$$f(n) = 1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \dots + n\cdot n!$$

Find the general form of $f(n)$

$f(1) = 1$

$f(2) = 5$

$f(3) = 23$

$f(4) = 119.$

$f(n) = (n+1)! - 1.$ ✓

Note that $f(n+1) = f(n) + (n+1)\cdot(n+1)!$

<u>Claim</u>

$$\underline{f(n) + (n+1)\cdot(n+1)! = (n+1)! - 1}$$

$$f(n) = (n+1)!\,((n+2)-(n+1)) - 1$$

$$f(n) = (n+1)! - 1$$

$$f(n) = (n+1)! - 1$$

S is a geometric sequence

The sum of first 6 terms of S is equal to 9 times the sum of first 3 terms.

The 7^{th} term of S is <u>360</u>.

Find the <u>1^{st}</u> term.

$$S = a, ar, ar^2, \ldots, ar^n$$

$$S_6 = \frac{a(1-r^6)}{1-r} \quad ; \; r \neq 1.$$

$$\frac{\underline{a}(1-r^3)}{1-r} \; S_3$$

$$\frac{\cancel{a}(1-r^6)}{\cancel{1-r}} = \frac{9\cancel{a}(1-\underline{r}^3)}{\underline{1-r}}$$

$$1-r^6 = 9(1-r^3)$$

$$\underline{1+r^3 = 9}$$

$$r^3 = 8$$

$\boxed{r = 2}$

$6\,a\,2 = 360$

45

~~90~~

~~360~~

~~16~~

8

$a =$

~~64~~

Consider graph of $y = e^x$
Determine which of the following is the greatest.

