Monte-Carlo and Empirical Risk Estimation of Indian Equity Returns

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1 Introduction

This project analyses downside risk in Indian equity markets using empirical distribution techniques, Monte-Carlo simulation and bootstrap validation.
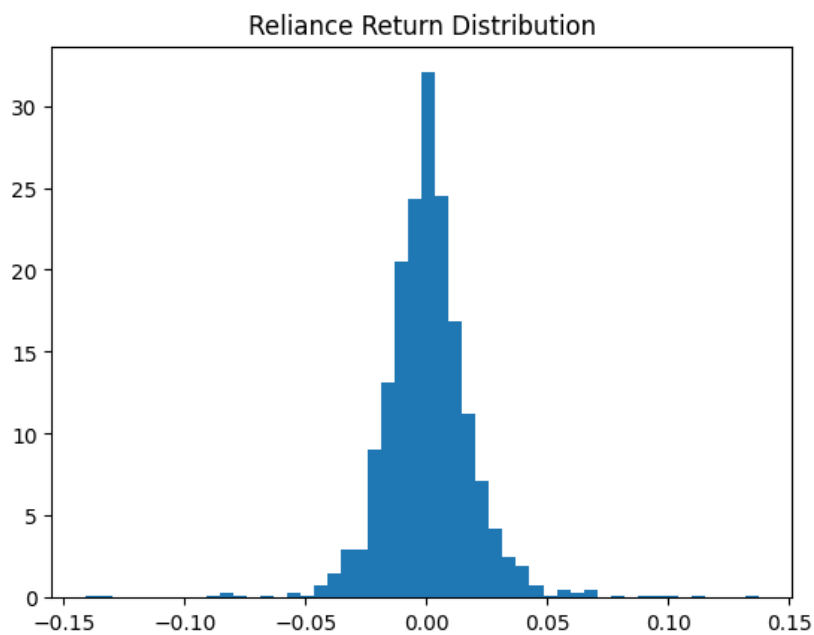

Figure 1: Distribution of Reliance daily log-returns.

2 Data Description

Daily closing prices of RELIANCE.NS from January 2019 to January 2024 were used. Log-returns were computed as
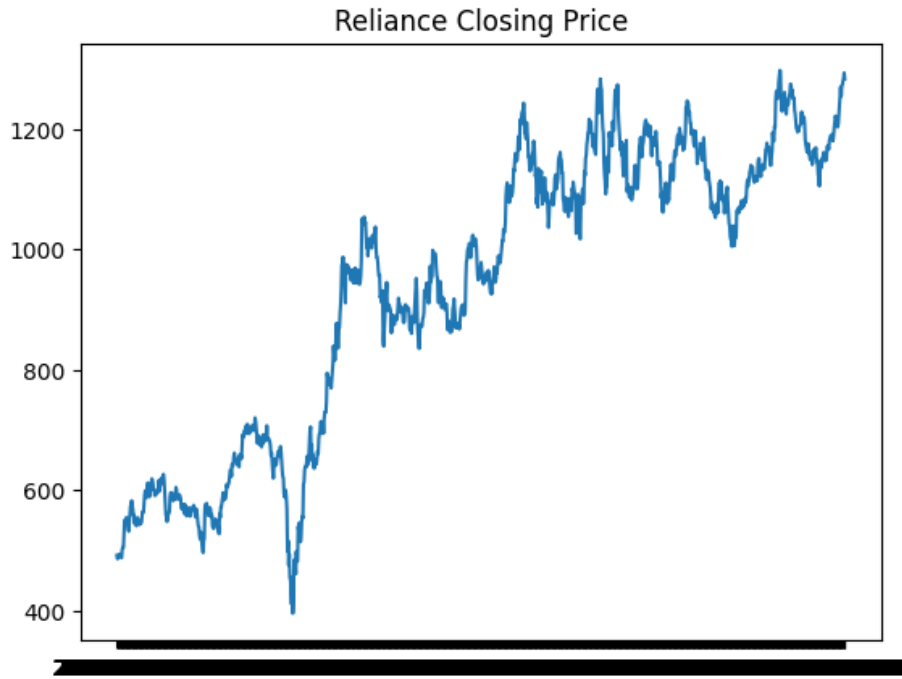
$$R_t = \log \frac{P_t}{P_{t-1}}.$$

Figure 2: Reliance closing price (2019–2024).

