**Boston University Questrom School of Business**

**MF 793 – Fall 2018**

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**Problem Set 6**

Due **Monday December 9th in class**

* Turn in a paper copy , no email submission accepted.
* To get a check, you need the discussion questions in HANDWRITING.
* The problem set is not long, It’s just that I explain everything!
* Doing this problem set gets you ready for the second exam, ***better believe it* !**
* Problems turned in after the deadline will not be graded.

**A Risk Metrics and a GARCH problems will be added**

**Problem 1: Understanding AR models**

a) You want to simulate an AR(1) series; unconditional mean 16, unconditional standard deviation of 6, and autocorrelation of 0.85. What intercept and noise standard deviation will you use?

b) Xt follows an AR(1): Xt = ϕ0 + ϕ1 Xt-1 + εt, ε ~ i.i.d. N( 0 , σε), Xt ∼ N(μ, σ )

The last observed value today is XT. Write the mean and standard deviation formulas for forecasts of XT+1, XT+2, XT+3, and X∞. Ignore parameter estimation uncertainty. That is, you need to write the theoretical formulas based on (μ, σ, ϕ0, ϕ1, σε).

c) For the parameters found in a) and XT = 24, fill in Table 1.

Table 1: Forecasting with the stationary AR(1)

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mean | Variance | Std. Dev. |
| XT | 24 | 0 | 0 |
| XT+1 |  |  |  |
| XT+2 |  |  |  |
| XT+3 |  |  |  |
| X∞ |  |  |  |

d) Use the *arima.sim* command to simulate 5,000 observations of a zero-mean AR(2) with parameters ϕ1=0.7 ϕ2=0.25, and of another AR(1) with parameters ϕ1=0.8 ϕ2=-0.25.

In Figure 2, panels a) and b) put their ACF function.

Compare the ACFs, what pattern do you notice?

R advice:

Use the Acf (not acf) command. When making two plots in one Figure, use: par(mfrow=c(1,2),mar=c(3,3,2,0.1),mgp=c(1.5,0.5,0))

The mar flag is crucial otherwise your plots look silly

**Problem 2: AR(1) on VIX or log(VIX)?**

vixsp-mon.csv contains the monthly VIX and S&P return. Use the data until March. 2009 included.

**a)** In a few hand-written lines, describe the VIX. For example, on Oct. 31st 2008, the VIX is 59.89. What is 59.89?

**b)** In **Figure 1**, put three plots, the time series of VIX, its ACF and Partial ACF.

Make the time series plot professional by using the ts command:

*ts.plot(ts(vix[beg:end,2],frequency=12,start=c(1990,1))*

Use the *Acf* command, not the *acf* command !

Do you think VIX is better described by a short AR or a short MA? Explain why

**c)** With the ***arma*** command, estimate an AR(1) on VIX. Look at the output with the summary command. You can put the estimates of the ϕs and σϵ in Table 1 at the end of the problem.

Use: summary(var1) # var1 is the name of your arma model like with lm

Get the residuals and the fitted values, see names(var1), for all you can get out of arma.

Understand why residual[1] and fitted.value[1], are what they are!

**d)** In **Figure 2**, plot the time series of the residuals in panel a), the **absolute values** of the residuals vs the fitted value in panel b) and a normal probability plot of the residuals in panel c). Compute the skewness of the residuals.

What do panels a) b) tell about the homoskedasticity of the residuals? What pattern do you see?

From panel c), does the distribution of the residuals look normal?

**e)** Shoot, you should have thought about that before running this regression! In **Figure 3**, put the normal probability plot of VIX in panel a), and of log(VIX) in panel b).

Compare the skewness of VIX and log(VIX)?

What is the number one thing to do to improve this regression / AR model ?

**f)** Estimate an AR(1) on log(vix). In **Figure 4**, repeat Figure 2 plots for the log model. Compute the skewness of the residuals. Compare Figure 4 to Figure 2. What model is better specified?

**g)** Given your parameter estimates, make forecasts for April 2009 VIX, using the vix AR, and the log(vix) AR model. Namely: give the forecast and a 50% confidence, LB and UB, interval for VIXT+1. Show your intermediate computation for the Log-AR. Fill-in **Table 1**.

In the table, f the Log-AR1 column forecasts must be for the VIX, not for the log(VIX) of course.

Hint: Remember the log-normal distribution?

To simplify things, you do **not** need to take parameter estimation error into account.

**Table 1:** Forecasting with the AR(1) vs. LogAR(1)

|  |  |  |
| --- | --- | --- |
|  | With AR1 | With Log-AR1 |
| Φ0 |  |  |
| Φ1 |  |  |
| σϵ |  |  |
| E(VIXT+1) |  |  |
| LB(VIXT+1) |  |  |
| UB(VIXT+1) |  |  |

A Risk Metrics and GARCH problem follow