Parameterized Complexity

Total marks: 40 (5% of total evaluation)

1. K_{10} is a clique (complete graph) on 10 vertices. Consider the following K_{10}-Collecting problem: Given a graph G and an integer k, parameter k, the goal is to find if there exists at least k vertex disjoint K_{10}s in G.

   (a) (5 points) Let $\mathcal{F}$ be a family of all the 10 sized vertex subsets of $V(G)$ that induce a K_{10} in G. Show that G contains a set of ℓ vertex disjoint K_{10}s if and only if $\mathcal{F}$ contains ℓ pairwise disjoint sets.

   (b) (10 points) Design a polynomial time algorithm to find a family $\mathcal{F}^\star \subseteq \mathcal{F}$ of size at most $\mathcal{O}(k^{10})$ such that $\mathcal{F}$ contains at least k pairwise disjoint sets if and if $\mathcal{F}^\star$ contains at least k pairwise disjoint sets. (Use Sunflower lemma)

   (c) (5 points) Show that if there exists a vertex $v \in V(G)$ such that v is not in any set in $\mathcal{F}^\star$, then (G, k) is a yes instance of K_{10}-Collecting if and only if $(G - v, k)$ is a yes instance of K_{10}-Collecting.

   (d) (5 points) Use (a), (b) and (c) to show that K_{10}-Collecting admits a kernel with $\mathcal{O}(k^{10})$ vertices. Write the running time analysis as well.

2. (15 points) Write Vertex Cover parametrized by solution size kernel in details with all the proofs and running time analysis from the lecture 7 of class.