# Discrete Structures (MA5.101)

**Quiz - 1 (Monsoon 2021)**

***International Institute of Information Technology, Hyderabad***

Time: 60 Minutes Total Marks: 30

Instructions: This is online examination.

Write at the top of your Answer book the following:

Discrete Structures (MA5.101)

Quiz - 1 (Monsoon 2021)

Date: 17-December-2021

Name:

Roll Number:

Submit your scanned hand-written Answer script in the moodle with the file name: RollNo_Quiz1_SecNo_17Dec2021.pdf

December 17, 2021

1. Choose the correct option for the following questions:

   (a) Let $S(1), S(2), S(3), \cdots, S(100)$ be a collection of 100 sets, where the cardinality of a set $S(i)$ is defined by $S(i) = i + 1$ and $S(i-1)$ is subset of $S(i)$, where $i = 2, 3, \cdots, 100$. Then, the cardinality of union of the all the sets $|S(1) \cup S(2) \cup \cdots \cup S(100)|$ is __________.

   (A) 99

   (B) 100

   (C) 101

   (D) 102

   Answer: C

   Explanation: Since all sets are subsets of S(100) therefore in union only elements of S(100) will come S(100) contains 101 elements.

   (b) Let $A$ and $B$ be two disjoint sets, and the bit strings of them are 1111100000 and 1010101010, respectively. Therefore, $A \cup B$ is __________.

   (A) 1010100000

   (B) 1010101101

   (C) 1111111100

   (D) 1111101010

   Answer: D

   Explanation: The bit string for the union is the bitwise OR of the bit strings.

   (c) Consider three sets $X$, $Y$, and $Z$, such that the sets $Y \cap Z$, $X \cap Y$, and $Z \cap X$ consists of 8, 7, and 7 elements, respectively. Then, the minimum element in set $X \cup Y \cup Z$ is __________.

(A) 8
(B) 14
(C) 22
(D) 15

Answer: A
Explanation: For minimum elements set Y and Z have 8 elements each and all of the elements are same, Also set X should have 7 elements which are already present in Y and Z. Thus $X \cup Y \cup Z \equiv X \equiv Y$.

(d) If $n(C \times D) = n(D \times C) = 64$, then which of the followings holds true?

(A) $n(C) = 2$, $n(D) = 32$
(B) $n(C) = 4$, $n(D) = 16$
(C) $n(C) = 8$, $n(D) = 8$
(D) None of the above

Answer: C
Explanation: n(C) should be equal to n(D) for $n(C \times D) = n(D \times C)$.

(e) Let $A_n$ and $B_n$ be two sets, where $A_n$ and $B_n$ represent all the factors of $n$ and all multiples of $n$ less than $1000$, respectively. Which of the below statement(s) is/are true?
i. $A_{108} \cap A_{84} = A_{12}$
ii. $B_{12} \cap B_{18} = B_{36}$
iii. $B_{12} \subset (B_6 \cap B_4)$
iv. $B_{12} \cup B_{18} = B_{36}$

(A) i, ii and iv only
(B) i, ii and iii only
(C) i and ii only
(D) None of them

Answer: B
Explanation: https://iim-cat-questions-answers.2iim.com/quant/arithmetic/set-theory/

(f) A class have 40 students, where 12 students are registered for both History and Geography and 22 registered for Geography. If the students of the class registered for at least one of the two subjects, then the number of students registered for only History and not Geography is ________.

(A) 30
(B) 10
(C) 18
(C) 28

Answer: C
Let A be the set of students who have registered for History and B be the set of students who have enrolled for Geography.
Then, $(A \cup B)$ is the set of students who have registered for at least one of the two languages. Because the students of the class have registered for at least one of the two languages, we will

not find anyone outside $(A \cup B)$ in this class. Therefore, $n(A \cup B)$ = number of students in the class So, $n(A \cup B)$ = 40

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
i.e., $40 = n(A) + 22 - 12$
Or $n(A) = 30$
$n(A)$ is the number of students who have registered for History. This number is the sum of those who have registered for only History and those who have enrolled for both the languages.

What we have to compute the number of students who have registered for only History. n(only History)= n(History) - $n(A \cap B)$
= 30 - 12 = 18.

(g) Let $X$, $Y$, and $Z$ be three sets, where $X$ contains all prime numbers and $Y$ contains all even prime numbers, whereas the set $Z$ contains all odd prime numbers. Then, which of the following statement(s) is/are true?

(A) $X \equiv Y \cup Z$
(B) $Y$ is a singleton set.
(C) $X \equiv Z \cup \{2\}$
(D) All of the mentioned

Answer: D
Explanation: 2 is the only even prime number

(h) A necessary and sufficient condition for $S + T = S \cup T$, where $S + T = (S \cap T') \cup (S' \cap T)$ is __________.

