BOOLEAN ALGEBRA

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Boolean algebra (switching algebra), is a mathematical system for the manipulation of variables that can have one of two values - “true” and “false.” In digital systems, these values are “on” and “off,” 1 and 0, or “high” and “low.”

Boolean Algebra is used to analyze and simplify the digital (logic) circuits. As it uses only the binary numbers i.e. 0 and 1, it is also called as **Binary Algebra** or **logical Algebra**.

BOOLEAN ALGEBRA

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Rule in Boolean Algebra

- Variable used can have only two values. Binary 1 for HIGH and Binary 0 for LOW.
- Complement of a variable is represented by an overbar or ‘.
- Eg: complement of variable B is represented as B’.

Logical ANDing of the two or more variable is represented by writing a dot between them such as A.B.C.

ORing of the variables is represented by a plus (+) sign between them. For example ORing of A, B, C is represented as A + B + C

Two key Concepts

1 Duality Theorem

- Replace each • (AND) with + (OR), + (OR) with • (AND),
- Replace each 0 with 1, and 1 with 0
- Leave the variables unchanged

All Boolean expressions have logical duals

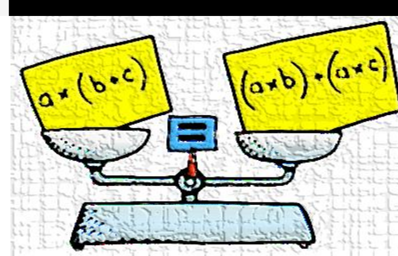
Any theorem that can be proved is also proved for its dual

2 De Morgan's Law -- "Break the line , Change the Sign"

- Replace each • (AND) with + (OR), + (OR) with • (AND),
- Replace each 0 with 1, and 1 with 0
- Replace all variables with their complements

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$



DUALITY THEOREM

Interchanging the logical **AND** operator with logical **OR** operator and **zeros with ones & ones with zeros**.

Examples:

Given Expression	Dual	Given Expression	Dual
$\overline{0} = 1$	$\overline{1} = 0$	$A.(A+B) = A$	$A + A.B = A$
$0.1 = 0$	$1 + 0 = 1$	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A+B} = \overline{A}.\overline{B}$
$A.0 = 0$	$A + 1 = 1$	$(A+C)(\overline{A}+B) = AB + A\overline{C}$	$AC + \overline{A}B = (A+B).(A+\overline{C})$
$A.B = B.A$	$A + B = B + A$	$A+B = AB + \overline{A}B + A\overline{B}$	$AB = (A+B).(\overline{A}+B).(A+\overline{B})$
$A.\overline{A} = 0$	$A + \overline{A} = 1$	$\overline{AB} + \overline{A} + AB = 0$	$((\overline{A+B})).\overline{A}.(A+B) = 1$

Exercises:

Find the dual of :

1. AB+AC
2. (A+B)+C =A+(B+C)
3. A'+0=A
4. A. (B.C) = (A.B). C

Basic Postulates

1. OR Relations (Logical Addition)

   0 + 0 = 0   0 + 1 = 1

   1 + 1 = 1   1 + 0 = 1

2. AND Relations (Logical Multiplication)

   0 . 0 = 0   0 . 1 = 0

   1 . 1 = 1   1 . 0 = 0

3. Complement Rules

   0’ = 1

   1’ = 0

Basic Theorems of Boolean Algebra

Single Variable Theorems

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Null Law

1) $A . 0 = 0$

2) $A + 1 = 1$

3) $A + 0 = A$

4) $A . 1 = A$

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Idempotence Law

5) $A + A = A$

6) $A . A = A$

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Involution Law

7) $A'' = A$

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Complementarity Law

8) $A + A' = 1$

9) $A . A' = 0$

Basic Theorems of Boolean Algebra

Multivariable Theorems

8 Commutative Law

10) $A + B = B + A$

11) $A . B = B . A$

9 Distributive Law

14) $A . (B + C) = AB + AC$

15) $A + (B . C) = (A + B) . (A + C)$

10 Associative Law

12) $A + (B + C) = (A + B) + C$

13) $A . (B . C) = (A . B) . C$

11 Absorption Law

16) $A + AB = A$

17) $A . (A + B) = A$

12 Third Distributive Law

18) $A + A'B = A + B$

19) $A . (A' + B) = A . B$

De Morgan's Theorem

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

"Break the line , Change the Sign, invert the bits"

Replace each • (AND) with + (OR), + (OR) with • (AND),

0 with 1, and 1 with 0

Replace all variables with their complements

Theorem 1 : (A+B)' = A' . B' ;

Complement of the sum is the product of the complements

Theorem 2 : (A.B)' = A' + B' ;

Complement of the product is the sum of the complements

De Morgan's law can be extended to any number of variables.

- **(A+B+....+Z)' = A' . B' . ... .Z'**
- **(A.B.C....Z)' = A' + B' + ...+Z'**

DE MORGAN'S THEOREM

A	B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}.\overline{B}$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

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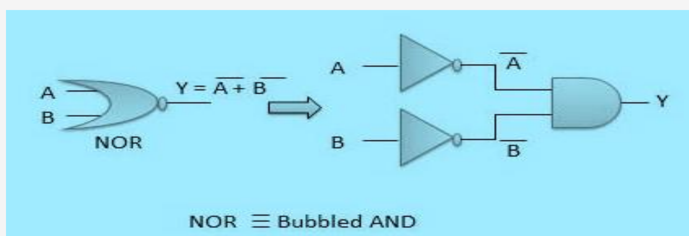
Theorem 1 : (A+B)’ = A’ . B’ ;

O/P bubbled OR = I/P bubbled AND

The LHS represents a **NOR gate** whereas the RHS represents an AND gate with inverted inputs(**Bubbled AND)**.



