CSE 401: Numerical Analysis - Fall 2016
Homework 1

Bhavesh Shrimali (NetID: bshrima2)

September 9, 2016

Problem 8

In order to show that the matrix is singular it is sufficient to prove that the determinant is equal to zero.

$$\text{i. e. } det(\mathbf{A}) = 0 \tag{1}$$

The determinant of the matrix can be computed along the first row

$$\begin{aligned}
&0.1 \cdot (0.45 - 0.48) - 0.2 \cdot (0.36 - 0.42) + 0.3 \cdot (0.32 - 0.35) \\
&= 0.003 - 0.012 + 0.009 \\
&= 0
\end{aligned}$$

Now In order to find out the solution of the system of equations $\mathbf{Ax} = \mathbf{b}$ we proceed with Gaussian-Elimination with partial pivoting

- $R_2 \to R_2 - 4 \cdot R_1$
$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0 & -0.3 & -0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix} \mathbf{x} = \begin{Bmatrix} 0.1 \\ -0.1 \\ 0.5 \end{Bmatrix}$$

- $R_3 \to R_3 - 7 \cdot R_1$
$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0 & -0.3 & -0.6 \\ 0 & -0.6 & -1.2 \end{bmatrix} \mathbf{x} = \begin{Bmatrix} 0.1 \\ -0.1 \\ -0.2 \end{Bmatrix}$$

- $R_3 \to R_3 - 2 \cdot R_2$
$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0 & -0.3 & -0.6 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x} = \begin{Bmatrix} 0.1 \\ -0.1 \\ 0 \end{Bmatrix}$$

- In essence, we can do another row operation $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 0.1 & -0.1 & -0.3 \\ 0 & -0.3 & -0.6 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x} = \begin{Bmatrix} 0 \\ -0.1 \\ 0 \end{Bmatrix}$$

Let

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

This implies, for

$$\begin{aligned} x_3 &= \lambda \\ x_2 &= \frac{1}{3} - 2\lambda \\ x_1 &= \lambda + \frac{1}{3} \qquad \forall \lambda \in \mathbb{R} \end{aligned} \tag{2}$$

Thus the solution set is described as

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \lambda \begin{Bmatrix} 1 \\ -2 \\ 1 \end{Bmatrix} + \begin{Bmatrix} 1/3 \\ 1/3 \\ 0 \end{Bmatrix} \qquad \forall \lambda \in \mathbb{R}$$

2

If Gaussian Elimination with partial pivoting is used to solve the given system the process would eventually fail at **back substitution**