Name: Rollno:

CSE340: Theory of Computation (Mid Semester Exam)

20th September, 2019

Total Number of Pages: 5 Total Points 75

Instructions

1. Read these instructions carefully.
2. Write you name and roll number on all the pages of the answer book.
3. Cheating or resorting to unfair means will be severely penalized.
4. Do not exchange question books or change the seat after obtaining question paper.
5. Using pens (blue/black ink) and not pencils. Do not use red pens for answering.
6. Every question should be done on a new page. Parts of the same question can be done on the same page. Failure to do this will result in a penalty of 5 marks.

Helpful hints

1. It is advisable to solve a problem first before writing down the solution.
2. The questions are *not* arranged according to the increasing order of difficulty. Do a quick first round where you answer the easy ones and leave the difficult ones for the subsequent rounds.

Question 1. (14 points) For each of the following languages state whether the language is regular (write R), not regular but context free (write C) or not context free (write N). No argument or proof required.

(a) $L_1 = \{a^i b^j c^k d^l \mid 2i = 3k, 4j = 5l\}$

Solution: N

(b) $L_2 = \{a^i b^j c^k d^l \mid i \leq j, k \leq l\}$

Solution: C

(c) $L_3 = \{a^i b^j c^k d^l \mid 2(k + l) \leq i + j \leq 3(k + l)\}$

Solution: C

(d) $L_4 = \{a^i b^j c^k d^l \mid (i - j) \mod 3 = (k - l) \mod 2\}$

Solution: R

(e) $L_5 = \{a,b,c\}^* \setminus \{a^ib^jc^k \mid i=j=k\}$

Solution: C

(f) $L_6 = (a^*b^*c^*) \cap \{a^ib^jc^kd^l \mid i=j=l\}$

Solution: R

(g) $L_7 = (a^*b^*c^*) \cup \{a^ib^jc^kd^l \mid i=j=l\}$

Solution: N

Question 2. (5 points) Give regular expression for the language

$$L = \{w \in \{0,1\}^* \mid w \text{ begins with 01 and ends with 10}\}.$$

Solution: $01(0+1)^*10 + 010$

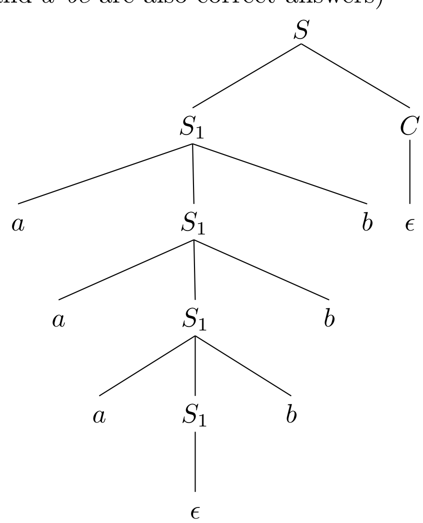
Question 3. Consider the CFG G given by the following production rules

$$\begin{aligned} S &\longrightarrow S_1C \mid AS_2 \\ S_1 &\longrightarrow aS_1b \mid \epsilon \\ S_2 &\longrightarrow bS_2c \mid \epsilon \\ A &\longrightarrow aA \mid \epsilon \\ C &\longrightarrow cC \mid \epsilon \end{aligned}$$

(a) (3 points) The language of G,
$L(G) = \{w \in \{a,b,c\}^* \mid$ $w = a^ib^jc^k$, $i=j$ **or** $j=k$ $\}$

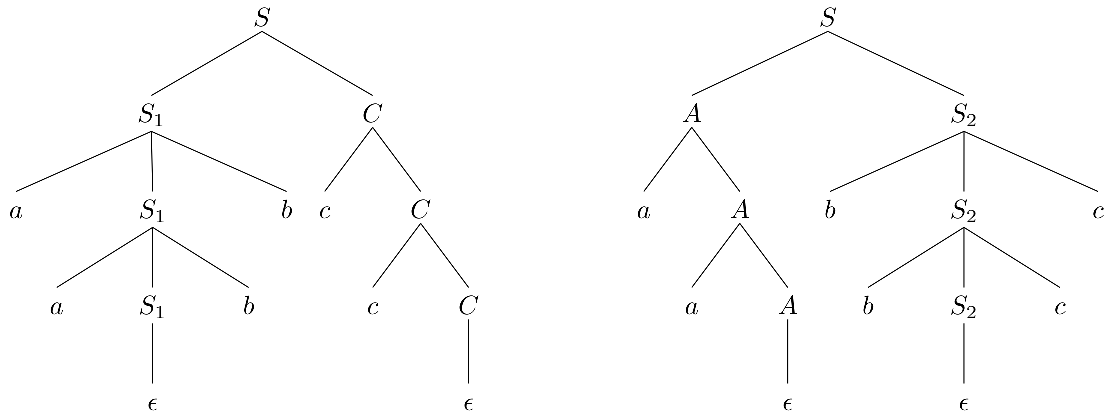
(b) (4 points) Give a string of length 6 in $L(G)$ that has a unique parse tree with respect to G. Draw the parse tree.

Solution: a^3b^3 (b^3c^3, abc^4 and a^4bc are also correct answers)



(c) (4 points) Give a string of length 6 in $L(G)$ that has two parse trees with respect to G. Draw the two parse trees.

