CSE 401: Numerical Analysis
HW 2

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Solution 3:

The question pertains to the process of Householder transformations applied on the following matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$$

The Householder matrix ($\mathbf{H}$) is given as follows:

$$\mathbf{H} = \mathbf{I} - 2\frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}}$$

1.

The rank of Matrix $\mathbf{A}$ is 3, implying that it is not rank-deficient, and hence we would need 3 Householder transformations to reduce it to an upper triangular matrix. The corresponding transformations are done as follows:

- Computing $\mathbf{H}_1$ $\mathbf{A}$

$$\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix}; \quad \mathbf{H}_1\mathbf{A} = \begin{bmatrix} -2 & -5 & -15 \\ 0 & 0 & -1.333 \\ 0 & 1 & 3.667 \\ 0 & 2 & 10.667 \end{bmatrix}$$

- Computing $\mathbf{H}_2$ $\mathbf{H}_1$ $\mathbf{A}$

$$\mathbf{v} = \begin{bmatrix} 0 \\ -2.2361 \\ 1.000 \\ 2.000 \end{bmatrix}; \quad \mathbf{H}_2\mathbf{H}_1\mathbf{A} = \begin{bmatrix} -2 & -5 & -15 \\ 0 & -2.2361 & -11.1803 \\ 0 & 0 & -1.9296 \\ 0 & 0 & -0.5259 \end{bmatrix}$$

- Computing $\mathbf{H}_3$ $\mathbf{H}_2$ $\mathbf{H}_1$ $\mathbf{A}$

$$\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ -3.9296 \\ -0.5259 \end{bmatrix}; \quad \mathbf{H}_2\mathbf{H}_2\mathbf{H}_1\mathbf{A} = \begin{bmatrix} -2 & -5 & -15 \\ 0 & -2.2361 & -11.1803 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus we can see that it takes a total of three Householder transformations to compute the **Q-R** factorization of **A**

2.

The first column of A after the first householder transformation becomes:

$$\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3.

The first column of the matrix **A** does not change on applying the second Householder transformation as can be seen from above.

4.

In principal, a total of 6 Givens' Rotations would be required to get the QR factorization of A.

- The first three for making the sub-diagonal entries of the first column equal to zero.
- The next two for making the sub-diagonal entries of the second column equal to zero.
- And the last for making the remaining sub-diagonal entry of the third column equal to zero.
