MAXTERMS

1

Maxterms (OR terms or Sum terms):

A literal or the logical sum (OR) of multiple literals within the logic system.

Eg: X X'+Y X+(Y+Z) A'+B+C

If the value of a variable is 1, then its complement is added else the variable is added as it is.

If A=0,B=1,C=1, then maxterm is A**+B'+C'**

Maxterms can also be written as M with a subscript which is decimal equivalent of given input combination.

Eg: A+B'+C' can be written as M_3

Given that each binary variable may appear normal (A) or complemented (A'), there are **2^n maxterms for n variables.**

Eg: 3 variables (A,B&C) produce **2^3=8** maxterms

Variables			Sum terms	Maxterm
A	B	C		M_i
0	0	0	A + B + C	M_0
0	0	1	A + B + C'	M_1
0	1	0	A + B' + C	M_2
0	1	1	A + B' + C'	M_3
1	0	0	A' + B + C	M_4
1	0	1	A' + B + C'	M_5
1	1	0	A' + B' + C	M_6
1	1	1	A' + B' + C'	M_7

POS & STANDARD POS

2

1. **Products-of-Sum (POS) Expression**
   - also called as **Conjunctive Normal Form**
   - when two or more sum terms are logically multiplied
   - 2-level OR-AND circuit
   - `F(x,y,z) = (x+y)(x+z)(y+z)`

2. – **Canonical Products-of-Sum (POS) Expression**
   - when a Boolean expression is represented purely as product of maxterms and every variable in the domain must appear in each term.
   - Eg: F = (X'+Y+Z).(X+Y'+Z')

Can also be represented as

F = Π (3,4)

PRODUCTS-OF-SUM (POS) FORM

3

- Two or more sum terms are logically multiplied
- Eg: (A+B) . (A+B+C)   A ‘ . (A’+B+C)
- In POS expression, a single overbar cannot extend over more than one variable; however, more than one variable in a term can have an overbar. Eg: a POS expression can have the term A’+B’+C’ but not (A+B+C)’
- In Standard POS expression, all the variables in the domain must appear in each sum term in the expression.

PRODUCTS-OF-SUM (POS) FORM

OR/AND IMPLEMENTATION OF POS FORM

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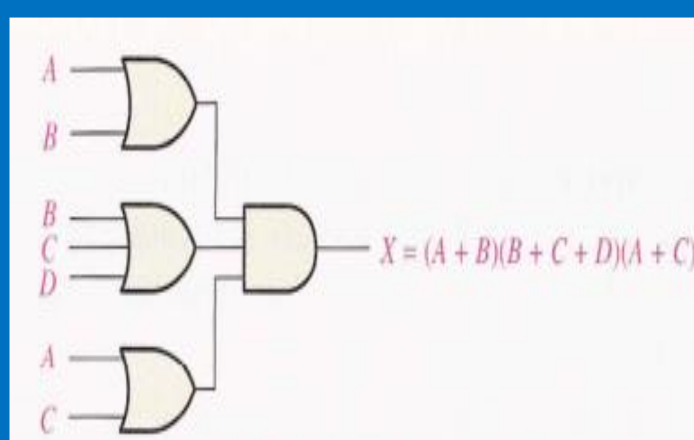
POS can be implemented using **2-level OR-AND** circuit

Implementation of the POS expression $(A + B)(B + C + D)(A + C)$.



Products-of-Sum (POS) Form

Maxterms:

1 - complements (X')

0 - true value (X)

POS Expression from Truth Table

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1. For a given expression, prepare a truth table for all possible combinations of inputs.
2. Take the product of all the maxterms which produces LOW output.

Express the POS form of the Boolean function F(A,B,C) for the truth table given:

F = (A+B+C) .

(A+B'+C').

(A'+B+C).

