# Reinforcement Learning

CS771: Introduction to Machine Learning

Purushottam Kar

# Announcements



**Doubt clearing session**: November 17 (Sunday), **6:00PM** KD101

**Practice end-sem paper** now available on website

**End-sem Exam**: November 18 (Monday), 4-7PM, L19, L20

*Syllabus is till whatever we cover today i.e. Nov 15 (Fri)*

*Bring your **institute ID card** with you – will lose time if you forget*

*Bring a **pencil, pen, eraser, sharpener** with you – we wont provide!*

*Answers to be written on question paper itself. If you write with pen and make a mistake, no extra paper. Final answer **must be in pen***

***Auditors cannot appear** for the mid-sem exam*

**Seating for end-sem exam**: assigned seating (announced on Piazza)

*Everyone has been assigned a lecture hall, and a seat number in that hall*

*All must go to their assigned hall – no extra space if you go to wrong hall*

# Reinforcement Learning



We have an environment $\mathcal{E}$ with which our ML/RL algo can interact

If we were to take a recsys example, a user may be our "environment"

*At any point of time* $t$*, env is in a certain* state $s_t \in \mathcal{S}$

The user has certain likes and certain dislikes

*Algo can perform an action* $a_t \in \mathcal{A}$ *from a set of allowed* actions

Recommend one of the products currently available for sale on the website

*In response,* $\mathcal{E}$ *changes its state to* $s_{t+1}$ *and gives algo a reward* $r_t$

Reward is a click/purchase, state change may be user discovering a new liked item

Goal: keep playing this game and maximize total reward $\sum_{t \geq 1} r_t$

Simplified form

*Current state of environment completely visible to algo – not true in RecSys*

*Next state* $s_{t+1}$ *a deterministic function of* $s_t, a_t$ *but this function is unknown*

*Reward* $r_t$ *depends only on next state* $s_{t+1}$ *e.g. if Mario jumps on a Koopa*

# Markovian Decision Process (MDP)



A more general formulation of this is an MDP

*Given action $a_t$ when in state $s_t$, next state/reward is not entirely deterministic*

*Next state is $s$ with prob $\mathbb{P}_{a_t}(s_t, s)$ - this distribution is not known to algo*

*Reward may depend on current + next state + action i.e. $r_t = R_{a_t}(s_t, s_{t+1})$*

A still more general formulation is a partially observed MDP (POMDP)

*In a POMDP, even the current state of $\mathcal{E}$ visible only partially to the algo*

**Policy**: a rule that tells us which action to take on which state $\pi: \mathcal{S} \rightarrow \mathcal{A}$

*RL algorithms try various ways to learn the best policy that maximizes reward*

*Definitely want to take an action $a_t$ such that $r_t$ is big but also want to ensure that we land on a state after taking this action so that $r_{t+1}, r_{t+2}, \ldots$ also big*

*Would be naïve to take action that gives large $r_t$ but subsequent rewards $0$ ☹*

# Value Function



Given a policy $\pi: \mathcal{S} \rightarrow \mathcal{A}$, we assign a *value* to this policy

*Tells us how much reward we get if we always play actions dictated by policy* $\pi$

*Assume for simplicity that state transitions and rewards are deterministic*

***Notation****: action* $a$ *in state* $s$ *takes us to state* $T(s,a)$ *and gives reward* $R(s,a)$

Notation: let $s_0 \coloneqq s$ be first state, $s_1 = T\big(s_0, \pi(s_0)\big)$ be state after playing according to $\pi$ for one step, $s_2 = T\big(s_1, \pi(s_0)\big)$ be state after playing using $\pi$ for two steps and so on …

*Possible defn of value* $R\big(s_0, \pi(s_0)\big) + R\big(s_1, \pi(s_1)\big) + R\big(s_2, \pi(s_2)\big) + \cdots$

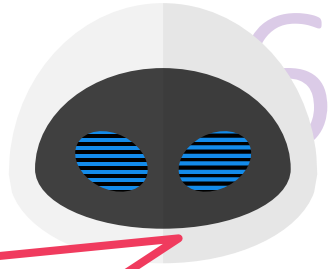
*Usually we care less about future rewards so* discount *them – let* $\gamma \in (0,1)$

$$V^{\pi}(s_0) \triangleq R\big(s_0, \pi(s_0)\big) + \gamma \cdot R\big(s_1, \pi(s_1)\big) + \gamma^2 \cdot R\big(s_2, \pi(s_2)\big) + \cdots$$

Note that we have $V^{\pi}(s_0) = R\big(s_0, \pi(s_0)\big) + \gamma \cdot V^{\pi}(s_1)$

*Called the* Bellman Equation. *For settings where state transitions and rewards are probabilistic, we replace the above using expectations*

As is obvious, RL algos try to learn policies that have high values for all states. For a given state $s$, $V^*(s)$ denotes the highest value obtainable when starting from that state (using any conceivable policy) i.e. $V^*(s) = \max_{\pi} V^{\pi}(s)$. Also let $\pi^*(s) = \arg\max_{\pi} V^{\pi}(s)$ denote the optimal policy which gets this value. Note that $V^*$ satisfies a recurrence $V^*(s) = \max_{a \in \mathcal{A}} \{R(s,a) + \gamma \cdot V^*(T(s,a))\}$



*by policy $\pi$*

*Assume for simplicity that state transitions and rewards are deterministic*

***Notation**: action $a$ in state $s$ takes us to state $T(s,a)$ and gives reward $R(s,a)$*

Not … $\pi$

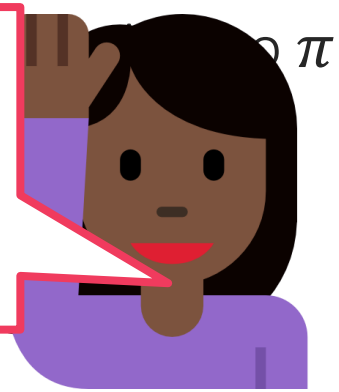
for …

*Possi…*

*Usua…*

$V^{\pi}(s_0) \triangleq R(s_0, \pi(s…$

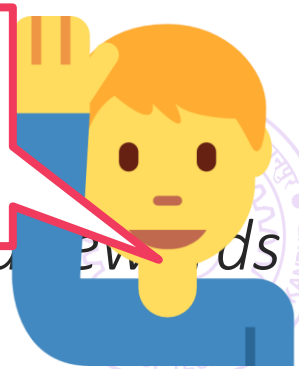
If someone could give us this magical optimal value function $V^*: \mathcal{S} \to \mathbb{R}$, then some math shows that we can come up with the best policy $\pi^*$ by simply defining $\pi^*(s) = \arg\max_{a \in \mathcal{A}} V^*(T(s,a))$. For probabilistic rewards and transitions, we use expectations in above definitions.



