Name: Rollno:

CSE340: Theory of Computation (Homework Assignment 1)

Due Date: 20th August, 2019 (in class)

Total Number of Pages: 1 Total Points 50

Question 1. (18 points) Give DFAs for the following languages.

(a) $A = \{x \in \{0,1\}^* \mid \#_0(x) \leq 2 \text{ and } \#_1(x) \geq 1\}$

(b) $B = \{x \in \{0,1\}^* \mid x \text{ does not contain the substring } 1011\}$

(c) $C = \{x \in \{0,1\}^* \mid x \text{ has at least 3 occurrences of 3 consecutive 1's with possible overlapping}\}$
(For example the string 11111 is in the language C.)

Question 2. (12 points) Give DFAs accepting the same language as the following regular expressions using the minimum number of states. Give reason why you cannot have a DFA with lesser number of states.

(a) $(0 + 1(01^*0)^*1)^*$

(b) $(000^* + 111^*)^*$

Question 3. (10 points) For languages L_1 and L_2 over Σ, define

$$\text{Mix}(L_1, L_2) = \{w \in \Sigma^* \mid w = x_1y_1x_2y_2 \ldots x_ky_k, \text{ where } x_1x_2 \ldots x_k \in L_1 \text{ and } y_1y_2 \ldots y_k \in L_2, \text{ each } x_i, y_i \in \Sigma^*\}.$$

Show that if L_1 and L_2 are regular then $\text{Mix}(L_1, L_2)$ is also regular.

Question 4. (10 points) Let Σ and Δ be two alphabets and let $h : \Sigma \to \Delta^*$. Extend h to be a function from Σ^* to Δ^* as follows:

$$\begin{aligned} h(\epsilon) &= \epsilon, \\ h(wa) &= h(w)h(a) \quad \text{where } w \in \Sigma^*,\ a \in \Sigma. \end{aligned}$$

(Such a function h is called a *homomorphism*.)

Now, for $L \subseteq \Sigma^*$,

$$h(L) = \{h(w) \in \Delta^* \mid w \in L\}.$$

Also, for $L \subseteq \Delta^*$,

$$h^{-1}(L) = \{w \in \Sigma^* \mid h(w) \in L\}.$$

(a) Prove that if $L \subseteq \Sigma^*$ is regular, then so is $h(L)$.

(b) Prove that if $L \subseteq \Delta^*$ is regular, then so is $h^{-1}(L)$.