

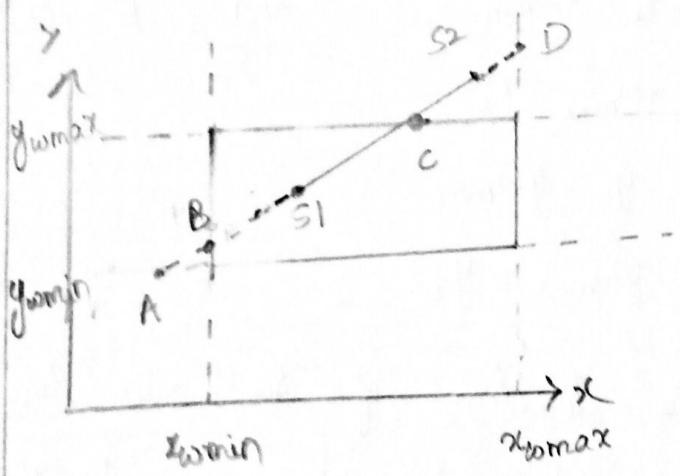
5/12/21

CGM END EXAM

Question 2

- a) Clipping is to display only the region of interest which is defined by a rectangle ( $x_{\min}, x_{\max}, y_{\min}, y_{\max}$ )

Liang-Barsky line clipping algorithm is used to clip line segments. It is a faster line clipping algorithm and is based on parametric line equations.



consider the following example,

the main idea of Liang-Barsky is to find nearest point to  $S_2$  from  $A, B, S_1$  ( $S_1$  in example) and to find nearest point to  $S_1$  from  $C, S_2, P$  ( $C$  in example).

we consider the parametric equations using which we decide whether a given line segment is visible inside a clipping window or not and if visible what part of it is visible.

$$x = x_1 + u \Delta x$$

where  $S_1(x_1, y_1), S_2(x_2, y_2)$

$$y = y_1 + u \Delta y$$

$\Delta x = x_2 - x_1, \Delta y = y_2 - y_1, u \in [0, 1]$

Point clipping condition in parametric form is given by

$$x_{w\min} \leq x_1 + u \Delta x \leq x_{w\max}, y_{w\min} \leq y_1 + u \Delta y \leq y_{w\max}$$

$$\Rightarrow u \times (-\Delta x) \leq x_1 - x w_{\min}$$

$$u \times (\Delta x) \leq x w_{\max} - x_1$$

$$u \times (-\Delta y) \leq y_1 - y w_{\min}$$

$$u \times (\Delta y) \leq y w_{\max} - y_1$$

In another form we can write this as,

$$u p_k \leq q_k \text{ where,}$$

$$p_1 = -\Delta x \quad q_1 = x_1 - x w_{\min} \quad \text{and} \quad k = 1, 2, 3, 4$$

$$p_2 = \Delta x \quad q_2 = x w_{\max} - x_1$$

$$p_3 = -\Delta y \quad q_3 = y_1 - y w_{\min}$$

$$p_4 = \Delta y \quad q_4 = y w_{\max} - y_1$$

We need to decide the direction of the given line segment to find the starting and ending edge. We do it based

on

$\Delta x, \Delta y$	Starting Edge	Ending Edge
$\Delta x > 0$	$x_{\min}$	$x_{\max}$
$\Delta x < 0$	$x_{\max}$	$x_{\min}$
$\Delta y > 0$	$y_{\min}$	$y_{\max}$
$\Delta y < 0$	$y_{\max}$	$y_{\min}$

If  $p_k = 0$  for  $k=1, 2$ , the line segment is parallel to the clipping edge and

a) If  $q_k < 0$  ( $k=1, 2$ ) the line is completely outside of clipping region and we discard it.

b) If  $q_k > 0$  ( $k=1, 2$ ) the segment is visible and we get intersections with window edges and decide the visible part.

Similarly for  $b_k = 0$  given  $k=3, 4$ .

The parameters of intersections of  $s_1 s_2$  with the starting edges are referred to as  $u_1'$ ,  $u_2''$  and

$$u_1 = \max \{ u_1', u_1'', 0 \}$$

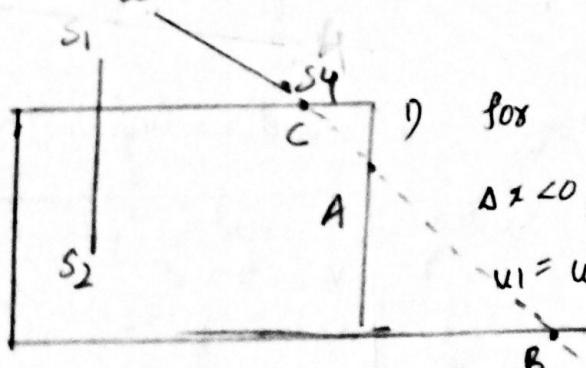
where  $u_1$  is the nearest clipping point to  $s_2$ .

