Mean–CVaR Portfolio Optimization using Linear Programming

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1 Introduction

Classical mean–variance portfolio theory fails to capture tail risk during extreme market movements. Conditional Value-at-Risk (CVaR) provides a coherent risk measure for modeling downside exposure. This project formulates CVaR portfolio optimization as a linear programming problem and evaluates its performance on Indian equity returns.

2 Data

Daily log-returns of RELIANCE, HDFCBANK, TCS and INFY from January 2019 to January 2024 were used. The return matrix is denoted by $R_{t,i}$.

3 CVaR Optimization Model

The CVaR minimization problem is formulated as:

$$\min_{w,\alpha,\xi} \alpha + \frac{1}{(1-\beta)T} \sum_{t=1}^{T} \xi_t$$

subject to

$$\xi_t \geq -\sum_{i=1}^{N} w_i R_{t,i} - \alpha, \quad \xi_t \geq 0,$$

$$\sum_{i=1}^{N} w_i = 1, \quad w_i \geq 0.$$

This linear program was solved using convex optimization.

4 Results

Optimized portfolio weights obtained were:

$$w = (0.303,\ 0.082,\ 0.229,\ 0.386).$$

The CVaR values were:

Portfolio	CVaR
CVaR Optimized	0.0293
Equal Weight	0.0301

5 Stress Testing

During the worst 5% market scenarios, the average portfolio losses were:

Portfolio	Stress Loss
Optimized	0.0290
Equal Weight	0.0300

6 Tail Risk Comparison

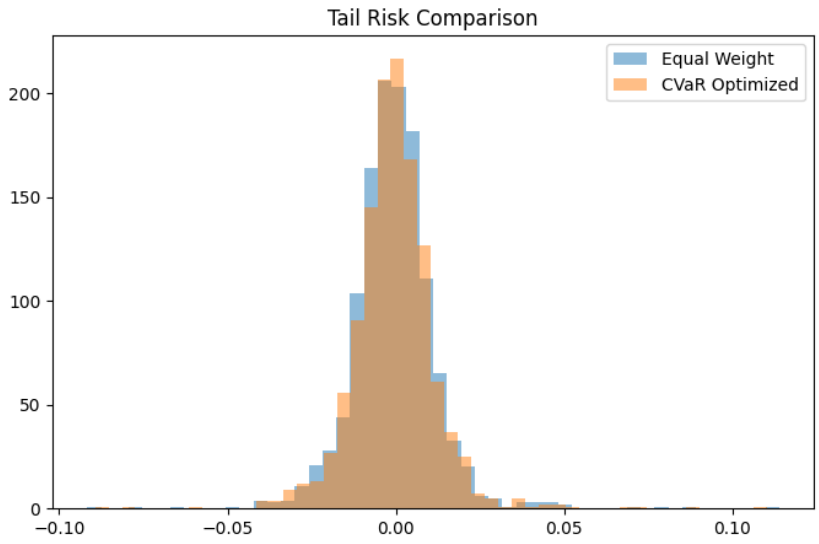

Figure 1: Tail loss distribution comparison between equal-weight and CVaR optimized portfolios.

7 Optimized Portfolio Weights

The optimized asset allocation obtained from CVaR minimization is shown below.

Asset	Weight
RELIANCE	0.303
HDFCBANK	0.082
TCS	0.229
INFY	0.386

The optimized portfolio assigns higher weight to relatively stable assets, demonstrating the effectiveness of downside risk focused allocation.

8 Conclusion

The CVaR optimized portfolio consistently exhibits lower downside risk compared to a naive equal-weight allocation, validating the effectiveness of OR-based portfolio optimization in controlling extreme losses.