Note that we have $V$…

*Called the* Bellman Equation. *For settings where state transitions and rewards are probabilistic, we replace the above using expectations*

We can also show that if someone gives us the optimal policy $\pi^*$ then we could easily obtain the optimal value function $V^*$ by using the Bellman equations



# Value and Policy Iteration Algos



**Method 1**: **Value Iteration** (used if #states, #actions is small)

*Get a good estimate* $\hat{V}(s)$ *of* $V^*(s)$ *for all* $s \in \mathcal{S}$ *using pure exploration*

*Use this to define a policy* $\hat{\pi}(s) = \arg\max_{a \in \mathcal{A}} \{\hat{V}(T(s,a))\}$

*Not very nice just as pure exploration was not nice for bandits*

*Not feasible if number of states is large or action sets are large*

**Method 2**: **Policy Iteration** (alternating optimization)

*Start with a random policy* $\pi$*, find its value function* $V^{\pi}(\cdot)$ *using Bellman eqns*

*Update the policy as* $\pi(s) \leftarrow \arg\max_{a \in \mathcal{A}} \{V^{\pi}(T(s,a))\}$

*Repeat till converged or else till budget allows*

*Also not feasible for large state/action spaces and wasteful*

These are unsafe algos – will sacrifice many Marios while learning ☹

*Becomes unacceptable if we replace Mario with a real robot/drone*

# Q Learning



Reduces RL to regression/multiclass problems

Define a $Q$ function as follows

$$Q(s,a) = R(s,a) + \gamma \cdot V^*\big(T(s,a)\big)$$

*Recall that this means that* $V^*(s) = \max_{a \in \mathcal{A}} Q(s,a)$

*Note that this also means that the optimal action is* $\arg\max_{a \in \mathcal{A}} Q(s,a)$

Use a powerful ML algo (kernels, deepnets) to do one of the following

*Given (vector representations of) a state $s$ and action $a$, predict $Q(s,a)$*

Works when too many (infinite) states as well as actions

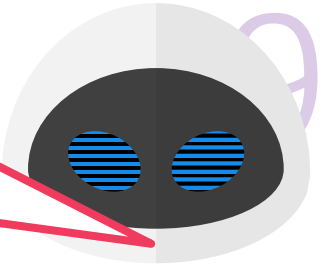
*Given (vector representation of) a state $s$, solve the multiclass problem of predicting the best action i.e.* $\arg\max_{a \in \mathcal{A}} Q(s,a)$

Works when too many (infinite) states but only a few actions

Note that this approach avoids value function/rewards altogether ☺

# Q Learning

Deep Q learning is a leading technique in reinforcement learning for several application domains – very convenient



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# Episodic RL



In certain settings, instantaneous rewards i.e. rewards being given on every single action taken by algo is infeasible

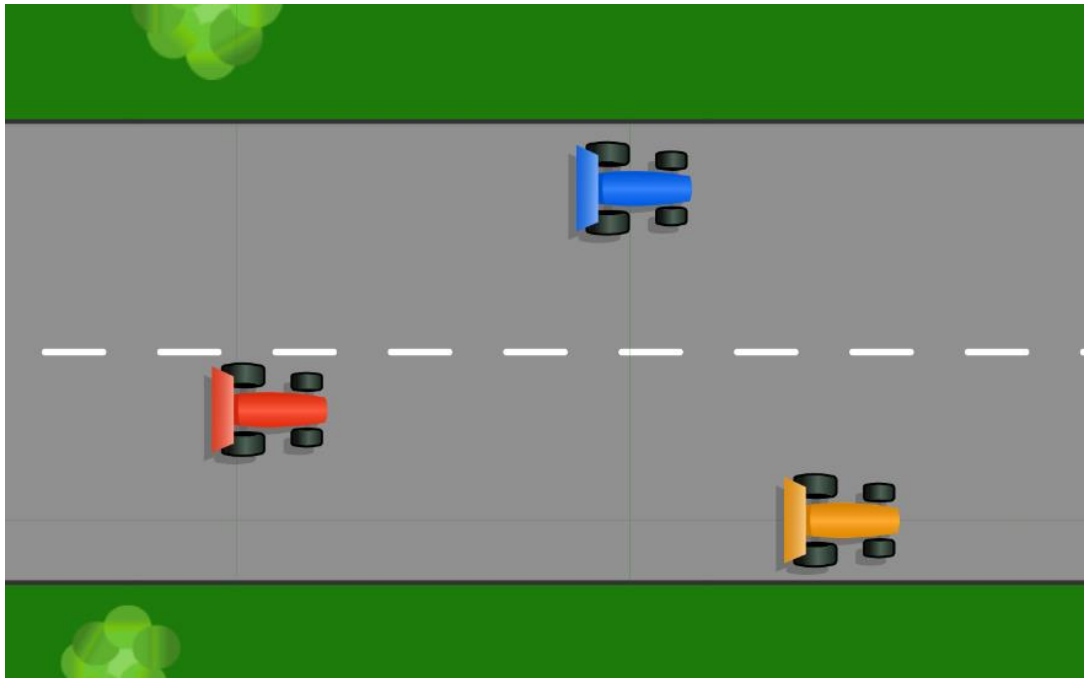
- *E.g. game of chess, reward is win/loss/tie only available at end of game*
- *E.g. Robot navigation, some instantaneous reward e.g. following a predefined trajectory, not falling into a ditch, but eventual reward only on reaching goal*

In such settings, usually there is a *terminal* or *goal* state

- *Entire or large bunch of reward given when reaching that state*
- *Period from start state to one such terminal state is called an* episode
- *We usually do not discount rewards in episodic tasks*

# RL for Playing Video Games





**States**: can be encoded as distance of our car from other cars and our current speed

More challenging to take image pixels as state and use a CNN to infer these distances first

**Actions**: move ↑/↓, speed ↑/↓, do nothing

**Rewards**: various forms can be offered

*Small instantaneous reward for surviving one frame*

*Small reward for overtaking a competitor car*

*Large penalty for crashing onto another car*

Since number of states is small here, various techniques are at least feasible here e.g. value iteration, policy iteration, Q learning

RL gained popularity for playing Atari games

HTTPS://WWW.ARCADEMICS.COM/GAMES/GRAND-PRIX

MNIH ET AL. PLAYING ATARI WITH DEEP REINFORCEMENT LEARNING, NIPS 2013

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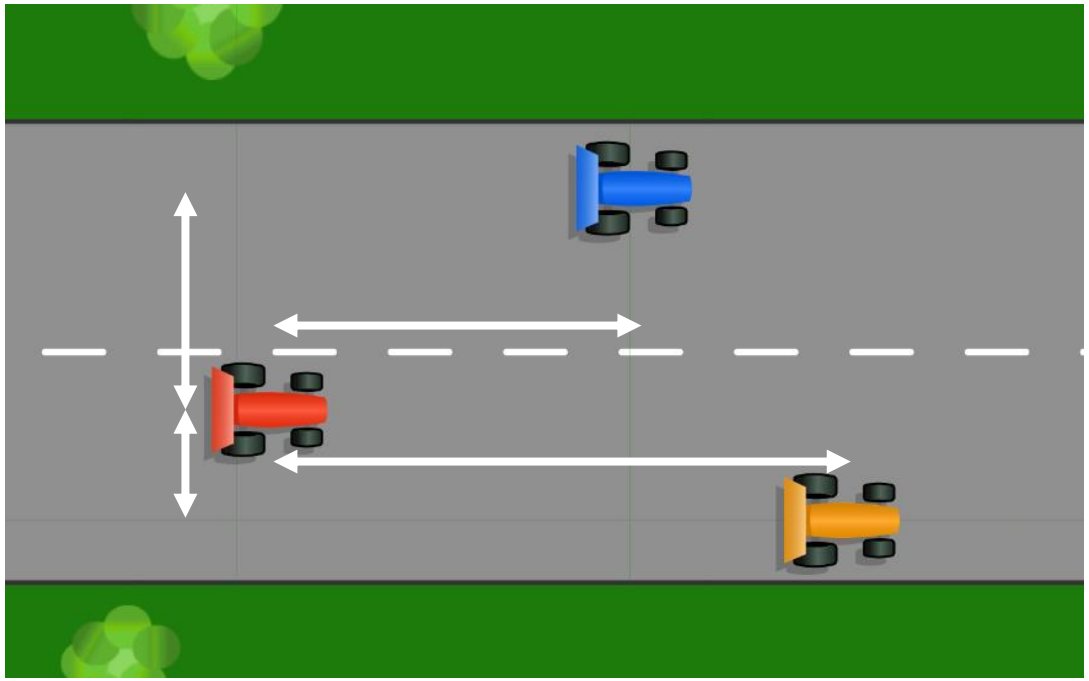

