Assignment 5
Bridge course: Data Structure and Practices
Indian Institute of Technology, Jodhpur

Topic- Minimum Spanning Tree & Shortest Path

Note: Write code for all the questions; the output should display the solution.

Instructions:

1. Plagiarism is not allowed. Also copying code from the internet or any other source is not allowed. If found, you will be given zero marks.

2. Submit all code in a single zip file with your roll number as file name.
Example: MP19CS001.zip

3. Inside the zip, every file should be named as illustrated below:

a. <roll_number>_<question_number>.<file_type>

b. Example: If your roll number is MP19CS001 and the solution is for question 1, then file name will be : MP19CS001_Q1.py

4. Include a readme file.

5. You are only allowed to code in C, C++, and python.

1. In this case, we wish to know if a tree T is the minimal spanning tree of a graph G. We aim to demonstrate that T is a minimal spanning tree of G if and only if T has an edge e that is one of the least cost edges in the cut (S, S’).
   A. Demonstrate that if a cut (S, S’) exists such that T contains no of its minimal cost edges, T cannot be a minimum spanning tree. **(10 points)**
   B. Show that if T has an edge e that is one of the minimal cost edges in the cut (S, S’), T must be a minimum spanning tree. **(10 points)**

2. Consider a graph G and a Minimum Spanning Tree T. These already exist in memory, in whichever form (adjacency list/array) you like. G now has a new edge e = (u, v) with the weight w(u, v).
   A. Create a method to update the tree T such that it is a Minimum Spanning Tree for G. Your method should complete in O(n) time. **(10 points)**
   B. Demonstrate the accuracy or correctness of your algorithm. **(10 points)**

3. You are responsible for dispatching ambulances to emergencies. You have k ambulances in your fleet parked in various places, and you need to transport them to an emergency as quickly as possible to assist. You have a map of Seattle represented as the adjacency list for a weighted, directed graph G with |V | vertices and |E| edges, where edge weights are positive values denoting the journey time from the source to the destination. (The number of ambulances, k, is much fewer than the number of vertices |V |.) You also have a list of vertices reflecting the locations of each of your ambulances and the vertex representing the location of the new emergency. Figure 1 depicts a sample graph.

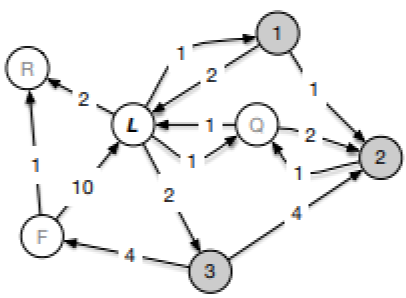

Figure 1 shows an example graph with k = 3 (highlighted vertices). L stands for emergency, while 1, 2, and 3 are ambulances. Other intermediate sites include F, R, and Q. 1-L, 2-Q-L, and 3-2-Q-L are the shortest paths from 1, 2, and 3 to L, respectively. suppose you can't change the presented graph and see what we can do.
   A. Explain how you would use Dijkstra's algorithm to find the shortest route from each ambulance to the emergency vertex. (You should output the whole path rather than simply how long it takes.) **(10 points)**
   B. How long does this procedure take? Provide a simplified, tight big-O in k, |V |, and/or |E| for the runtime. **(10 points)**
   C. How much more room is required for this process? Provide a simplified, tight big-O in terms of k, |V |, and/or |E| for memory utilization. Consider the space

necessary for Dijkstra's algorithm's information table (tight-O (|V |)) and the space required for the output. **(10 points)**