(A'+B+C')

Inputs			Output	Maxterms
A	B	C	X	M_n
0	0	0	0	m_0 = A+B+C
0	0	1	1	m_1 = A+B+C'
0	1	0	1	m_2 = A+B'+C
0	1	1	0	m_3 = A+B'+C'
1	0	0	0	m_4 = A'+B+C
1	0	1	0	m_5 = A'+B+C'
1	1	0	1	m_6 = A'+B'+C
1	1	1	1	m_7 = A'+B'+C'

Products-of-Sum (POS) Form

Algebraic Method

Algebraic Method for Maxterm expansion

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1. Simplify the given expression using appropriate theorems/rules.
2. Convert the given expression in POS form by applying the rule: X+YZ = (X+Y).(X+Z)
3. In each term, if any variable is missing, add that term with (missingterm . missingterm') factor.
4. Simplify the expression until we get POS terms
5. Remove all the duplicates

PRODUCTS-OF-SUM (POS) FORM

ALGEBRAIC METHOD

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Solution The domain of this POS expression is A, B, C, D. Take one term at a time. The first term, $A + \overline{B} + C$, is missing variable D or $\overline{D}$, so add $D\overline{D}$ and apply rule 12 as follows:

$$A + \overline{B} + C = A + \overline{B} + C + D\overline{D} = (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})$$

The second term, $\overline{B} + C + \overline{D}$, is missing variable A or $\overline{A}$, so add $A\overline{A}$ and apply rule 12 as follows:

$$\overline{B} + C + \overline{D} = \overline{B} + C + \overline{D} + A\overline{A} = (A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})$$

The third term, $A + \overline{B} + \overline{C} + D$, is already in standard form. The standard POS form of the original expression is as follows:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D) =$$
$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

PRODUCTS-OF-SUM (POS) FORM

Exercises:

1. Convert $F(X,Y,Z) = \Pi(0,2,5)$

2. Convert $F(A,B,C,D) = \Pi(0,9,12,15)$

Shorthand Maxterm Notation

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If $F = \Pi(0,1,4,5,7)$, this specifies that output F is product of $0^{th}, 1^{st}, 4^{th}, 5^{th}$ and 7^{th} maxterms.

Ie $F = M_0 . M_1 . M_4 . M_5 . M_7$

M_0 -> 000 -> X + Y + Z

M_1 -> 001 -> X + Y + Z'

M_4 -> 100 -> X' + Y + Z

M_5 -> 101 -> X' + Y + Z'

M_7 -> 111 -> X' + Y' + Z'

F = (X+Y+Z) . (X + Y + Z'). (X' + Y + Z). (X'+ Y + Z'). (X' + Y' + Z')

Conversion between Canonical Forms

Complement of a function

The Complement of a function expressed as the sum of minterms equals the sum of minterms missing from the original function. This is because the original function is expressed by those minterms that make the function equal to 1, while its complement is a 1 for those minterms for which the function is a 0.

Eg: $F(A,B,C) = \Sigma(1,4,5,6,7) = m_1+m_4+m_5+m_6+m_7$

has a complement that can be expressed as **$F'(A,B,C) = \Sigma(0,2,3) = m_0+m_2+m_3$**

Conversion between Canonical Forms

Standard SOP to Standard POS

$F(A,B,C) = \Sigma(1,4,5,6,7) = m_1+m_4+m_5+m_6+m_7$ has a complement that can be expressed as

$F'(A,B,C) = \Sigma(0,2,3) = m_0+m_2+m_3$

To convert from one canonical form to another, **interchange the symbol** and **list those numbers missing from the original form.**

If we take the complement of F',

$$\begin{aligned} \mathbf{F''} &= (m_0+m_2+m_3)' \\ &= (m_0)'.(m_2)'.(m_3)' \\ &= M_0.M_2.M_3 \\ &= \Pi(0,2,3) \end{aligned}$$

$M_i = m_i'$

Exercise:

$F(X,Y,Z) = \Pi(0,2,4,5)$ is in POS form, find its SOP?

Conversion between Canonical Forms

Standard SOP to Standard POS

Convert the SOP expression to an equivalent POS expression: A'B'C' + A'BC' + A'BC + AB'C +ABC

Solution : **The given SOP Expression contains the terms**

A'B'C' + A'BC' + A'BC + AB'C + ABC

000 010 011 101 111

m_0 m_2 m_3 m_5 m_7

Since there are three variables in the domain of the given expression, there are total of 2^3 possible combinations. The given SOP expression contains 5 of these combinations, so the POS must contain the other 3 which are 001(m_1), 100(m_4),110(m_6)

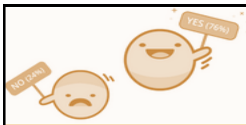
So the equivalent POS expression is F = (A+B+C') . (A'+B+C) . (A'+B'+C)

Conversion between Canonical Forms

Standard SOP to Standard POS

Identify each of the following expression as SOP, Standard SOP, POS or Standard POS.

