Sensitivity of mountain range options prices

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Abstract: Mountain range options are a particular class of multi-asset options for which no closed form formula for valuation exists and Monte Carlo simulation should be used. In this paper, we conducted an analysis of the sensitivity of mountain range options prices to changes in various risk factors, such as correlation coefficients, underlying prices, volatilities, risk-free rate, and time to expiration. We found numbers of non-typical and nonlinear dependencies in options valuation.

Keywords: mountain range option; sensitivity; structured products; Altiplano; Atlas; Everest; Himalaya; Kilimanjaro.

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1 Introduction

Mountain range options were introduced in 1998 by Société Générale. The options were a new class of exotic multiple-asset options and differed significantly from options known from popular handbooks, e.g., (Haug, 2006). They are considered as a combination of basket and range options. Through the basket option property they can capture a multiple underlying asset property, and through the range option property they can have a different time structure. There are five following types of mountain range options: Altiplano, Atlas, Everest, Himalaya and Kilimanjaro. The details of their construction are in the next sections. All of these options are traded over the counter by banks and other financial institutions. They are rather a component of structured products than independent securities. They also gained in popularity during the global crisis after the collapse of Lehman Brothers in September of 2008, when investors withdrew capital from risk markets to relatively safe instruments such as bonds or structured products. Competition on this market forced financial engineers to find more and more sophisticated products, such as structures with embedded mountain range options.

The literature on mountain range options is scarce. There are only a few papers dealing with this topic, and among the most popular are Quessette (2002), Overhaus (2002) and Bouzoubaa and Osseiran (2010).

The prices of mountain range options are determined by a number of variables, similarly as in the typical multi-asset options case. The aim of this paper is to conduct an analysis of the sensitivity of mountain range options prices to changes in various risk factors, such as correlation coefficients, underlying prices, volatilities, risk-free rate and time to expiration. Mountain range options cannot be priced using closed-form formulae and are instead valued using Monte Carlo simulation methods.

The rest of the paper is organised as follows. First, we provide an overview of the Monte Carlo algorithm for multi-asset option valuation. Then, in Section 3, we describe the data used in the study. Sections 4 to 8 present particular options and their price sensitivity. We discuss the following types of options: Altiplano, Atlas, Everest, Himalaya, and Kilimanjaro. Finally, Section 9 presents the conclusions.

2 Monte Carlo algorithm for multi-asset option valuation

In multi-asset option pricing it is necessary to simulate paths of multi-dimensional geometric Brownian motion. It can be specified through a system of SDEs of the form (see Glasserman, 2003):

$$\frac{dS_i(t)}{S_i(t)} = rdt + \sigma_i dX_i(t), \quad i = 1, \ldots, n, \tag{1}$$

where

$S_i(t)$	i^{th} of n underlyings at time $t \in [t_0, T]$, where t_0 is the initial date and T is the maturity date
r	risk-free interest rate
σ_i	volatility of the i^{th} underlying asset
$X_i(t)$	is a standard one-dimensional Brownian motion and $X_i(t)$ and $X_j(t)$ have correlation ρ_{ij}.

Thus, the prices of underlying at any time $t_k < T$ have the following representation:

$$S_i(t_k) = S_i(t_{k-1})e^{\left(r-\frac{1}{2}\sigma_i^2\right)(t_k-t_{k-1})-\sqrt{t_k-t_{k-1}}X_i(t_k)}. \qquad (2)$$

To generate correlated random variables it is convenient to use Cholesky factorisation. It allows to construct vector-correlated variables **X** as the transformation **X** = **T** · **Y**, where **Y** is a vector of independent random variables and **T** is a triangular matrix with zeros in the upper right corners (above the diagonal) fulfilling the relationship **R** = **T** · **T′** for the correlation matrix **R**.

After having generated prices of underlying assets it is easy to calculate the payoff of any option. The discounted mean of payoffs is an estimator of the option price (Hull, 2006).

3 Test data

Analysis of each type of mountain range option is divided into two sections. The first section introduces the exact definition of the option and focuses on its payoff structure. The second section analyses the sensitivity of the theoretical option price obtained by various input parameters as scenarios of market changes. Due to the basket character of mountain range options, the simulations are carried out under the *ceteris paribus* assumption. The inclusion of multiple assets to the basket requires separate consideration of the effects of individual risk factors to changes in option price. The basic set of parameters is presented below. The correlation matrix is as follows

$$\mathbf{R} = \begin{bmatrix} 1 & & & & \\ 0.8 & 1 & & & \\ 0.5 & 0.3 & 1 & & \\ 0 & 0.1 & -0.1 & 1 & \\ 0.8 & -0.3 & -0.5 & 0.1 & 1 \end{bmatrix}.$$

In our analysis we focus on the dependence of asset 1 on the others. The matrix shows that asset 1 is highly positively correlated with asset 2 and highly negatively correlated with asset 5. The correlation coefficient between assets 1 and 3 is rather weak, and there is no correlation between assets 1 and 4. The volatilities of each asset are equal to 30%. A risk-free interest rate corresponds to the cost of carry and is equal to 5%. We use the same interest rate for all assets, although in practice interest rates can differ when assets from different markets are considered. All products have a 2.5-year time period to expire and half-year moments of sampling. Initial values of underlying assets are 100. This is a typical value for options embedded in structured products, where only changes of the underlying are important to determine the payoff value.

4 Altiplano option

The Altiplano option is a multi-asset derivative with payoff based on the returns of n assets composing the underlying basket. It entitles the holder to receive a large fixed coupon C at maturity T, provided that none of the assets in the basket have fallen below a predetermined barrier H during a given time period. This observation period is usually started at the inception date and ends at the maturity date, but it can also be a specific time sub-period of the option's lifetime. If one of the components in the set of the chosen underlying crosses the downside barrier, then the payoff is computed differently, usually by participation in a call-type payoff. Here we consider the most common case, in which the Altiplano payoff is given (Bouzoubaa and Osseiran, 2010):

$$\text{payoff}(T) = \eta \cdot \varphi \cdot N \cdot \max\left(0, \sum_{i=1}^{n} \frac{S_i(T)}{S_i(0)} - X\right) + (1-\eta) \cdot C, \tag{3}$$

where

N face value

n number of assets

$S_i(t)$ closing price of asset i at time t

X strike price

C coupon

φ participation coefficient

η binary variable equal to the condition set for the index:

$$\eta = \begin{cases} 1, & \text{if } \min\limits_{\substack{1 \le i \le n \\ t_1 \le t \le t_n}} \left(\dfrac{S_i(t)}{S_i(0)}\right) \le H, \\ 0, & \text{otherwise,} \end{cases}$$

H barrier

t_1, t_2 the beginning and end of the observation period.

