Discrete Structures (MA5.101)

Mid Semester Examination (Monsoon 2021)

International Institute of Information Technology, Hyderabad

Time: 90 Minutes | Total Marks: 30

Instructions: This is online examination.

Write at the top of your answer book the following:

Discrete Structures (MA5.101)

Mid Semester Examination (Monsoon 2021)

Date: 10-Jan-2022

Name:

Roll Number:

Submit your scanned hand-written answer script in the moodle with the file name: RollNo_MidSem_SecNo_10Jan2022.pdf

NOTE: No email submissions for the answer scripts are allowed even if you are facing Internet issues or moodle problem from your end. In that case, viva-voce of mid semester examination will be considered later for fair evaluations to all.

10 January 2022

1. (a) We know that the composite gf of any two injections $f : S \to T$ and $g : T \to V$ is an injection. Extend the definition of the composite gf to the case in which the domain of g contains the codomain of f. Prove or disprove:

   (i) the composite gf of any two injections is an injection.

   (ii) the composite gf of any two surjectios is a surjection.

   (b) A set S is said to be *infinite* if there is a one-to-one correspondence (bijection) between S and a proper subset of S. Using this definition, prove that the set of real numbers is infinite.

   [(2.5 + 2.5) + 5 = 10]

2. (a) Let A, B, and C be the subsets of a set U with the properties that $B \cap C = \emptyset$ and $A = B \cup C$. Construct a bijection map $b : \mathcal{P}(A) \to \mathcal{P}(B) \times \mathcal{P}(C)$, where $\mathcal{P}(X)$ represents the power set of a given set X.

   (b) Construct a truth table for each of these compound propositions:

   (i) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

   (ii) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

   [5 + (2.5 + 2.5) = 10]

3. (a) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent by a series of logical equivalences.

(b) (i) Build a digital circuit that produces the output $(a \vee \neg c) \wedge (\neg a \vee (b \vee \neg c))$, where input bits a, b, and c are given.

(ii) Show that $(a \vee b) \wedge (\neg a \vee c) \rightarrow (b \vee c)$ is a tautology.

[5 + (2.5 + 2.5) = 10]

**************** **End of Question Paper** ********************