# Formula Vehicle Drag and Lift Coefficients

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A critical aspect of Formula 1 racing is the vehicle's aerodynamics, including the lift and drag which affect the vehicle's ability to maintain high speed through turns and straights. In this paper, an optimization solution has been developed to optimize these factors to minimize lap time on a given track. Using a Monte Carlo simulation, an optimal lift-drag ratio can be determined. Then a regression model is used to model the coefficient of lift versus lap time. This regression model is then used with reliability-based design optimization methods to identify the optimum coefficient of lift that minimizes lap time.

#### **Nomenclature**

 $C_L$  = coefficient of lift  $C_D$  = coefficient of drag L/D = lift-drag ratio

LT = lap time

 $a_1$  = regression coefficient 1  $a_2$  = regression coefficient 2  $a_3$  = regression coefficient 3  $F_L$  = force caused by lift  $F_D$  = force caused by drag

F = total force on aerodynamic surfaces

 $\rho = \text{density of air}$  v = velocity of air

A = frontal cross-sectional area of vehicle

 $\sigma_y$  = yield stress of the wing

 $\beta$  = reliability index

 $\beta_t$  = target reliability index k = penalty multiplier

 $\alpha$  = step size

## I. Introduction

In the modern era of Formula 1, where technological innovation and engineering precision define the competition, achieving the optimal balance in a car has become a critical priority for all teams. The challenge lies in creating a vehicle that is not only the fastest on the grid but also perfectly suited to the driver's preferences and reliable enough to endure the demands of a race. Teams must fine-tune multiple aspects of performance, including brakes, suspension, and power units, to gain a competitive edge. Among these, aerodynamic balance is pivotal in influencing the car's speed, stability, and overall efficiency on the track. This study delves into the intricate science of aerodynamic balance, focusing on two key parameters: the coefficient of lift and the coefficient of drag.

The coefficient of lift  $(C_L)$  determines the amount of downforce generated by the car, enhancing grip and cornering performance, particularly on tracks with numerous sharp turns. On the other hand, the coefficient of drag  $(C_D)$  represents the air resistance that the car encounters as it speeds down straights. While a higher  $C_L$  is beneficial for improving traction, a lower  $C_D$  is crucial for minimizing resistance and achieving higher speeds. These parameters, however, are inversely related—a fundamental limitation dictated by physics. Increasing  $C_L$  invariably results in a proportional increase in  $C_D$ , and vice versa, which forces teams to make strategic decisions based on the characteristics of each racetrack. For instance, tracks with long straights, such as Autodromo Nazionale Monza, prioritize a low  $C_D$  to maximize top speed, while circuits with numerous corners, such as Paul Ricard, demand a higher  $C_L$  to ensure optimal grip.

In this study, we aim to explore the trade-off between  $C_L$  and  $C_D$  to identify the optimal aerodynamic setup for three distinct tracks: Spa-Francorchamps, Autodromo Nazionale Monza, and Paul Ricard. These tracks were chosen due to their varying configurations, which provide a comprehensive understanding of aerodynamic balance under different racing conditions. For each track, we will simulate 1,000 iterations of a custom MATLAB program, varying  $C_L$  and  $C_D$  values to determine the optimum lift-drag ratio (L/D). This ratio represents the ideal aerodynamic balance tailored to each circuit's unique demands.

Once the simulations are complete, we will analyze the relationship between  $C_L$  and lap time (LT) using linear regression. This step enables us to establish a mathematical model that predicts lap performance based on aerodynamic parameters. With this model in place, we can apply optimization algorithms to further refine the values of  $C_L$  and  $C_D$ , ensuring they meet the specific requirements of each track. Additionally, we will evaluate the structural integrity of the rear wing under the forces generated by  $C_L$  and  $C_D$ . By defining a limit state function based on wing failure due to aerodynamic stresses, we incorporate Reliability-Based Design Optimization (RBDO) into our methodology. This approach ensures that the optimized values of  $C_L$  and  $C_D$  to maximize performance and maintain structural safety.

The results section will present the optimized values of  $C_L$ ,  $C_D$ , and L/D for each track, along with a detailed discussion of the physical reasoning behind these outcomes and the minimized LT. By analyzing the interplay between aerodynamic forces, lap times, and structural constraints, we aim to provide a comprehensive understanding of how teams can strategically tailor their aerodynamic setups for maximum performance. This study highlights the critical role of physics and engineering in shaping Formula 1 strategies and offers insights into the delicate balance that defines success in this highly competitive sport.