DE MORGAN'S THEOREM

A	B	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

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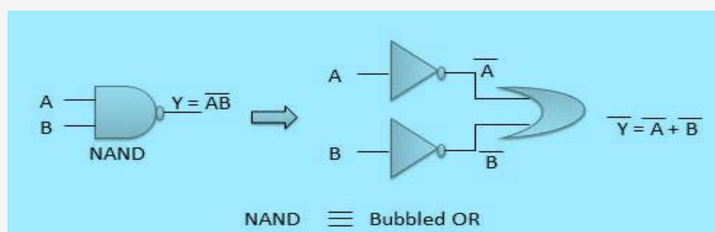
Theorem 2 : (A.B)' = A' + B' ;

O/P bubbled AND = I/P bubbled OR

The left hand side (LHS) represents a **NAND gate** whereas the RHS represents an OR gate with inverted inputs(**Bubbled OR**).



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Example 1

$$\overline{AB + CD}$$

↓ Breaking bar in middle

Notice the grouping maintained with parentheses → $\overline{(AB)}\;\overline{(CD)}$

↓ Breaking both bars in middle

Correct answer: $(\bar{A} + \bar{B})(\bar{C} + \bar{D})$

Example 2

$$\overline{\overline{A + BC} + \overline{A\bar{B}}}$$

↓ Breaking longest bar

$$\overline{\overline{(A + BC)}}\;\overline{\overline{(A\bar{B})}}$$

↓ Applying identity $\overline{\overline{A}} = A$ wherever double bars of equal length are found

$$(A + BC)(A\bar{B})$$

Apply Demorgan's theorem to the expression,

1. $\overline{A \cdot (A+C)}$
2. $\overline{(\overline{A + B\,\overline{C}}) + D(\overline{E + \overline{F}})}$
3. $\overline{A\,\overline{B}.(A + C)} + \overline{A}\,B.\overline{(A + B + \overline{C})}$

We can find the complement of a function using De Morgan's theorem

Examples:

$$F(X,Y,Z) = (XY) + (\overline{X}Z) + (Y\overline{Z})$$
$$\overline{F}(X,Y,Z) = \overline{(XY) + (\overline{X}Z) + (Y\overline{Z})}$$
$$= (\overline{XY})(\overline{\overline{X}Z})(\overline{Y\overline{Z}})$$
$$= (\overline{X}+\overline{Y})(X+\overline{Z})(\overline{Y}+Z)$$

$$F = \overline{X}Y\overline{Z} + \overline{X}\,\overline{Y}Z$$
$$\overline{F} = \overline{\overline{X}Y\overline{Z} + \overline{X}\,\overline{Y}Z}$$
$$= (\overline{\overline{X}Y\overline{Z}}) \bullet (\overline{\overline{X}\,\overline{Y}Z})$$
$$= (X+\overline{Y}+Z) \bullet (X+Y+\overline{Z})$$

De Morgan's Theorem

Exercises:

Find the complement of :

1. F = (AB + AC')'
2. F = A B' C' + A' B' C + A B' C + A B C'
3. F=(A+B)•(A'+C)

Evaluation of Boolean Expressions using Truth Table

- Verify X'Y + XY' +X'Y' = (X'+Y') using truth table.

X	Y	X'	Y'	X'Y	XY'	X'Y'	X'Y + XY' + X'Y'	X' + Y'
0	0	1	1	0	0	1	1	1
0	1	1	0	1	0	0	1	1
1	0	0	1	0	1	0	1	1
1	1	0	0	0	0	0	0	0

- Verify (xyz)' = x'+y'+z'using truth table

x y z	(xyz)'	x'	y'	z'	x'+y'+z'
0 0 0	1	1	1	1	1
0 0 1	1	1	1	0	1
0 1 0	1	1	0	1	1
0 1 1	1	1	0	0	1
1 0 0	1	0	1	1	1
1 0 1	1	0	1	0	1
1 1 0	1	0	0	1	1
1 1 1	0	0	0	0	0

Algebraic Method of Evaluation

- **Prove by algebraic manipulation that A'B'+AB = (AB'+A'B)'**

(A.B'+A'.B)' = (A.B')'. (A'.B)'		**De Morgan's**
	= (A'+B) . (A+B')	**De Morgan's**
	= A'A+A'B'+AB+BB'	**Distributivity**
	= A'B'+AB	**Identity**

- **Prove (A+B)(A+C)=A+BC**

$$
\begin{aligned}
(A+B)(A+C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1 + C + B) + BC \\
&= A \cdot 1 + BC \\
&= A + BC
\end{aligned}
$$

Algebraic Method of Evaluation

3. Reduce F = A[B+C'.(AB+AC')']

= AB+AC'((AB)'.(AC')')

= AB+AC'((A'+B').(A'+C))

=AB+AC'((A'A'+A'C+A'B'+B'C))

= AB+AC'A'+AA'CC'+AA'B'C'+AB'CC'

= AB+0 = AB

4. Reduce F = (A+(BC)')' . (AB'+ABC)

= (A'BC) . (AB'+ABC)

=AA'BB'C+AA'BBCC

=0

5. Reduce F = A+B[AC+(B+C')D]

= A+B[AC+BD+C'D]

= A+ABC+BBD+BC'D

= A(1+BC) + BD(1+C')

= A.1 + BD.1

= A+BD

EXERCISES

1. Give the dual of the Boolean expression: (x+y).(x'+z').(y+z)
2. Using truth table, prove that AB + BC+ CA'=AB+CA'
3. Prove the idempotence law and complementarity law with the help of a truth table.
4. Prove that X.(X+Y)=X by algebraic method