CS203B : Mathematics for Computer Science - III
CSE, IIT Kanpur

Practice sheet 2

The topics are:

- Random variable and expected value
- Linearity of expectation

1. **Coin and dice**

   A player throws a fair die and simultaneously flips a fair coin. If the coin lands heads, then she wins twice, and if tails, then one-half of the value that appears on the die. Determine her expected winnings.

2. **Coin toss to get a head**

   There is a coin which, when tossed, gives head with probability p. Outcome of each toss is independent. Let X be the random variable for the number of times we need to toss the coin till we get a head. What is $\mathbf{E}[X]$?

3. **red and blue balls again**

   There is a bag containing r red balls and b blue balls. We take out balls uniformly randomly and without replacement. Assume $r \geq 5$. What is the expected number of blue balls that appear between 3rd red ball and 4th red ball ? What is the expected number of blue balls left after all the red balls have been taken out.

4. **Sum of samples**

   An urn contains n balls numbered 1,2,...,n. We remove k balls at random (without replacement) and add up their numbers. Find the expected value of this final number.

5. **Surviving couples**

   Of the $2n$ people in a given collection of n couples, exactly m die. Assuming that the m have been picked at random, find the expected number of surviving couples.

6. **Magnet blocks**

   A total of n bar magnets are placed end to end in a line with random independent orientations. Adjacent like poles repel, ends with opposite polarities join to form blocks. Find the expected number of blocks of joined magnets.

7. **Stick break**

   Given a stick with n joints. The stick is dropped from certain height. During the fall, each joint breaks with probability p independent of other joints. What is the expected number of pieces ? What is the expected number of pieces with 4 joints ?

8. **two urns with white and black balls**

   Urn 1 contains 5 white and 6 black balls, while urn 2 contains 8 white and 10 black balls. Two balls are randomly selected from urn 1 and are then put in urn 2. If 3 balls are then randomly selected from urn 2, compute the expected number of white balls in the trio

Note: There are a few questions in this sheet which were asked during the lectures.