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# RL for Adaptive Control



RL is widely used for controlling motors and other actuators in AE, ME applications

*Extremely challenging to fly a helicopter upside down*

*Required modifications to helicopter (8kgs) e.g. more powerful rotor, compass, gyroscope, accelerometer etc*

**States**: feedback from all these sensors

**Actions**: rotors controlling pitch (fw/bw, left/right), thrust, turning (yaw)

Human pilot was asked to demonstrate

*Was used to learn state transitions $P_{sa}$ i.e. the MDP*

**Rewards**: deviation of helicopter from set trajectory (Q learning used)

Several applications to robotics as well

NG ET AL, AUTONOMOUS INVERTED HELICOPTER FLIGHT VIA REINFORCEMENT LEARNING, EXPERIMENTAL ROBOTICS IX, 2006

# RL for Program Repair



```
1  #include<stdio.h>
2  int main(){
3    float ti, tax;
4    scanf ( "%f" ; &ti);
5    if(ti<200001){
6      printf("ti=0");}
7    else if(200000<ti && ti<500001){
8      tax=0.1*(ti-200000);
9      printf("%.2f", tax);}
10   else if(500000<ti && ti<1000001){
11     tax=30000+0.2*(ti-500000);
12     printf ( "%.2f" , tax ) ;
13   else if(ti>1000000){
14     tax=130000+0.3*(ti-1000000);
15     printf("%.2f", tax);}
16   return 0;}
```

$e_1$

$e_2$

**States**: encoding of program as string of tokens and posn of cursor within that string

**Actions**: two types of actions

- Navigation*: move the cursor around (these actions do not change the program at all)*
- Edit*: insert/delete/replace token at cursor posn*

**Rewards**: episodic learning

- *Full reward given when all errors removed*
- *Partial reward for removing one error*

*NN used to get low-dim representation of states*

*Linear models used to predict the best action by treating it as a multi-class problem*

GUPTA ET AL, DEEP REINFORCEMENT LEARNING FOR SYNTACTIC ERROR REPAIR IN STUDENT PROGRAMS, AAAI 2019

# RL for Planning and Routing



VALADARSKY ET AL, LEARNING TO ROUTE WITH DEEP RL, NIPS 2017

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Optimize route of delivery vehicle so that food/items can be delivered to maximum users in single round trip

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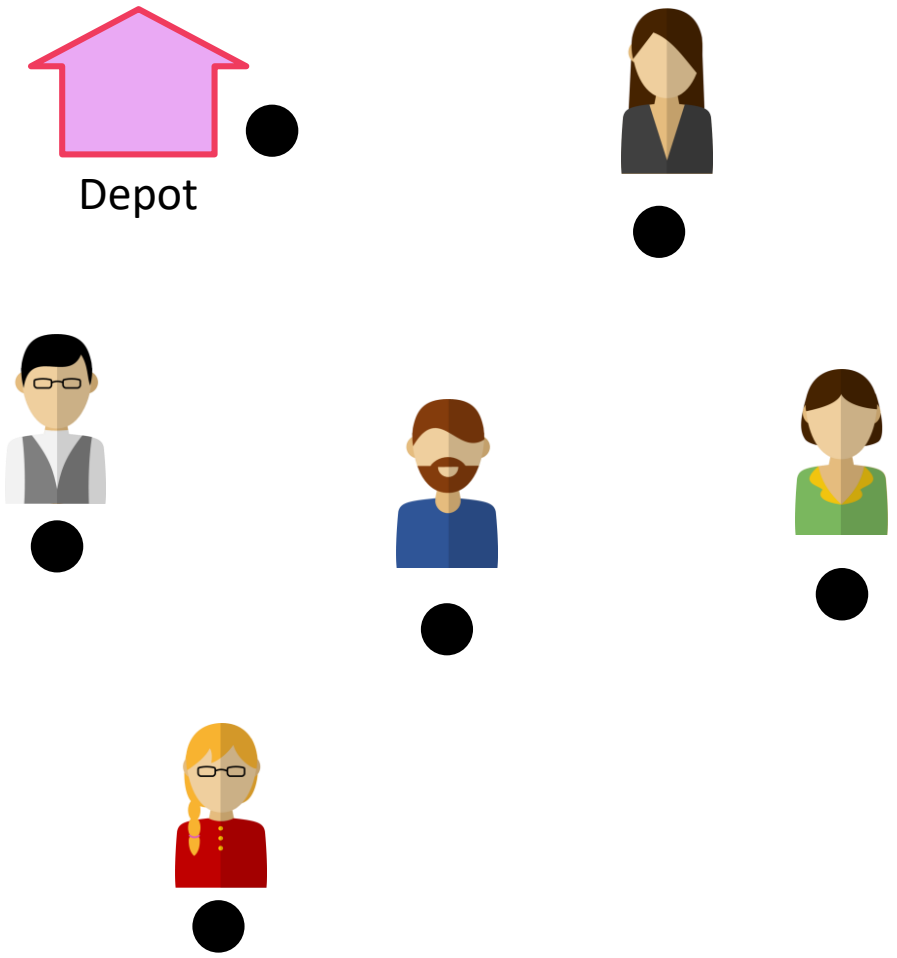


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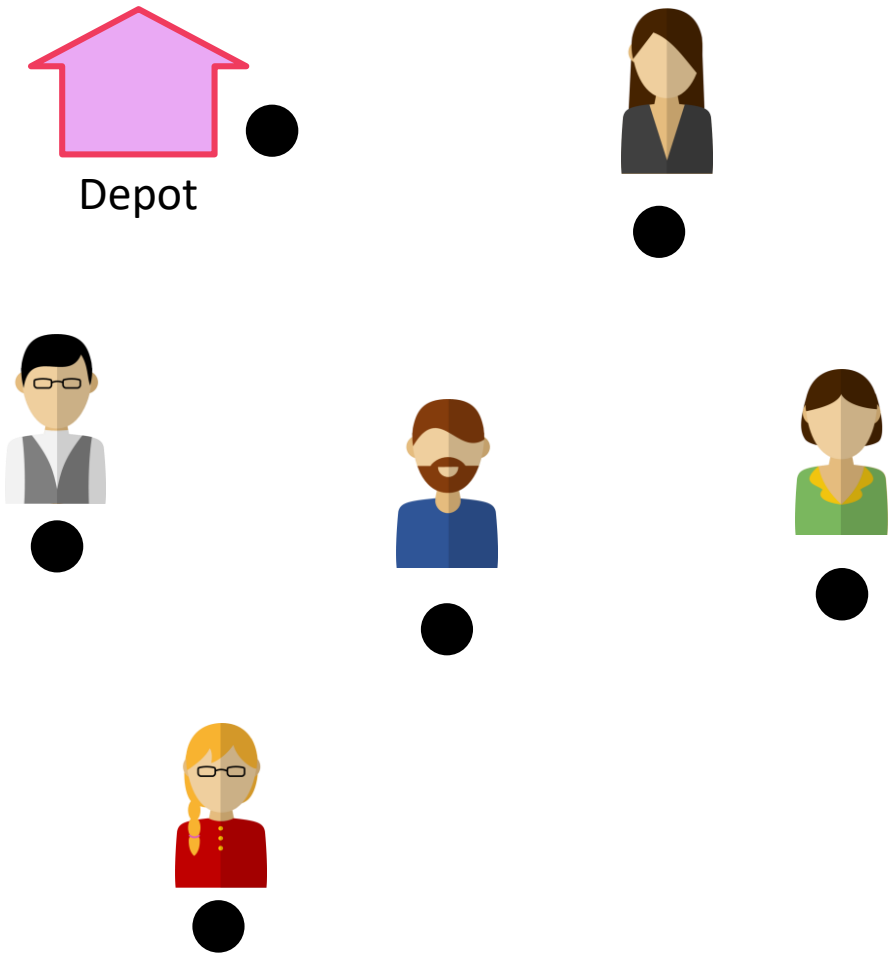


