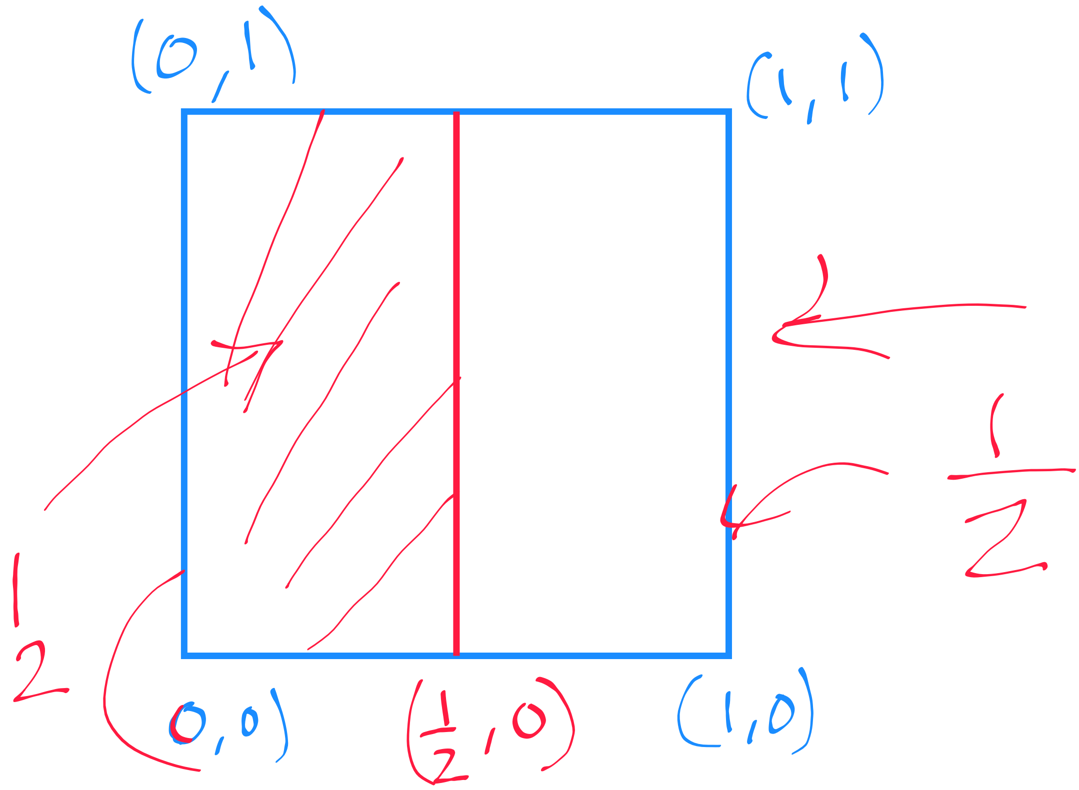