## II. Theory

### A. Optimization of Lift-Drag Ratio

The first step in identifying the optimum values of the coefficient of lift and the coefficient of drag is to find the optimal lift-drag ratio for a given track. Once this is completed, the lap time can be minimized in terms of only one variable, coefficient of lift, and a regression model can be developed to perform optimization procedures on.

Initially, random variables are created for  $C_L$  and  $C_D$  are created with ranges  $-10 \le C_L \le -2.5$  and  $-1.6 \le C_D \le -0.7$  to be used in a Monte Carlo simulation. These random variables will then be run through a lap time simulation based on the open-source code provided by OpenLAP-Lap-Time-Simulator that will use the vehicle and track files provided by the user to identify predicted lap times for each combination of coefficients.

$$L/D = \frac{C_L}{C_D} \tag{1}$$

Using Equation (1), the lap times can be distributed based on their respective lift-drag ratios. Plotting this relationship shows an exponential relationship with many outliers. By refining the data and removing excessive outliers, this process uses this relationship to identify an exponential regression model between the lift-drag ratio and correlating lap time. Due to the exponential pattern, the optimal lift-drag ratio is considered the point of minimal lift-drag for minimal lap time which can be found where  $\left|\frac{d(L/D)}{d(LT)}\right| = 1$ .

## B. Regression of Lift Coefficient versus Lap Time

$$C_D = C_L \cdot (L/D)^{-1} \tag{2}$$

Once the optimum lift-drag ratio has been identified, the coefficients of drag will be replaced by values that directly relate to the coefficient of lift using Equation (2). Due to the accuracy of this process and the duration required to run the lap time simulation, these coefficients will be sorted evenly, and a small number will be picked evenly throughout the set. These values of  $C_L$  and  $C_D$  will be run through the lap time simulation to find matching lap times for each set.

$$LT = a_1 + a_2 C_L + a_3 C_L^2 (3)$$

Because of the use of L/D to get the new values for  $C_D$  based on  $C_L$ , the model for minimizing lap time is now only a function of one variable:  $C_L$ . Lap time, LT, can be modeled by the quadratic function displayed in Equation (3).

$$[a] = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad [LT] = \begin{bmatrix} LT_1 \\ LT_2 \\ ... \\ LT_n \end{bmatrix} \qquad [C_L] = \begin{bmatrix} 1 & C_{L,1} & C_{L,1}^2 \\ 1 & C_{L,2} & C_{L,2}^2 \\ ... & ... & ... \\ 1 & C_{L,n} & C_{L,n}^2 \end{bmatrix}$$
(4)

$$[a] = ([C_L]' \cdot [C_L])^{-1} \cdot [C_L]' \cdot [LT]$$

$$(5)$$

It is then possible to find the regression model. By setting up the matrices according to Equation (4) and using Equation (5), the regression coefficients of the model can be found and applied to develop the minimization function to be used in future steps to optimize  $C_L$  to minimize lap time.

## C. Optimizing Coefficient of Lift

By using functional approximation to minimize lap time by optimizing the coefficient of lift as seen in Equation (3), we have the unconstrained optimization function. This optimization statement, however, needs to incorporate structural constraints to become a reliability-based design optimization (RBDO) solution.

$$f(C_L) = a_1 + a_2 C_L + a_3 C_L^2 \tag{6}$$

$$LT(C_L) = f(C_L) + k \cdot \sum g(C_L) \tag{7}$$

To embed the RBDO in this study, we need to define a limit state function. By making  $f(C_L)$  in Equation (6) the open optimization statement and  $g(C_L)$  the limit state function, the new optimization statement considers the optimum  $C_L$  to minimize LT considering the forces caused by lift and drag and the structural reliability of the vehicle's aerodynamic surfaces. The limit state function that is considered is the failure of the rear wing, representing the primary aerodynamic surface of the vehicle, due to the forces exerted by lift and drag. The stresses considered here are defined by the Von Misses Criteria.

