Discrete Structures (MA5.101)

Quiz - 2 (Monsoon 2021)

International Institute of Information Technology, Hyderabad

Time: 60 Minutes Total Marks: 30

Instructions: This is online examination.

Write at the top of your answer book the following:

Discrete Structures (MA5.101)

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Date: 27-Dec-2021

Name:

Roll Number:

Submit your scanned hand-written answer script in the moodle with the file name: RollNo_Quiz2_SecNo_27Dec2021.pdf

December 27, 2021

1. Choose the correct option for the following questions:

   (a) Let a set A have 11 district elements, then the number of binary relations on A are __________.

   (A) 121

   (B) 2^{11}

   (C) 121^2

   (D) 2^{121}

   (b) The rank of smallest equivalence relation on a set S, where S contains 12 distinct elements is __________.

   (A) 12

   (B) 144

   (C) 132

   (D) 156

   (c) Let S be a set of the elements $\{1, 2, 3, 4, 5\}$. The transitive closure of the relation $\{(0, 1), (1, 2), (2, 2), (3,$ on the set S is __________.

   (A) $\{(0, 1), (1, 2), (2, 2), (3, 4)\}$

   (B) $\{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$

   (C) $\{(0, 1), (1, 1), (2, 2), (5, 3), (5, 4)\}$

   (D) $\{(0, 1), (0, 2), (1, 2), (2, 2), (3, 4), (5, 3), (5, 4)\}$

   (d) Let a relation R is defined on $\mathbb{Z}$ (set of integers) as ${}_xR_y$ iff $x + y$ is even and R is known as __________.

(A) an equivalence relation with one equivalence class
(B) an equivalence relation with three equivalence classes
(C) an equivalence relation with two equivalence classes
(D) an equivalence relation

(e) Let S be a set have n elements and R be a binary relation on the set S. Then, the time complexity for computing the transitive closure of R should be __________.

(A) $O(n)$
(B) $O(n^3)$
(C) $O(n^{(n+3/2)})$
(D) $O(\log n)$

(f) Let a relation R is defined as $R = \{(x, y)|y = x - 1,\ \&\ x, y \in \{1, 2, 3\}\}$. Then, the reflexive transitive closure of R is __________.

(A) $\{(x, y)|x \geq y\ \&\ x, y \in \{1, 2, 3\}\}$
(B) $\{(x, y)|x = y\ \&\ x, y \in \{1, 2, 3\}\}$
(C) $\{(x, y)|x > y\ \&\ x, y \in \{1, 2, 3\}\}$
(C) $\{(x, y)|x \leq y\ \&\ x, y \in \{1, 2, 3\}\}$

(g) A partial order $\leq$ is defined on the set $S = \{x, b_1, b_2, \cdots b_n, y\}$ as $x \leq b_i$ for all i and $b_i \leq y$ for all i, where $n \geq 1$. The number of total orders on the set S which contain the partial order $\leq$ is __________.

(A) $n + 4$
(B) $n!$
(C) n^2
(D) n^3

(h) Let a relation R on $\mathbb{Z}$ and define as $(a, b) \in R|a \geq b^2$. Then R is __________.

(A) Not transitive
(B) Antisymmetric
(C) Symmetric
(D) Not reflexive

(i) For $x, y \in \mathbb{Z}$ defined as $x|y$, which means that x divides y is a relation which does not satisfy __________.

(A) reflexive and symmetric relations
(B) symmetric relation
(C) transitive relation
(D) irreflexive and symmetric relation

(j) For $x, y \in R$ defined as $x = y$, which means that $|x| = |y|$. If $[x]$ is an equivalence relation in R, then the equivalence relation for $[17]$ is __________.

(A) $\{, \cdots, -11, -7, 0, 7, 11, \cdots\}$
(B) $\{, \cdots, -17, 0, 17, \cdots\}$
(C) $\{2, 4, 9, 11, 15, \cdots\}$
(D) $\{-17, 17\}$

$[10 \times 1 = 10]$

2. Consider the set $S = \{1, 2, 3, 4\}$ and a relation R defined in it as $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$.

   (i) If the symmetric closure of R is exist then find out, else why not.

   (ii) What about the transitive closure of R.

[5 + 5 = 10]

3. (a) Prove that a relation R defined on a set A is an equivalence relation, if and only if R is reflexive and such that ${}_aR_b$ and ${}_bR_c$ imply ${}_cR_a$, for $a, b, c \in A$.

   (b) Let S be a set and let R be a binary relation on S. Then, prove or disprove $R \cup R^{-1}$ is the smallest symmetric relation containing the relation R.

[5 + 5 = 10]

****************** End of Question Paper **********************