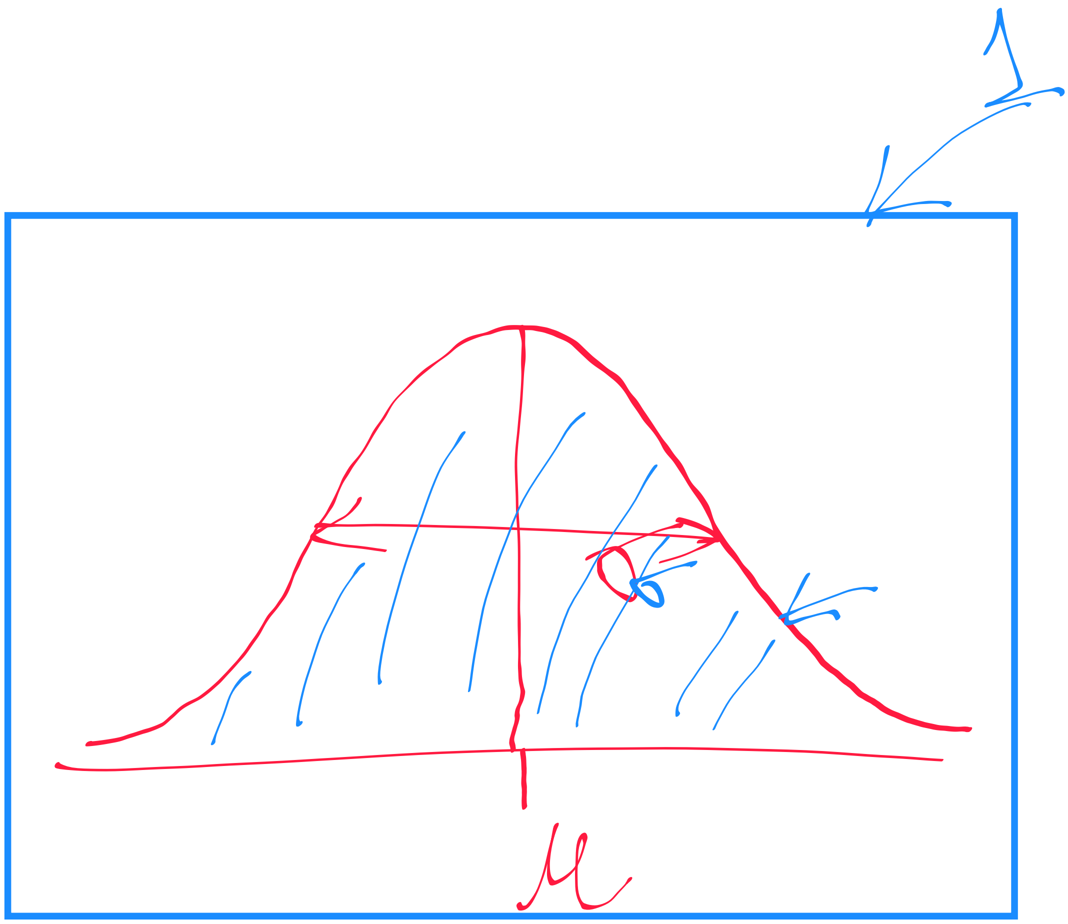
Monte Carlo

