Assignment-2

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1 Q.1

Here is the pseudo code

main

{

Input the values of n and k (given n must be greater than or equal to k else it is not defined)

declare a variable

call a recursive-function to find the value of $\binom{n}{k}$ and equate to the variable (pass the value of n and k as parameters)

print the value of the variable

}

function

{

declare a variable

if (value of k is equal to zero)

return 1

(base case)

else

return (call the function with n-1 , k-1 as parameters and then multiply it by n and divide by k)

}

The above code has just k+1 recursive calls and each recursive call takes a constant time t thus total time is (k+1)*t which is a polynomial in k

2 Q.2

2.1

[1,1,2,3,5,8] is a set of six numbers from 1 to 8 such that if we pick up any three numbers,we can observe that no triangle would be formed out of it(as it is not satisfying the condition $a + b > c$)

2.2

The sum of any two numbers taken from [k,2k) is always greater than 2k, thus selecting any three numbers from [k,2k) would always form a triangle as sum of two numbers selected would always be greater than than third number selected (following $a + b > c$)

2.3

We need to show that there exists 7 numbers from [1,8] such that they form a triangle.So let us first divide [1,8] into the intervals [1,2),[2,4),[4,8) and take two numbers from each of these three intervals.Based on the above property , now if we take one number from any of the intervals we can say that a triangle will always be formed.

3 Q.3

3.1

Two elements of a chain cannot have the same height . We can observe through the vertical connected lines in a Hasse diagram of a poset. Any two elements of the chain must be connected by vertical lines (directly or indirectly via different nodes) and thus they would have a difference of atleast one height.Thus the set of all elements of height h in a poset form an antichain.

3.2

If the largest chain in a poset has size r then the number of elements with different height are r.Thus using the previous property (set of all elements of height h in a poset form an antichain) we can say that P can be partitioned into r antichains .

4 Q.4

Let us first divide the range into two parts, [1,10) and [10,400]. Inorder to find number of primes we can find number of composites and then subtract from total. As we know that a composite number is always divisible by a prime not exceeding its square root, therefore we need to find all composite numbers in the range [10,400] and having a prime factor not exceeding 20(thus the primes are 2,3,5,7,11,13,17,19) We suppose that B1 is the set of numbers divisible by 2, similarly B2 be set divisible by 3, B3 is the set divisible by 5 and so on till B8(set divisible by 19)

B1= 10,12,14,16,18,20,..........400 $|B1| = 196$
B2= 12,15,18....................399 $|B2| = 130$
B3= 10,15,20....................400 $|B3| = 79$

B4= 14,21,28....................399 $|B4| = 56$

B5= 11,22,33....................396 $|B5| = 36$

B6= 13,18,24....................396 $|B6| = 30$

B7= 17,34,51....................391 $|B7| = 23$

B8= 19,38,57....................399 $|B8| = 21$

Similarly we need to find all composite numbers divisible by any two primes,then any three and so on till we find numbers divisible by all primes less than 20(in the range [10,400])

Now applying inclusion-exclusion principle,total number of composites

Let A1 be composites divisible by one prime ,A2 be the composites divisible by two primes,and so on till A8 divisible by all primes less than 20

A1= 571 (sum of all sets of B found earlier)

A2= 323

A3= 75

A4= 6

cardinality of A5,A6,A7,A8 would be zero as their product is exceeding 400.

Thus number of composites are A1-A2+A3-A4+A5-A6+A7-A8 (i.e

$$\sum_{i=1} \cup Ai$$

) =317

number of primes in [1,10) = 4

Hence the no.of primes = 4 + 391-(317) = 78