CSE 401: Numerical Analysis
HW 5

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Solution 3: Equivalent First Order System

The equivalent first order linear ODE systems, for the given higher order ODEs are as follows:

(a) Van der Pol Equation:

$$y'' = y'(1 - y^2) - y$$

The equivalent first order system can be given by making the following substitution

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} y \\ y' \end{Bmatrix}$$

Thus we have

$$\mathbf{y}' = \begin{Bmatrix} y_1' \\ y_2' \end{Bmatrix} = \begin{Bmatrix} y_2 \\ y_2(1 - y_1^2) - y_1 \end{Bmatrix}$$

We can therefore rewrite the system in the following form

$$\boxed{\mathbf{y}' = \mathbf{f}(t, \mathbf{y}) \quad ; \quad \mathbf{y}' = \begin{Bmatrix} y_1' \\ y_2' \end{Bmatrix} \qquad \mathbf{f}(t, \mathbf{y}) = \begin{Bmatrix} y_2 \\ y_2(1 - y_1^2) - y_1 \end{Bmatrix}}$$

(b) Blasius Equation

Given ODE

$$y''' = -yy''$$

We again make a similar substitution, as above, illustrated below

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} y \\ y' \\ y'' \end{Bmatrix}$$

Thus our equivalent first order ODE system is

$$\boxed{\mathbf{y}' = \mathbf{f}(t, \mathbf{y}) \quad ; \mathbf{y}' = \begin{Bmatrix} y_1' \\ y_2' \\ y_3' \end{Bmatrix} \qquad \mathbf{f}(t, \mathbf{y}) = \begin{Bmatrix} y_2 \\ y_3 \\ -y_1 \cdot y_3 \end{Bmatrix}}$$

(c) Newton's Second Law-Two Body Problem

Given ODE system

$$\begin{Bmatrix} y_1'' \\ y_2'' \end{Bmatrix} = -\frac{GM}{(y_1^2 + y_2^2)^{3/2}} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

Let

$$k(y_1, y_2) = \frac{GM}{(y_1^2 + y_2^2)^{3/2}}$$

Using this we make the substitution given below:

$$\mathbf{y} = \begin{Bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{Bmatrix} = \begin{Bmatrix} y_1 \\ y_1' \\ y_2 \\ y_2' \end{Bmatrix} \implies \mathbf{y}' = \begin{Bmatrix} y_1' \\ y_1'' \\ y_2' \\ y_2'' \end{Bmatrix} = \begin{Bmatrix} y_{12} \\ -ky_{11} \\ y_{22} \\ -ky_{21} \end{Bmatrix}$$

Thus our corresponding linear first order system is given by

$$\boxed{\mathbf{y}' = \mathbf{f}(t, \mathbf{y}) \quad ; \qquad \mathbf{y}' = \begin{Bmatrix} y_{11}' \\ y_{12}' \\ y_{21}' \\ y_{22}' \end{Bmatrix} \qquad \mathbf{f}(t, \mathbf{y}) = \begin{Bmatrix} y_{12} \\ -k(y_{11}, y_{21}) \cdot y_{11} \\ y_{22} \\ -k(y_{11}, y_{21}) \cdot y_{21} \end{Bmatrix}}$$