The Annapurna option has a similar construction. It gives the option holder a payoff provided that none of the stocks within the basket reaches a predetermined barrier during a given time period.

Table 1 The terms of an Altiplano option

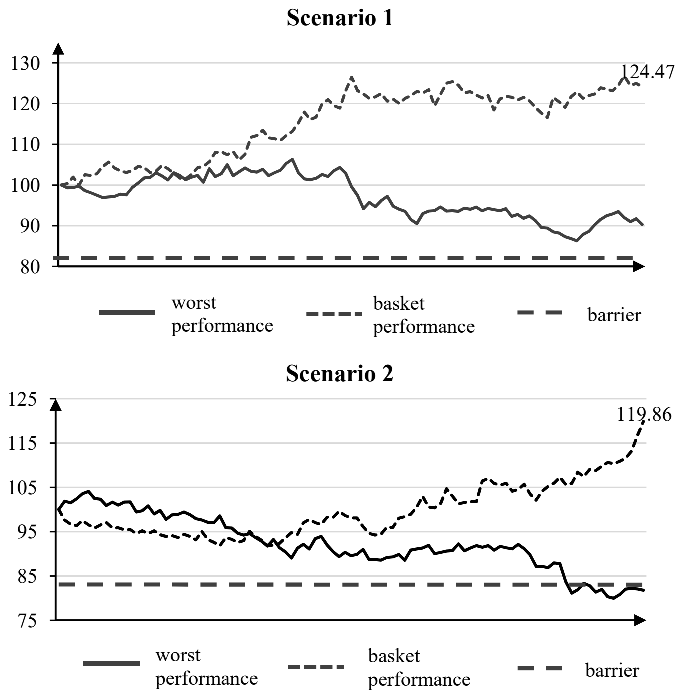
Underlying basket	5 blue chip stocks
Tenor	2.5 year
Barrier (daily observation)	85%
Coupon (C)	100%
Call type	Basket
Strike price (X)	100%
Participation (φ)	110%

The terms of the 2.5-year Altiplano option are shown in Table 1. Figure 1 shows two different asset return scenarios: the dashed lines indicate the basket performance, whereas the solid lines constitute the worst asset performance registered over time. In the first scenario the barrier of 85% is never crossed during the life of the option. Here the option holder receives payment at the maturity equal to 100% of the notional. In the second scenario at least one of the assets composing the basket has been below 85% of its initial level. In this case, the Altiplano option pays 110% participation in a basket call payoff:

$$100\% \cdot (119.86\% - 100\%) = 21.16\%$$

of the face value.

Figure 1 Two scenarios showing the basket performance as well as the worst performance observed during the life of Altiplano option



Source: Own calculation

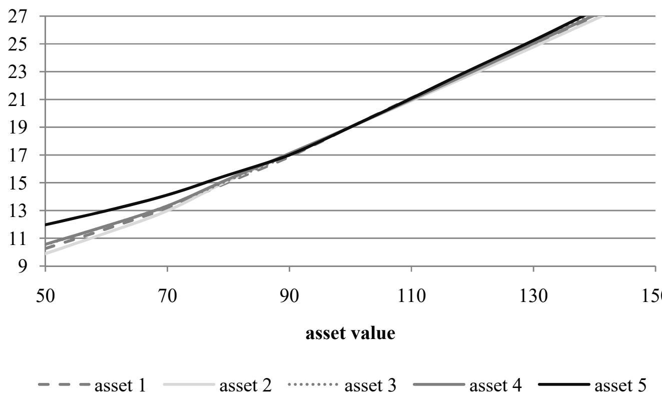
4.1 *Sensitivity of the Altiplano option price*

In Figure 2, we can notice that the option price is an increasing function of the underlying asset price and that the growth rate is similar for each asset. Close to the value of 100 the differences are negligible and the price profiles are almost linear. The slope of these profiles can be interpreted as a partial delta of the option. Here the slope is equal to about 1.8, hence the sensitivity of option to asset price changes is quite high. This could be explained by the barrier feature of the Altiplano option. If one of the underlying crosses the barrier then the payoff is calculated differently. The most convex profile is for asset 5, which is negatively correlated with the others.

The Altiplano option price is an increasing function of the volatility for positively correlated assets (Figure 3). The dependence of option price-volatility is almost linear for assets 1–3. For uncorrelated asset 4 the volatility initially does not affect the option price (to a value of about 0.3), afterwards the interrelation is linear and similar to that for assets

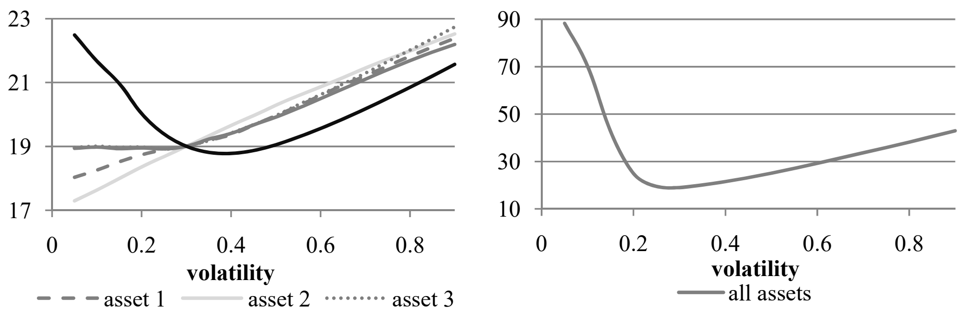
1–3. The negative correlation for asset 5 causes a convexity in valuation. When the overall market becomes unstable, the option price falls very quickly (Figure 3) and then rises slowly.

