S520 Fall 2021
Problem Set 4 Solutions

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1.

Ans:

(a) $p^5 * (1-p)^2 * C(7,5) = 0.5^5 * (1-0.5)^2 * C(7,5) =$ **0.164**

(b) F(2)=

$$\begin{cases} (0.5)^0 * (0.5)^7 & \text{for } x = 0 \\ 0.5)^1 * (0.5)^6 & \text{for } x = 1 \\ 0.5)^2 * (0.5)^5 & \text{for } x = 2 \\ 0.5)^3 * (0.5)^4 & \text{for } x = 3 \\ 0.5)^4 * (0.5)^3 & \text{for } x = 4 \\ 0.5)^5 * (0.5)^2 & \text{for } x = 5 \\ 0.5)^6 * (0.5)^1 & \text{for } x = 6 \\ 0.5)^7 * (0.5)^0 & \text{for } x = 7 \end{cases}$$

P(X=2)= P(X≤2)-P(X<2)= **0.2265**

(c) E(X)=X.P(X)=n*p=7*0.5=**3.5** var(x)=n*p*(1-p)=**1.75**, std=sqrt(n*p*(1-p))= **1.322**

(d) P(All 4 heads in 1st 4 trails)=P(A), P(All 4 tails in the first 4 trails)=P(B) As P(A∩B) = 0, **NotIndependent events**

2.

Ans:

(a) P(R)=0.02, P(A/R)=0.4, S=50,000. E(X)=n*p=50,000*0.2=**1000**, var(x)=**980**, std=**31.30**

(b) E(X)=50,000*0.4=**400**, VAR(X)=50,000*0.008*(1-0.008)=396.8, std=**19.91**

(c) 1- pbinom(x=419,n=50,000,p=0.008)= **0.1647013**

3.

Ans:

(a) P(each)=1/5, n=25. E(X)=n*p=25*1/5= **5**

(b) P(X>7)=1-P(X≤7)= 1- pbinom(7,25,1/5)=**0.1091228**

(c) P(X≥0)=1-P(X=0)=1- pbinom(0,20,0.1091228) = **0.9008353**

4.

Ans:

(a)

$$\int_{20}^{40} 1/20\, dx$$

= 40/20-20/20= **1**. Therefore, F is a PDF (non-negative)

(b) CDF= F(y)=

$$\begin{cases} 0 & \text{for } y < 20 \\ \frac{y}{20} - 1 & \text{for } 20 \leq y \leq 40 \\ 1 & \text{Otherwise} \end{cases}$$

(c) P(30<X<50)= P(X≤50)-P(X≤30)= 1-1/2= **0.5**

(d)

$$\int x f(x) dx$$

= (1600-400)/40= 1200/40= **30**

(e) var(x)= (b-a)^2/12 = 400/12 = **33.33** std = **5.7735**

5.

Ans:

(a) None

(b)

$$\int_1^2 (2x-2)\,dx$$

=(4-4)-(1-2)= **1** It's Non-negative and therefore is PDF.

(c) P(1.50<X<1.75)= P(X≤1.75)-P(X≤1.5)= **0.3125**

6.

Ans:

(a) 0+

$$\int_0^{1.5} (cx)\,dx$$

+

$$\int_{1.5}^3 c(3-x)\,dx$$

=> c=1/2.25= **0.444**

(b) E(X)= Since it is asymmetric graph, our expected value is mid-point which is =**1.5**

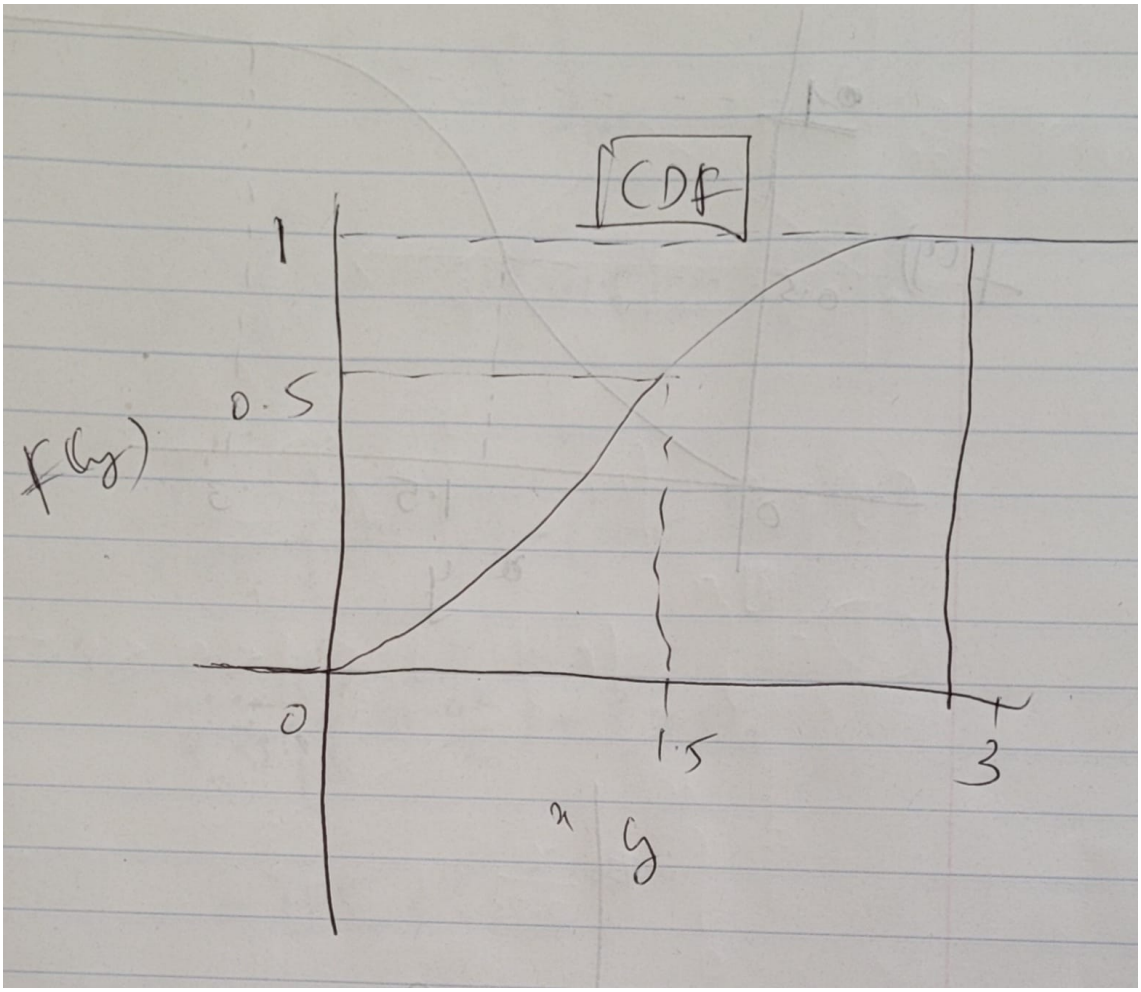
(c) P(X>2)=

$$\int_2^3 C(3-X)\,dx$$

= 3C-5C/2= C/2= **0.222**

(d) **The variance of Y is greater than that of variance of X.**

(e) **Please see the below image in the next page for the graph.....**


CDF
1
0.5
F(y)
0
1.5
3
y