Date: 03|04|2025

[Linear algebra for Machine learning & data science]

week 1

~~linear equation~~ (1 power) $\boxed{xa+b=c}$

$\boxed{m \times speed + b = output}$

$$w_1x_1^{(1)} + w_2x_2^{(1)} + w_3x_3^{(1)} + \cdots + w_nx_n^{(1)} + b = y^{(1)}$$

Here superscript 1 denotes (one example) or you could say no. of examples eg.

different ← x_1^1 & x_1^2

$$w_1x_1^{(2)} + w_2x_2^{(2)} + w_3x_3^{(2)} + \cdots + w_nx_n^{(2)} + b = y^{(2)}$$

Here superscript 2 represent 2nd example of dataset

similarly

$$w_1x_1^{(m)} + w_2x_2^{(m)} + \cdots + w_nx_n^{(m)} + b = y^{(m)}$$

where m = mth example of the dataset

$w \rightarrow$ weights $(w_1, w_2, w_3 \ldots w_n)$ weights of n features

$b \rightarrow$ bias term $\quad n \rightarrow$ no. of features

$y \rightarrow$ output (label) $\quad x \rightarrow$ features (like $x_1 \rightarrow f1 \ldots$)

There all equations forms a "system of linear equations"

⇑

This ~~model~~ is saying "Give me a set of x's and I can estimate a value for y because I have a model that tells me what all the W's & B's are"

Instead of writing this long form we can also say this:

$[y^{(1)}\ y^{(2)} \ldots y^{(m)}]$

matrix

$$[w_1\, w_2\, w_3 \ldots w_n] \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} & \cdots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} & \cdots & x_n^{(2)} \\ x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & x_4^{(3)} & \cdots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & x_4^{(m)} & & x_n^{(m)} \end{bmatrix}$$

vector $\qquad\qquad$ ↑ vector

$$W \cdot X + b = \hat{Y}$$

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I have a vector of weights called W i.e made up of w_1 w_2 ... and I multiply that by each row of features X in my matrix of features which I now call X & then I add a bias term & set that all equal to $\hat{y}$ which is a vector of my target variable

1. System of sentences behaves a lot like system of equations ~~sentences~~ (to give information)

System 1

The dog is black
The cat is orange
→ $\boxed{\text{complete system}}$
Two sentences → Two piece of information
– non-singular system

In this system (example) you assume you only have one dog and one cat & they're both of only one color.

System 2

The dog is black
The dog is black
→ $\boxed{\text{Redundant system}}$ — singular sys.
two sentences → same information
one piece of information

S1 → The dog is black, The cat is orange, The bird is red
→ complete, non-singular

S2 → The dog is black, The dog is black, The bird is red
→ redundant, singular

S3 → The dog is black × 3
→ Redundant, singular

S4 → The dog is black, The dog is white, The bird is red → contradictory, singular

Here S_3 is more redundant than S_2

System 3

The dog is black
The dog is white
→ $\boxed{\text{contradictory}}$ sen...
2 piece of informa...
(cause you only ha... col...

A non-singular system is a system ... many pieces of information as ... most informative system it ...

A singular system is less informative than a non-singular.

A system can carry as much no. of sentences (2, 3 ... n)

Is there a measure of how redundant a system is → yes
It is called a rank

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2. System of linear equations

system of equations | solution

$a+b+c=10$ | $a=3$
$a+2b+c=15$ | $b=5$
$a+b+2c=12$ | $c=2$

→ (non-singular) complete system

(unique solution)

(singular)

$a+b=10$
$2a+2b=20$

→ Redundant system

$\boxed{\text{solution}}$ → infinitely many solution

$a=8$ $b=2$ or
$a=5$ $b=5$ or
$a=8.3$ $b=1.7$ or
$a=0$ $b=10$

→ infinitely many solution

cause you don't have enough information

(singular)

$a+b=10$
$2a+2b=24$

→ contradictory system

$\boxed{\text{solution}}$ → no solution..

System 1
$a+b+c=10$
$a+b+2c=15$
$a+b+3c=20$

Infinitely many sols. (Redundant) (less)
$c=5$, $a+b=5$
$(0,5,5), (1,4,5)\ldots$

System 2
$a+b+c=10$
$a+b+2c=15$
$a+b+3c=18$

NO solutions (Contradictory)
From eq 1 & 2 $c=5$
From eq 2 & 3 $c=3$

System 3
$a+b+c=10$
$2a+2b+2c=20$
$3a+3b+3c=30$

Infinitely many sols. (Redundant) (more)
Any 3 no's that add to 10 work $(0,0,10)$ $(2,7,1)\ldots$

