

CS203B : Mathematics for Computer Science - III

CSE, IIT Kanpur

Practice sheet 3

1. On independent random variables

Let X and Y be independent random variables taking values from the positive integers $\{1, 2, \dots\}$, and having the same mass function $f(i) = 2^{-i}$ for $i = 1, 2, \dots$. Find

- (a) $\mathbf{P}(\min(X, Y) \leq x)$.
- (b) $\mathbf{P}(X = Y)$.
- (c) $\mathbf{P}(X \text{ divides } Y)$.

2. A game of coin toss

Alice and Bob play a game - They take turns to toss a fair coin. First Alice tosses, then Bob tosses, then Alice, and so on. Whoever gets the first head is the winner. Find out the probability that Alice wins.

3. Geometric random variables

There is a coin which gives head with probability p . Using conditional expectation, calculate the expected number of coin tosses to get the first head.

4. Visible Sticks

There are n sticks. No two sticks have the same height. We arrange them in a uniformly random order along a line. What is the expected number of sticks that will be visible from left ? Solve this problem by using the following random variable:

X_i is 1 if stick at i th position is visible, and 0 otherwise.

5. Alternate way to calculate expected number of visible sticks

There are n sticks. No two sticks have the same height. We arrange them in a uniformly random order along a line. What is the expected number of sticks that will be visible from left ? Solve this problem by using the following random variable:

X_i is 1 if i th tallest stick is visible, and 0 otherwise.

6. Balls into bins

We throw n balls randomly uniformly and independently into m bins. What is the expected number of empty bins ?

Note: There are a few questions in this sheet which were asked during the lectures.