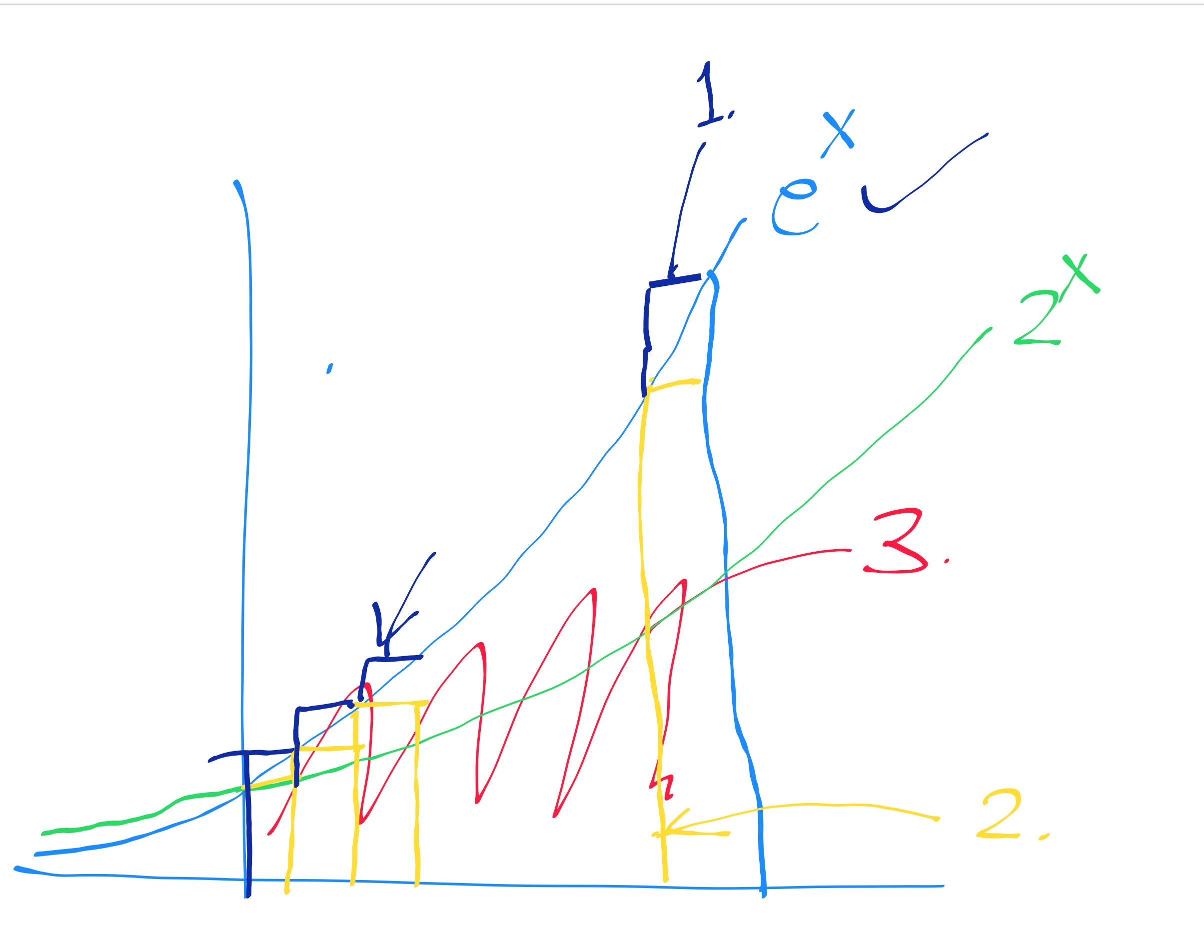
1. $\sum_{n=1}^{100} e^n$

2. $\sum_{n=1}^{100} e^{n-1}$

100

3. $\int_{0} e^{x}\,dx$

4. $\int_{0}^{160} 2^{x}\,dx$



0 94 100

The monic polynomial

$p(x) = x^3 + ax^2 + bx + c$ has roots α, β, γ; where

$$\alpha^2 + \beta^2 + \gamma^2 = 85 \ \&$$

$$p'(\alpha) + p'(\beta) + p'(\gamma) = 87.$$

Find $\alpha\beta + \beta\gamma + \gamma\alpha$.

Note that,

$$p(x) = (x-\alpha)(x-\beta)(x-\gamma).$$

$$p'(x) = (x-\beta)(x-\gamma) + (x-\alpha)\big((x-\gamma)+(x-\beta)\big)$$

$$= (x-\beta)(x-\gamma) + (x-\alpha)(x-\gamma) + (x-\alpha)(x-\beta)$$

$$p'(\alpha) + p'(\beta) + p'(\gamma)$$

$$= (\alpha-\beta)(\alpha-\gamma) + (\beta-\alpha)(\beta-\gamma) + (\gamma-\alpha)(\gamma-\beta)$$

$$= \alpha^2 - \cancel{\alpha\gamma} - \cancel{\beta\alpha} + \cancel{\beta\gamma} + \beta^2 - \cancel{\beta\gamma} - \alpha\beta + \cancel{\alpha\gamma}$$

$$+ \gamma^2 - \gamma\beta - \alpha\gamma + \cancel{\alpha\beta}.$$

$$\Rightarrow 87 = 85 - \alpha\beta - \gamma\beta - \alpha\gamma$$

$$\Rightarrow \alpha\beta + \beta\gamma + \alpha\gamma = -2$$

Find the coefficient of x in

$$\underline{(1+x)^0} + (1+x)^1 + \overset{80}{(1+x)}^2 + \dots + (1+x)$$

$$(x+1)^n = \sum_{r=0} \binom{n}{r} x^r$$

$$\binom{n}{1} = n$$

$$1 + 2 + 3 + \dots + 80$$

$$= \frac{\overset{40}{\cancel{80}} \cdot 81}{\cancel{2}}$$

$= 3240$

Find $A \in \mathbb{N}_{>0}$ such that exactly two of the following statements are true.