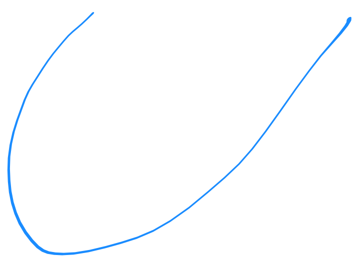


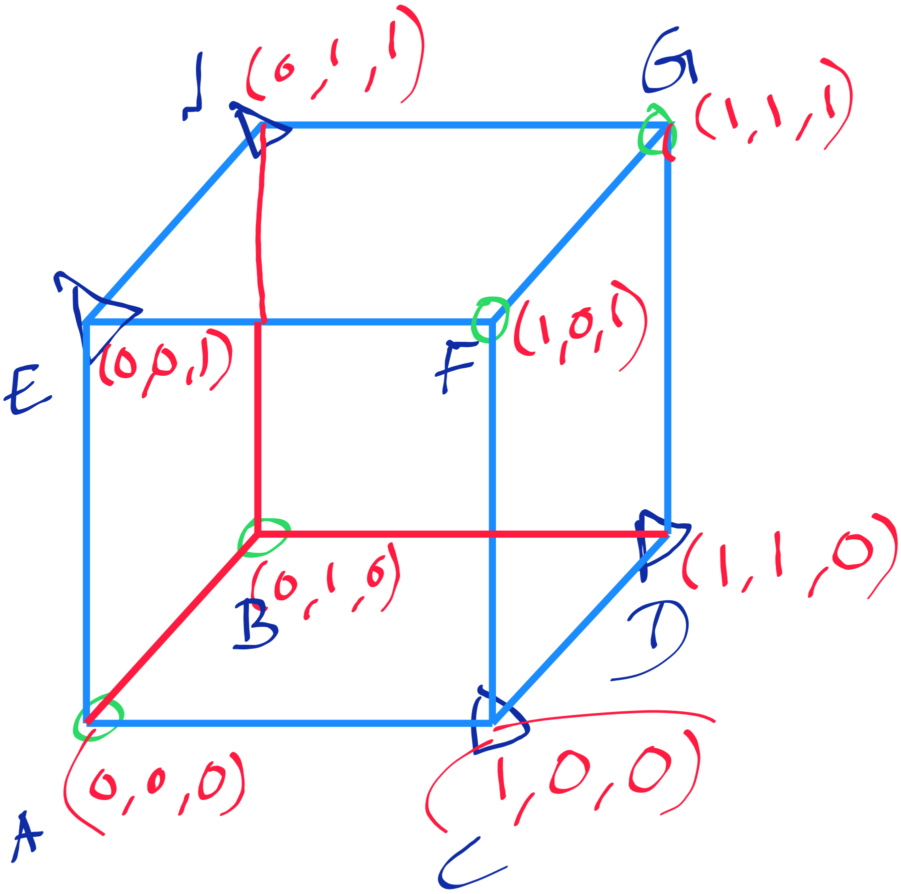
$A = (x, y)$

Monte Carlo Simulation



(A,B)





$$12 / \binom{8}{4} = 0{,}17$$

Cambridge

AP = $a_0, a_2, \ldots\ldots =$

$(a+id)_{i=0}$

$$GP = g_1, g_2, \ldots \ldots = (gr^i)_0^\infty$$

$$g = g_1 = a_1 = a$$

$$gr = g_2 = a_4 = a + 3d \rightarrow ①$$

$$gr^2 = g_3 = a_6 = a + 5d \rightarrow ②$$

$$\frac{g}{1-r} = \sum_0^\infty gr^i = 12.$$

$$\Rightarrow a = 12 - 12r \rightarrow ③$$

$$① - ② \Rightarrow a(r - r^2) = -2d.$$

$$③ \Rightarrow (12 - 12r)(r - r^2) = -2d.$$

$$\Rightarrow d = (6 - 6r)(r^2 - r)$$

$\hookrightarrow$ 4

3 $\quad$ 1, 4 $\Rightarrow (12-12r)r =$

$(12-12r)$

$+ 3(6-6r)(r^2-r)$

$\Rightarrow 2r = 2 + 3r^2 - 3r.$

$\Rightarrow 3r^2 - 5r + 2 = 0.$

$\Rightarrow r = \dfrac{5 \pm \sqrt{25-24}}{6}$

$= \dfrac{5 \pm 1}{6}$

$= 1, \dfrac{2}{3}$

$r = 1$ ✗

$r = 2/3$

$\Rightarrow a = 4$

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The curve S has equation

$$y = px^2 + 6x - q$$

where p and q are constants.

S has a line of symmetry at $x = -\frac{1}{4}$ and touches the x-axis at exactly one point.

What is the value of $p + 8q$?