3. What is a linear equation?

linear	non-linear
$a+b=10$	$a^2+b^2=10$
$2a+3b=15$	$\sin(a)+b^5=15$
$3.4a-48.99b+2c=122.5$	$2^a-3^b=0$
	$ab^2+\frac{b}{a}-\frac{3}{b}-\log(c)=4^9$

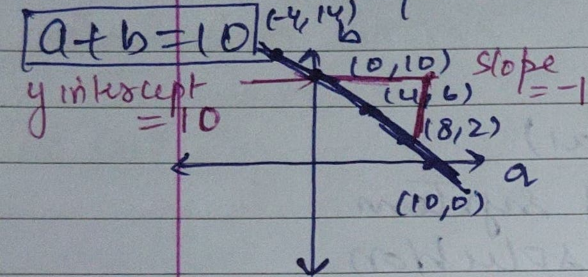
4. System of equations as lines & planes

If two variables visualize them ~~as~~ as lines on plane
If three variables they are plane in space. If more variables they look like high dimensional things.

Since linear equations can be represented as lines then systems of linear equations can be represented as arrangements of lines on the plane.

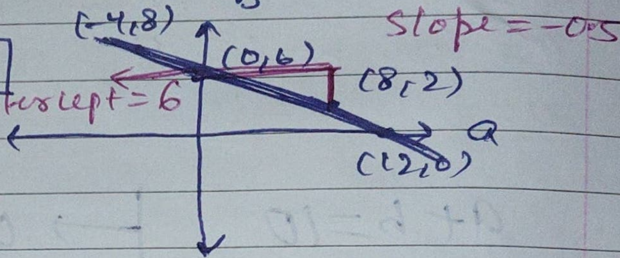
linear equation $\rightarrow$ line

$\boxed{a+b=10}$

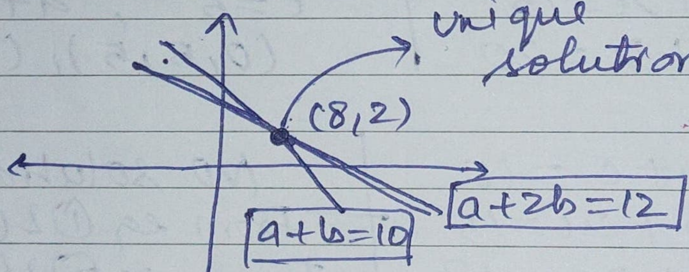


$\boxed{a+2b=12}$

y intercept = 6

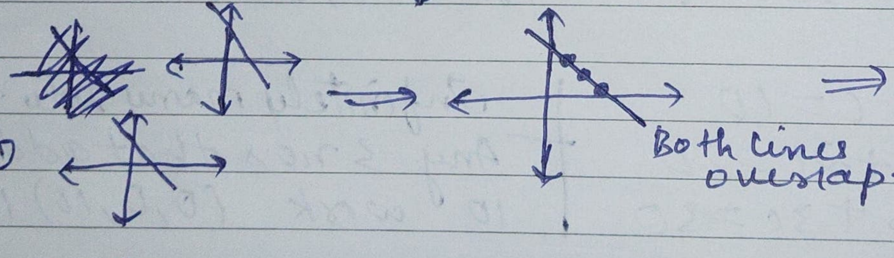


system of linear equation



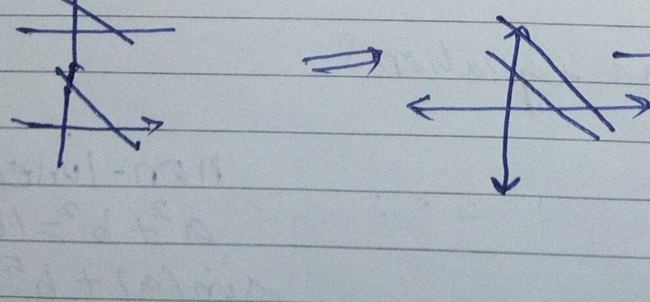
(Complete) sentence
$\Rightarrow$ non-singular

$a+b=10$
$2a+2b=20$



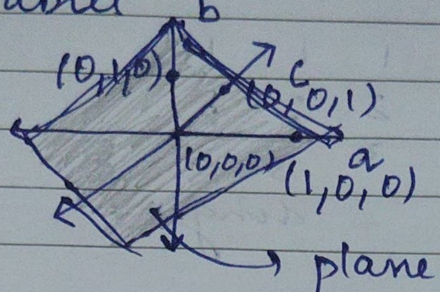
$\Rightarrow$ Infinitely many sol. (Redundant)
$\Rightarrow$ singular.