Similarly  $s_1 s_2$  intersection with ending edges are referred to as  $u_2'$ ,  $u_2''$  and  $u_2 = \min \{ u_2', u_2'', 0 \}$

1. If  $u_1 < u_2$ ,  $x = x_1 + \Delta x \times u$   
 $y = y_1 + \Delta y \times u$  ( $u_1 < u < u_2$ )

This defines the visible part.

2. If  $u_1 > u_2$  then the line is invisible.



for line  $s_3, s_4$ .

$$\Delta x < 0, \Delta y > 0 \Rightarrow \text{starting} = (x_{\text{max}}, y_{\text{min}})$$

$$u_1 = u_3 = 0 \text{ and } u_2 = u_4 < 0$$

$$\therefore u_1 > u_2$$

$s_3 s_4$  is invisible.

Example

2) for the line  $s_1 s_2$ ,  $p_1 = p_2 = 0$  but  $q_1 > 0, q_2 > 0$

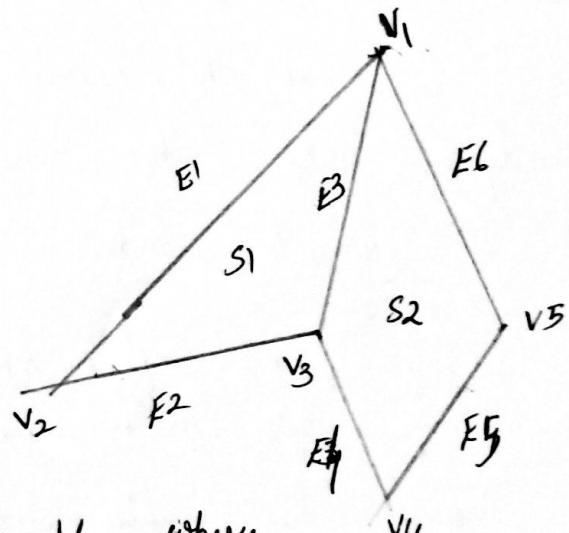
$\therefore$  the line is visible and we find out the visible part by considering the intersections.

b) Polygon surfaces ~~representations~~ are the surfaces that are used for Boundary representation of a 3D object.

Polygon tables are used to represent polygon surfaces.

The Polygon tables contains three tables.

- 1) Vertex table
- 2) Edge table
- 3) Polygon - surface table



\* consider the following example where,  
the polygon consists of two surfaces  $S_1, S_2$ .

\* The vertex table stores the vertex information, i.e.,  $x, y, z$  coordinates for each vertex.

vertex table	
$v_1$ :	$x_1, y_1, z_1$
$v_2$ :	$x_2, y_2, z_2$
$v_3$ :	$x_3, y_3, z_3$
$v_4$ :	$x_4, y_4, z_4$
$v_5$ :	$x_5, y_5, z_5$

- \* Edge table is used to store information of the edges in terms of vertices.
- \* The polygon surface tables which describes the surface stores the surface information for each surface present in the polygon.
- \* In the given example,  $s_1$  is covered by Edges  $E_1, E_2$  and  $E_3$  and hence is represented as  $s_1: E_1, E_2, E_3$ .

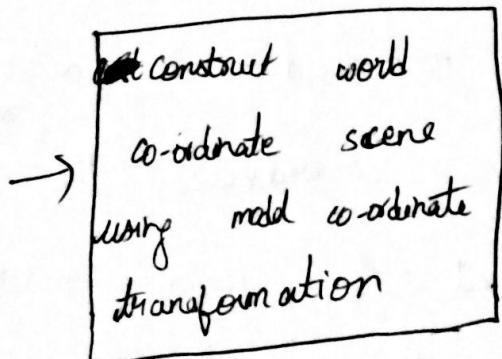
Edge table
$E_1: v_1, v_2$
$E_2: v_2, v_3$
$E_3: v_3, v_1$
$E_4: v_3, v_4$
$E_5: v_4, v_5$
$E_6: v_5, v_1$