Figure 2 Sensitivity of Altiplano option to underlying assets



Source: Own calculations

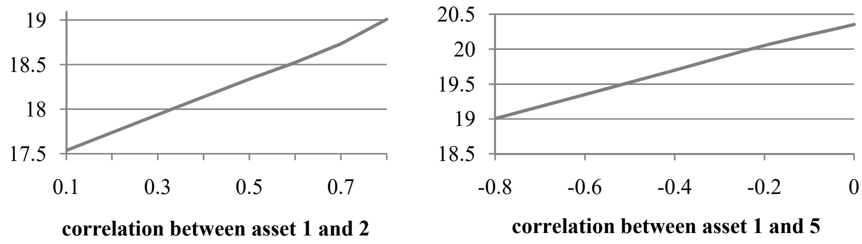
Figure 3 Sensitivity of Altiplano option to volatility



Source: Own calculations

When we analyse the influence of the correlation coefficients on the Altiplano option value, presented in Figure 4, it is easy to notice that a higher correlation between the assets implies a higher option value. This is a normal phenomenon in every basket option. The spread between minimum and maximum option values is not impressive, with quite a large range of correlation changes. The reason for such low sensitivity is that in our simulations only one pair of correlations may change. In practice, due to various events the whole structure of correlations changes rather than only one pair. Thus changes in the option value are much higher than in our case, especially if the volatility risk changes in the same time period. We can also note stronger sensitivity of the option value to changes in the positive correlation coefficient than in the negative case.

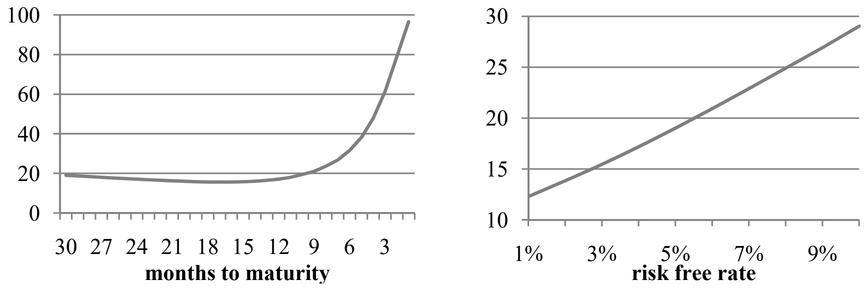
Figure 4 Sensitivity of Altiplano option to correlation between positive (left chart) or negative (right chart) and highly correlated assets



Source: Own calculations

The impact of passing time on the valuation is not typical (Figure 5, left). For the first two years, elapsed time reduces the option premium, but in the last year the price sharply increases. The reason for this behaviour is decreasing uncertainty about to cross the barrier. Let us remember not to exceed the barrier results in the payment of a specific coupon, in our case 100%.

Figure 5 Sensitivity of Altiplano option to time to maturity and to risk free interest rate



Source: Own calculations

Sensitivity of the Altiplano option premium to changes in the risk-free interest rate is linear (Figure 5, right). In our example the slope of the price function is equal to 1.87, therefore an increase of 1 percentage point of the risk-free rate causes an increase in the option premium of 1.87. This could be interpreted as a rho value.

5 Atlas option

The atlas option is a multi-asset derivative with payoff linked to the basket of n assets and computed as follows: at maturity the n_1 worst-performing assets as well as the n_2 best performers are removed from the underlying basket. The payoff at maturity is based on average performance from inception of the remaining assets composing the heart of the basket:

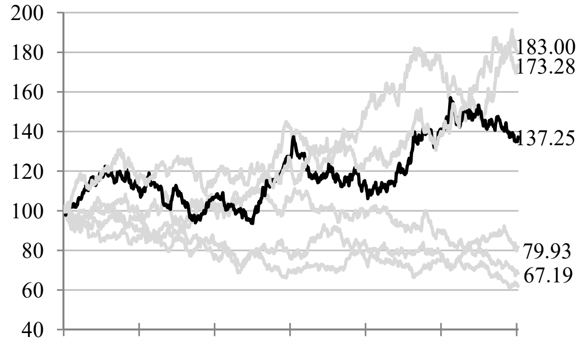
$$\text{payoff} = N \cdot \max\left[0, \left(\frac{1}{n-(n_1+n_2)} \sum_{i=n_1+1}^{n-n_2} \frac{S_i(T)}{S_i(0)}\right) - X\right], \tag{4}$$

where

N	face value
n	number of assets
n_1	number of worst-performing assets
n_2	number of best-performing assets
$S_i(t)$	closing price of asset i at time t
X	strike price.

The terms n_1 and n_2 are constrained by the condition: $n > n_1 + n_2$.

Figure 6 Scenario showing the returns of the underlying assets as well as the asset selection process of the atlas option



Source: Own calculations

In Figure 6, we can see a scenario showing the returns of five assets from the basket and $n_1 = n_2 = 2$. The graph emphasises the asset selection mechanism of the Atlas option. The solid line represents the return of the only one asset that constitutes the final basket. Here the level of the remaining asset is equal to 137.25%, the strike price is 100%, then the payoff is equal to:

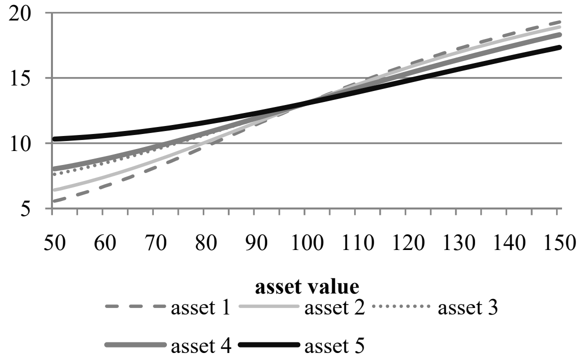
$$137.25\% - 100\% = 37.25\%$$

of the face value.

5.1 Sensitivity of the Atlas option price

The option price is an increasing function of the underlying asset price (Figure 7). Even though the charts have similar shapes, its concavity and convexity are different. The most sensitive is the option price to the value of positively correlated assets. Asset 5, with a negative correlation, has lower sensitivity. The deltas have the highest values at the strike because the fastest rising and falling assets are removed from the basket at maturity and, finally, do not touch the payment of the Atlas option.