3 Distribution Fitting

Estimated parameters of Reliance returns are

$$\hat{\mu} = 0.000766, \quad \hat{\sigma} = 0.0191.$$

The Kolmogorov–Smirnov test rejects normality with p-value 2.76×10^{-6}, indicating heavy tails.
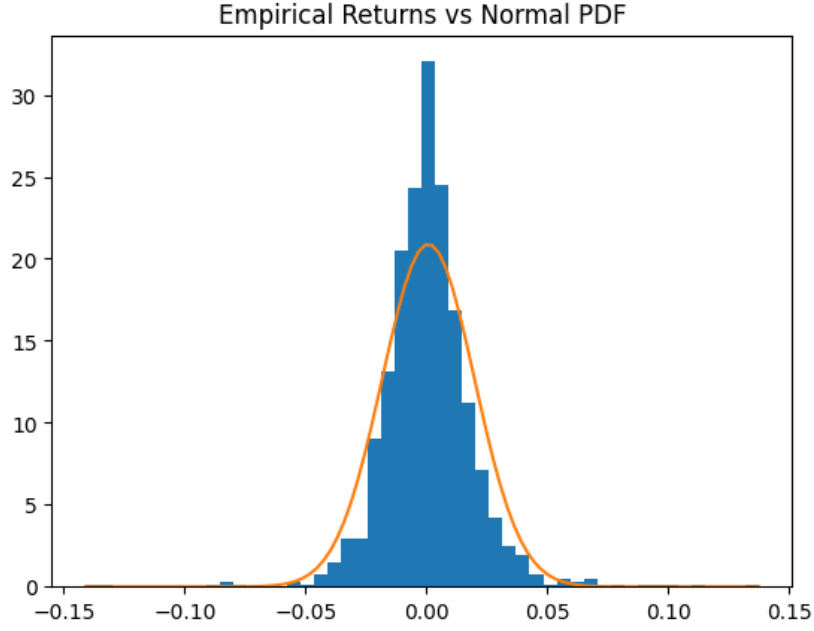
Figure 3: Empirical return distribution with fitted normal density.

4   Empirical Risk Estimation

Value-at-Risk at level $\alpha = 0.95$ is defined as

$$VaR_{0.95} = -q_{0.05}(R),$$

and Expected Shortfall as

$$ES_{0.95} = -E(R \mid R \leq -VaR).$$

Empirical estimates were

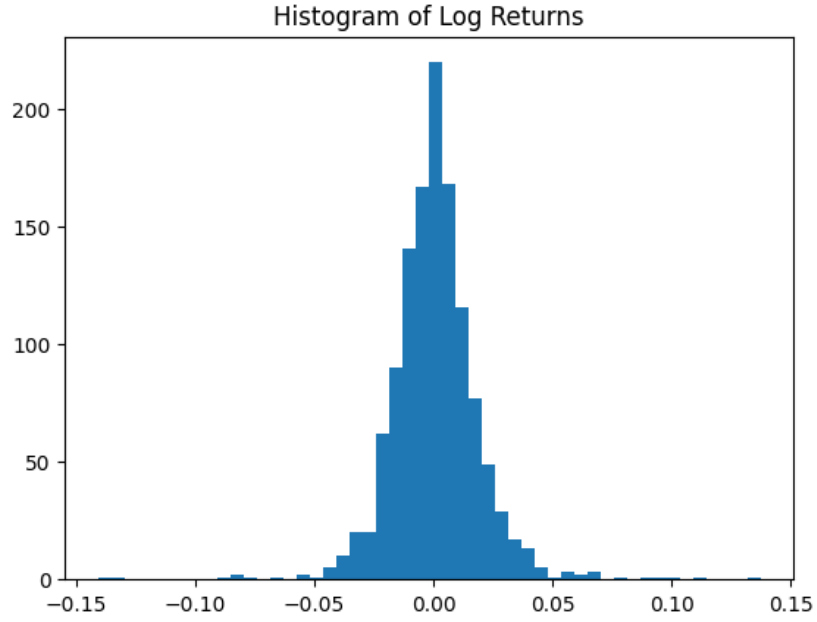$$VaR_{0.95} = 0.0247, \quad ES_{0.95} = 0.0405.$$

Figure 4: Histogram of Reliance log-returns.

