CSE 401: Numerical Analysis
HW 7

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Solution 2:

Given ODE

$$u'' = u^3 + t \quad ; \qquad a < t < b$$

subject to boundary conditions

$$u(a) = \alpha \quad ; \qquad u(b) = \beta$$

(a):

The Euler's method for an ODE

$$\mathbf{y}' = \mathbf{f}(t, \mathbf{y})$$

is illustrated via the iterative scheme given below:

$$\mathbf{y}_{i+1} = \mathbf{y}_i + h \cdot \mathbf{f}(t_i, \mathbf{y}_i) \tag{1}$$

Let $u'(a) = s$, hence for a single step

$$h = b - a \quad ; \qquad t_i = a \quad ; \qquad \mathbf{y}_0 = \begin{Bmatrix} \alpha \\ s \end{Bmatrix}; \qquad \mathbf{y}_1 = \begin{Bmatrix} \beta \\ u_1' \end{Bmatrix}$$

Rewriting the system of equations in the iterative form given by (1)

$$\begin{bmatrix} \beta \\ u_1' \end{bmatrix} = \begin{Bmatrix} \alpha \\ s \end{Bmatrix} + (b - a) \cdot \begin{Bmatrix} s \\ a + \alpha^3 \end{Bmatrix}$$

Thus the algebraic relation for the initial slope, to be determined, is given by:

$$\boxed{\boldsymbol{\beta} = \boldsymbol{\alpha} + (\mathbf{b} - \mathbf{a})\mathbf{s}}$$

(b):

The initial value of the slope obtained from a single step of Euler's method is the slope of the **straight-line** joining the end points (the independent variable (t) and the value of the solution at those points $u(t)$ of the domain.

$$\boxed{\mathbf{s} = \frac{\boldsymbol{\beta} - \boldsymbol{\alpha}}{\mathbf{b} - \mathbf{a}}}$$