$$F = \sqrt{F_L^2 + F_D^2} \tag{8}$$

$$F_L = \frac{1}{2} \cdot \rho \cdot v^2 \cdot A \cdot C_L \tag{9}$$

$$F_D = \frac{1}{2} \cdot \rho \cdot v^2 \cdot A \cdot C_D \tag{10}$$

Equation (8) above shows the total force that will be exerted on the wing that will potentially cause failure. It consists of Equations (9) and (10) which will use the givens that are fixed for this process and defined in the vehicle specifications. It is important to note that  $\rho$  and A are constants fixed in the vehicle specifications while v is the average velocity of the vehicle on a track, generalized for this solution process. Using Equation (2), Equations (9) and (10) can be related to make Equation (8) a function of  $C_L$ .

$$g_1(C_L) = F - \sigma_v \le 0 \tag{11}$$

$$g_2(C_L) = \beta_t - \beta \le 0 \tag{12}$$

Considering the yield stress of the wing,  $\sigma_y$ , the limit state function is given in Equation (11). By using this limit state function in the Hasofer & Lind method, we can get the functions reliability index,  $\beta$ . Using the industry standard for the target reliability index for structural problems,  $\beta_t = 4$ , which defines the probability of failure at less than 1%, we can add an industry structural constraint to our optimization solution:  $\beta \ge \beta_t$ . This constraint will form equation (12).

With all the constraints defined for the optimization solution, we can develop an optimization algorithm. The Exterior Penalty method and the Steepest Gradient Descent method can be used on the completed equation (7) to optimize LT subject to the constraints mentioned following the algorithm:

- 1. Define the initial value of  $C_L$  (i.e. the highest sample value). Loop 1
- 2. Use Equation (2) to create a corresponding  $C_D$ .
- 3. Develop the limit state function shown in Equation (11).
- Use the Hasofer & Lind Method to calculate β.
- 5. Use the Exterior Penalty method to formulate the minimization function seen in Equation (7).
- 6. Run the Steepest Gradient Descent method to optimize the value of  $C_L$  assuming a step size of  $\alpha = 0.05$ .
- 7. Repeat steps 3-6 until the value of  $C_L$  converges to a tolerance of  $\varepsilon = 10^{-6}$ .

Using this algorithm after the optimal L/D value is found and the regression model for  $C_L$  versus LT is defined will provide the optimum values of  $C_L$ ,  $C_D$ , L/D, and LT. Performing this process on any lap will provide the user with these optimal aspects of their vehicle as shown below for the three tracks included in this paper.

## III. Demonstration Example

#### A. Problem Statement

Given the necessary data, find the minimum amount of time it will take for a formula vehicle to perform a lap at the Autodromo Nazionale Monza racetrack in Italy by altering the coefficients of lift and drag assuming that the optimal values are within the ranges of  $-10 \le C_L \le -2.5$  and  $-1.6 \le C_D \le -0.7$  using a sample size of 10.

#### **B.** Solution

1. Create n=10 variables for the lift and drag coefficients.

$C_L$	-10.0	-4.85	-6.35	-3.67	-4.28	-2.50	-2.62	-7.86	-4.43	-9.14
$C_D$	-1.60	-0.87	-1.12	-1.14	-0.70	-1.00	-1.00	-0.75	-0.80	-1.04

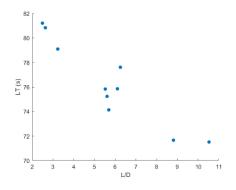
2. Run the lap time simulator to find corresponding lap times and use Equation (1) to find L/D.

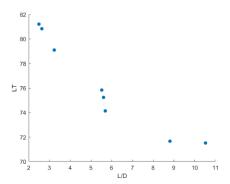
L/D	6.25	5.61	5.69	3.23	6.11	2.49	2.63	10.5	5.52	8.81
LT	77.6	75.2	74.1	79.1	75.8	81.2	80.8	71.5	75.8	71.7

3. Sort the data to be increasing in terms of L/D.

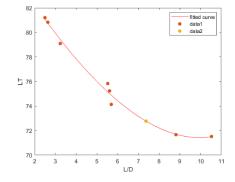
L/D	2.49	2.63	3.23	5.52	5.61	5.69	6.11	6.25	8.81	10.5
LT	81.2	80.8	79.1	75.8	75.2	74.1	75.8	77.6	71.7	71.5

4. Refine the data to remove outliers.





5. Create exponential regression model and identify where  $\left|\frac{d(L/D)}{d(LT)}\right| = 1$  and set as optimal lift-drag ratio (shown in gold).



$$L/D^* = 7.37$$

6. Make new coefficient of drag terms that are subject to the relationship shown in Equation (2) and sort in terms of ascending coefficients of lift.

