Problem Statement:

A doubly reinforced rectangular beam has a width (b) of 240 mm and an overall depth (D) of 500 mm. The limiting stresses in concrete (σ_{cbc}) and steel (σ_{st}) are 5 N/mm² and 230 N/mm², respectively. Determine the areas of tension steel (A_{st}) and compression steel (A_{sc}) required to resist a bending moment (M_u) of 80 kN-m. Assume the effective cover (d') for compression steel and the cover for tension steel are both 40 mm. The modular ratio (m) is given as 19.

(1) Given Data:

- Width of the beam (b) = 240 mm
- Overall depth of the beam (D) = 500 mm
- Limiting compressive stress in concrete (σ_{cbc}) = 5 N/mm²
- Limiting tensile stress in steel (σ_{st}) = 230 N/mm²
- Bending moment (M) = 80 kN-m = 80×10^6 N-mm
- Effective cover (d') for compression steel = 40 mm
- Modular ratio (m) = 19

(2) Determine the Effective Depth (d):

Effective depth (d) = Overall depth (D) - Cover for tension steel
$d = 500 - 40 = 460$ mm

(3) Determine the Depth of the Neutral Axis (x_c) for a Singly Reinforced Balanced Section:

For a balanced section, the strain in concrete and steel reaches their maximum permissible values simultaneously. The depth of the neutral axis (x_c) can be found using the strain compatibility relationship:
$\frac{x_c}{d-x_c} = \frac{\sigma_{cbc}/E_c}{\sigma_{st}/E_s} = \frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{m\cdot\sigma_{cbc}}{\sigma_{st}} Substitutingthegivenvalues: x_c = \frac{m\cdot\sigma_{cbc}}{m\cdot\sigma_{cbc}+\sigma_{st}} \cdot d = \frac{19\times5}{19\times5+230} \times 460$

$$x_c = \frac{95}{95+230} \times 460 = \frac{95}{325} \times 460 = 0.2923 \times 460 = 134.46 \text{ mm}$$

(4) Calculate the Limiting Moment of Resistance (M_{lim}) of a Singly Reinforced Balanced Section:

$$46.49 \times 10^6 = A_{sc} \times 66.74 \times 420$$

$$A_{sc} = \frac{46.49 \times 10^6}{66.74 \times 420} = \frac{46490000}{28030.8} = 1658.5 \text{ mm}^2$$

(9) Calculate the Area of Tension Steel (A_{st}):

The total area of tension steel (A_{st}) is the sum of the area of tension steel required for the balanced section (A_{st1}) and the additional tension steel required to balance the compression steel (A_{st2}).

- Area of tension steel for the balanced section (A_{st1}):

$$A_{st1} = \frac{M_{lim}}{\sigma_{st} \cdot (d - x_c/3)} = \frac{33.51 \times 10^6}{230 \times (460 - 134.46/3)} = \frac{33510000}{230 \times 415.18} = \frac{33510000}{95491.4} = 350.9 \text{ mm}^2$$

- Additional area of tension steel (A_{st2}):
  This is required to balance the compressive force developed by the compression steel:

$$A_{st2} \cdot \sigma_{st} = A_{sc} \cdot \sigma_{sc}$$

$$A_{st2} = \frac{A_{sc} \cdot \sigma_{sc}}{\sigma_{st}} = \frac{1658.5 \times 66.74}{230} = \frac{110698.69}{230} = 481.3 \text{ mm}^2$$

- Total area of tension steel (A_{st}):

$$A_{st} = A_{st1} + A_{st2} = 350.9 + 481.3 = 832.2 \text{ mm}^2$$

(10) Summary of Steel Areas:

- Area of compression steel (A_{sc}) = 1658.5 mm²
- Area of tension steel (A_{st}) = 832.2 mm²

The limiting moment of resistance can be calculated based on the compressive force in concrete: $M_{lim} = \frac{1}{2} \cdot \sigma_{cbc} \cdot b \cdot x_c \cdot (d - \frac{x_c}{3}) Substitutingthevalues : M_{lim} = \frac{1}{2} \times 5 \times 240 \times 134.46 \times (460 - \frac{134.46}{3})$

$$M_{lim} = 600 \times 134.46 \times (460 - 44.82)$$

$$M_{lim} = 80676 \times 415.18 = 33506.5 \times 10^3 \text{ N-mm}$$

$$M_{lim} = 33.51 \text{ kN-m}$$

(5) Determine if the Beam Needs to be Doubly Reinforced:

The applied bending moment ($M = 80$ kN-m) is greater than the limiting moment of resistance of a singly reinforced balanced section ($M_{lim} = 33.51$ kN-m). Therefore, the beam needs to be **doubly reinforced.**

(6) Calculate the Moment to be Resisted by the Compression Steel and Additional Tension Steel (M_2):

$$M_2 = M - M_{lim} = 80 - 33.51 = 46.49 \text{ kN-m} = 46.49 \times 10^6 \text{ N-mm}$$

(7) Determine the Stress in the Compression Steel (σ_{sc}):

The depth of the neutral axis ($x_c = 134.46$ mm) is greater than the effective cover to the compression steel ($d' = 40$ mm). Therefore, the compression steel yields. The strain in compression steel is proportional to the strain in concrete at that level.
The stress in compression steel (σ_{sc}) can be calculated as:
$\sigma_{sc} = m \cdot \sigma_{cbc} \cdot \frac{x_c - d}{x_c} Substitutingthevalues : \sigma_{sc} = 19 \times 5 \times \frac{134.46-40}{134.46} = 95 \times \frac{94.46}{134.46} = 95 \times 0.7025 = 66.74 \text{ N/mm}^2$
However, the maximum stress in compression steel cannot exceed the limiting stress in steel ($\sigma_{st} = 230$ N/mm²). In this case, $66.74 \text{ N/mm}^2 < 230 \text{ N/mm}^2$, so the calculated stress is valid based on linear elastic behavior.

(8) Calculate the Area of Compression Steel (A_{sc}):

The moment resisted by the compression steel and the additional tension steel (M_2) is given by: $M_2 = A_{sc} \cdot \sigma_{sc} \cdot (d - d') Substitutingthevalues : 46.49 \times 10^6 = A_{sc} \times 66.74 \times (460 - 40)$