CSE 401: Numerical Analysis
HW 3

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Solution (6):

The Newton's method for computing the square root of a number can be formulated as follows.

$$f(x) = x^2 - y$$

(a):

- Choose a sample input, say y, whose square root is to be found
- Start the iteration, with an initial trial, say x_0, then $x_{k+1} = F(x_k)$ and repeat till the root is within a specified tolerance

The Newton's method is equivalent to a fixed point iteration problem where the function $g(x)$ is determined using the approximation of the function by the tangent at the iteration point.

$$\begin{aligned} x_{k+1} &= x_k - \frac{f(x)}{f'(x)} \\ &= x_k - \frac{x_k^2 - y}{2x_k} \end{aligned}$$

(b):

Since Newton's method for finding the square root, provided that $y \neq 0$, would not have $f'(x) = 0$, at $x = x_{sol}$, where x_{sol} is the exact root, we can safely assume that it is going to have quadratic convergence. Thus the number of accurate digits in the approximate solution doubles at each iteration

- First Iteration: 8 bits
- Second Iteration: $2 \cdot 8$ bits = 16 bits
- Third Iteration: $2 \cdot 16$ bits = 32 bits $\leftarrow$ corresponding to **24**.
- Fourth Iteration: $2 \cdot 32$ bits = 64 bits $\leftarrow$ corresponding to **53**.

Hence it takes 3 and 4 iterations for getting an accuracy of 24 bits and 53 bits respectively