$C_L$	-10.0	-9.14	-7.86	-6.35	-4.85	-4.43	-4.28	-3.67	-2.62	-2.50
$C_D$	-1.36	-1.24	-1.07	-0.86	-0.66	-0.60	-0.58	-0.50	-0.36	-0.34

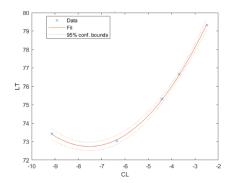
Identify a small subset (i.e. 5) evenly distributed samples from the sets of lift and drag coefficients.

$C_L$	-9.14	-6.35	-4.43	-3.67	-2.50
$C_D$	-1.24	-0.86	-0.60	-0.50	-0.34

Use Equation (5) to identify the regression coefficients.

$a_1$	87.6
$a_2$	3.93
$a_3$	0.26

Plug the regression coefficients into Equation (3) to determine the minimization function.



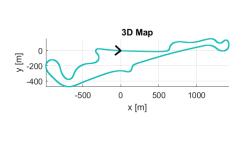
$$LT = 87.6 + 3.93C_L + 0.26C_L^2$$

10. Use the algorithm to optimize  $C_L$  using the External Penalty and the Steepest Gradient Descent methods to find the optimum values for  $C_L$  and  $C_D$ .

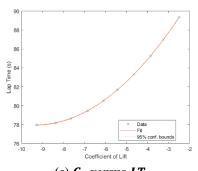
$C_L$	-7.54
$C_D$	-1.02
LT	72.7

IV. **Results** 

Based on the process laid out and supported in the theory section above, this optimization process was applied to the three tracks previously mentioned based on an arbitrary list of vehicle specifications. After this process was completed for Paul Ricard, Spa-Francorchamps, and Autodromo Nationale Monza using 1000 iterations, the following observations can be made:



6 8 Lift-to-Drag Ratio



(a) Track Map.

(b) L/D versus LT.

(c)  $C_L$  versus LT.

Figure 1: Paul Ricard Simulation Results.

The layout of the Spa track has been simulated using its given coordinates, as shown in Figure 1a. Displayed in Figure 1b is the lift-drag ratio versus lap time data and the exponential regression model that matches the data. In Figure 1c is the coefficient of lift results plotted against lap time.

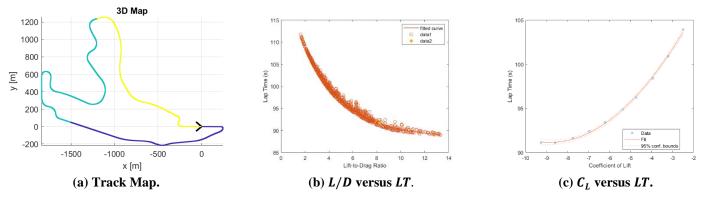


Figure 2: Spa-Francorchamps Simulation Results.

Figure 2a shows the map of Spa-Francorchamps and displays the coordinates in terms of meters. Figure 2b displays the results of the exponential model of the lift-drag ratio versus the lap times. Figure 2c plots the quadratic regression model of the coefficient of lift versus lap time.

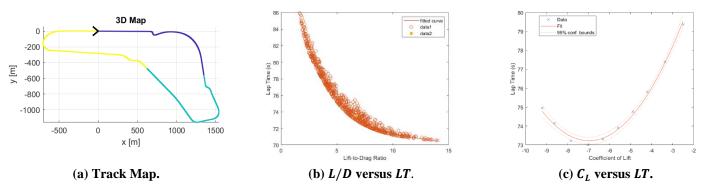


Figure 3: Autodromo Nazionale Monza Simulation Results.

Figure 3a shows the track of Autodromo Nazionale Monza, filled with straightaways and very few turns. Figure 3b shows the relationship between lift-drag ratio and lap time as well as the regression model to fit. Shown in Figure 3c is the regression model to identify the optimum coefficient of lift to minimize lap time.

Track	L/D	$C_L$	$C_D$	LT (s)
Paul Ricard	8.33	-9.36	-1.12	77.956
Spa-Francorchamps	8.23	-8.84	-1.07	91.152
Autodromo Nazionale Monza	6.73	-6.98	-1.04	73.2253

Table 1: Optimum Values.