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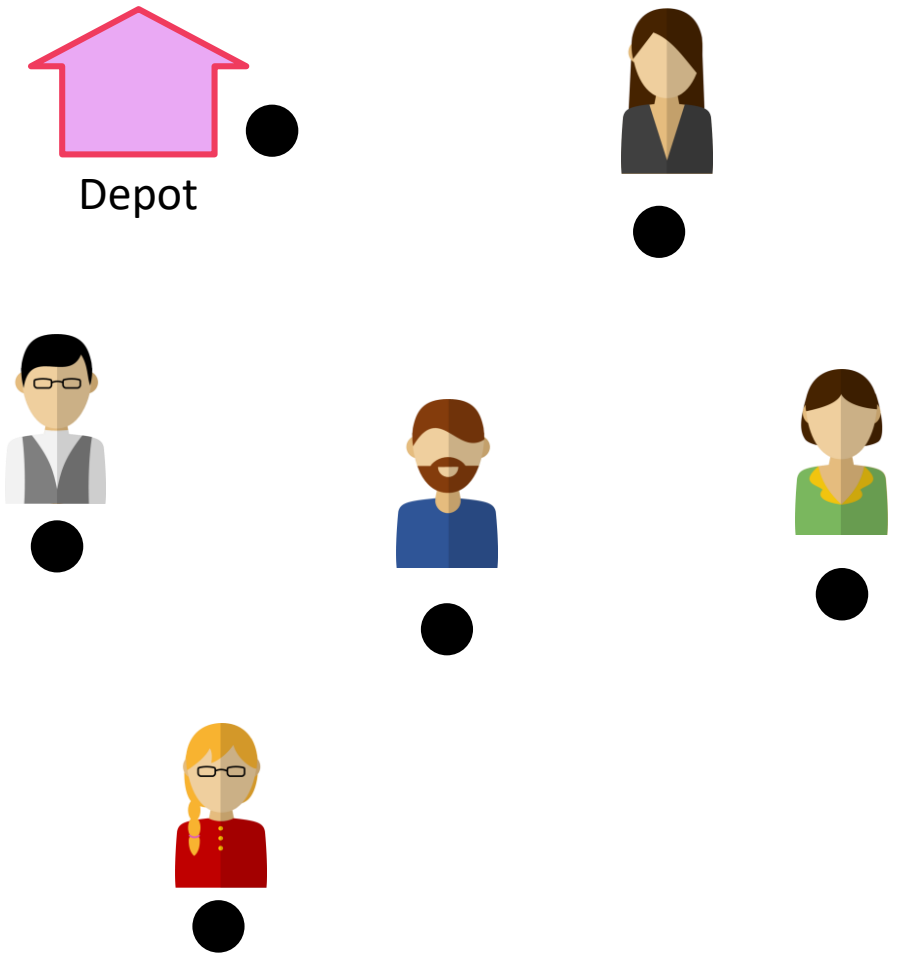


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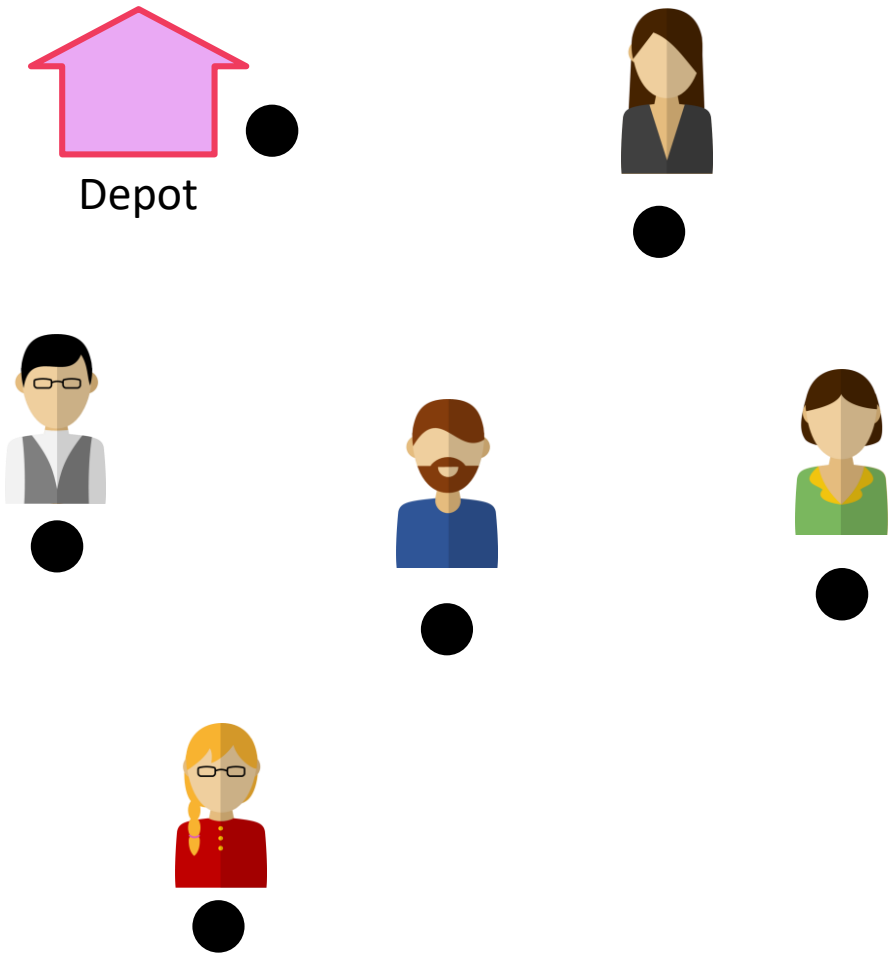


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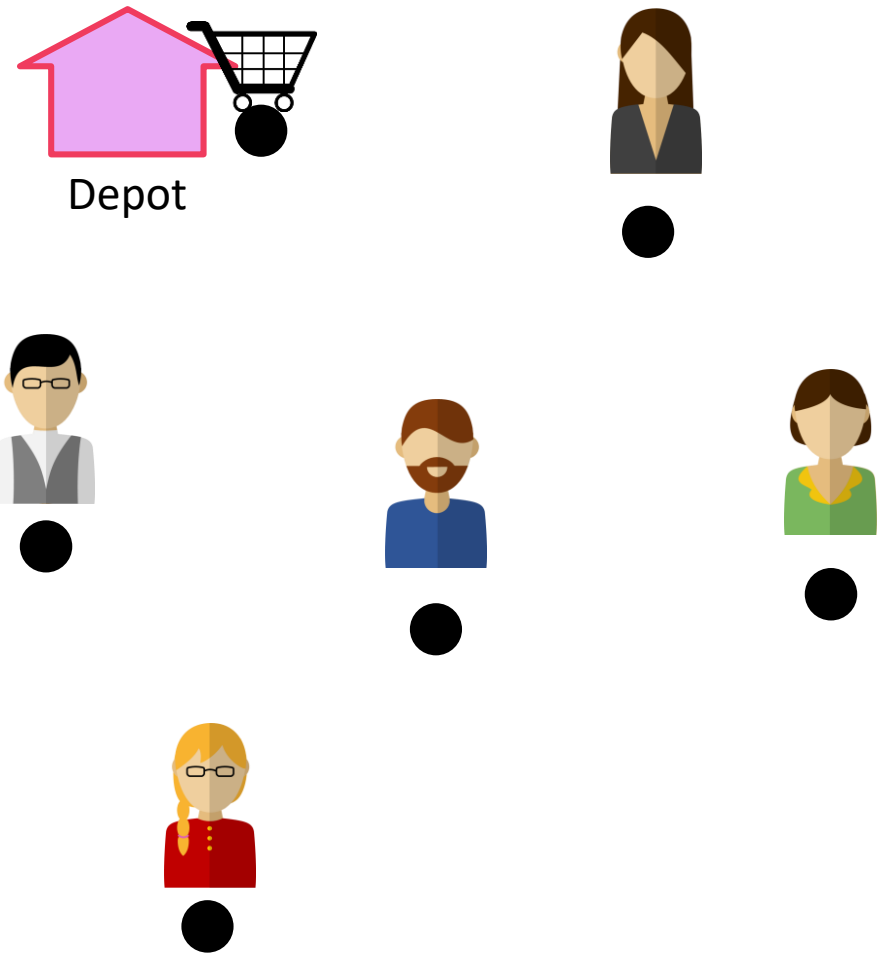


