q-3:- Mathematically derive the average runtime complexity of the non random pivot o version of quick sort.

let's the array have n elements. if We devide array in two parts based on pivot.

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

b) for Best case:-
if suppose the partitioning algorithm produce a 9 to 1 proportional split the recurrence will be
$O(n)$ is time complexity for partitioning step

$$T(n) = T\left(9n/10\right) + T\left(n/10\right) + \theta(n)$$

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + \theta(n)$$

expand

$$T\left(\frac{9n}{10}\right) = T\left(\frac{9n}{100}\right) + \left(\frac{81n}{100}\right)$$

$$+ \left(\frac{81n}{1000}\right) + \left(\frac{729n}{1000}\right) + \ldots$$

let's establish general pattern

$$n\left(\frac{9}{10}\right)^k \approx 1 \quad \& \quad k \approx \log_{10/9} n$$