Assignment 1

1. Prove that any boolean formula F consisting of $\wedge, \vee, \neg$ operators can be converted into an equivalent formula in Conjunctive Normal Form as well as Disjunctive Normal form. (Hint:Use DeMorgan's Laws)

2. Let $n \geq 2$ and $a_1, a_2, \ldots a_n$ be positive real numbers satisfying $a_1 a_2 \ldots a_n = 1$. Use mathematical induction to prove that $a_1 + a_2 + \ldots + a_n \geq n$.

3. For a natural number n let $Euler(n) = \{a \in \mathbb{Z} | 1 \leq a \leq n, gcd(a,n) = 1\}$ and let $\phi(n) = |Euler(n)|$. E,g, if n=10, then Euler(n)=1,3,7,9 and $\phi(n) = 4$. Obtain Formula for $phi(n)$.

4. Prove that boolean NAND operation is universal (i.e. any boolean formula involving $\wedge, lor, \neg$ can be converted to equivalent formula involving only NAND operations.

Assignment 2

1. Suppose S is any set of $n+1$ natural numbers among $\{1, 2, 3, \ldots, 2n\}$.
   (a) Show that S must have two distinct numbers a, b such that $a|b$
   (b) Show that S must have two distinct numbers a, b such that $gcd(a,b) = 1$

2. State and Prove divisibility tests for primes 13, 41 (By divisibility test we mean an algorithm which is significantly simpler than carrying out actual division, e.g. divisibility test of 9 is: number is divisible by 9 iff sum of digits of n is divisible by 9, note that we plan to apply divisibility test for large numbers). (Hint: $13|1001, 41|99999$).

3. Consider a circular track on which there are n petrol pumps $P_1, \ldots, P_n$ located at arbitrary positions. Assume that pump P_i has L_i litters of petrol for $i = 1$ to n. The total quantity of fuel at all the pumps together is just enough to complete a traversal round the track with a car with unit fuel efficiency. Prove that there always exist a pump where one can start with a (unit fuel efficiency) car with empty fuel tank and complete the traversal round the track (in clockwise or anticlockwise direction). Note that car collects all the fuel available at any pump which it encounters during the traversal (assume cars fuel tank is large enough to hold all the fuel available at all the pumps).

Assignment 3

1. State and prove Inclusion Exclusion Principle (for n sets). Explain with an example.

Assignment 4

2. Prove that product of any consecutive k integers is divisible by $k!$.

3. Explain what is meaning of modular inversion. Prove that modular inverse of number a modulo n exists iff gcd(a,n)=1.

4. Suppose a class contain 84 students. We want to partition students into groups such that each group contains exactly 4 students. How many distinct ways are there to do this?

Assignment 5

1. Let $S = \{1, 2, \ldots, n\}$. Let $T = \{(A, B) | A \subset B \subseteq S\}$ where $A \subset B$ means A is strict subset of B. Find out $|T|$ as a function of n.

2. How many times digit 7 is used if you write down all the natural numbers between 1 and 10000?

3. During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games total. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games (Hint: Use Pigeon Hole principle).

4. Write a recursive program in C language to enlist all subsets of size r of $\{1, 2, \ldots, n\}$. n, r will be given as input and expected output is list of all subsets of size r. E.g. if $n = 3, r = 2$, possible output can be $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$. Note that the ordering in which you wish to enumerate the subsets is your choice. Also, algorithm shouldn't use following approach: go through all subsets and display only subsets of size r. Implement a most efficient algorithm you can think of. You are expected to add program and sample output as a solution for this problem.