1. AB + A'BD +A'CD'
2. (A+B'+C).(A'+B+C')
3. A'BC +ABC'
4. A(A+C')(A+B)



Boolean Functions

1

Boolean function or Switching Function:

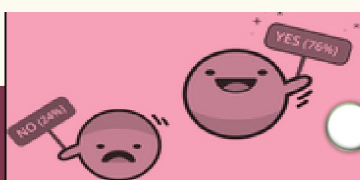
consists of an algebraic expression formed with binary variables, the constants 0 and 1, the logic operation symbols, parenthesis, and an equal sign.

Eg: $F(X,Y,Z) = X + Y' Z + 1$

The variable in true or complemented form (eg: X or Z') is called as a **Literal**

Domain of a Boolean expression is **set of literals** contained in the expression.

Eg: Domain of AB + C is A, B, and C



Boolean Functions

2

A function can be specified or represented in any of the following ways:

- Truth table
- Logic Circuit
- Boolean expression
- SOP (Sum Of Products)
- POS (Product of Sums)
- Canonical SOP
- Canonical POS

Minterms

Minterms (Product terms or AND terms)

They are a literal or the logical product (AND) of multiple literals within the logic system.

Eg: X XY XYZ X'YZ' A'BC

Variables with a value 0 can be represented by its complement.

Eg: If x=0,y=1,z=0, then minterm is **x ' y z '**

Minterms can also be written as m with a subscript which is decimal equivalent of given input combination . Eg: m_2

Given that each binary variable may appear normal (x) or complemented (x'), there are **2^n minterms for n variables.**

Eg: 2 variables (X & Y) produce 2^2 = 4 minterms

x	y	minterm	designation
0	0	x'y'	m_0
0	1	x'y	m_1
1	0	xy'	m_2
1	1	xy	m_3

x	y	z	minterm	designation
0	0	0	x'y'z'	m_0
0	0	1	x'y'z	m_1
0	1	0	x'yz'	m_2
0	1	1	x'yz	m_3
1	0	0	xy'z'	m_4
1	0	1	xy'z	m_5
1	1	0	xyz'	m_6
1	1	1	xyz	m_7

SOP & Canonical SOP

1 Sum-of-Products (SOP) Expression

- also called as **Disjunctive Normal Form**
- two or more product terms are logically added together
- Eg: X'.Y +XY' X.Y+Z X+Y

F(x,y,z) = xy + xz + yz

2 Canonical Sum-of-Products (SOP) Expression

- when a Boolean expression is represented purely as sum of minterms (product terms) and every variable in the domain must appear in each term.
- Eg: F = AB'C+A'B'C+ABC

Can also be represented as F = Σ (1,5,7)

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Sum of Products (SOP) Form

- SOP: Two or more product terms are logically added together.
- Eg: AB + ABC   BC + CDE + B'CD'   A + A'BC

- In SOP expression, a single overbar cannot extend over more than one variable; however, more than one variable in a term can have an overbar.
- Eg: an SOP expression can have the term A'B'C' but not (ABC)'

- SOP can be implemented using **2-level AND-OR circuit**

Minterms:

0 - complements (X')

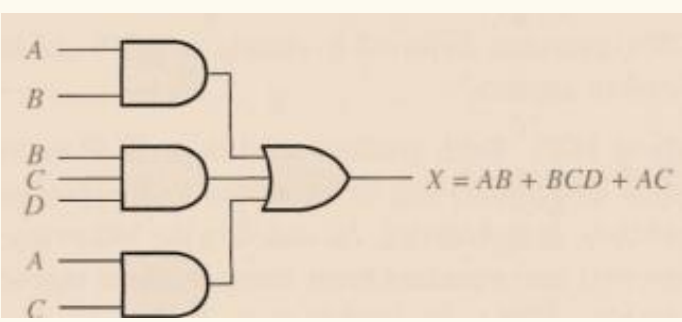
1 - true value (X)

6

SOP FORM

AND/OR IMPLEMENTATION OF SOP FORM

Implementation of the SOP expression $AB + BCD + AC$.