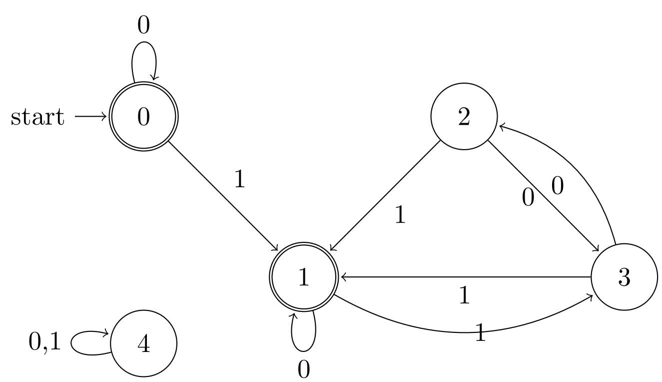
Solution: $a^2b^2c^2$



Question 4. Let $Q = \{0, 1, 2, 3, 4\}$. Consider the DFA $D = (Q, \{0, 1\}, \delta, 0, \{0, 1\})$, where

$$\delta(q, i) = (q^3 + qi + i) \bmod 5.$$

(a) (4 points) Construct the state diagram of D.



Solution:

(b) (5 points) Minimize D.

Solution: Initial table (Note that we do not include 4 since 4 is not reachable from the start state)

0			
-	1		
-	-	2	
-	-	-	3

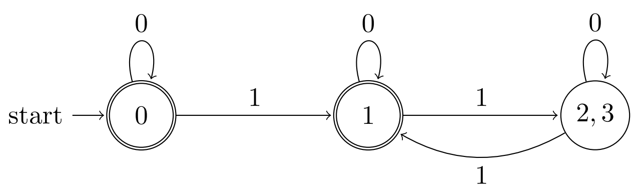
After first pass (marked all pairs consisting of one accept state and one non-accept state).

0			
-	1		
$\times$	$\times$	2	
$\times$	$\times$	-	3

After second pass.

0			
$\times$	1		
$\times$	$\times$	2	
$\times$	$\times$	-	3

At this stage no further new pairs can be marked, hence the algorithm terminates. The minimised DFA is as follows:



(c) (3 points) Give the simplest possible description of $L(D)$.

$L(D) = \{w \in \{0,1\}^* \mid$ w **has 0 or an odd number of 1's** $\}$

Question 5. (10 points) Give a CFG for the language

$$L = \{a^i b^j c^k \mid j \leq i + k \leq 2j, i, j, k \geq 0\}.$$

For each variable used in your CFG, describe the language generated by the variable.

Solution:

$$\begin{aligned} S &\longrightarrow T_1 T_2 \mid a T_1 b T_2 c \\ T_1 &\longrightarrow a T_1 b \mid aa T_1 b \mid \epsilon \\ T_2 &\longrightarrow b T_1 c \mid b T_1 cc \mid \epsilon \end{aligned}$$

- Language generated by T_1 is $\{a^i b^{j_1} \mid j_1 \leq i \leq 2j_1\}$
- Language generated by T_2 is $\{b^{j_2} c^k \mid j_2 \leq k \leq 2j_2\}$
- Language generated by S is $\{a^i b^j c^k \mid j \leq i + k \leq 2j\}$

Question 6. (5 points) Prove that no infinite subset of the language $L = \{0^n 1^n \mid n \geq 0\}$ is regular.

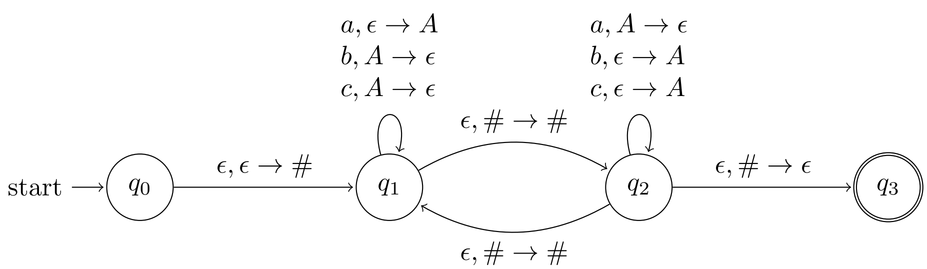
Solution: Given $p > 0$, set $w = 0^q 1^q$ where q is some number greater than p and $0^q 1^q$ is in the language. Since it is an infinite subset, therefore existence of such a string is guaranteed. Rest of the proof is similar to the proof that L is not regular using pumping lemma.

Question 7. (8 points) Give the transition diagram of a PDA that accepts the following language

$$L = \{w \in \{a, b, c\}^* \mid \#_a(w) = \#_b(w) + \#_c(w)\}.$$

Give a short explanation of the utility of all the states in your PDA.

Solution:



The PDA stays at q_1 if for the substring read so far say y, $\#_a(y) \geq \#_b(y) + \#_c(y)$. If $\#_a(y) \leq \#_b(y) + \#_c(y)$ the PDA shifts to q_2.

Question 8. (10 points) Let $A \subseteq \{0, 1\}^*$ and let

$$A' = \{xy \mid x1y \in A\}.$$

That is, A' contains all strings obtained from a string in A by deleting exactly one 1. Show that if A is regular, then A' is also regular (give the construction only).

Solution: Let $D = (Q, \{0, 1\}, \delta, q_0, F)$ be a DFA for A. We construct an NFA $N = (Q', \{0, 1\}, \delta', q'_0, F')$ for A', where $Q' = Q \times (\alpha, \beta)$, $q'_0 = (q_0, \alpha)$, $F' = \{(f, \beta) \mid f \in F\}$ and δ' is defined as follows:

$$\begin{aligned}
\delta'((q, \alpha), a) &= \{(\delta(q, a), \alpha)\} \\
\delta'((q, \beta), a) &= \{(\delta(q, a), \beta)\} \\
\delta'((q, \alpha), \epsilon) &= \{(\delta(q, 1), \beta)\}
\end{aligned}$$

Essentially, states of the form (q, α) corresponds to the automata not having skipped a 1 yet and states of the form (q, β) corresponds to the fact that the automata has skipped a 1.