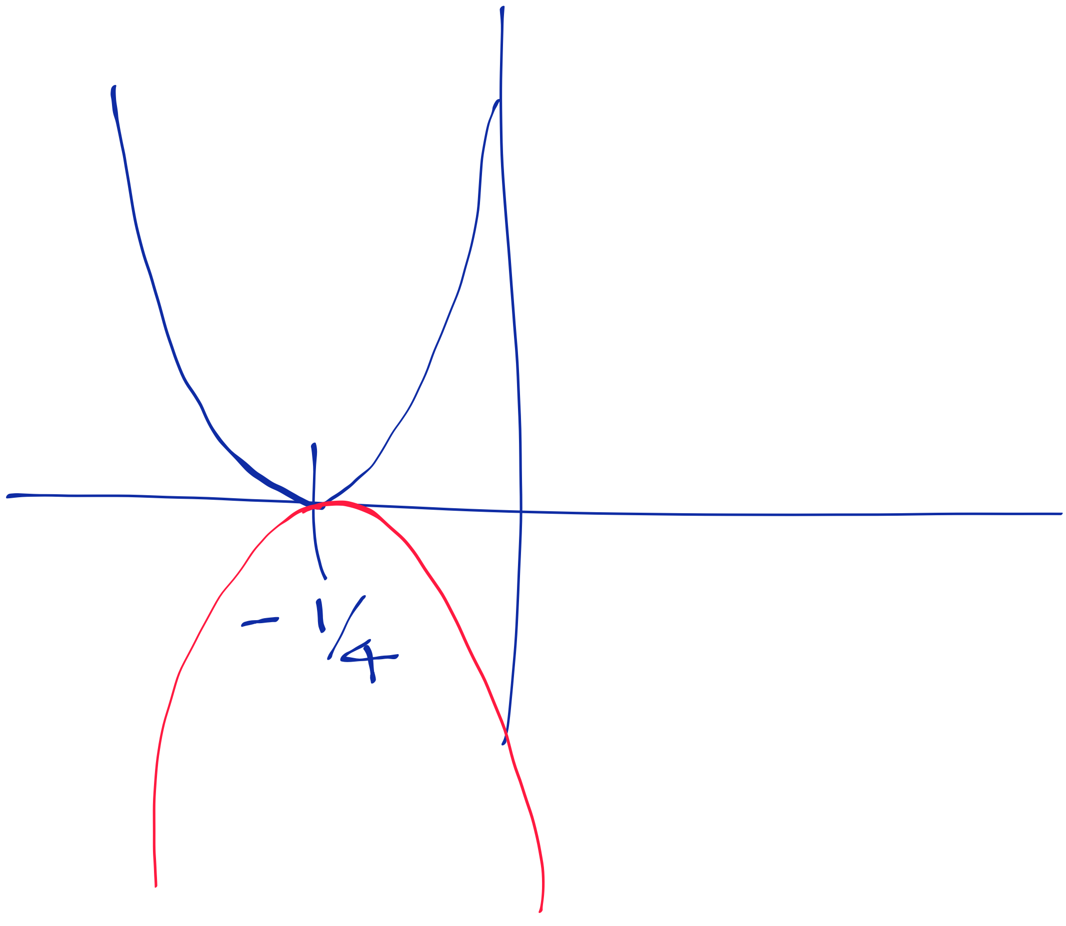
A 6

B 18

C 21

D 25

E 38



−1/4

$$y = px^2 + 6x - q$$

$$y' = 2px + 6.$$

$$0 = y'(-1/4) = \frac{-p}{2} + 6.$$

$$\Rightarrow p = 12$$

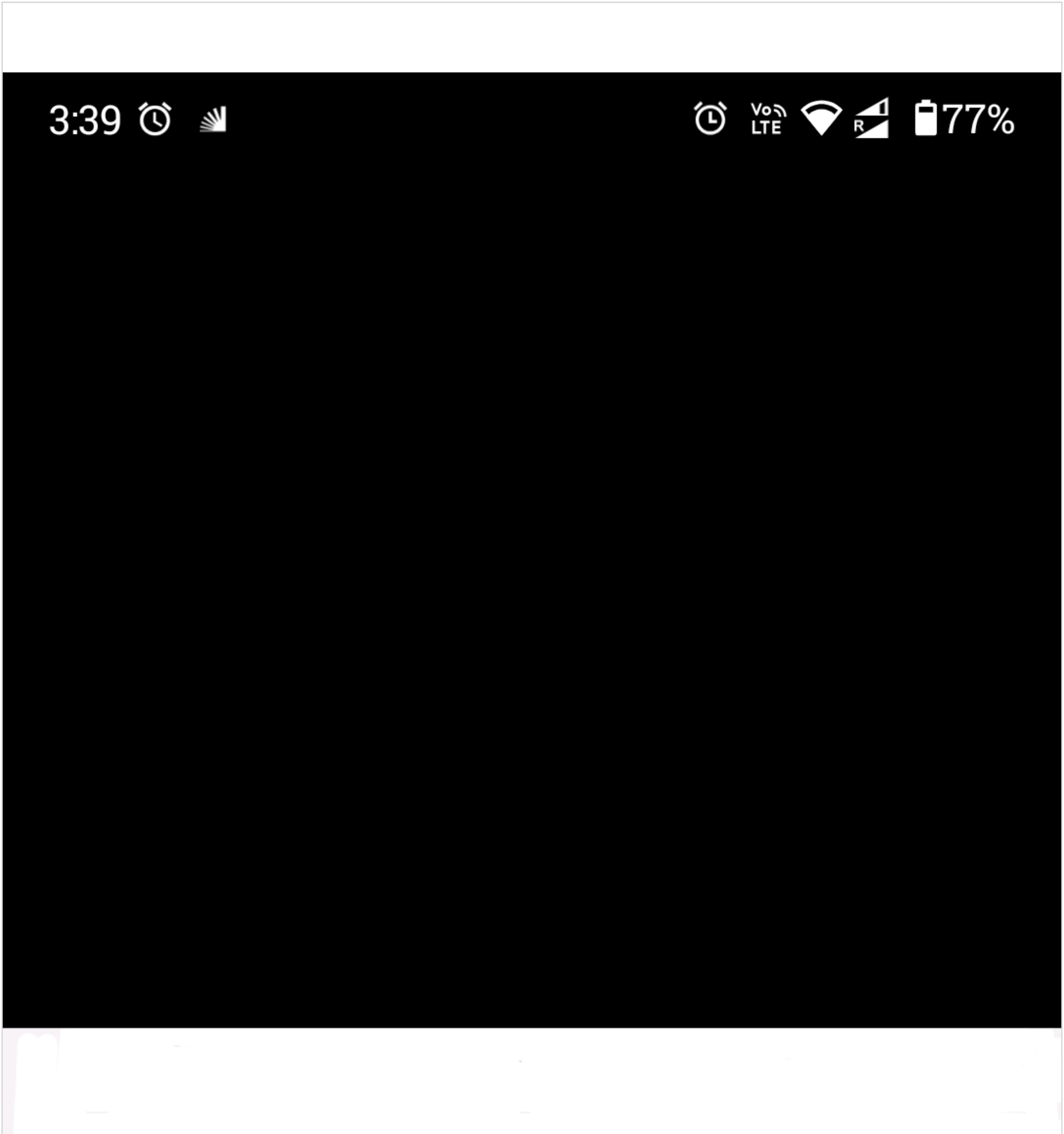
$$y(-1/4) = 0$$

$$\Rightarrow \frac{p}{16} + \frac{6}{(-4)} - q = 0$$

$$\Rightarrow \frac{3}{4} - \frac{6}{\ } = q\,4$$

$$\Rightarrow q = -3/4$$

$$\Rightarrow p + 8q = 12 - 6$$
$$= 6$$



Find the particular solution to the differential equation $(x-2)\frac{dy}{dx} = xy$ that passes through the point $(0, 1)$.

$$\frac{dy}{y} = \frac{x}{x-2}\,dx$$

$$\Rightarrow \ln|y| = x + \int \frac{2}{x-2}\,dx$$

$$\Rightarrow \ln|y| = x + 2\ln|x-2| + C$$

$$2 \Rightarrow y = C_1 e^x \cdot (x-2)$$

$$\Rightarrow y(0) = 1 \Rightarrow 1 = C_1 \cdot 4$$

$$\Rightarrow C_1 = 1/4$$

$$2$$

$$\Rightarrow y = \frac{1}{\;} e^x (x-2) \; 4$$

Determine $a, b \in \mathbb{N}_{\geq 0}$ s.t.

