Functions Tutorial (Solutions)

3.

a. One-one

b. Onto

c. Onto

d. One-one

e. Both

f. Both

4.

If you are using the definition of function as a special kind of relation such that every element in domain is related to a unique element in codomain, then your function $f : A \to B$ is a subset of $A \times B$ such that

(x, y) *and* $(x, y_1) \in f \Rightarrow y = y_1$

and $x \in A \Rightarrow \exists y \in B$ such that $(x, y) \in f$.

Now can you get the equivalence you need?

OK. In all details, if $f = g$ i.e. if $f \subseteq g$ and $g \subseteq f$ then,

if $a \in dom(f), \exists b \in Bs.t.\ (a, b) \in f$

Thus, $(a, b) \in f$ and hence $(a, b) \in g$ and hence $a \in dom(g)$ and also $g(a) = b = f(a)$. Thus, $dom(f) \subseteq dom(g)$ and $f(a) = g(a)$ on $dom(f)$.

Similarly, you show that $dom(g) \subseteq dom(f)$ and $f(a) = g(a)$ on $dom(g)$.

The converse is proved along similar lines.

5.

$(f \circ g)(x) = \sqrt{3x + 1}$

$(g \circ f)(x) = 3\sqrt{x} + 1$

And hence, $f \circ g \neq g \circ f$

6.

$g \circ f : \mathbb{Q}^+ \to \mathbb{R}$ and $(g \circ f)(x) = 2 + \frac{1}{x}$ for $x \in \mathbb{Q}^+$

P.S. note the domain of this composition.

10.

Statement is false

Counter-example:

$A = B = \mathbb{N}, C = \{1\}, f(n) = n + 1$

Thus f is one-one

$f(A - C) = \mathbb{N} - \{1, 2\}$ and $B - f(C) = \mathbb{N} - \{2\}$

And hence we can see that $B - f(C) \nsubseteq f(A - C)$

11.

Let the length and breadth of the rectangle be x and y, and let $g : \mathbb{R}^+ \to \mathbb{R}^+$ such that $y = g(x)$

Since area is equal to it's perimeter, we get $xy = 2(x + y)$

and hence, $y = \frac{2x}{x-2}$

But also, $x = \frac{2y}{y-2}$

Thus, we can write $x = g^{-1}(y)$ and then we can see that $g = g^{-1}$

Hence, the function g is an involution.

Also, $f(x) = \frac{2x}{x-2}$ for $x \in \mathbb{Z}^+$