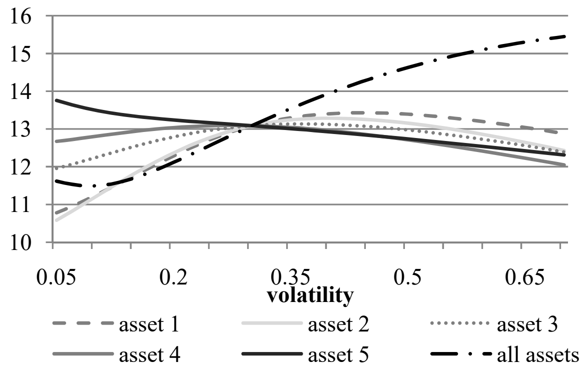
Figure 7 Sensitivity of Atlas option to underlying assets



Source: Own calculations

As we can observe in Figure 8 for positively correlated assets, the plot of the sensitivity option to the volatility has a parabolic shape. Initially, volatility increases and the option price increases as well, and after reaching the maximum the rise in volatility results in a decline in option premium. This type of dependency is the result of the selection of scheme assets – assets with high volatility will be thrown out of the basket. The volatility function for asset 5, with a negative correlation, behaves differently as it is a decreasing function of volatility. When the overall market becomes unstable, the option price rises sharply (see the dashed line in Figure 8).

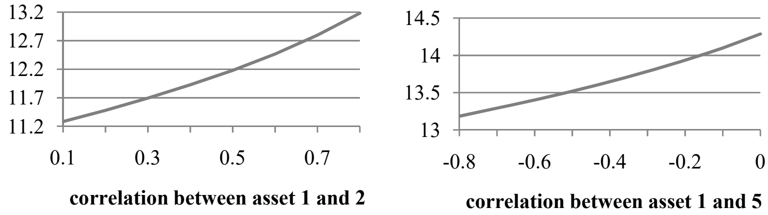
Figure 8 Sensitivity of Atlas option to volatility



Source: Own calculations

The impact of the correlation coefficients for the Atlas option value is presented in Figures 9. The dependence between positively and strongly correlated assets is typical – the plot is increasing and convex (Figure 9, left). However, negatively correlated assets are characterised by an inverse relationship, i.e. decreasing and convex (Figure 9, right).

Figure 9 Sensitivity of atlas option to correlation between positive (left chart) or negative (right chart) and highly correlated assets

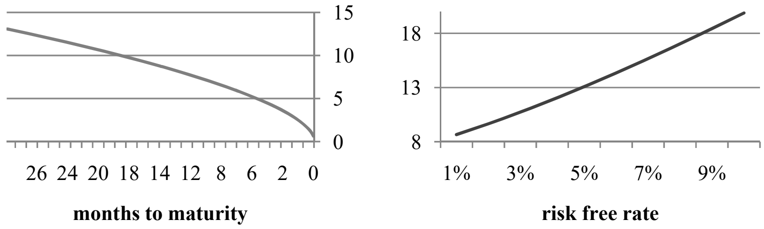


Source: Own calculations

In Figure 10 (left panel), we analysed the impact of passing time on the valuation, and this relationship is standard as it reduces the option premium.

Sensitivity of the Atlas option premium to changes in the risk-free interest rate is almost linear (Figure 10, right). In our case the slope of the price function is equal to 1.25, therefore its increase of 1 percentage point causes an increase in the option premium of 1.25. As was already mentioned, this could be interpreted as a rho value.

Figure 10 Sensitivity of atlas option to time to maturity and to risk free interest rate



Source: Own calculations

6 Everest option

The Everest option is not an option in the strict sense because within certain parameters it may result in negative cash flow as a long position at maturity. Moreover, the Everest option is not path-dependent, and payoff is linked to the worst-performing asset at maturity from the basket and specific coupon:

$$payoff(T) = N \cdot \left[C + \min_{i=1,2,\ldots,n} R_j(T)\right], \tag{5}$$

where

N face value

C coupon

$R_i(T)$ return of asset j at maturity (T):

$$R_i(T) = \frac{S_i(T)}{S_i(0)} - 1,$$

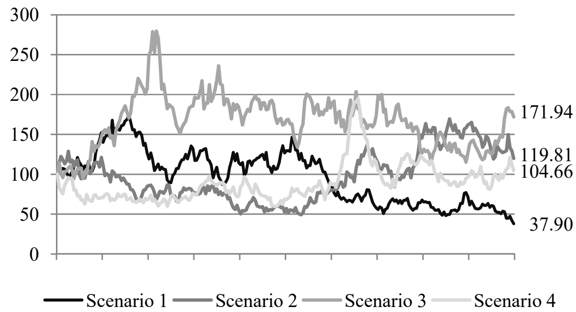
$S_i(t)$ closing price of asset i at time t.

It is interesting to note that if the coupon is smaller than 100% then with the return of the worst asset equal to –100% the holder of the Everest option should pay to the writer.

In Figure 11, we can see four scenarios showing the returns of the worst-performing assets at maturity. Table 2 clarifies the payoff scenarios of the Everest option according to cases presented in Figure 11. For example, the performance of the worst-off in the first scenario at maturity is equal to 37.90% of its initial level. If the coupon is equal to 100%, then the payoff is equal to:

$$100\% + (37.90\% - 100\%) = 37.90\%$$

Figure 11 Scenarios showing the worst performing assets returns in a basket



Source: Own calculations

Table 2 Payoff scenarios of an Everest option

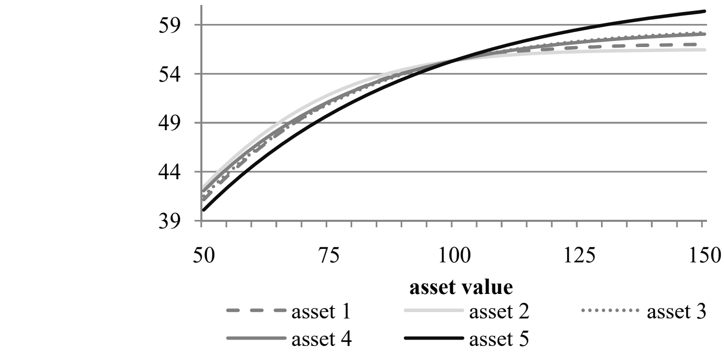
Scenario	*S(T)*	*R(T)*	*payoff*
1	37.90%	–62.1%	37.90%
2	119.81%	19.81%	119.81%
3	171.94%	71.94%	171.94%
4	104.66%	4.66%	104.66%

6.1 Sensitivity of the Everest option price

The option price is an increasing and concave function of the value of assets (Figure 12). Sensitivity to changes in the price of the underlying decreases because, ultimately, it is the worst of the components that determines the amount of the payment. Once again, sensitivity behaves differently for assets 5 (which are negatively correlated).

