Quarter Square Multiplication

1. $xy = -\frac{(x-y)^2}{4} + \frac{(x+y)^2}{4} \longrightarrow (*)$

$2 \cdot 3$
$x \cdot t = 6$

n	0	1	2	3	4	5
$\lfloor n^2/4 \rfloor$	0	0	1	1	4	6

Gives all xy when $x-y,\ x+y \in [0, 5]$

Eg. $2 \cdot 2 = -0 + 4$

$x, y \in \mathbb{Z} \Rightarrow (x+y)^2 \equiv (x-y)^2 \ (4) \leftarrow$ Video $\longrightarrow$ (1)

$\boxed{\begin{array}{l} a \equiv b\ (n) \\ n \mid a-b \end{array}}$

$(x+y)^2 = 4q + r$
$(x-y)^2 = 4q_1 + r_1$ } where $r, r_1 \in [0, 4)$

(1) $\Rightarrow r = r_1$

$\Rightarrow \frac{(x+y)^2}{4} = q + \frac{r}{4}$ & $\frac{(x-y)^2}{4} = q_1 + \frac{r}{4} \longrightarrow$ (2)

$\Rightarrow \lfloor (x+y)^2/4 \rfloor = q$ & $\lfloor (x-y)^2/4 \rfloor = q_1$

$\Rightarrow \lfloor (x+y)^2/4 \rfloor - \lfloor (x-y)^2/4 \rfloor = \underline{q - q_1} \longrightarrow$ (3)

(3), (*), (2) $\Rightarrow$ LHS of (3) $= xy$.