Discrete Structures (MA5.101)

Quiz - 2 (Monsoon 2021)

International Institute of Information Technology, Hyderabad

Time: 60 Minutes Total Marks: 30

Instructions: This is online examination.

Write at the top of your Answer book the following:

Discrete Structures (MA5.101)

Quiz - 2 (Monsoon 2021)

Date: 27-Dec-2021

Name:

Roll Number:

Submit your scanned hand-written Answer script in the moodle with the file name: RollNo_Quiz2_SecNo_27Dec2021.pdf

December 29, 2021

1. Choose the correct option for the following questions:

   (a) If A and B are symmetric relations, then the relation $A \cup B$ is __________.

   (A) Reflexive

   (B) Symmetric

   (C) Transitive

   (D) None of these

   Answer: (B)

   (b) Let $S = \{0, 1, 2, 3, 4\}$, a partition be $P = \{\{0, 2\}, \{1\}, \{3, 4\}\}$, and a relation R induced by a partition which is an equivalence relation. Then, the ordered pairs in R is __________.

   (A) $\{(0, 0), (2, 2), (0, 2), (2, 0), (1, 1), (3, 3), (4, 4), (3, 4), (4, 3)\}$

   (B) $\{(0, 0), (2, 4), (4, 2), (0, 2), (2, 0), (1, 1), (3, 3), (4, 4), (3, 4), (4, 3)\}$

   (C) $\{(0, 0), (2, 3), (3, 2), (0, 2), (2, 0), (1, 1), (3, 3), (4, 4), (3, 4), (4, 3)\}$

   (D) $\{(0, 0), (1, 2), (2, 1), (0, 2), (2, 0), (1, 1), (3, 3), (4, 4), (3, 4), (4, 3)\}$

   Answer: (A)

   Explanation: Then equivalence classes are: $\{0, 2\} = [0] = [2]$, $\{1\} = [1]$, and $\{3, 4\} = [3] = [4]$

   (c) Let R be a relation on a set S, and R is symmetric as well as transitive relation. Then, R will be an equivalence relation if __________.

   (A) $\forall\, a \in S \nexists b$ such that xRy

   (B) for some $a \in S \exists b$ such that xRy

   (C) $\forall\, a \in S \exists b$ such that xRy

(D) None of above

Answer: (C)

(d) Let $\mathbb{R}$ be a set of real numbers, and define a binary relation R on $\mathbb{R}\times\mathbb{R}$: $\forall(x, y), (z, w) \in \mathbb{R}\times\mathbb{R}$, $(x, y)R(z, w)$ if and only if either $x < z$ or both $x = z$ and $y \leq w$. The, R is __________.

(A) Reflexive, Symmetric and transitive

(B) Reflexive, antisymmetric and not transitive

(C) Reflexive, antisymmetric and transitive

(D) A partial order relation

Answer: (D), (C)

(e) Let R be a relation on the set $\mathbb{Z}$ (set of all integers) and define as follows: $\forall x, y \in \mathbb{Z}, xRy$ if and only if $(x + y)$ is divisible by 2. Then, R is __________.

(A) A partial order relation

(B) A symmetric relation

(C) An antisymmetric relation

(D) An antisymmetric but not partial order relation

Answer: (B)
Explanation: No. The relation is symmetric, hence it cannot be a partial order.

(f) Let a set S contains 23 elements, and then the cardinality of symmetric relations can be __________.

(A) 2^{529}

(B) 2^{506}

(C) 2^{276}

(C) 2^{253}

Answer: (C)

(g) Let $A = \{0, 1, 2, 3\}$ and define the relations R, S, and T on A as follows:
i. $R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$,
ii. $S = \{(0, 0), (0, 2), (0, 3), (2, 3)\}$,
iii. $T = \{(0, 1), (2, 3)\}$
Then, which of the following statement is/are correct(s):

(A) R reflexive, symmetric, and transitive

(B) R reflexive and not transitive.

(C) S is not reflexive

(D) T is transitive

Answer: (B), (C), (D)

(h) Let a relation R on $\mathbb{Z}$ and define as $(a, b) \in R | a \geq b^2$. Then R is __________.

(A) Not transitive

(B) Antisymmetric

(C) Symmetric

(D) Not reflexive

Answer: (B), (D)
(D) (Not reflexive because we can't have (2,2).)
Not symmetric because if we have (9,3), we can't have (3,9).
(B) Antisymmetric, because each integer will map to another integer but not in reverse (besides 0 and 1).
Is transitive because if $a \geq b^2$ and $b \geq c^2$, then $a \geq c^2$

(i) Let m and n be integers and let d be a positive integer. m is congruent to n modulo d is define by $m \equiv n \pmod{d}$ iff $d|(m-n)$. Then, which of the following statement(s) is/are true:

(A) $12 \equiv 7 \pmod{5}$
(B) $6 \equiv -8 \pmod{4}$
(C) $3 \equiv 3 \pmod{7}$
(D) None of the above

Answer: (A), (C)

(j) Let $S = \{0, 1, 2, 3, 4\}$ and define a relation R on S as $R = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}$. Then, the distinct equivalence classes are __________.

(A) $\{0, 4\}$ and $\{1, 3\}$
(B) $\{0, 4\}, \{1, 3\}$, and $\{2\}$
(C) $\{1, 3\}$ and $\{2\}$
(D) None of the above

Answer: (B) Thus the distinct equivalence classes of the relation are $\{0, 4\}, \{1, 3\}$, and $\{2\}$.

$[10 \times 1 = 10]$

2. Consider the set $A = \{a, b, c\}$ and a relation R defined in it as $R = \{(a, a), (b, b), (c, c), (b, c), (c, a)\}$.

(i) Find the symmetric closure of R.
(ii) Find the transitive closure of R.

[5 + 5 = 10]

3. (a) Let $\epsilon = 0.0005$, and let R_ϵ be the relation defined as $\{(x, y) \in \mathbb{R}^2 : |x - y| < \epsilon\}$, where R_ϵ could be interpreted as the relation "approximately equal". Prove or disprove that R_ϵ is reflexive, symmetric, and transitive.

Answer: Reflexive: For all $x \in X, |x - x| = 0 < \epsilon$. Symmetric: For all $x, y \in \mathbb{R}, |x - y| = |y - x|$. So, if $|x - y| < \epsilon$, then $|y - x| = |x - y| < \epsilon$

(b) Give an example of a relation on the set of positive integers which is

(i) symmetric and reflexive, but not transitive
(ii) reflexive and transitive, but not symmetric

[5 + (2.5 + 2.5) = 10]

**************** **End of Question Paper** *******************