CSE 401: Numerical Analysis
HW 5

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November 2, 2016

Solution 2:

Given data points

$$(-1,1),(0,0),(1,1)$$

Using Monomial Basis function:

The monomial basis functions are given by the following formula

$$\phi_k(x) = x^{k-1} \implies \phi_1(x) = 1 \ ; \quad \phi_2(x) = x \ ; \quad \phi_3(x) = x^2$$

And the corresponding Vandermonde matrix is given by

$$\mathbf{V} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Let the corresponding weights be denoted by w_1, w_2, w_3 and therefore we get the corresponding linear system

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \implies \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence the interpolating polynomial

$$\boxed{m(x) = x^2}$$

Using Lagrange Basis functions:

The lagrange basis functions are given by the following formula

$$\phi_k(x) = \prod_{j=1, j\neq k}^{j=n} \frac{x - x_j}{x_k - x_j} \implies \phi_1(x) = \frac{(x)(x-1)}{2} \ ; \quad \phi_2(x) = (x+1)(x-1) \ ; \quad \phi_3(x) = \frac{(x+1)(x)}{2}$$

And the corresponding Coefficient matrix is given by

$$\mathbf{V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let the corresponding weights be denoted by w_1, w_2, w_3 and therefore we get the corresponding linear system

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \implies \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence the interpolating polynomial

$$\boxed{m(x) = \frac{(x-1)(x)}{2} + \frac{(x+1)(x)}{2} = x^2}$$

Using Newton Basis functions:

The newton basis functions are given by the following formula

$$\phi_k(x) = \prod_{j=1}^{k-1}(x - x_k) \implies \phi_1(x) = 1\ ; \quad \phi_2(x) = x + 1\ ; \quad \phi_3(x) = (x)(x+1)$$

And the corresponding Coefficient matrix is given by

$$\mathbf{V} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

Let the corresponding weights be denoted by w_1, w_2, w_3 and therefore we get the corresponding linear system

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \implies \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Hence the interpolating polynomial

$$\begin{aligned} m(x) &= 1 \cdot 1 - 1 \cdot (x+1) + 1 \cdot (x^2 + x) = 1 - x - 1 + x^2 + x \\ &\boxed{= x^2} \end{aligned}$$

Remarks:

It can be seen that all of the above basis functions give a unique polynomial. This follows from the fact that the interpolant of degree $n - 1$ to n data points is unique and hence all the three methods should give a unique polynomial. Thus for '3' points a polynomial of degree '2' should be unique, and this is verified above.