$a+b=10$
$2a+2b=24$



$\rightarrow$ parallel lines $\Rightarrow$ no solutions (Contradictory)
$\Rightarrow$ singular

Three Variables



$a+b+c=1$

Points

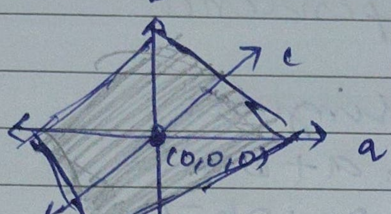
$(0,0,1)=1$

$(1,0,0)=1$

$(0,1,0)=1$

Three points define a plane

An entire plane that passes through those three points is the set of sol^n of the eq^n $a+b+c=1$



$3a-5b+2c=0$

$3(0)-5(0)+2(0)=0$

Where the constant of the eq^n is 0 ~~to~~ the plane must go through origin

5. A geometric notion of singularity

In singular the lines are $\parallel$ or overlapping. In conclusion the constants in the system don't matter when it comes to determine if the system is singular or non-singular

6. Singular vs. non-singular matrixes

matrices arise from different places in math, In this case they will arise from the coefficients in a system of eq^n.

System 1

$a+b=0$

$a+2b=0$

$$\begin{array}{cc} a & b \\ 1 & 1 \\ 1 & 2 \end{array}$$

non-singular system
– non singular matrix (Unique sol^n)

System 2

$a+b=0$

$2a+2b=0$

$$\begin{array}{cc} a & b \\ 1 & 1 \\ 2 & 2 \end{array}$$

singular system
– singular matrix (Infinitely many sol^n)

system 1
$a+b+c=0$
$a+2b+c=0$
$a+b+2c=0$

$$\begin{matrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{matrix}$$

non-sing

system 2
$a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$

$$\begin{matrix}1 & 1 & 1\\ 1 & 1 & 2\\ 1 & 1 & 3\end{matrix}$$

sing

system 4
$a+b+c=0$
$2a+2b+2c=0$
$3a+3b+3c=0$

$$\begin{matrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{matrix}$$

sing

7. linear dependence & Independence

non-singular
$a+b=0$
$a+2b=0$

$$\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$

no eq^n is a multiple of the other one

No row is a multiple of the other one

$\boxed{\text{Rows are linearly independent}}$

singular
$a+b=0$
$2a+2b=0$

second eq^n is a multiple of the first one

$$\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}$$

2nd row is a multiple of 1st row

$\boxed{\text{Rows are linearly dependent}}$

$a=1 \longrightarrow a+0b+0c=1$
$b=2 \qquad 0a+b+0c=2$
$a+b=3 \qquad a+b+0c=3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$$

Row3 = Row1 + Row2

Row3 depends on Row1 & Row2

$\boxed{\text{Rows are linearly dependent}}$ → Singular matrix

$a+b+c=0 \longrightarrow a+b+c=0$
$2a+2b+2c=0 \qquad +\ 2a+2b+2c=0$
$3a+3b+3c=0 \qquad \overline{3a+3b+3c=0}$

System
highly singular matrix with lot of dependency.

$$\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$$

Row3 = Row1 + Row2
&
$2\times R_1 = R_2$
$3\times R_1 = R_3$

∴ $\boxed{\text{Rows are linearly dependent}}$

$a+b+c=0$
$a+b+2c=0$
$a+b+3c=0$

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 2\\ 1 & 1 & 3\end{bmatrix}$$ → singular matrix

Here Row2 is the avg of sum of Row1 & Row3

$a+b+c=0$
$\underline{a+b+3c=0}$
$2a+2b+4c=0 \div 2 = a+b+2c=0$

Rows are linearly dependent

Examples ①

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{matrix}$$

Row3 = $3R_1 + 2R_2$
⇒ singular (dependent)

②

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{matrix}$$

Row3 = $R_1 - R_2$
⇒ singular (dependent)

③

$$\begin{matrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{matrix}$$

No relations
Non-singular (independent)

④

$$\begin{matrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{matrix}$$

$R_3 = 2R_1$ singular. (dependent)

8- Determinant For singular $D=0$ & For non-singular $D \neq 0$

Non-singular matrix

$$\begin{matrix} 1 & 1 \\ 1 & 2 \end{matrix}$$

Determinant ⇒ $2-1=\boxed{1}$

Singular matrix

$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix}$$

Determinant ⇒ $2-2=\boxed{0}$

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

matrix is singular if
$a \ b * K = c \ d$
$aK = c$
$bK = d$
$\frac{c}{a} = \frac{d}{b} = K$
$bc = ad \Rightarrow \boxed{ad - bc = 0}$ Determinant

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix}$$

For 3x3 matrix

$1(4-1) - 1(2-1) + 1(1-2)$
$3-1-1=1$

If all the elements under the main diagonal is zero then the Determinant is the product of all elements of the main diagonal

$$\begin{matrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{matrix}$$