Table 1 shows the results for the optimum values for each track. While the geometry of each track makes the lap times impossible to compare, the variation of L/D,  $C_L$ , and  $C_D$  are evident. A noteworthy observation is the presence of negative signs for  $C_L$  and  $C_D$  values, indicating their respective directions: negative  $C_L$  represents downforce, while negative  $C_D$  signifies resistance opposing the vehicle's motion.

## V. Discussion

#### A. Ranges for the Coefficients of Lift and Drag

One of the most critical aspects of this process is the initial setup of the random variables for the coefficients of lift and drag. Based on the research paper, "Aerodynamic studies of 2021 F1 car," by Max Taylor, computational fluid dynamic tests support that modern geometries and manufacturing processes cannot yield a vehicle that extends beyond the ranges of  $-3 \le C_L \le -2.5$  and  $-1.6 \le C_D \le -1.0$ . These were the first ranges that were used to develop a solution. However, these ranges proved ineffective after significant experimentation had been performed.

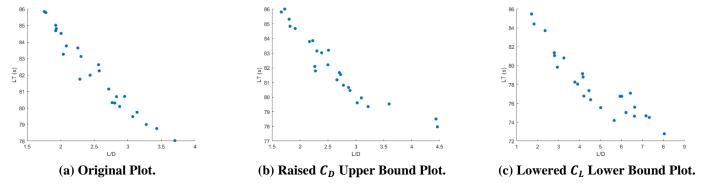


Figure 4: Lift-Drag Ratio versus Lap Time Plots for Different Ranges.

Figure 4a shows the L/D versus LT plots using the raw data after random variables within these ranges and has an apparent linear or quadratic pattern, suggesting that further experimentation is necessary to better define the optimal model to find a regression pattern that can help identify the optimal lift-drag ratio. To better understand the pattern of this relationship, experimentation was performed to identify how the input value ranges affected the results.

Since it is clear, that L/D leads to a lower lap time at high values, the ranges were altered to accommodate a better visual of the optimal point by raising the coefficient of lift range, lowering the coefficient of drag range, or both. In Figure 4b, the upper bound of the coefficient of drag was raised to -0.7, which is commonly recognized as a realistic lower bound for F1 cars on the track. This results in a model that is bad at reaching exceptional lap times, but does create a far more exceptional model. After a significant amount of tests at this range, it was suggestive that the relationship was a quadratic or exponential model.

In order to properly examine results in every case, another few tests were performed using the original range for the coefficient of drag while the lower bound of the lift coefficient dropped to -10.0. Displayed in Figure 4c, it is clear that while this method reached exceptionally low lap times, the predictive model based on these ranges are subpar. While these results to provide the minimal values for lap times, it is incredibly important to note that with current technologies and manufacturable geometries, these lift-drag ratios are impossible to accomplish and this process will become more effective as that changes.

After updating the ranges in for both coefficients, it was determined that the ranges of  $-10 \le C_L \le -2.5$  and  $-1.6 \le C_D \le -0.7$  create an accurate model that found significantly low values for lap time. The results of this model are shown in the results above for each of the three tracks. Experiments with this range suggested that the model of L/D versus LT is exponential.

#### **B.** Simulation Method for Lap Time

The lap time simulator that was constructed from the open-source code created by Michael Halkiopoulos uses data sets that determine the lap and vehicle specifications that are used to calculate lap time. In its original form, this code uses pre-set values for the coefficients of lift and drag. In this process, it was critical to run a significant number of random variables through the simulation to create enough data to develop a regression model for L/D versus LT and  $C_L$  versus LT. For ease of development, this was performed using completely randomized values to perform a Monte Carlo simulation, which worked significantly with a significant number of random variables. In the future, this code could profit from an improvement in this sample gathering process.

The use of Latin Hyper Cube sampling in future iterations of this process to develop a well spread dispersion of these coefficients could improve the evenness of experimentation. It is critical that some randomizing process is used as it is critical to get different values for L/D that are comprised of higher and lower values for both coefficients.