$$\frac{a+b}{2} - \sqrt{ab} = 1.$$

$$\Rightarrow \left(\sqrt{a} - \sqrt{b}\right)^2 = 2$$

$$\Rightarrow \sqrt{a} - \sqrt{b} = \pm\sqrt{2} \longrightarrow$$

1

$x = \sqrt{a}$, $y = \sqrt{b}$

$$\Rightarrow a = b \pm 2\sqrt{2}\,y + 2.$$

$$y^2 \pm 2\sqrt{2}\,y$$

$$\Rightarrow 2\sqrt{2}\sqrt{b} \in \mathbb{Z}.$$

1

$$\Rightarrow 2\sqrt{2}\sqrt{a} \in \mathbb{Z} \longleftarrow$$

$$2\sqrt{2}\sqrt{b} = A$$

$$\Rightarrow 8b = A^2$$

$$\Rightarrow b = A^2/8.$$

$$a = B^2/8.$$

$$\Rightarrow A = 4k, \quad B = 4l.$$

$$\Rightarrow b = 2k^2, \quad a = 2l^2$$

$$\sqrt{2}k - \sqrt{2}l = \pm\sqrt{2}$$

$$\Rightarrow k - l = \pm 1.$$

$$\Rightarrow k = l \pm 1.$$

$$\Rightarrow (a, b) = \underline{(2l^2, 2(l \pm 1)^2)}$$

$= (2\ell^2, 2\ell^2 \pm 4\ell + 2)$.

$2\ell^2 \pm 2\ell + 1 - 2(\ell \pm 1)\ell$

$2 = 2\ell^2 \pm 2\ell + 1 - 2\ell^2 \mp$

2ℓ.

$= 1$

$(0, 2)$ $(2, 0)$ $(2, 8)$

$f(x) + f(x-1) = x^2 \leftarrow$

$f(6) = 9$, $f(9) = ?$

$f(9) = 81 - f(8) =$

$\cancel{81} - 24$

$\cancel{= 57}$

$f(8) = 64 - f(7) = \cancel{64 - 40}$

$$f(7) = 49 - 9 = 40$$

There is no perfect square in

$S = 11, 111, 1111, 11111, \ldots$

$$(2k+1)^2 \equiv 1 \ (4)$$

$$a \in S \Rightarrow a \equiv 3 \ (4)$$

What about,

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Find the real non-zero solution to the equation

$$\frac{2^{(9^x)}}{8^{(3^x)}} = \frac{1}{4}$$

A $\log_3 2$

B $2\log_3 2$

C 1

D 2

E $\log_2 3$

F $2\log_2 3$

$$\frac{3^{2^x}}{9^{2^x}} = \frac{1}{9}$$

x

$$9^x - 3 \cdot 3 = -2$$

$$3^x = y$$

$$y^2 - 3y + 2 = 0$$

$$(y-2)(y-1) = 0$$

$$\Rightarrow 3^x = 2 \quad \text{or} \quad 3^x = 1$$

$$\Rightarrow x = \log_3 2 \quad \text{or} \quad x = 0$$

GMO - Germany

Determine all primes p for which the system

$$\begin{aligned} p+1 &= 2x^2 \\ p^2+1 &= 2y^2 \end{aligned}$$

has a solution in integers x, y.

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Show translation

What is the coefficient of the x^2 term in the expansion of the following?

$$\frac{x^2 - 1}{\sqrt{x+1}}$$

Hint:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

where n is rational and $|x| < 1$.

A. $\frac{3}{8}$

B. $\frac{5}{8}$

C. $-\frac{3}{8}$

D. $-\frac{5}{8}$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \cdots$$

$$(x^2-1)(1+x)^{-1/2} = x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^5 + \cdots$$

$$- \left(1 - \frac{1}{2}x + \frac{3}{8}x^2 + \cdots\right)$$

$$\Rightarrow \text{Coeff of } x^2 = 1 - \frac{3}{8}$$

$$= 5/8$$