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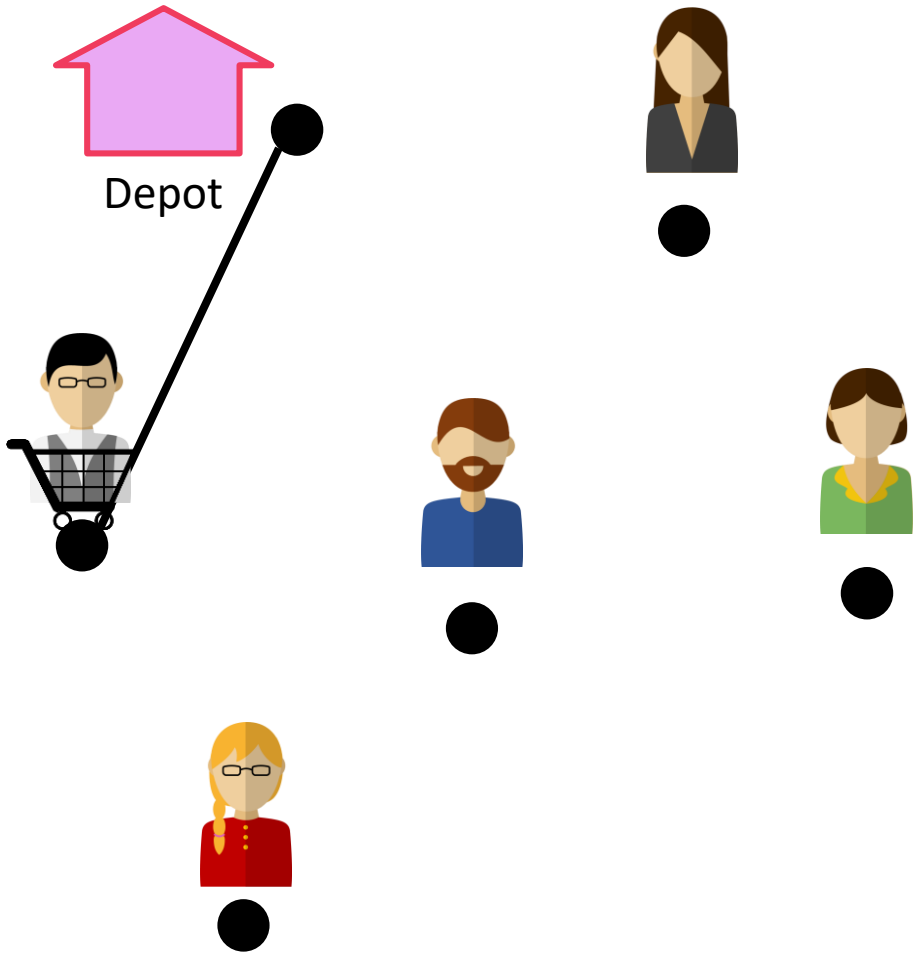


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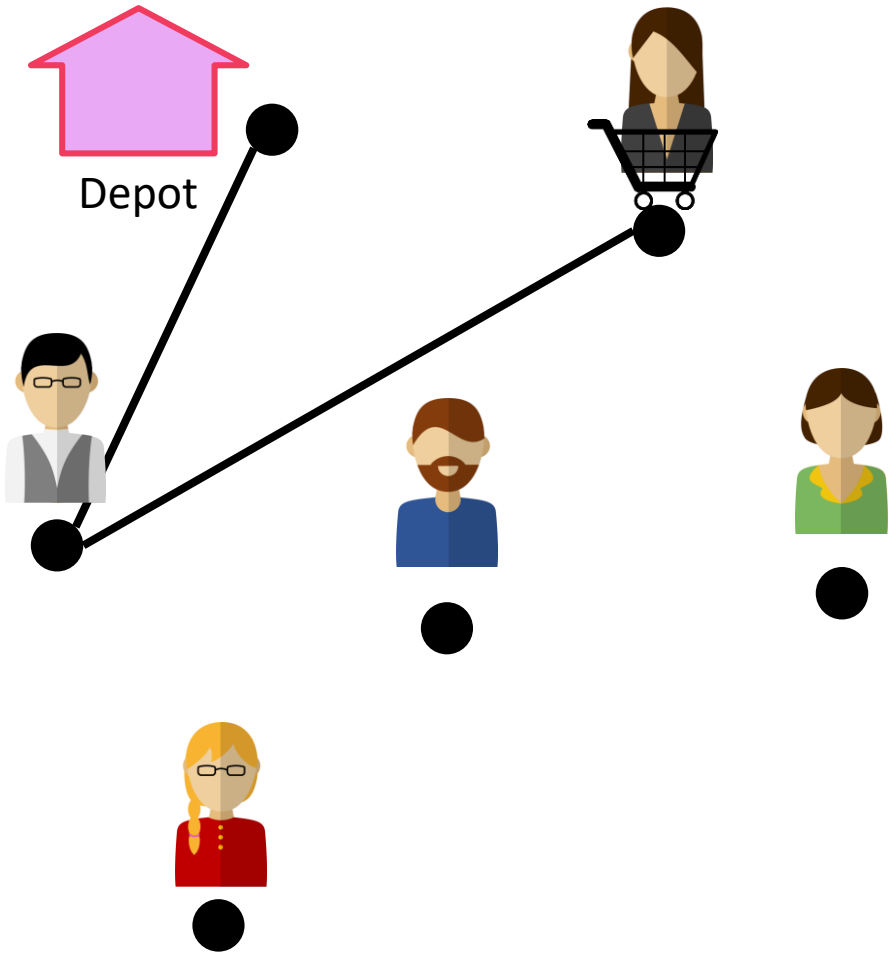


