CSE 401: Numerical Analysis - Fall 2016
Homework 1

Bhavesh Shrimali (NetID: bshrima2)

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Problem 7

1

The determinant of the matrix is given as

$$\begin{vmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{vmatrix} = \epsilon^2$$

2

It can be readily seen from the calculation of the determinant that the corresponding range required on ϵ

$$-\epsilon_{mach} < \epsilon < \epsilon_{mach} \tag{1}$$

where ϵ_{mach} denotes the machine epsilon. It can be seen that for all values satisfied by (1) the matrix would indeed be stored as

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

which is singular.

3

The L-U Factorization can be readily computed knowing that all the diagonal entries in L are equal to 1. This permits the following representation for L and U respectively

$$\mathbf{L} = \begin{bmatrix} 1 & 0 \\ \dots & 1 \end{bmatrix} \text{ and } \mathbf{U} = \begin{bmatrix} \dots & \dots \\ 0 & \dots \end{bmatrix}$$

Now we start with the given matrix ($\mathbf{A}$) and try to reduce it to $\mathbf{U}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix}$$

Now $R_2 \to R_2 + (\epsilon - 1) \cdot R_1$ which gives, as per rule,

$$\mathbf{U} = \begin{bmatrix} 1 & 1+\epsilon \\ 0 & \epsilon^2 \end{bmatrix} \text{ and } \mathbf{L} = \begin{bmatrix} 1 & 0 \\ 1-\epsilon & 1 \end{bmatrix}$$

4

The computed value of **U** would be singular whenever ϵ^2 is less than ϵ_{mach} that is,

$$-\sqrt{\epsilon_{mach}} < \epsilon < \sqrt{\epsilon_{mach}}$$

Note: The above inequality holds true for a given **U** i.e. if given a **U** then the only way it can be singular is when (2) holds. If epsilon (ϵ) is assumed to be strictly positive then we can ignore the negative part of the inequality, i.e.

$$0 \leq \epsilon < \sqrt{\epsilon_{mach}} \tag{2}$$

Now if U is reduced from the given **A** then in the very first step when **A** is stored, it is singular if the epsilon(ϵ) chosen is such that it is less than the machine epsilon (ϵ_{mach}) and corresponding **U** is also singular. To summarize:

- If it is the computed **U** as the only thing we need to be concerned about, disregarding any previous calculations, then

$$0 \leq \epsilon < \sqrt{\epsilon_{mach}} \tag{3}$$

- If we are concerned about determining **U** from the given **A** and all the steps involved therein, then

$$0 \leq \epsilon < \epsilon_{mach} \tag{4}$$

  Because if ϵ is less than ϵ_{mach} then **A** becomes singular and correspondingly **U** obtained from **A** is singular