The ARAS (Additive Ratio Assessment System) method is a multi-criteria decision-making (MCDM) technique used to evaluate and rank options based on multiple criteria, often in cases where multiple conflicting attributes or criteria need to be considered. The method is particularly useful in evaluating fuzzy numbers, such as triangular fuzzy numbers (TFNs), commonly used to handle uncertainty and imprecision.

Here's how the ARAS method can be applied to find the rank of options with triangular fuzzy numbers (TFNs):

Steps of the ARAS Method with TFNs:

1. **Define the Decision Matrix**: Create a decision matrix where each option has multiple criteria represented by TFNs. Each element in the matrix will be a TFN in the form $\widetilde{A}_{ij} = \left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u}\right)$, where a^{l}, a^{m}, and a^{u} are the lower, middle, and upper values of the triangular fuzzy number.
2. **Normalize the Matrix**: Normalize the fuzzy decision matrix to make the values comparable across different criteria. For TFNs, the normalization is done differently for benefit and cost criteria:
   - **Benefit criteria**: $\widetilde{N}_{ij} = \frac{\widetilde{A}_{ij}}{\widetilde{A}_{j}^{\max}}$
   - **Cost criteria**: $\widetilde{N}_{ij} = \frac{\widetilde{A}_{j}^{\min}}{\widetilde{A}_{ij}}$
   - where $\widetilde{A}_{j}^{\max}$ and $\widetilde{A}_{j}^{\min}$ are the maximum and minimum fuzzy values for each criterion j.
3. **Weighted Normalization**: Multiply each normalized value by the corresponding criterion weight w_j, to get the weighted normalized matrix $\widetilde{S}_{ij}$: $\widetilde{S}_{ij} = w_j \cdot \widetilde{N}_{ij}$
4. **Determine Overall Score for Each Option**: For each option i, calculate the total score S_i by summing all weighted normalized values: $S_i = \sum_{j=1}^{n} \widetilde{S}_{ij}$
5. **Rank the Options**: Compare the overall scores S_i of each option. The higher the score, the better the rank.

In the context of TFNs, you may need to defuzzify the scores (e.g., by calculating the centroid of each fuzzy number) to obtain crisp values for easier comparison and ranking.

Here is the data extracted from the first four rows and columns of your table:

	C1	C2	C3	C4
A1	10	22	9	90
A2	0.33	9	3	70
A3	8	13	8	70
A4	15	28	5	90

This is a subset of your data focusing on the first four criteria (C1-C4) for the first four alternatives (A1-A4). Let me know if you'd like further steps or additional data processing!

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To calculate the Triangular Fuzzy Numbers (TFNs) with a 5% deviation for the data you've provided, we'll follow these steps:

TFN Representation:

A TFN is typically represented as $\left(a^l, a^m, a^u\right)$, where:

- a^l is the lower bound of the TFN.
- a^m is the middle value (the original value).
- a^u is the upper bound of the TFN.

Given a 5% deviation, the TFN can be calculated as:

- $a^l = a^m - (5\% \cdot a^m)$
- $a^u = a^m + (5\% \cdot a^m)$

Let's apply this to the first four rows and columns.

Step-by-Step Calculation:

For A1:

1. **C1 (10)**:
    - $a^l = 10 - 0.05 \times 10 = 9.5$

- $a^u = 10 + 0.05 \times 10 = 10.5$
- TFN: $(9.5, 10, 10.5)$

2. **C2 (22)**:
   - $a^l = 22 - 0.05 \times 22 = 20.9$
   - $a^u = 22 + 0.05 \times 22 = 23.1$
   - TFN: $(20.9, 22, 23.1)$
3. **C3 (9)**:
   - $a^l = 9 - 0.05 \times 9 = 8.55$
   - $a^u = 9 + 0.05 \times 9 = 9.45$
   - TFN: $(8.55, 9, 9.45)$
4. **C4 (90)**:
   - $a^l = 90 - 0.05 \times 90 = 85.5$
   - $a^u = 90 + 0.05 \times 90 = 94.5$
   - TFN: $(85.5, 90, 94.5)$

For A2:

1. **C1 (0.33)**:
   - $a^l = 0.33 - 0.05 \times 0.33 = 0.3135$
   - $a^u = 0.33 + 0.05 \times 0.33 = 0.3465$
   - TFN: $(0.3135, 0.33, 0.3465)$
2. **C2 (9)**:
   - $a^l = 9 - 0.05 \times 9 = 8.55$
   - $a^u = 9 + 0.05 \times 9 = 9.45$
   - TFN: $(8.55, 9, 9.45)$
3. **C3 (3)**:
   - $a^l = 3 - 0.05 \times 3 = 2.85$
   - $a^u = 3 + 0.05 \times 3 = 3.15$
   - TFN: $(2.85, 3, 3.15)$
4. **C4 (70)**:
   - $a^l = 70 - 0.05 \times 70 = 66.5$
   - $a^u = 70 + 0.05 \times 70 = 73.5$
   - TFN: $(66.5, 70, 73.5)$

For A3:

1. **C1 (8)**:
   - $a^l = 8 - 0.05 \times 8 = 7.6$
   - $a^u = 8 + 0.05 \times 8 = 8.4$

- TFN: $(7.6, 8, 8.4)$

2. **C2 (13)**:
   - $a^l = 13 - 0.05 \times 13 = 12.35$
   - $a^u = 13 + 0.05 \times 13 = 13.65$
   - TFN: $(12.35, 13, 13.65)$
3. **C3 (8)**:
   - $a^l = 8 - 0.05 \times 8 = 7.6$
   - $a^u = 8 + 0.05 \times 8 = 8.4$
   - TFN: $(7.6, 8, 8.4)$
4. **C4 (70)**:
   - $a^l = 70 - 0.05 \times 70 = 66.5$
   - $a^u = 70 + 0.05 \times 70 = 73.5$
   - TFN: $(66.5, 70, 73.5)$

For A4:

1. **C1 (15)**:
   - $a^l = 15 - 0.05 \times 15 = 14.25$
   - $a^u = 15 + 0.05 \times 15 = 15.75$
   - TFN: $(14.25, 15, 15.75)$
2. **C2 (28)**:
   - $a^l = 28 - 0.05 \times 28 = 26.6$
   - $a^u = 28 + 0.05 \times 28 = 29.4$
   - TFN: $(26.6, 28, 29.4)$
3. **C3 (5)**:
   - $a^l = 5 - 0.05 \times 5 = 4.75$
   - $a^u = 5 + 0.05 \times 5 = 5.25$
   - TFN: $(4.75, 5, 5.25)$
4. **C4 (90)**:
   - $a^l = 90 - 0.05 \times 90 = 85.5$
   - $a^u = 90 + 0.05 \times 90 = 94.5$
   - TFN: $(85.5, 90, 94.5)$

Summary of TFNs for the First Four Rows and Columns:

	C1 (TFN)	C2 (TFN)	C3 (TFN)	C4 (TFN)
A1	(9.5, 10, 10.5)	(20.9, 22, 23.1)	(8.55, 9, 9.45)	(85.5, 90, 94.5)

	C1 (TFN)	C2 (TFN)	C3 (TFN)	C4 (TFN)
A2	(0.3135, 0.33, 0.3465)	(8.55, 9, 9.45)	(2.85, 3, 3.15)	(66.5, 70, 73.5)
A3	(7.6, 8, 8.4)	(12.35, 13, 13.65)	(7.6, 8, 8.4)	(66.5, 70, 73.5)
A4	(14.25, 15, 15.75)	(26.6, 28, 29.4)	(4.75, 5, 5.25)	(85.5, 90, 94.5)

Let me know if you'd like to proceed with any further steps!

To rank the alternatives (A1, A2, A3, A4) using the **ARAS (Additive Ratio Assessment System)** method based on the calculated **Triangular Fuzzy Numbers (TFNs)**, let's follow the steps:

Step 1: Normalize the Fuzzy Decision Matrix

The first step in the ARAS method is normalizing the decision matrix. The normalization for each element is performed by dividing each TFN value by the sum of all corresponding TFN values for that criterion.

Normalization formula:

- For **benefit criteria** (higher values are better):

$$\widetilde{N}_{ij} = \frac{\widetilde{A}_{ij}}{\sum_{i=1}^{m} \widetilde{A}_{ij}}$$

- For **cost criteria** (lower values are better):

$$\widetilde{N}_{ij} = \frac{\min \widetilde{A}_{ij}}{\widetilde{A}_{ij}}$$

Here we'll assume all criteria (C1, C2, C3, C4) are **benefit criteria**.

1. Calculate the total sum of each criterion: For each criterion C_j, calculate the sum of the triangular fuzzy numbers across all alternatives.

	C1 (TFN)	C2 (TFN)	C3 (TFN)	C4 (TFN)
A1	(9.5, 10, 10.5)	(20.9, 22, 23.1)	(8.55, 9, 9.45)	(85.5, 90, 94.5)
A2	(0.3135, 0.33, 0.3465)	(8.55, 9, 9.45)	(2.85, 3, 3.15)	(66.5, 70, 73.5)

	C1 (TFN)	C2 (TFN)	C3 (TFN)	C4 (TFN)
A3	(7.6, 8, 8.4)	(12.35, 13, 13.65)	(7.6, 8, 8.4)	(66.5, 70, 73.5)
A4	(14.25, 15, 15.75)	(26.6, 28, 29.4)	(4.75, 5, 5.25)	(85.5, 90, 94.5)

Let's first calculate the sum for each criterion:

Sum of C1 (TFN):

- Lower sum $= 9.5 + 0.3135 + 7.6 + 14.25 = 31.6635$
- Middle sum $= 10 + 0.33 + 8 + 15 = 33.33$
- Upper sum $= 10.5 + 0.3465 + 8.4 + 15.75 = 35.013$

Sum of C2 (TFN):

- Lower sum $= 20.9 + 8.55 + 12.35 + 26.6 = 68.4$
- Middle sum $= 22 + 9 + 13 + 28 = 72$
- Upper sum $= 23.1 + 9.45 + 13.65 + 29.4 = 75.6$

Sum of C3 (TFN):

- Lower sum $= 8.55 + 2.85 + 7.6 + 4.75 = 23.75$
- Middle sum $= 9 + 3 + 8 + 5 = 25$
- Upper sum $= 9.45 + 3.15 + 8.4 + 5.25 = 26.25$

Sum of C4 (TFN):

- Lower sum $= 85.5 + 66.5 + 66.5 + 85.5 = 304$
- Middle sum $= 90 + 70 + 70 + 90 = 320$
- Upper sum $= 94.5 + 73.5 + 73.5 + 94.5 = 336$

2. Normalize each TFN: Now, we will normalize each element of the matrix by dividing it by the total sum for that criterion.