1. $A+82$ is square
2. The last digit of A is 5
3. $A-7$ is square

$$A+82 = a^2 \quad \& \quad A-7 = b^2$$

$$\Rightarrow A = a^2 - 82 = b^2 + 7$$

(2) $\Rightarrow (a-b)(a+b) = 89.$

$$\Rightarrow a+b = 89$$
$$a-b = 1$$

$$\Rightarrow 2a = 90$$
$$\Rightarrow a = 45 \rightarrow (1)$$

$$b = 44.$$

$$A = 2025 - 82$$

$$= 1943$$

Prove a) $A+82$ is square then $A \in \mathbb{N}_{>0}$ A does not end with 5.

b) $A - 7$ is square number

then A doesn't end with 5.

$$A + 82 = a^2$$

Suppose $A = 10k + 5$.

$$a^2 = A + 82 = 10k + 5.$$

$$\Rightarrow a^2 \equiv 5 \ (10)$$

0, 1, 4, 9,

Determine $a, b, c \in \mathbb{N}_{>0}$ where $p, p+2$ are primes

& $2^a p^b = (p+2)^c - 1$.

Note $p = 3$, $(a, b, c) = (3, 1, 2)$

is a solution.

$p \equiv 2\,(3)$ can't be a solution

$p \equiv 1\,(3)$ sol $\Rightarrow$ a is odd.

Maths - Oxford - UK

There is a straight line that is normal to the curve $y = x^3 - kx$ at two different points if and only if

(a) $k \geq \sqrt{3}$,

(b) $k^2 \geq 3$,

(c) $k^2 \geq 1$,

(d) $k \geq 1$,

(e) $k \geq \sqrt{3}$ or $k \leq -1$.

A normal line has a slope $= \dfrac{-1}{3x^2-k}$ at $(x, y(x))$.

So at $x=a$ a normal line has slope $\dfrac{-1}{3a^2-k}$.

If a, b are two points with same normal line then

$$3a^2-k = 3b^2-k.$$

$$\Rightarrow a^2 = b^2$$

$$a \neq b \Rightarrow a = -b.$$

Now, normal line pass

through a & b. Hence

Normal line = line through a, b.

Hence both slopes are same.

$$\Rightarrow \frac{-1}{3a^2-k} = \frac{(a^3-ka)-(-a^3+ka)}{a-(-a)}$$

$$\Rightarrow \frac{-1}{3a^2-k} = \frac{2a^3-2ka}{2a}$$

$$\Rightarrow \frac{-1}{3a^2-k} = a^2-k$$

$$\Rightarrow 3a^4 - 3a^2k - ka^2 + k^2 + 1 = 0$$

We take $a^2 = u$

$\Rightarrow 3u^2 - 4ku + (k^2+1)$.

$\hookrightarrow$ A

We know a exist & hence

A has soln.

$\Leftrightarrow \Delta = 16k^2 - 12(k^2+1)$
≥ 0

$\Leftrightarrow 4k^2 - 3k^2 - 3 \geq 0$

$\Leftrightarrow k^2 \geq 3$

$\Leftrightarrow k \geq \sqrt{3}$

Maths - Oxford - UK

The point A has coordinates $(3,4)$. The origin $(0,0)$ and the point A both lie on the circumference of a circle $\mathcal{C}$. The diameter of $\mathcal{C}$ through A also meets $\mathcal{C}$ at another point B. The distance between B and the origin is 10. It follows that the coordinates of B could be either