Polygon surface table
$s_1: E_1, E_2, E_3$
$s_2: E_3, E_4, E_5, E_6$

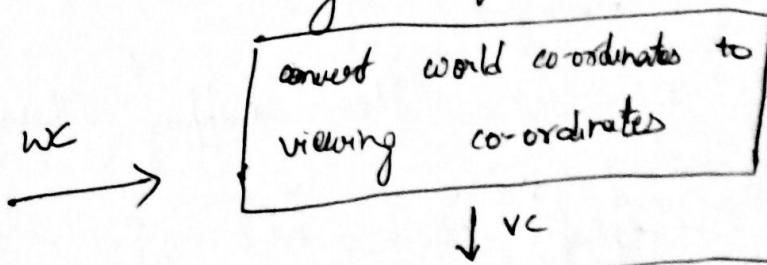
### Question 1

a) 2D viewing pipeline

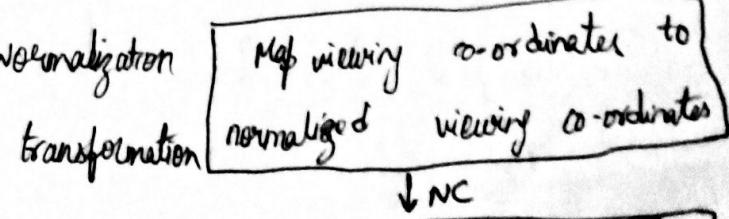
modelling transformation



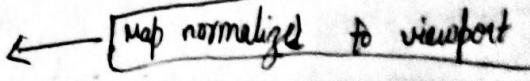
viewing transformation



normalization transformation



(device co-ordinates) DC



- \* In the first component, the coordinates in which individual objects are created are called model co-ordinates when several objects are assembled into a single scene, they are described by world co-ordinates. So, we set up the co-ordinates of a scene to display.
- \* In the second component, viewing transformation is done in which world co-ordinates which we created earlier are transformed into the coordinate system of camera and viewer. The world co-ordinates become viewing co-ordinates (VC).
- \* In the third step, these viewing co-ordinates are normalized to get normalized viewing co-ordinates (NV). This is done to ensure that the viewing process is independent of any output devices. Clipping is generally done for normalized viewing co-ordinates.
- \* Finally, after mapping these normalized co-ordinates to a specific device we get device co-ordinates. The normalized viewport co-ordinates are mapped to device co-ordinates. The objects that lie outside viewport will be clipped and will not be displayed.

b) glMatrixMode(GL\_MODELVIEW); // defining matrix mode  
 glLoadIdentity(); // loads identity matrix  
 glColor3f(0.0, 0.0, 1.0); // color set to blue.  
 glRecti(50, 100, 200, 150); // displaying blue rectangle  
 glColor3f(0.0, 1.0, 0.0); // color set to green  
~~glTranslatef(250, 250, 0.0); // translation.~~  
 glRotatef(45.0, 0.0, 0.0, 1.0); // rotate 45° around z-axis  
 glRecti(300, 100, 450, 150); // display green rotated rectangle  
 side by side.

Question 3

- a) There are many visible surface detection methods. one of the methods is depth(z) buffer method. To represent a non-transparent object on 2D plane to make it realistic we need to remove the non-visible planes from diagram. Every point on polygon (3D) is represented on a 2D new plane, each point is compared with depth(z) values. We will use normalized z values (0,1). Here we use two buffer areas Depth(z) buffer, Refresh Buffer.

Depth buffer contains depth value of each  $(x, y)$  position.

Refresh buffer contains intensity value at each  $(x, y)$  position.

steps :-

1) Initiating Intensity and depth values

$$A(x, y) \text{ depth}(x, y) = z_{\max} \quad (z_{\max} = 1)$$

$$\text{refresh}(x, y) = I_B \quad (I_B \text{ is Background Color})$$

2) calculating on every position of polygon surface.

a) calculate z-depth for each position  $(x, y)$  of the polygon.

b) Here we will consider least value so that object or closer is visible to us.

If  $z < \text{depth}(x, y)$  then :

$$\text{depth}(x, y) = z$$

$$\text{refresh}(x, y) = I_s(x, y)$$

where  $I_s$  is the projection intensity value of the surface at  $(x, y)$  which has minimal value of  $z$  at current current iteration

$$z = \frac{-Ax - By - D}{C}$$

for plane

$$Ax + By + Cz + D = 0$$

## b) 3D Geometric transformations

i) 3D translation: same as 2D, add z-axis and z-co-ordinate  
 Homogeneous co-ordinates contains four co-ordinates and  
 $4 \times 4$  matrix is multiplied to apply transformation.

A position  $p = (x, y, z)$  in 3D is translated to a location

$p' = (x', y', z')$  by adding translation distance  $tx, ty, tz$ .

$$x' = x + tx, \quad y' = y + ty, \quad z' = z + tz.$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$p' = Tp.$$

### (ii) 3D rotation.

co-ordinate axis rotation - radio, yaxis, zaxis rotation.

rotation about an axis that is parallel to one of  
 co-ordinate axes.

### General 3D rotation

rotation about an arbitrary axis.

→ first translate  $p_i$  to origin.

→ rotate so the arbitrary axis is aligned with one of co-ordinate axes and perform desired rotation and rotate back.

3D Scaling - scale object relative to  $(0, 0, 0)$

All vectors are scaled from origin.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$