#### C. Identification of the optimal Lift-Drag Ratio

After experimentation had been performed to find the optimal ranges for the lift and drag coefficients to create a valid L/D versus LT regression model, finding the optimal lift-drag ratio that minimized lap time becomes a complicated problem. Since there is an exponential decay model relationship between the two factors, the lap time will decrease as the lift-drag ratio increases to infinity. Because it is easiest to accomplish a low lift-drag ratio, it is critical to define an acceptable value for  $\left|\frac{d(L/D)}{d(LT)}\right|$ . This will change as a user aims to focus on either a lower lap time and better technical performance (a lower value of  $\left|\frac{d(L/D)}{d(LT)}\right|$ ) or a lower lift-drag ratio which is easier to manufacture (a higher value of  $\left|\frac{d(L/D)}{d(LT)}\right|$ ). For this process and the results shown above, a value of  $\left|\frac{d(L/D)}{d(LT)}\right| = 1$  was used to minimize lap time at an equal rate as the lift-drag ratio is minimized.

### D. Regression Modeling for Coefficient of Lift vs Lap Time

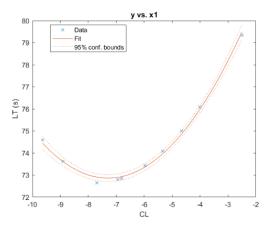


Figure 5: Quadratic Regression Model between  $C_L$  and LT.

To optimize  $C_L$  to minimize LT, it was critical to define the minimization function. Once the optimal L/D value has been obtained and the new values of  $C_D$  have been found, it is necessary to run the lap time simulator these values. Using the determined ranges for the coefficient of lift and coefficient of drag, it is clear as shown in Figure 5 that a quadratic model is the most accurate for the relationship between lift coefficient and lap time. After experimenting sufficiently, it was discovered that using these refined values for  $C_L$  and  $C_D$  to find lap times, a regression model could be determined using the processes from Equations (4) and (5) that consistently has an  $R^2$  value above 0.995, suggesting an extremely accurate model given the data. Because of this reliability and the consideration that the lap time simulator takes significant time to run ( $\sim$ 2 seconds per iteration), it was determined that running only a small number of sets for lift and drag coefficients, 10 in most cases, that were equally distributed between the range of samples provided enough sample data for an equally accurate quadratic regression model to be developed. This greatly reduces the amount of time necessary to find a regression model for coefficient of lift versus lap time.

## E. Optimization Method

The Steepest Gradient Descent Method was chosen to simplify the optimization process while ensuring maximum efficiency while finding the optimum value for  $C_L$ . The step size was kept as low as 0.05 to achieve a balance between accuracy and computational effectiveness.

The value of  $\beta$  obtained in each iteration of the Hasofer & Lind Method consistently approaches 12, significantly exceeding the set value of  $\beta_t = 4$ . This suggests that the structure might appear overdesigned for the stresses induced by lift and drag forces. However, upon closer examination, it becomes evident that the rear wing structure is also designed to withstand additional stresses, including those caused by high temperatures due to high velocity and bending stresses from supporting part of the car's weight. Therefore, it would be incorrect to conclude that the structure is overdesigned; instead, it fulfills the constraints set for this design. As a future extension of this project, we can refine our assumptions by accounting for all relevant stresses, not just those from lift and drag forces, and develop a comprehensive Limit State Function. This would allow us to better define the constraints and perform a more robust Reliability-Based Design Optimization for this study.

### F. Analysis of Results

The map in Figure 1a shows that the Paul Ricard racetrack is dominated by turns, suggesting that we can expect the optimum values to favor a high coefficient of lift. Figure 1b illustrates the results from the Monte Carlo simulation and exponential regression modeling for the lift-drag ratio specific to the Paul Ricard track. This plot displays how L/D versus LT follows an exponential trend after the removal of outliers. From the regression model based on these results, the optimum lift-drag ratio has been derived, serving as a critical input for next steps in the optimization process. Figure 1c presents the quadratic regression results that relates  $C_L$  to LT. For Paul Ricard, this data had an  $R^2$  value of 0.999, making it an extremely reliable model. Achieving a precise fit for Cl vs. lap times is essential, as the resulting equation directly influences the optimization process. A strong correlation ensures greater confidence in the outcomes of the optimization analysis.