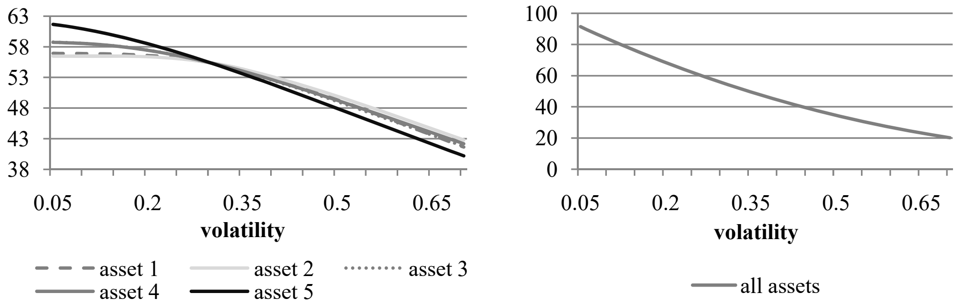
As we can see in Figure 13, the option price is a decreasing function of the volatility. It is not a surprising property if we consider the method of calculation of the payoff. High volatility means high probability of a low price of the worst asset. When the overall market becomes unstable, the option price falls sharply (Figure 13, right). This dependence is close to quadratic.

Figure 12 Sensitivity of Everest option to underlying assets



Source: Own calculations

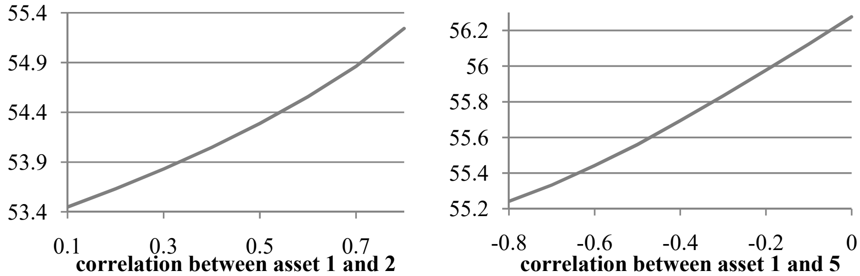
Figure 13 Sensitivity of Everest option to volatility



Source: Own calculations

The impact of the correlation coefficients for the Everest option value is presented in Figure 14. The price of the option is an increasing function of the correlation. The interrelation is weak, and a change in the correlation from an absolute value of 0.8 to 0 causes a change in the option premium by about 2% of its initial value.

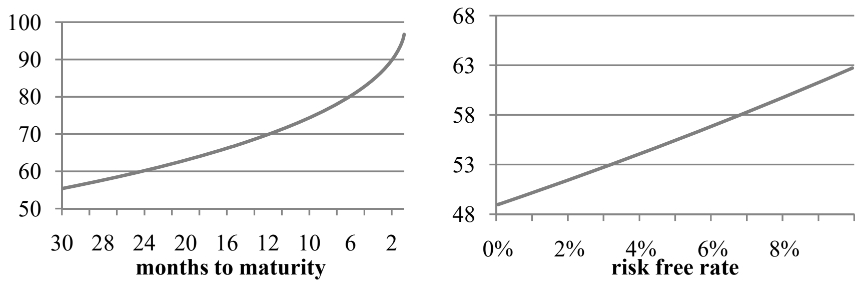
Figure 14 Sensitivity of Everest option to correlation between positive (left chart) or negative (right chart) and highly correlated assets



Source: Own calculations

The impact of passing time is unusual (Figure 15, left), and elapsed time increases the price, especially at the end of option life. The reason for this behaviour is the decreasing uncertainty about selection of the worst asset.

Figure 15 Sensitivity of Everest option to time to maturity and to risk free interest rate



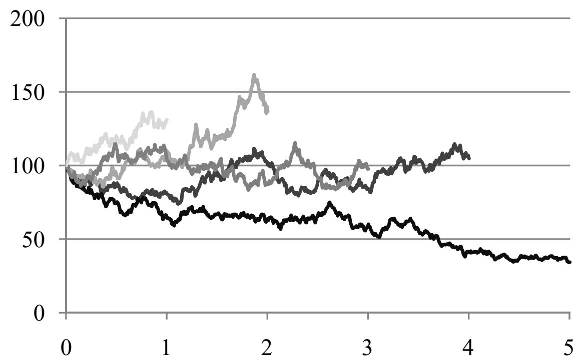
Source: Own calculations

Sensitivity of the Everest option premium to changes in the risk-free interest rate is linear and quite weak (Figure 15, right). In our case, the slope of the price function (rho value) is equal to 0.13.

7 Himalaya option

The Himalaya option is a strong path-dependent option of the European type. In this option the final payoff depends on the returns of the best-performing assets from the basket at predefined sampling dates. The best return R^* at each sampling date is remembered, but this asset is removed from the portfolio and is not taken into account at the next sampling moments. The option is not only path-dependent but also exhibits strong path-dependence due to the removal attribute. The way of describing the best returns can be presented with the numerical example and illustrated in Figure 16 and Tables 3 and 4.

