is the largest Q that is a terminal substring of *Q0a* = *a,* so *f(Q0, a)* = Q1

A is the largest Q that is a terminal substring of *Q3a* = $a^3$*bx,* so *f(Q3, x)* = Q0

| | $a$ | $b$ | $x$ |
|---|---|---|---|
| $Q_0$ | $Q_1$ | $Q_0$ | $Q_0$ |
| $Q_1$ | $Q_2$ | $Q_0$ | $Q_0$ |
| $Q_2$ | $Q_2$ | $Q_3$ | $Q_0$ |
| $Q_3$ | $P$ | $Q_0$ | $Q_0$ |

(a) Pattern matching table




b Pattern matching graph

Fig 1.25 Pattern matching table and graph

Although Q1 = *a* is a terminal substring of *Q2a* = $a^3$, we have *f(Q2, a)* = Q2 because Q2 is also a terminal substring of *Q2a* = $a^3$ and Q2 is larger than *Q1*. We note *that f(Qi, x)* = Q0 for any *Q,* since *x* does not appear in the pattern *P* Accordingly, the column corresponding to *x* is usually omitted from the table.

**Pattern matching Graph**

The graph is obtained with the table as follows.

First, a node in the graph corresponding to each initial substring Qi of *P.* The Q's are called the *states* of the system, and Q0 is called the *initial* state.

Second, there is an arrow (a directed edge) in the graph corresponding to each entry in the table. Specifically, if

*f(Qi, t) = Qj*

then there is an arrow labeled by the character *t* from *Qi* to *Qj*

For example, f(Q2, *b) = Q3* so there is an arrow labeled *b* from Q2 to *Q3*

For notational convenience, all arrows labeled *x* are omitted*,* which must lead to the initial state *Qo.*

The second pattern matching algorithm for the pattern P = *aaba.*

- Let T = T1 T2 T3 ... TN denote the n-character-string text which is searched for the pattern P. Beginning with the initial state Q0 and using the text T, we will obtain a sequence of states S1, S2, S3, ... as follows.

- Let S1 = Q0 and read the first character T1. The pair (S1, T1) yields a second state S2; that is, F(S1, T1) = S2, Read the next character T2, The pair (S2, T2) yields a state S3, and so on.

There are two possibilities:

1. Some state SK = P, the desired pattern. In this case, P does appear in T and its index is K - LENGTH(P).
2. No state S1, S2, ... , SN +1 is equal to P. In this case, P does not appear in T.

---

Algorithm: (PATTERN MATCHING) The pattern matching table F(Q1, T) of a pattern P is in memory, and the input is an N-character string T = T1 T2 T3 …… TN. The algorithm finds the INDEX of P in T.

1. [Initialize] set K: =1 ans S1 = Q0
2. Repeat steps 3 to 5 while SK ≠ P and K ≤ N
3. Read TK
4. Set SK+1 : = F(SK, TK) [finds next state]
5. Set K: = K + 1 [Updates counter] [End of step 2 loop]
6. [Successful ?]

   If SK = P, then

   INDEX = K – LENGTH (P)

   Else

   INDEX = 0

   [End of IF structure]
7. Exit.

---