By analyzing Figure 2a, it is evident that the racetrack of Spa-Francorchamps is considerably evenly placed considering dominance of straightaways and turns, suggesting that it will have moderate L/D that balance the coefficients of lift and drag without favoring one of the other with a slight bias towards turns and a higher lift coefficient. Figure 2b, showing the lift-drag ratio's exponential relationship with lap time is extremely helpful. It shows that in the case of Spa, with more straightaways than Paul Ricard, the exponential model has a faster decay which causes the point at which  $\left|\frac{d(L/D)}{d(LT)}\right| = 1$  to be met by a lower value of L/D. As the regression model shown in Figure 2c has an  $R^2$  value of 0.997, the model is still considered extremely precise and can be relied upon

to produce accurate results for optimizing the coefficient of lift. While this plot is very similar to Figure 1c, it is noteworthy that the minimum  $C_L$  crests at a higher coefficient of lift.

Autodromo Nazionale Monza is notorious for being full of straightaways with minimal turns as displayed in Figure 3a. Because of this, it is evident that the optimal values for such a track will heavily favor a minimal drag coefficient over a high lift coefficient. The results of the Monte Carlo simulation and resulting exponential regression model for L/D versus LT is clear when comparing Figures 3b and 1b. As the track is dominated more by straightaways than turns, the exponential decay rate increases, leading the optimum lift-drag ratio to be lower. Based on the results of these three tracks, this is a reliable hypothesis for all tracks and should be considered by racing engineers. Figure 3c also shows a pattern relative to the results of Paul Ricard and Spa-Francorchamps. In a track that is highly dominated with straightaways like Autodromo Nazionale Monza, the minimum lap time is met by a considerably lower lift coefficient than in cases dominated by turns. With an  $R^2$  value of 0.995, it is evident that while the model is still extremely reliable, straighter tracks lead to a less reliable model between coefficient of lift and lap time.

Referencing the data in Table 1, it is extremely easy to see that the predicted patterns were confirmed. In a track that favors turns, such as Paul Ricard, it is extremely critical to maximize the coefficient of lift. In tracks that are more straightaway-oriented, it is important to minimize the coefficient of drag. By analyzing the data, we can also see that the lift-drag ratio also follows a pattern depending on whether a track favors straightaways or turns. In tracks that favor straightaways more, the lift-drag ratio decreases quickly as the optimum coefficient of drag drops at a considerable rate while the coefficient of drag drops at a much lower rate. Comparing the two most radical cases,  $C_L$  drops by 2.38 while  $C_D$  drops by 0.08 leading to a drop in L/D by 1.6 or nearly 20% from Paul Ricard to Autodromo Nazionale Monza.

### VI. Conclusion

This reliability-based design optimization solution to identify for any given track and vehicle the optimum coefficients of lift and drag is capable of reliably providing engineers with predictive data to help optimize configuration and vehicle geometry to minimize lap time. It proves that while optimal lift-drag ratios for most tracks are beyond the realm of possibility based on current technologies, patterns can be predicted to help suggest areas to focus on in a vehicle's aerodynamic surfaces to improve performance and aerodynamic balance. While there are several considerations that can improve future drafts of this optimization solution, the algorithm created and laid out in this paper provides a reliable basis for this kind of aerodynamic optimization.

## VII. Contribution

#### James

- Took lead in generating dataset and setting ranges for coefficients of lift and drag.
- Created Monte Carlo Simulation to identify corresponding lap times.
- Developed exponential regression model for L/D versus LT.
- Defined method to find optimum L/D.
- Formulated method to identify quadratic regression model for  $C_L$  versus LT.
- Written Sections:
  - Abstract
  - Nomenclature
  - Theory
  - o Demonstration Example
  - o Discussion
  - Conclusion
- Formatted document

#### Bhavin

- Developed RBDO process including establishing External Penalty Method.
- Used Hasofer & Lind process to find reliability.
- Defined Algorithm for using Steepest Gradient Method to find optimum  $C_L$ .
- Written Sections
  - Introduction
  - Theory
  - o Discussion
  - Contribution

## References

 $Michael\ Halkiopoulos\ (2024).\ OpenLAP-Lap-Time-Simulator\ (https://github.com/mc12027/OpenLAP-Lap-Time-Simulator/releases/tag/V01.00),\ GitHub.$ 

Taylor, M. (2022, January 17). Aerodynamic studies of a 2022 F1 car. Max Taylor - Aerodynamics and Motorsport Engineering. https://maxtayloraero.com/2022/01/17/aerodynamicstudies-of-a-2022-f1-car/