Figure 16 Scenario showing the returns of the underlying assets and an asset selection process of the Himalaya option structure



Source: Own calculations

Table 3 Simulated asset prices

Asset	*Start*	*Period 1*	*Period 2*	*Period 3*	*Period 4*	*Period 5*
1	100	80.61	100.38	84.19	**104.75**	
2	100	64.34	63.88	60.14	41.46	**34.29**
3	100	109.87	**136.72**			
4	100	103.73	87.73	**97.72**		
5	100	**131.13**				

Table 4 Simulated returns

Asset	R^*	*Period 1*	*Period 2*	*Period 3*	*Period 4*	*Period 5*
1	4.75%	–19.39%	0.38%	–15.81%	**4.75%**	
2	–65.71%	–35.66%	–36.12%	–39.86%	–58.54%	**–65.71%**
3	36.72%	9.87%	**36.72%**			
4	–2.28%	3.73%	–12.27%	**–2.28%**		
5	31.13%	**31.13%**				

Source: Own calculations

In the literature (see Jørgensen, 2008), there is a number of various types of Himalaya options. These use different modifications of the payoff function. In our research we assumed

$$\text{payoff}(T) = \max\left\{\frac{1}{N}\sum_{k=1}^{n} R^*(t_k), 0\right\}, \tag{6}$$

where

n number of assets and number of sampling dates simultaneously

t_k subsequent time periods at which returns are selected

T maturity date ($T = t_N$)

$R^*(t_k)$ highest returns at sampling dates.

When we turn to our example the payoff can be calculated in the following way:

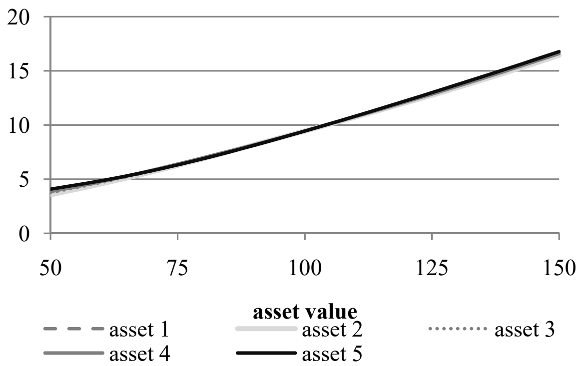
$$\text{payoff} = \max\left\{\frac{1}{5}(31.13\% + 36.72\% - 2.28\% + 4.75\% - 65.71\%).0\right\} = 0.92\%$$

of the face value.

7.1 Sensitivity of the Himalaya option price

The option price is an increasing function of the underlying asset price. As we can observe in Figure 17, the growth rate is similar for each asset. The differences are negligible and the price profiles are almost linear especially close to the value of 100. In our case, deltas are equal to about 0.13. Such a result seems to indicate relatively low sensitivity of the option to asset changes. In the standard European call option a delta close to zero means the option is out of money. The reason for this could be explained by the basket feature of the Himalaya options. This is because the i^{th} asset is only one of five factors affecting the payoff. On the other hand, it must also be a matter of the removal feature of the option and cutting the life time of the removal asset. Although it is not clearly seen in Figure 17, the most convex profile is for asset 5, which is negatively correlated with the others.

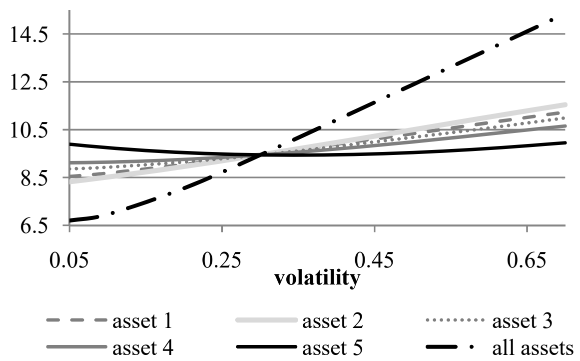
Figure 17 Sensitivity of Himalaya option to underlying assets



Source: Own calculations

As opposed to vanilla options, the Himalaya option price does not have to be an increasing function of the volatility. It is strongly a matter of the correlation structure in the basket. Until the correlations are positive for a majority of the assets, the dependence of option price-volatility seems to be positive and linear. Weakening the correlation structure and finally negative dependence, caused by asset 5, produces nonlinearity in valuation. When the overall market becomes volatile, the option price rises very quickly (see the dashed line in Figure 18). Because the dependence is close to linear, it informs us that a positive one-percentage change in market volatility impacts the value of the option with an increase of 0.14.

Figure 18 Sensitivity of Himalaya option to volatility

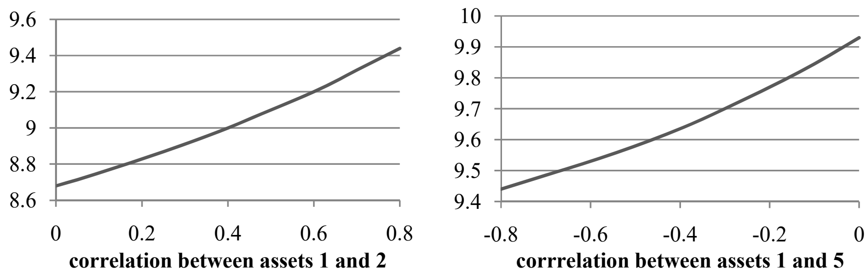


Source: Own calculations

Just as in the typical basket option, a higher correlation between assets implies a higher option value, but this interrelation is rather weak (Figure 19).

The impact of passing time on the valuation is not surprising. It is obvious that it lowers the option premium. In Figure 20, we analysed only passing time to the first removal. It is interesting that the option loses almost half of its value in the first period.

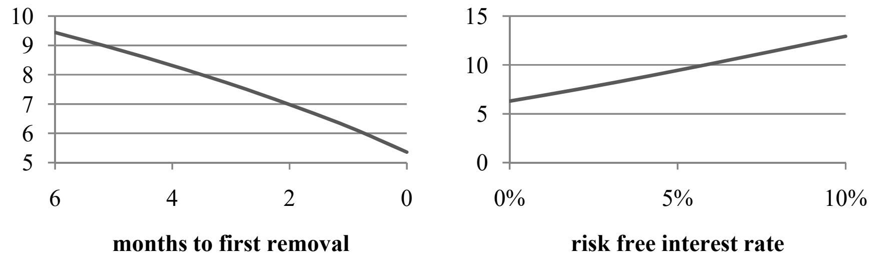
Figure 19 Sensitivity of Himalaya option to correlation between positive (left chart) or negative (right chart) and highly correlated assets



Source: Own calculations

Sensitivity of the Himalaya option premium to changes in the risk-free interest rate is linear (see Figure 20, right). In our example, the slope of the price function is equal to 0.67. This means that at any interest rate level its increase of 1 percentage point causes an increase in the option premium of 0.67.

