Parameterized Complexity

Total marks: 30 (5% of total evaluation)

1. In the parametrized problem Dominating Set parameterized by vertex cover, given a graph G (G does not have any isolated vertices), a set $X \subseteq V(G)$ of size k, such that X is a vertex cover of G, parameter k, the goal is to find a set $S \subseteq V(G)$ of minimum size such that for every vertex $v \in V(G)$, $(N(v) \cup \{v\}) \cap S \neq \emptyset$, that is either $v \in S$ or at least one neighbour of v is in S.

   (a) (10 points) Show that Dominating Set parameterized by vertex cover admits a kernel by designing a kernelization algorithm. Prove the correctness of all your reduction rules and observations/lemmas, and show that your kernelization algorithm runs in polynomial time.

   (b) (5 points) Design an algorithm to solve Dominating Set parameterized by vertex cover that runs in time $\mathcal{O}(2^{k^2} n^{\mathcal{O}(1)})$. Prove correctness of your algorithm and explain running time analysis.

2. In the parametrized problem Deletion to Degree One (DDO for short), given a graph G, and an integer k, parameter k, the question is to decide if there exists a set $S \subseteq V(G)$ of size at most k such that degree of every vertex in the graph $G - S$ is at most 1.

   (a) (10 points) Show that DDO admits a polynomial kernel with $\mathcal{O}(k^3)$ vertices. Prove the correctness of all your reduction rules and observations/lemmas, and show that your kernelization algorithm runs in polynomial time.

   (b) (5 points) Design an algorithm to solve DDO that runs in time $\mathcal{O}(2^{\mathcal{O}(k \log k)} n^{\mathcal{O}(1)})$. Prove the correctness of your algorithm and explain running time analysis.