(A) $S \cup T = \emptyset$
(B) $S \cap T = \emptyset$
(C) $S \cap T \neq \emptyset$
(D) None of these

Answer: (B)

(i) Let $A$ and $B$ be two set, and defined as $A = \{x | x \text{ is even number and} x < 15\}$ and $B = \{y | y \in Z_{13} - \{0\}\}$. Then, the symmetric diffence $A \triangle B$ is __________.

(A) $\{1, 3, 5, 7, 9, 11, 14\}$
(B) $\{1, 3, 5, 7, 9, 11, 13\}$
(C) $\{1, 3, 5, 7, 8, 9, 10, 11\}$
(D) $(Z_{13} - \{0\}) \cup \{1, 3, 5, 7, 9, 13\}$

Answer: (A)

(j) Let $X$, $Y$, and $Z$ be any sets such that $X \oplus Z = Y \oplus Z$, where $X \oplus Y = (X - Y) \cup (Y - X)$. Then, the relation set between $X$ and $Y$ is __________.

(A) $X \cup Y = Y$
(B) $X \cap Y \neq \emptyset$
(C) $X \cap Y = \emptyset$
(D) $X = Y$

Answer (D) $X = Y$

[$10 \times 1 = 10$]

2. Applying the principle of inclusion-exclusion, find the number of positive integers $\leq 3000$ and divisible by $3, 5$ or $7$.

[8]

Answer:

Example:- (Q2) (12)

Find the number of positive integers $\leq 3000$ and divisible by 3, 5, or 7.

Soln:- Let $A$, $B$, and $C$ denote the sets of positive integers $\leq 3000$ and divisible by 3, 5, and 7, respectively.

Then,

$A = \{x \in N \mid x \leq 3000$ and divisible by $3\}$,

$B = \{x \in N \mid x \leq 3000$ and divisible by $5\}$ an

$C = \{x \in N \mid x \leq 3000$ and divisible by $7\}$.

By the inclusion-exclusion principle,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

$$= \lfloor 3000/3 \rfloor + \lfloor 3000/5 \rfloor + \lfloor 3000/7 \rfloor - \lfloor 3000/(3\times5) \rfloor - \lfloor 3000/(5\times7) \rfloor - \lfloor 3000/(7\times3) \rfloor + \lfloor 3000/(3\times5\times7) \rfloor$$

$$= 1000 + 600 + 428 - 200 - 85 - 142 + 28$$

$= 1629$, is the required number of positive integers $\leq 3000$ and divisible by 3, 5, or 7. □

3. If $\mathcal{P}(A)$ is the power set of a set $A$, then for any two sets $A$ and $B$, prove or disprove the following:

(a) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

(b) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

(c) $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$

[4 + 4 + 4 = 12]

Answer:

**Problem 1.6.** *Prove or disprove the following:*

(i) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

(ii) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

(iii) $\mathcal{P}(A \setminus B) = \mathcal{P}(A) \setminus \mathcal{P}(B)$

(Q3)

**Solution:**

(i) $X \in \mathcal{P}(A \cap B)$

$\Leftrightarrow X \subseteq A \cap B$

$\Leftrightarrow X \subseteq A$ and $X \subseteq B$

$\Leftrightarrow X \in \mathcal{P}(A)$ and $X \in \mathcal{P}(B)$

$\Leftrightarrow X \in \mathcal{P}(A) \cap \mathcal{P}(B)$

Hence $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$, so the result is proved.

(ii) The result is not true.

Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$

Then $A \cup B = \{1, 2, 3, 4\}$, $C = \{1, 4\} \in \mathcal{P}(A \cup B)$

but $C \notin \mathcal{P}(A)$ and $C \notin \mathcal{P}(B)$

so that $C \notin \mathcal{P}(A) \cup \mathcal{P}(B)$.

$\therefore \mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$.

(iii) The result is not true. Choose $A$ and $B$ as in (ii) above. Then $A \setminus B = \{1\}$. So $\mathcal{P}(A \setminus B) = \{\phi, \{1\}\}$, $\{1, 2\} \in \mathcal{P}(A) \setminus \mathcal{P}(B)$ but $\{1, 2\} \notin \mathcal{P}(A \setminus B)$.

Hence $\mathcal{P}(A \setminus B) \neq \mathcal{P}(A) \setminus \mathcal{P}(B)$.

**************** **End of Question Paper** ********************

1 c

2 d

3 a

4 abc or a or b or c

5 c

6 c

7 d or abcd

8 b

9 a

10 d or ad

Updated Q1. Answers !