Figure 20 Sensitivity of Himalaya option to time to maturity and to risk free interest rate



Source: Own calculations

8 Kilimanjaro option

The Kilimanjaro option, similarly to the Himalaya option, is a path-dependent option of the European type. It pays two types of incomes to the holder. These are fixed periodic coupons C which are mostly higher than the market rate of interest, and the final coupon at maturity depends on performance of the assets that constitute the heart of the underlying basket and are calculated from the payoff function. Throughout the life m times respectively the worst assets are removed from the portfolio and are not taken into account at the next sampling moments. At maturity, m best-performing assets are removed from the underlying portfolio. The $n - 2m$ remaining assets are called the heart of the portfolio and their final values are taken into account to determine the payoff value or the final coupon. Generally, we can describe the payoff function using the following formula:

$$\text{payoff}(T) = \frac{1}{n-2m} \sum_{i=1}^{n-2m} R_i^*(T) \tag{7}$$

where

n number assets

m number assets removed at sampling dates and at maturity separately ($n > 2m$)

T maturity date

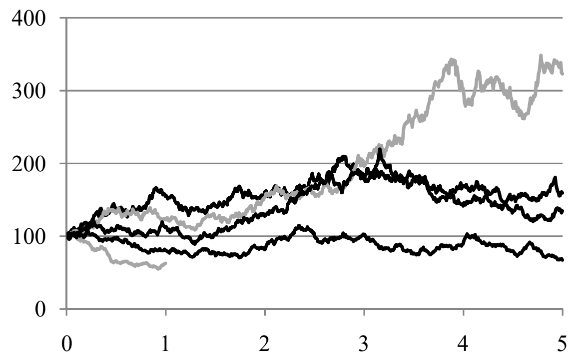
$R_i^*(T)$ returns from the heart of the portfolio at maturity.

Note that the Kilimanjaro option is not rightly an option because of the unconditional character of its payoff. Theoretically, it can have negative performance and it cannot protect invested capital. The entire income for the buyer of a Kilimanjaro option is the sum of the sequence regular coupons C and final payoff:

$$\text{income} = \sum_{k=1}^{T} C(t_k) + \text{payoff}(T). \tag{8}$$

The way of describing income from the Kilimanjaro option can be presented with a numerical example and illustrated as in Figure 21 and Table 5. This is a very simple case if there is only one sampling moment at date 1.

Figure 21 Scenario showing the returns of the underlying assets and an asset selection process of the Kilimanjaro option structure



Source: Own calculations

Figure 21 Scenario showing the returns of the underlying assets and the asset selection process of the Kilimanjaro option structure. The black line means assets from the heart of the portfolio. If we assume coupons at the level of 2.5% per period, then the entire holder's income is equal to[1]

$$income = 5 \cdot 2.5\% + \frac{1}{3}(59.41\% - 32.76\% + 34.82\%) = 32.99\%$$

of the face value.

Table 5 Simulated asset prices and returns

Asset	*Start*	*Period 1*	*Maturity*	R^*
1	100	62.86		
2	100	154.61	**159.41**	**59.41**
3	100	123.09	323.20	
4	100	80.84	**67.24**	**–32.76**
5	100	111.80	**134.82**	**34.82**

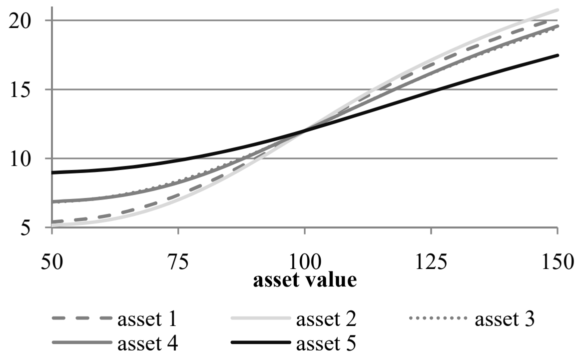
Source: Own calculations

8.1 Sensitivity of the Kilimanjaro option price

In our simulations, for convenience we assume that the coupon rate of the option is exactly equal to the risk-free interest rate. As was mentioned above, it is mostly higher than the market rate, thus our option valuations can be seen as underpriced. This simplification should not, however, affect the magnitude of change in the option premium.

The option price is an increasing function of the underlying asset price. Although the option price profiles illustrated in Figure 22 have a similar shape, its slope is quite different. The option price is most sensitive when changes take place in positively correlated assets. The lowest sensitivity of the option premium takes place due to a change in the negatively correlated asset price. Delta Greek, being the first derivative of the option premium with respect to the underlying asset is positive, but otherwise than in the vanilla European call case it has the highest value at the strike. The lower sensitivity of the premium after crossing the strike is a matter of construction of the Kilimanjaro option. The fastest rising assets are removed from the basket at maturity date and are not taken to the heart of the underlying assets and, finally, do not affect payment of the Kilimanjaro option.

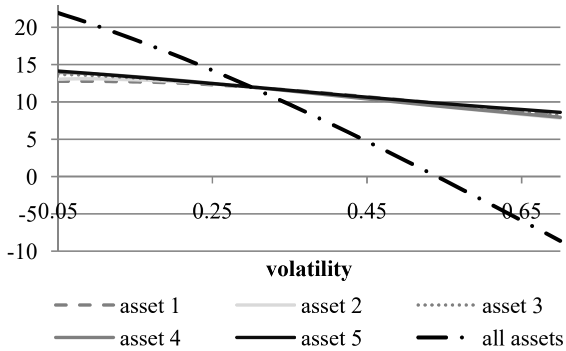
Figure 22 Sensitivity of Kilimanjaro option to underlying assets



Source: Own calculations

The option price is a decreasing and concave function of the volatility. As can be observed in Figure 23, the decline rate is not large and is similar for each asset. When the overall market becomes volatile, the option price falls rapidly (see the dashed line in Figure 23) to negative values. The negative dependence between the option premium and volatility has to be caused by a lack of the capital guarantee of the Kilimanjaro option. The option does not have conditional payoff and an increase in volatility can lead to negative valuation.

