CSE 401: Numerical Analysis
HW 4

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October 19, 2016

Solution 4:

Given function

$$f(x,y) = \frac{1}{2}(x_1^2 - x_2)^2 + \frac{1}{2}(1 - x_1)^2$$

(a):

The critical points of the given function are given by

$$\boldsymbol{\nabla} f = \begin{Bmatrix} x_1 - 2x_1(x_2 - x_1^2) - 1 \\ x_2 - x_1^2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \implies (x_1, x_2) = \left[\{1, 1\}\right]$$

The hessian matrix, $\mathbf{H}(\mathbf{x}, \mathbf{y})$, is given by

$$\begin{bmatrix} 6x_1^2 - 2x_2 + 1 & -2x_1 \\ -2x_1 & 1 \end{bmatrix}$$

The hessian matrix evaluated at the critical point $(1, 1)$ is

$$\mathbf{H}(1,1) = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

The corresponding eigenvalues are given by

$$\begin{vmatrix} 5 - \lambda & -2 \\ -2 & 1 - \lambda \end{vmatrix} = 0 \implies \lambda_1 = 0.1716\ ; \quad \lambda_2 = 5.8284$$

Thus the hessian matrix is positive definite implying that the point $(1, 1)$ is a local minimum.

(b):

The corresponding iteration is described as follows:

$$\mathbf{H}_f(\mathbf{x}_k) s_k = -\nabla f(\mathbf{x}_k)$$

$$-\nabla f(\mathbf{x}_k) = \begin{bmatrix} -9 \\ 2 \end{bmatrix} \quad ; \quad \mathbf{H}_f(\mathbf{x}_k) = \begin{bmatrix} 21 & -4 \\ -4 & 1 \end{bmatrix}$$

which gives

$$s_k = \begin{bmatrix} -0.2 \\ 1.2 \end{bmatrix}$$

thus

$$x_{k+1} = \begin{Bmatrix} 1.8 \\ 3.2 \end{Bmatrix}$$

(c):

In order to see if it is a good step, we compute

$$\nabla f(\mathbf{x}_k)^T s_k = -4.2 < 0$$

The vector obtained after the first iteration does come close to the true solution in one of the components. Hence it is a good step in the sense of descent direction.

(d):

While we know that the exact solution to the problem is $x = [1.0,\ 1.0]^T$, the last iteration in Newton's method, though bringing x_1 close to it's exact solution, takes x_2 farther than the initial guess. Hence in this sense it is a bad step. We can also compute the Euclidean norm of the difference between the true solution and current iterate.

$$||x_1 - x||_2 = \sqrt{2}$$
$$||x_2 - x||_2 = \sqrt{5.48}$$

which clearly indicates that the euclidean norm of the distance increases. Hence it is a bad step.