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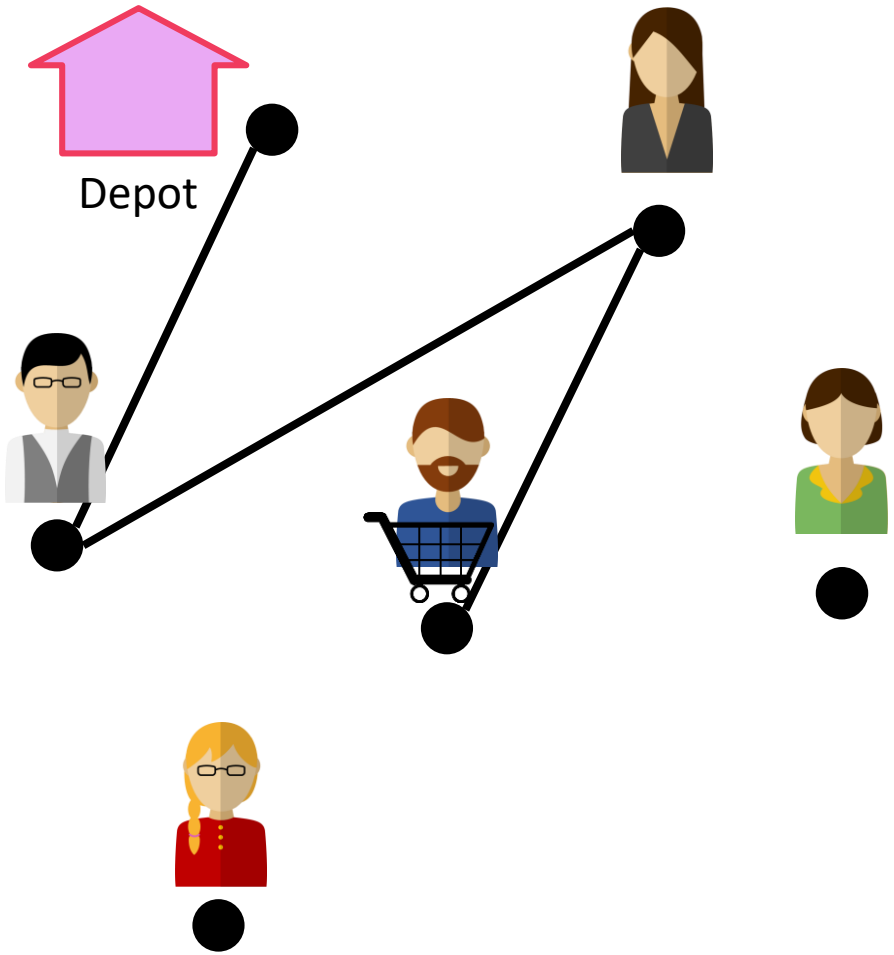


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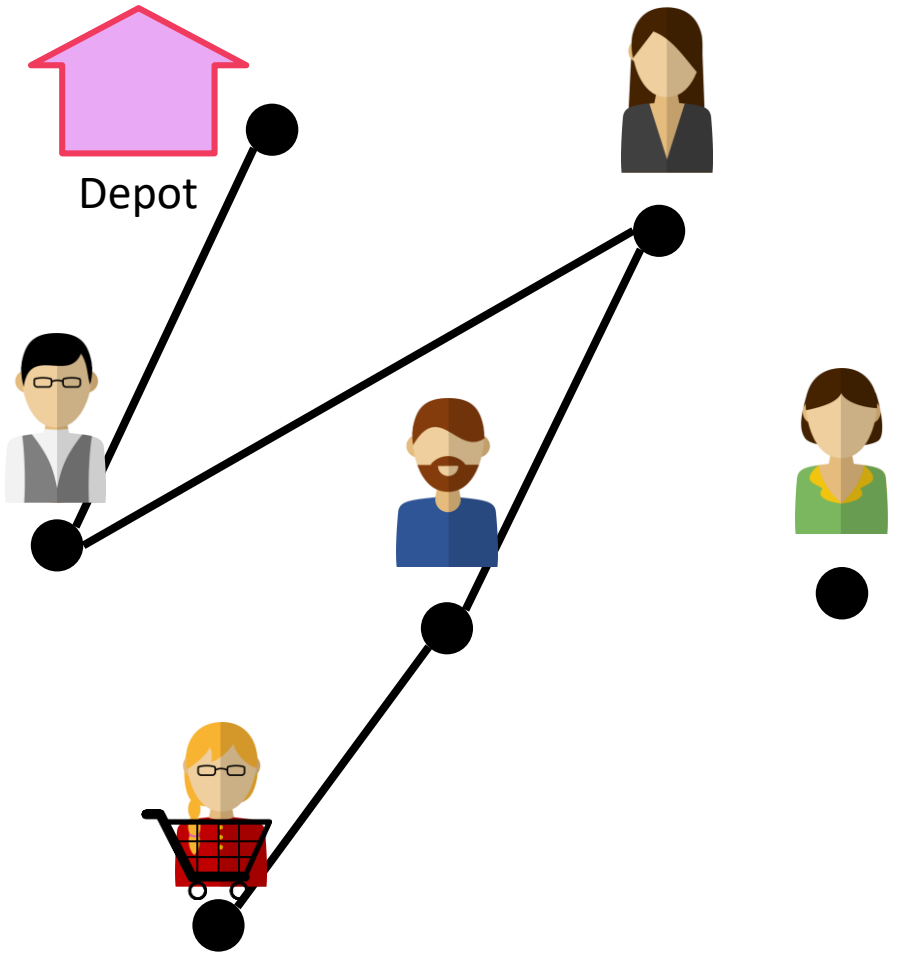


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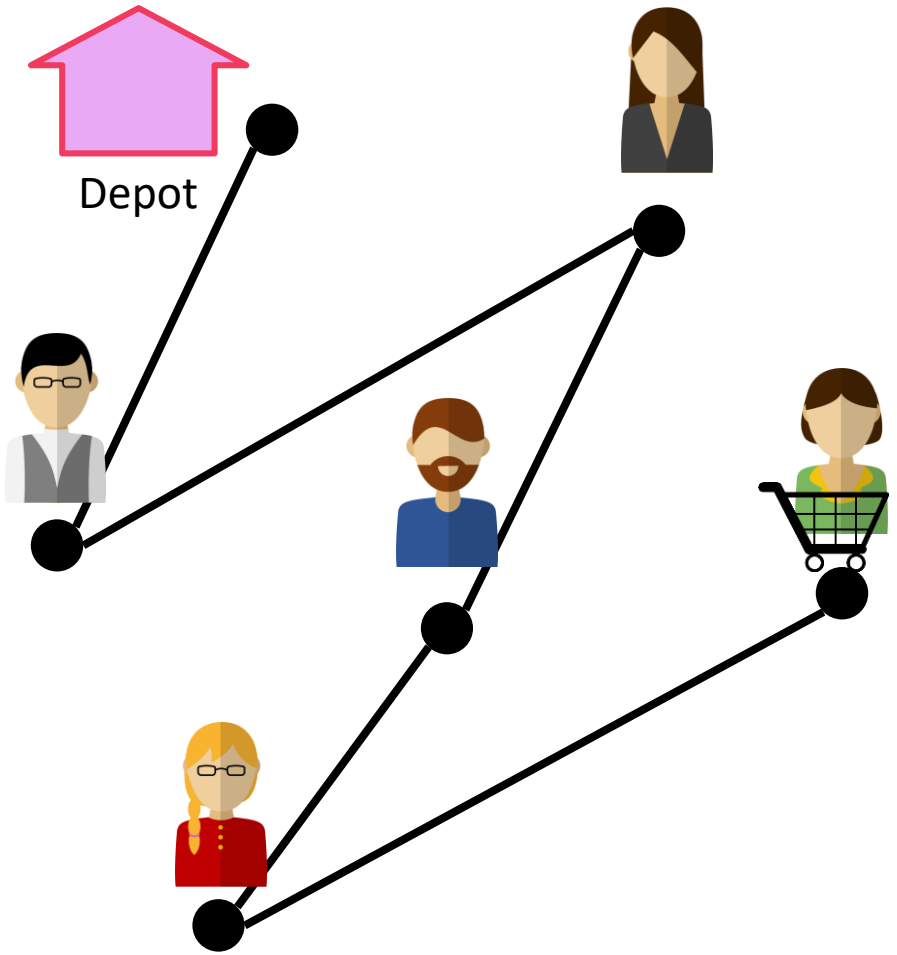