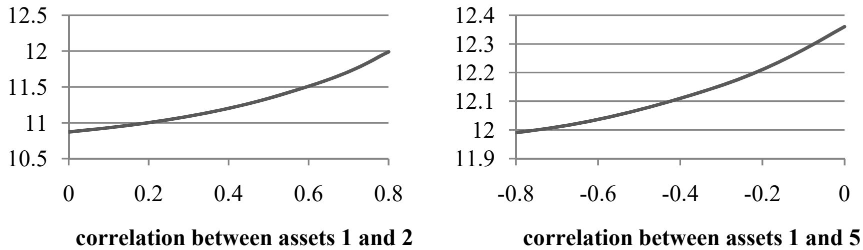
Figure 23 Sensitivity of Kilimanjaro option to volatility



Source: Own calculations

The impact of correlation coefficients for the Kilimanjaro option value is presented in Figure 24. Similarly as with the other mountain range options, the price of the Kilimanjaro option is an increasing and convex function of the correlation coefficient between the underlying assets. The spread between minimum and maximum option values is much higher in the case of assets with a strong positive correlation relative to assets with a strong negative correlation. In fact, changes in option value due to changes in correlation between negatively correlated assets are insignificant (see Figure 24, right). When the correlation weakens from –0.8 to 0, the option price rises by only 3% of its initial value. This is a very small rise with such a high change in the correlation.

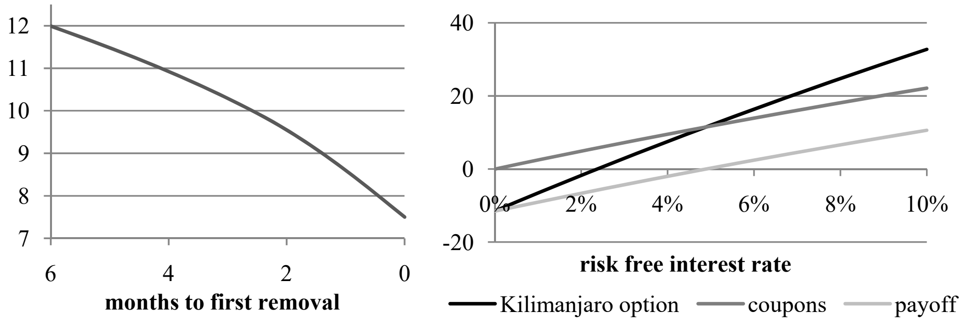
Figure 24 Sensitivity of Kilimanjaro option to correlation between positive (left chart) or negative (right chart) and highly correlated assets



Source: Own calculations

The Kilimanjaro option, similarly as the Himalaya option, has a removal feature. This causes almost the same sensitivity of both options to a change in the passage of time remaining to maturity. It is well visible when one compares Figures 20 and 25.

Figure 25 Sensitivity of Kilimanjaro option to time to maturity and to risk free interest rate



Source: Own calculations

The Kilimanjaro option price is a sum of the coupons' present value and the present value of payoff value. The option combines the features of bond and exotic security; hence its sensitivity to changes in the risk-free interest rate is higher than in the other mountain range options. Similarly as in the case of the Himalaya option's sensitivity, the Kilimanjaro option is an almost linear function of the interest rate, but its slope is much higher. In our case the slope is equal to 4.42, and an increase in the option price is 6.5 times higher than in the Himalaya option case. It is important to notice that the Kilimanjaro options can be negatively valued when interest rates are very low.

9 Conclusions

Before one begins investment in such complicated securities as mountain range options, it is important to fully understand their construction, risk factors and pricing method. Due to their portfolio character and the necessity of analysing under the *ceteris paribus* assumption, it is not easy to find their universal properties. Each of the five option types has a different construction and thus different sensitivity to risk factors.

The Altiplano option price is an increasing function of the underlying asset price and the growth rate is similar for each asset. Moreover, the option price is an increasing function of the volatility for positively correlated assets. The negative correlation for the asset causes convexity in valuation by the volatility. When the overall market becomes unstable, the option price falls very quickly and then rises slowly. The impact of passing time on the valuation is not typical. At the beginning, elapsed time reduces the option premium, but in the last years the price increases sharply. The increase in the risk-free interest rate causes a linear rise of the option price.

The Atlas option price is an increasing function of the underlying asset price. The option price is most sensitive to the value of positively correlated assets. Sensitivity of the option price to volatility has a parabolic shape. Initially, an increase in volatility increases the option price, and after reaching the maximum rise in volatility there is a decline in the option premium. For assets with a negative correlation it is a decreasing function of volatility. When the overall market becomes volatile, the option price rises sharply. The impact of passing time on the valuation is standard, it reduces the option premium.

The Everest option price is an increasing and concave function of the value of assets. Additionally, the option price is a decreasing function of the volatility. When the volatility of the overall market rises, the option price falls sharply. The impact of passing time is unusual, as elapsed time increases the price, especially at the end of option life. Sensitivity of the Everest option premium to changes in the risk-free interest rate is linear and quite weak.

The Himalaya option price is an almost linear function of the underlying asset price. Furthermore, option sensitivity to that risk factor is approximately the same for each asset in the portfolio. The volatility of individual assets slightly influences the price of the option, but when the volatility of the entire market rises the option price rises substantially. The option is strongly sensitive to the passage of time. It loses almost half of its value to the first removal moment. The increase of the risk-free interest rate causes a linear rise of the option price.

The Kilimanjaro option price is a nonlinear and increasing function of the underlying asset price. The asset which is negatively correlated with the other assets has the least effect on the option price. Similarly to the Himalaya option, the volatility of individual assets slightly influences the price of the option, although volatility of the entire market has a strong influence on the option price. Note that the option price is a decreasing function of volatility. Analogously to the Himalaya option, the Kilimanjaro option is strongly sensitive to the passage of time. Sensitivity of the option to a change in the interest rate is quite different. The Kilimanjaro option has a bond component, thus it is strongly dependent on that risk factor. The increase of the coupon interest rate has to cause a strong rise of the option price.

The impact of the correlation coefficient for all types of analysed options is the same. The price of the option is an increasing function of the correlation.

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Notes

1 We ignore here the changes in the value of money over time.