Tutorial 4

1 Contrapositive Proofs

Prove the following using Contrapositive:

1. For all integers n, if n^2 is not divisible by 7, then n is not divisible by 7.
2. If m and n are positive integers such that $mn = 100$ then $m \leq 10$ or $n \leq 10$
3. If x is a real number such that $0 < x < 1$ then $x > x^2$

2 Equivalence Proofs

Prove the following equivalences

1. Let $x, y \in \mathbb{R}$. Prove that $|x + y| = |x| + |y|$ if and only if $xy > 0$,
2. For all positive integers n and m, $m|n$ and $n|m$ if and only if $m = n$

3 Contradiction Proofs

Prove the following using contradiction.

1. Prove that the set $A = \{\frac{n-1}{n} : n \in \mathbb{Z}^+\}$ does not have a largest element.
2. If the mean of four distinct number is $n \in \mathbb{Z}$, then at least one of the integers is greater than $n + 1$.
3. Prove that there is no rational number r such that $2^r = 3$

4 Existence Proofs

Prove the following existence proofs.

1. A conference is being attended by 367 people. Prove that there exists at least two people born with same date of birth.
2. Let $\{b_1, b_2.., b_b\}$ be a set of integers such that $\sum b_k^2 < n$. Prove that at least one of the integers in the set is zero.