CS203B : Mathematics for Computer Science - III
CSE, IIT Kanpur

Practice sheet 4

Poisson Random Variable

We discussed in the class how Poisson random variable gets formulated in a natural way for some random experiments in real world. You are advised to revisit the lecture slides to fully understand it. Moreover, a Binomial random variable with parameters (n, p)can be approximated using Poisson random variable with $\lambda = np$ if n is too large and p is too small. The following problems should be helpful for you to get a better understanding of Poisson random variable and its application for approximating a Binomial random variable.

1. **Insect eggs**
   The number of eggs laid on a tree leaf by an insect of certaian type is a Poisson random variable with parameter λ. However, such a random variable can only be observed if it is positive, since if it is 0, then we cannnot know that such an insect was on the leaf. If we let Y denote the observed number of eggs, then
   $$\mathbf{P}(Y = i) = \mathbf{P}(X = i|X > 0)$$
   where X is Poisson with parameter λ. Find $\mathbf{E}[Y]$.

2. **Plot of Poisson random variable**
   Let X be a Poisson random variable with parameter λ. Show that $\mathbf{P}(X = i)$ increases monotonically and then decreases monotonically as i increases, reaching its maximum when i is the largest integer not exceeding λ.
   Hint: Consider $\mathbf{P}(X = i)/\mathbf{P}(X = i - 1)$.

3. **Abandoned cars on the highway**
   Suppose that the average number of cars abandoned weekly on a certain highway is 2.2. Approximate the probability that there will be

   - no abandoned car in the next week.
   - at least 2 abandoned cars in the next week.

   State your assumptions.

4. **Marriage partners sharing birthdays** Approximately 80,000 marriages took place in the state of New York in 1988. Estimate the probability that for at least one of these couples

   - both partners were born on April 30;
   - both partners celebrated their birthday on the same day of the year.

   State your assumptions.

5. **Typographical mistakes** The expected number of typographical errors on a page of a certain magazine is 0.2. What is the probability that the next page you read contains (a) 0 and (b) 2 or more typographical errors ? Explain your reasoning !

Random bits

1. **Simulating a fair coin using an unfair coin**

   You are given an unfair coin. The coin gives heads with probability $0 < p < 1/2$. But you want a fair coin for an experiment. How can you use the unfair coin to simulate a fair coin ? What is the expected no. of coin tosses of the unfair coin to simulate one toss of a fair coin ?

2. **Simulating an unfair coin using a fair coin**

   Let p be a given positive number satisfying $0 < p < 1/2$. You are given a fair coin. But for your experiment, you want an unfair coin that gives head with probability p and tail with probability $1-p$. How can you use the fair coin to simulate this unfair coin ? What is the expected no. of coin tosses of the fair coin to simulate one toss of the unfair coin ?

3. **Few random bits**

   Recall the problem of apple discussed in the class. Suppose Shyam should be given the apple with probability $p = 13/512$ and Kabir should be given the apple with probability $1 - p = 499/512$. By taking the *lazy* approach of generating random bits (discussed in the class), compute the expected number of coin tosses needed to distribute the apple between Shyam and Kabir.

4. **Random Sample and Random Permutation**

   Let A be a set of n distinct elements. We discussed an algorithm that computes a uniformly random sample of size k from A. We also discussed an algorithm that computes a uniformly random permutation of A. Establish their correctness.

Note: There are a few questions in this sheet which were asked during the lectures.