EXERCISES

1. AB+A'C+BD
2. AB'C+AB+AC

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Conversion of a general expression to SOP Form

Standard SOP Form (Canonical SOP)

- Any logic expression can be changed into SOP form by applying Boolean algebra techniques.
- Eg: A(B+CD) can be converted into SOP form by applying the distributive law as AB + ACD
- Exercises:
  1. Convert (A+B)(B+C+D)
  2. Convert A'BC' + (A+B')(B+C'+AB')

- In **standard SOP expression**, all the variables in the domain must appear in each product term in the expression.
- Two methods to **generate standard (Canonical) SOP terms**
  - Using Truth table
  - Using algebraic method

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SOP Form

Example- Algebraic Method

Algebraic Method for Minterm expansion

1. Convert the given expression in SOP form
2. In each term, if any variable is missing, multiply that term with (missingterm +missingterm') factor.
3. Expand the expression
4. Remove the duplicates

Example:

Convert F=X+Y to minterms

F = X + Y = X.1+Y.1

= X.(Y+Y')+Y.(X+X')

= XY+XY'+XY+X'Y

= XY+XY'+X'Y

Convert the following Boolean expression into standard SOP form:

$$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$$

The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\bar{B}C$, is missing variable D or $\bar{D}$, so multiply the first term by $D + \bar{D}$ as follows:

$$A\bar{B}C = A\bar{B}C(D + \bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$$

In this case, two standard product terms are the result.

The second term, $\bar{A}\bar{B}$, is missing variables C or $\bar{C}$ and D or $\bar{D}$, so first multiply the second term by $C + \bar{C}$ as follows:

$$\bar{A}\bar{B} = \bar{A}\bar{B}(C + \bar{C}) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

The two resulting terms are missing variable D or $\bar{D}$, so multiply both terms by $D + \bar{D}$ as follows:

$$\begin{aligned}\bar{A}\bar{B} &= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B}C(D + \bar{D}) + \bar{A}\bar{B}\bar{C}(D + \bar{D}) \\ &= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\bar{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$$

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SOP FORM

EXAMPLE-

ALGEBRAIC METHOD

EXERCISES

1. AB’+A’C+A’
2. ABC+A’B’+C

SOP Expression from Truth Table

1. For a given expression, prepare a truth table for all possible combinations of inputs.
2. Add all the minterms which produces HIGH output.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

F = A’BC + AB’C + ABC ‘ + ABC

= Σ(3,5,6,7)

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SOP FORM

Minterms:

0 - complements (X’)

1 - true value (X)

A Boolean function F defined on 3 variables – A, B & C is 1 iff odd number of one inputs. Draw the truth table for the function and express it in canonical SOP form.

Hint: The output Z is one only for odd number of one inputs. Draw the truth table and then add all the minterms corresponding to the high output

Input			Output
A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

F = A'B'C + A'BC' + AB'C '+ ABC

= Σ(1,2,4,7)

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SOP FORM

Minterms:

0 - complements (X')

1 - true value (X)

EXAMPLE-
TRUTH TABLE METHOD

Shorthand Minterm Notation

1. **First of all, copy original terms**
2. **Substitute 0's for complements and 1's for true letters.**
3. **Express the decimal equivalent as subscript of m.**

Example:

Find the minterm designation of **X Y' Z'**

Copy original term - X Y' Z'

Substitute 1's & 0's - 1 0 0

Decimal Equivalent of 100 is 4, Thus X Y' Z' = $\mathbf{m_4}$

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SOP FORM

Example

A B C ' D '

1 1 0 0

$\mathbf{m_{12}}$

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SOP Form

Converting Shorthand Notation to minterms

1. **Find binary equivalent of decimal subscript**
2. **For every 1's write the variable as it is and for 0's write variable's complemented form.**

Example:

Convert F = Σ(0,1,2,5) into canonical SOP form

$$F = m_0 + m_1 + m_2 + m_5$$

000 001 010 101

X'Y'Z'+X'Y'Z + X'YZ' + XY'Z

Exercises

1. $F = m_3 + m_4 + m_5$
2. F = Σ(7,12,15)