Representation of Signed Numbers

Module 1

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Storing integers in Binary

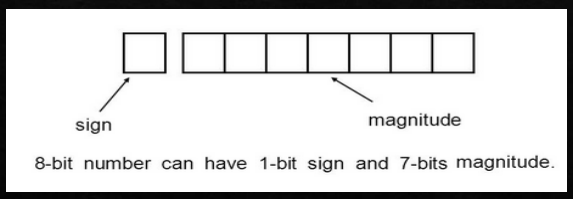
- Integers can be positive or negative.
- Any representation of an integer must have the sign as part of the representation of the number.
- In digital systems that operate with binary numbers the plus/minus sign of a number can be represented using the 0's and 1's

Methods for Representing Signed Numbers

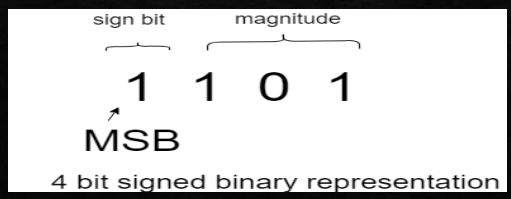
- Three **types of representations** for signed numbers
  - Sign & Magnitude Representation
  - Diminished Complement [(r-1)'s Complement]
  - True Complement [r's Complement]
- Representation of a positive number in all these 3 forms is same.
- Only the representation of negative number will differ in each form.

Sign & Magnitude Representation

- **Simplest method used to represent positive and negative integers.**
- **Binary numbers are represented with a separate sign bit along with the magnitude. In this system, the first bit (the MSB) of the number is used to represent the sign of the number.**
- **The MSB is the sign bit (0 for positive and 1 for negative) and the remaining digits represents the magnitude or value of the number.**



8-bit number can have 1-bit sign and 7-bits magnitude.



4 bit signed binary representation

Examples

+108

- Binary equivalent of 108 is 1101100. (7 bits represents magnitude).
- Since it is positive number, the sign bit (MSB) is set as zero.
- So, +108 = 01101100

-108

- Binary equivalent of 108 is 1101100. (7 bits represents magnitude).
- Since it is a negative number, the sign bit (MSB) is set as one.
- So, -108 = 11101100

Examples

1011 0111

- **The first bit, the MSB, is 1 indicating the integer is negative.**
- **The next 7 bits 011 0111 represent the magnitude of the integer.**
- **Binary 011 0111 has decimal value 32 + 16 + 4 + 2 + 1 = 55.**
- **So, 10110111 = -55**

Disadvantages:

1. During arithmetic operations, the sign bit also has to be considered along with the magnitude, which creates complication in the implementation level.
2. Zero has two representations [+0 as 0000_2 and -0 as 1000_2]

Complements

- Complements are used to simplify the subtraction operation and for logical manipulation. **Two types of complements** for each *base-r* system.
  - Diminishing radix complement ((r-1)'s complement)
  - Radix complement (r's complement)

		Decimal	Binary	Octal	Hexadecimal
r's	complement	10's	2's	8's	16's
r - 1's	complement	9's	1's	7's	15's

- The (r-1)'s complement of a number in any number system with base r can be found out by subtracting every single digit of a number by r-1.
- The r's complement of a non-zero number in any number system with base **r** can be calculated by adding **1** to the LSB of its **(r-1)'s** complement.

Complements – Decimal Number System

Decimal Number Complement – 9's & 10's

9's complement is obtained by subtracting each digit from 9;

10's complement = 9's complement +1.