5 Monte-Carlo Simulation

Assuming

$$R \sim N(\hat{\mu}, \hat{\sigma}^2),$$

10,000 simulated returns were generated giving

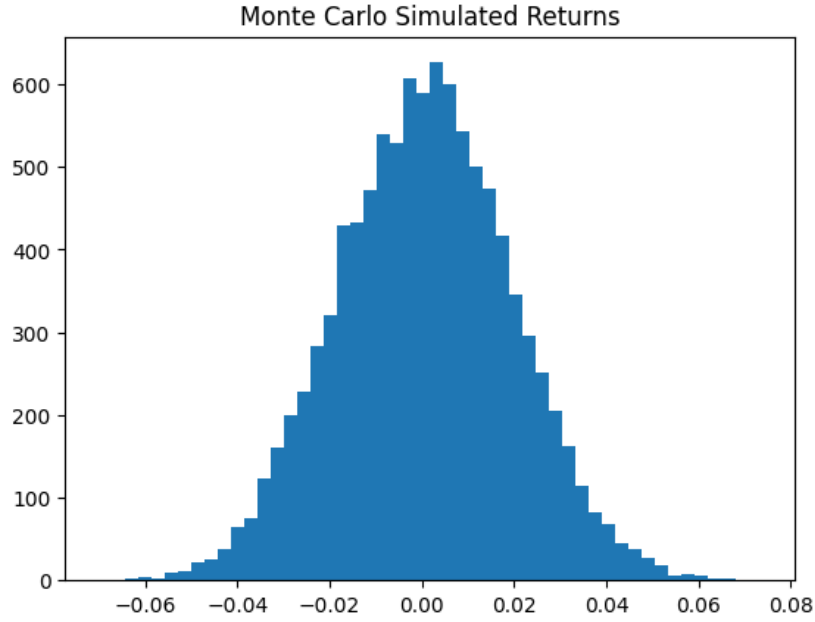$$VaR_{0.95}^{MC} = 0.0307, \quad ES_{0.95}^{MC} = 0.0379.$$

Figure 5: Monte-Carlo simulated return distribution.

6 Bootstrap Validation

Bootstrap resampling produced a 95% confidence interval

$$VaR_{0.95} \in [0.0226, 0.0278],$$

showing estimator stability.

7 Portfolio Diversification

An equally weighted portfolio of Reliance and HDFC Bank was formed:

$$R_p = 0.5R_1 + 0.5R_2.$$

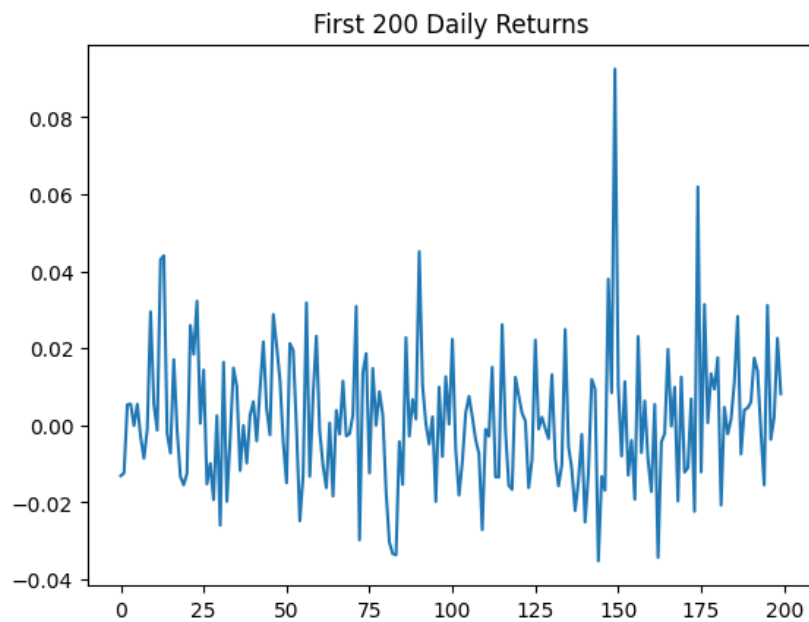The portfolio VaR was 0.0208, lower than both individual stocks, confirming diversification benefits.

Figure 6: First 200 daily log-returns of Reliance.

8 Conclusion

Empirical and model-based risk measures differ significantly, highlighting the importance of non-parametric techniques in quant risk modelling.