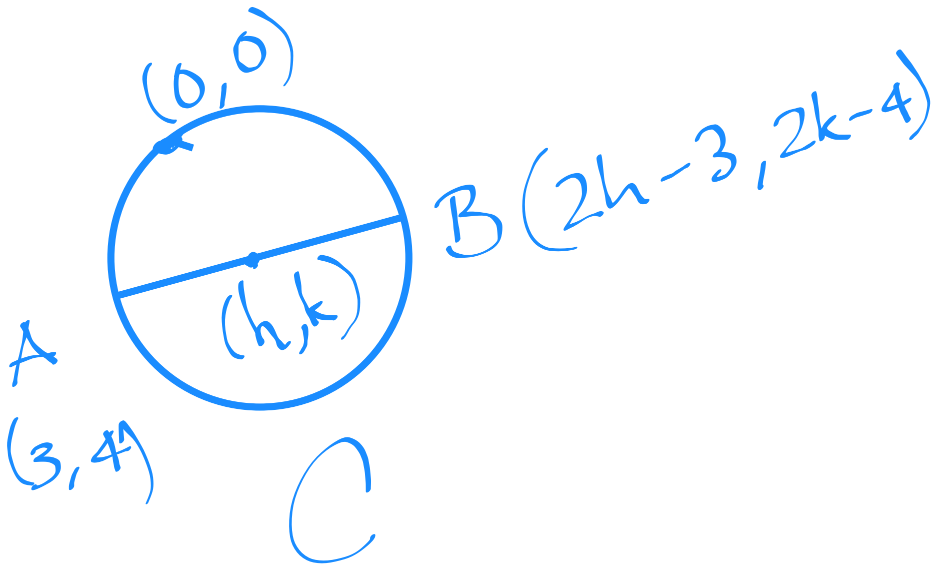
(a) $\left(-5\sqrt{2}, 5\sqrt{2}\right)$ or $\left(5\sqrt{2}, -5\sqrt{2}\right)$,

(b) $(-4, 3)$ or $(4, -3)$,

(c) $\left(-5, 5\sqrt{3}\right)$ or $\left(5, -5\sqrt{3}\right)$,

(d) $(-8, 6)$ or $(8, -6)$,

(e) $\left(-5\sqrt{3}, 5\right)$ or $\left(5\sqrt{3}, -5\right)$.



(0,0)
B (2h-3, 2k-4)
(h,k)
A
(3,4)
C

Note $h^2+k^2=r^2 \rightarrow (1)$

$(3-h)^2+(4-k) = r \rightarrow (2)$

$(1) - (2) \Rightarrow 9-6h+16-2k=0$

$\hookrightarrow (3)$

Note, $B=(2h-3,\ 2k-4)$

$\Rightarrow (2h-3)^2+(2k-4)^2=10^2$

//

$\hookrightarrow$ 4

[Dist(Origin, B) = 10]

3, 4 $\Rightarrow (h, k) = \left(+\frac{5}{2}, 5\right)$

or

$\left(\frac{11}{2}, -1\right)$

$\Rightarrow B = (-5-3, 6)$

$= (-8, 6)$

ee6JvsPEU4r9pQc

or

$$B = (11 - 3, -2 - 4)$$
$$= (8, -6)$$

Elton Papanikolla • 2nd
3h •

Show that:

$$\int_0^{\infty} x^n e^{-kx}\,dx = \frac{n!}{k^{n+1}} \quad \textbf{for } k > 0$$

Put $u = kx$,

$$x = u/k$$

$$\Rightarrow dx = du/k.$$

$$I_n(k) = \int_0^\infty \left(\frac{u}{k}\right)^n e^{-u} \frac{du}{k}$$

$$= \frac{1}{k^{n+1}} \underbrace{\int_0^\infty u^n e^{-u}\, du}_{J_n}.$$

$$\overset{IBP}{=} \frac{1}{k^{n+1}} \left(u^n \left(-e^{-u}\right)\Big|_0^\infty - n\int_0^\infty \left(-e^{-u}\right) u^{n-1}\, du \right)$$

$$= \frac{1}{k^{n+1}} \left(-u^n e^{-u}\Big|_0^\infty + n\int_0^\infty e^{-u} u^{n-1}\, du \right)$$

$$\Rightarrow J_n = n J_{n-1}.$$

∞ $-u$ du

$$\Rightarrow J_n = n!\, J_1 = n! \int_0^{\infty} e^{-u}\, du$$

$$= n!$$