1. Find the 9's & 10's Complement of 5982

9's complement

$$\begin{array}{r} 9999 \\ -\ 5982 \\ \hline 4017 \end{array}$$

10's complement

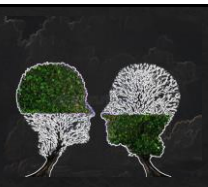
$$\begin{array}{r} 9999 \\ -\ 5982 \\ \hline 4017 \\ +\ 1 \\ \hline 4018 \end{array}$$

2. Find the 9's and 10's complement of 2496

9's Complement = 9999-2496 = 7503;

10's Complement =7503+1=7504

Complements – Octal Number System

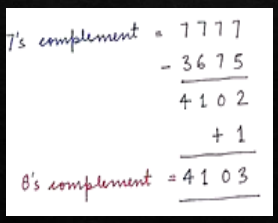


Octal Number Complement – 7's & 8's

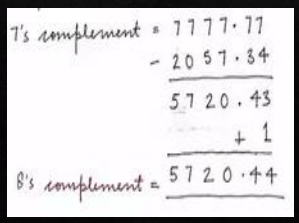
7's is obtained by subtracting each digit from 7;

8's complement = 7's complement +1.

1) 3675



2) 2057.34



3) Find the 7's and 8's complement of 562

7's complement = 777- 562 = 215;

8's Complement =215+1=216

Complements – Hexadecimal Number System

Hexadecimal Number Complement – 15's & 16's

15's is obtained by subtracting each digit from 15

16's complement = 15's complement +1.

1) 15AD

15	15	15	15
-1	-5	-A	-D
E	A	5	2

2) Find the 15's and 16's complement of 3BF.

15's comp.= 15 15 15 – 3 B F = C 4 0;

16's =C 4 0+1=C 4 1

Complements – Binary Number System

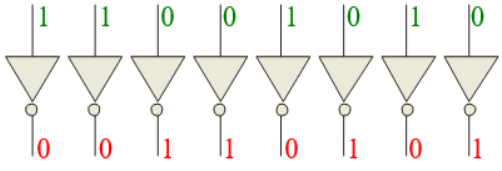
- In binary number system we have the 1's and 2's complement
- The 1's and 2's complement of a binary numbers are important because they permit the representation of negative numbers.
- The method of 2's complement arithmetic is commonly used in computers to handle negative numbers.

1's Complement

The 1's complement of a binary number is just the inverse of the digits. To form the 1's complement, change all 0's to 1's and all 1's to 0's.

For example, the 1's complement of 11001010 is 00110101

In digital circuits, the 1's complement is formed by using inverters:



A positive number in the 1's complement is represented by the same way as the positive sign-magnitude number.

A negative number is the 1's complement of the corresponding positive number.

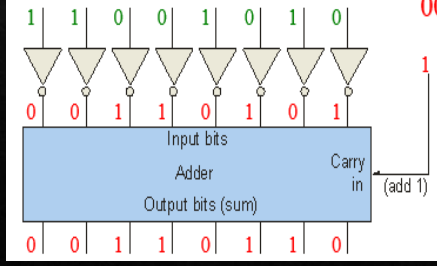
+25 = 00011001
-25 = 11100110

2's Complement

The 2's complement of a binary number is found by adding 1 to the LSB of the 1's complement.

Recall that the 1's complement of 11001010 is 00110101 (1's complement)

To form the 2's complement, add 1: $\frac{+1}{00110110}$ (2's complement)



A positive number in 2's complement form is represented by same way as in the sign-magnitude and 1's complement form.

A negative number is the 2's complement of the corresponding positive number.

+25 = 00011001
1's(-25) = 11100110
2's(-25) = 11100111

Exercises

Determine the 1's complement of each binary number.

a) 00011010 b) 11110111 c) 10001101

Determine the 2's complement of each binary number.

a) 00010110 b) 11111100 c) 10010001

Represent in Sign-magnitude, 1's and 2's complement form (use 8-bits)

a) 12 b) -9 c) -18

Quiz

Any signed negative binary number is recognised by its _____

a) MSB
b) LSB
c) Byte
d) Nibble

Quiz

Convert the following decimal number to 8-bit binary.
187

A. 10111011_2
B. 11011101_2
C. 10111101_2
D. 10111100_2

Quiz

Convert the following binary number to decimal.

01011_2

A. 11

B. 35

C. 15

D. 10

Quiz

Convert the binary number 1001.0010_2 to decimal.

A. 90.125

B. 9.125

C. 125

D. 12.5