For C1:

- $\widetilde{N}_{A1,C1} = \left(\frac{9.5}{31.6635}, \frac{10}{33.33}, \frac{10.5}{35.013}\right) = (0.300, 0.300, 0.300)$
- $\widetilde{N}_{A2,C1} = \left(\frac{0.3135}{31.6635}, \frac{0.33}{33.33}, \frac{0.3465}{35.013}\right) = (0.010, 0.010, 0.010)$
- $\widetilde{N}_{A3,C1} = \left(\frac{7.6}{31.6635}, \frac{8}{33.33}, \frac{8.4}{35.013}\right) = (0.240, 0.240, 0.240)$
- $\widetilde{N}_{A4,C1} = \left(\frac{14.25}{31.6635}, \frac{15}{33.33}, \frac{15.75}{35.013}\right) = (0.450, 0.450, 0.450)$

For C2:

- $\widetilde{N}_{A1,C2} = \left(\frac{20.9}{68.4}, \frac{22}{72}, \frac{23.1}{75.6}\right) = (0.305, 0.305, 0.305)$
- $\widetilde{N}_{A2,C2} = \left(\frac{8.55}{68.4}, \frac{9}{72}, \frac{9.45}{75.6}\right) = (0.125, 0.125, 0.125)$

- $\widetilde{N}_{A3,C2} = \left(\frac{12.35}{68.4}, \frac{13}{72}, \frac{13.65}{75.6}\right) = (0.180, 0.180, 0.180)$
- $\widetilde{N}_{A4,C2} = \left(\frac{26.6}{68.4}, \frac{28}{72}, \frac{29.4}{75.6}\right) = (0.390, 0.390, 0.390)$

For C3:

- $\widetilde{N}_{A1,C3} = \left(\frac{8.55}{23.75}, \frac{9}{25}, \frac{9.45}{26.25}\right) = (0.360, 0.360, 0.360)$
- $\widetilde{N}_{A2,C3} = \left(\frac{2.85}{23.75}, \frac{3}{25}, \frac{3.15}{26.25}\right) = (0.120, 0.120, 0.120)$
- $\widetilde{N}_{A3,C3} = \left(\frac{7.6}{23.75}, \frac{8}{25}, \frac{8.4}{26.25}\right) = (0.320, 0.320, 0.320)$
- $\widetilde{N}_{A4,C3} = \left(\frac{4.75}{23.75}, \frac{5}{25}, \frac{5.25}{26.25}\right) = (0.200, 0.200, 0.200)$

For C4:

- $\widetilde{N}_{A1,C4} = \left(\frac{85.5}{304}, \frac{90}{320}, \frac{94.5}{336}\right) = (0.281, 0.281, 0.281)$
- $\widetilde{N}_{A2,C4} = \left(\frac{66.5}{304}, \frac{70}{320}, \frac{73.5}{336}\right) = (0.219, 0.219, 0.219)$
- $\widetilde{N}_{A3,C4} = \left(\frac{66.5}{304}, \frac{70}{320}, \frac{73.5}{336}\right) = (0.219, 0.219, 0.219)$
- $\widetilde{N}_{A4,C4} = \left(\frac{85.5}{304}, \frac{90}{320}, \frac{94.5}{336}\right) = (0.281, 0.281, 0.281)$

Step 3: Calculate the Utility Degree (S)

For each alternative, sum the normalized TFN values across all criteria:

Let's sum the normalized TFN values:

For A1:

- $S_{A1} = (0.300 + 0.305 + 0.360 + 0.281) = 1.246$

For A2:

- $S_{A2} = (0.010 + 0.125 + 0.120 + 0.219) = 0.474$

For A3:

- $S_{A3} = (0.240 + 0.180 + 0.320 + 0.219) = 0.959$

For A4:

- $S_{A4} = (0.450 + 0.390 + 0.200 + 0.281) = 1.321$

Step 4: Rank the Alternatives

Rank the alternatives based on their utility degree S_i. The higher the value of S_i, the better the alternative:

Alternative	Utility Degree (S)	Rank
A4	1.321	1
A1	1.246	2
A3	0.959	3
A2	0.474	4

Final Ranking:

- **A4** is ranked 1st.
- **A1** is ranked 2nd.
- **A3** is ranked 3rd.
- **A2** is ranked 4th.

Let me know if you need further clarification or help!