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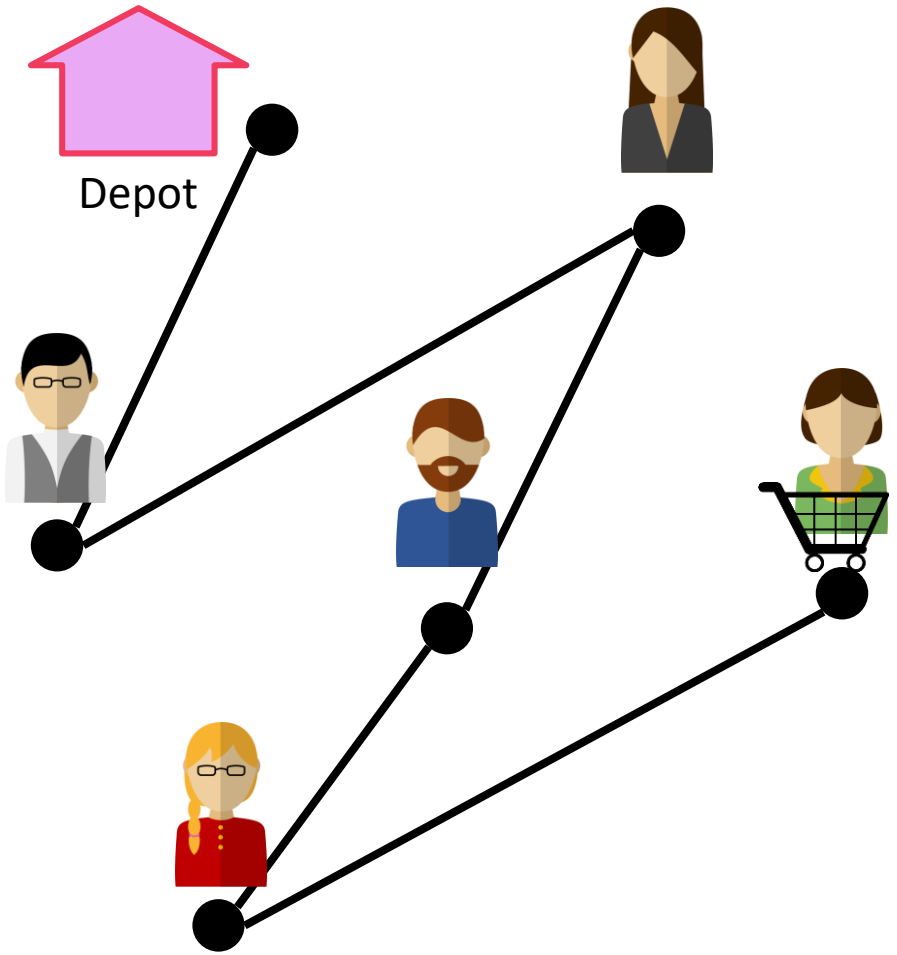


One of the earliest applications of RL

*Boyan and Littman, NIPS 93 – proposed Q learning*

Recent works: delivery vehicle and network routing

Optimize route of delivery vehicle so that food/items can be delivered to maximum users in single round trip

**States**: encoding of user locations, (remaining) demand for each user, (current) traffic conditions

*Our actions change the state by fulfilling (some) user demands but can be assumed to not change traffic conditions*

**Actions**: given state, find next user to serve

**Rewards**: instantaneous rewards

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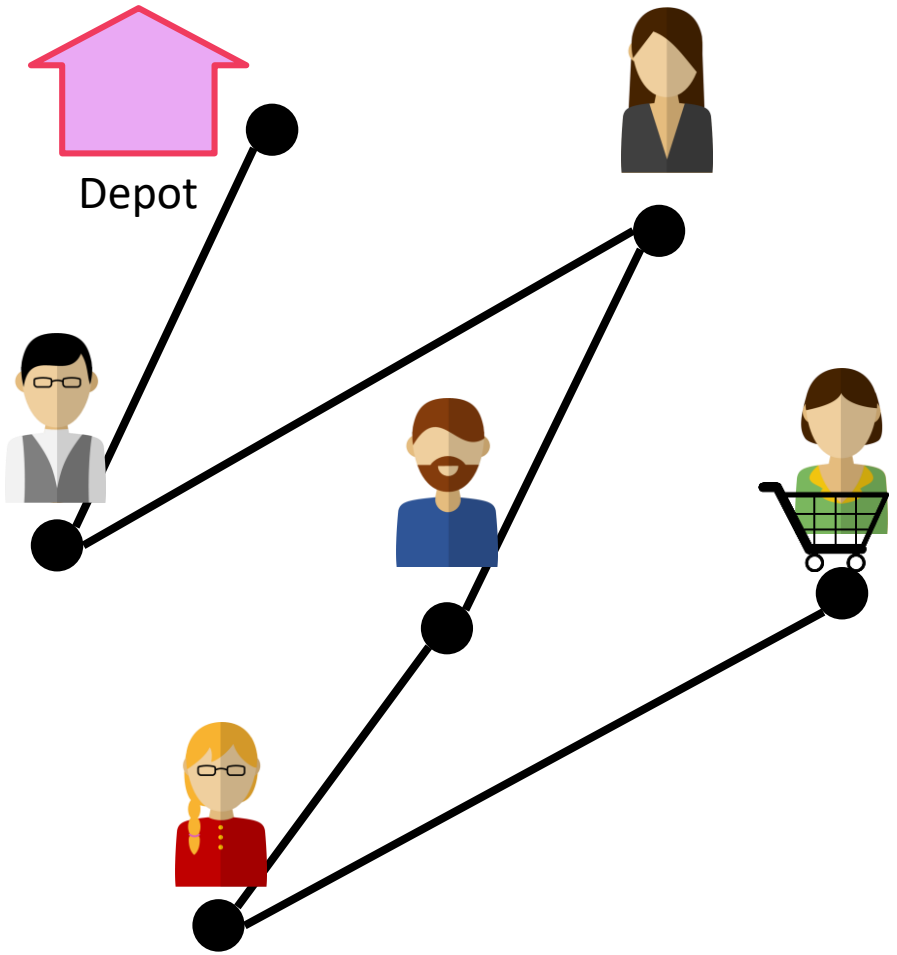


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Positive reward for amount of demand fulfilled

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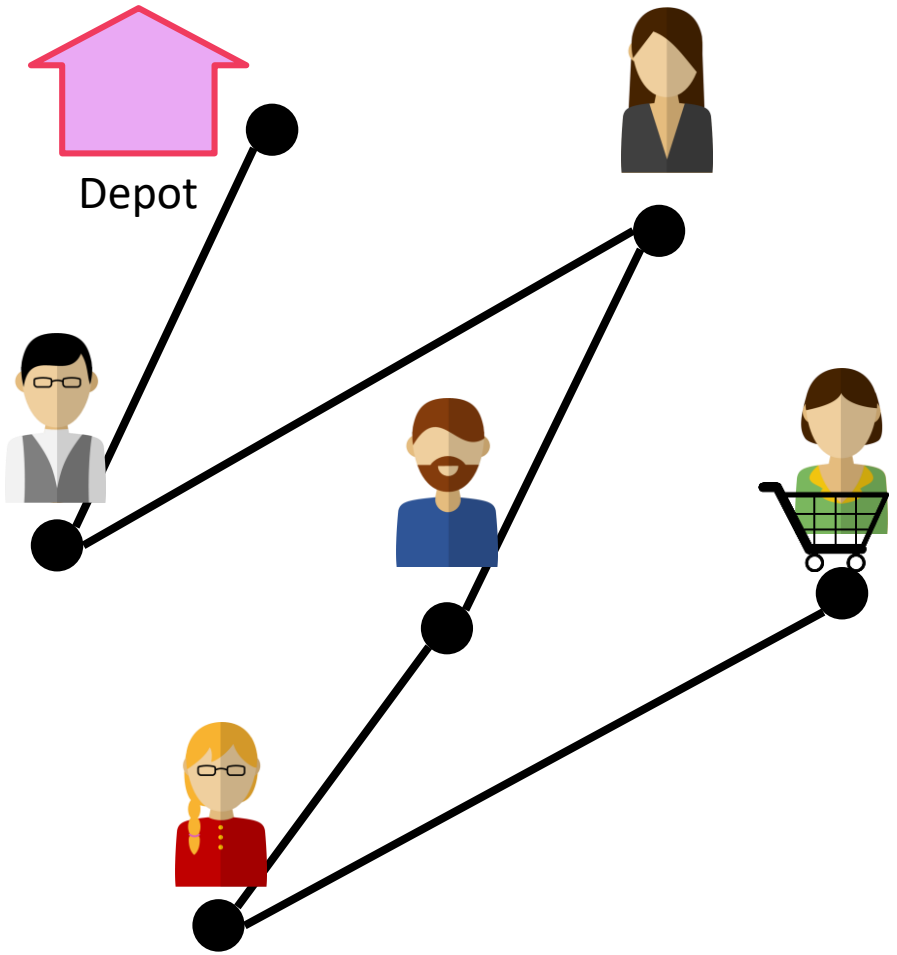


