

Problem set 4

S520 Fall 2021

Upload your answers through the Assignments tab on Canvas by 11:59 pm, Thursday 23rd September.

Trosset question numbers refer to the hardcover textbook. Show working (answers only will not get full credit.) You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources. You may (and sometimes have to) use R; you must state your R code, where relevant, for full credit. (For now you can just write out/copy-paste your R code into your document; you don't have to use Markdown unless you want to.)

1. (From the Spring 2017 midterm. You might want to do this question by hand, rather than use R, as practice for your midterm.) I toss a coin seven times, independently. The coin is fair, so the probability of getting a head is 0.5 for each toss. Let X be a binomial random variable representing the number of heads in the seven tosses.
 - (a) What is the probability I get exactly five heads?
 - (b) Find $F(2)$, the cumulative distribution function of X at 2.
 - (c) Find the expected value and standard deviation of X .
 - (d) Are the events “I get all heads on the first four tosses” and “I get all tails on the last four tosses” independent? Explain why or why not.
2. The response rate to telephone polls is around 2%. Of people who respond to telephone polls, currently about 40% approve of the President.

Suppose we perform a telephone poll by calling a random sample of 50,000 registered voters.

 - (a) Let X be a binomial random variable representing the number of people who respond to the telephone poll. What are the expected value and standard deviation of X ?
 - (b) Let Y be a binomial random variable representing the number of people who respond to the telephone poll and approve of the President. What are the expected value and standard deviation of Y ?
 - (c) Use R to find the probability that at least 420 people respond to the telephone poll and approve of the President. Give your R code.
3. Trosset chapter 4.5 exercise 14.

Hint: You can define two different binomial random variables here. Let X be the number of matches a particular receiver gets right; then X is $\text{Binomial}(25, 1/5)$. Let Y be the number of receivers that get a score of more than 7; then Y is $\text{Binomial}(20, p)$. (What's p ?)

4. Let X be a uniform random variable with probability density function (PDF)

$$f(x) = \begin{cases} \frac{1}{20} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Verify that f is a PDF. That is, check that $f(x)$ is always non-negative and that the area under $f(x)$ is 1.
 - (b) Find the CDF of X , $F(y)$, for all y .
 - (c) Find $P(30 < X < 50)$.
 - (d) Find the expected value of X .
 - (e) Find the variance and standard deviation of X . (Hint: You can use calculus if you wish, but it'll much easier to use a formula for the variance and SD of a uniform random variable.)
5. Trosset exercise 5.6.2, parts (b) and (c). You don't have to submit part (a), but it's worth drawing it for your own reference. (Suggestion for part (c): If you don't want to do calculus, you can find this probability as the difference between the areas of two triangles: one to the right of 1.5, and one to the right of 1.75. It might be easier to just do the integration, though.)
6. Trosset exercise 5.6.3. (Note: All S520 students must do this question; the one student in S350 does not have to do this question.)