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Penalty for amount of time taken to do so (traffic etc)

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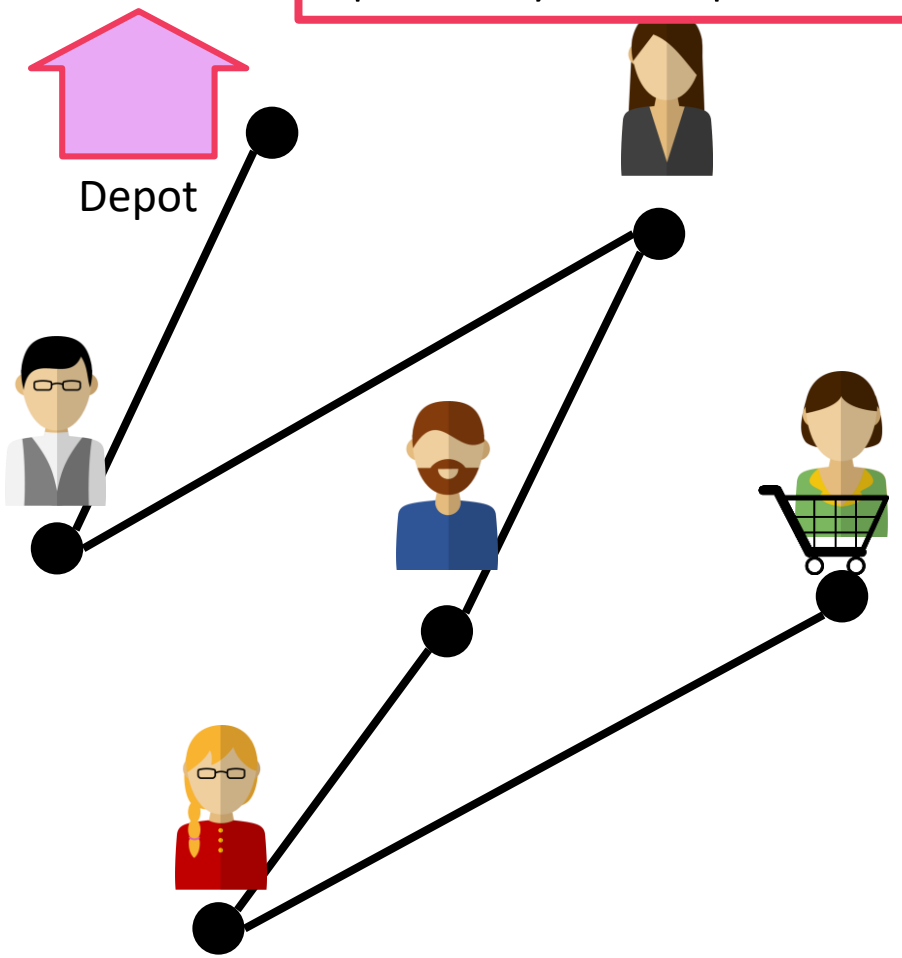
# RL for

Numerous other applications in economics, finance, governance. Basically in any situation where "environment" changes dynamically, possibly in response to our actions, RL is a valuable candidate algo

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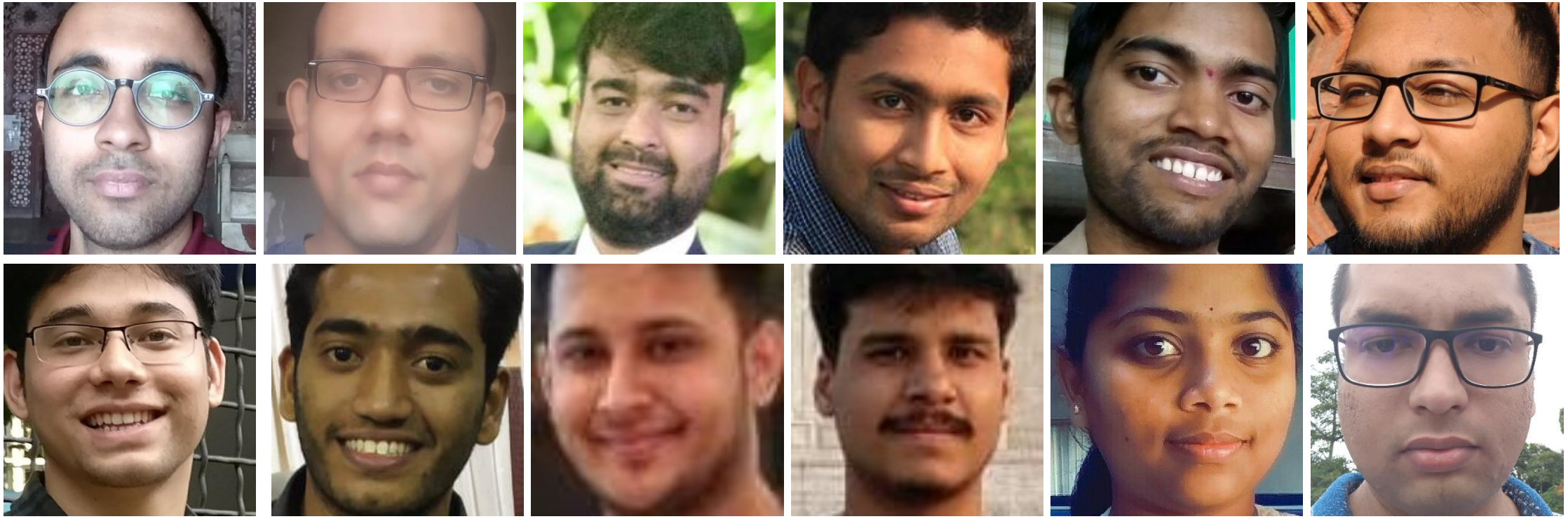
# Other Learning Paradigms

- Learning with incomplete/deficient/semi/active supervision
- Learning with *drift* (test data slightly different from train data)
- Learning multiple-tasks (generalization of multi-label learning)
- Learning with multiple views (several feature rep of same data pt)
- Learning with adversarial train/test data (robust ML)
- Learning under resource constraints (tiny/edge ML for IOT devices)
- Learning with online/streaming data
- Have seen glimpses of some of these in the course but only just ☺

# Acknowledgements



Course Team:



**CSE Office staff members**: Mr Prashant Kumar Sahu, Mr Rajesh Kumar, Mr Ranjan Kumar, Mr Anubhav Kumar Arya, Mr Amit Kumar Bharti

**Piazza, Gradescope** teams: invaluable for large courses such as ours