

CSE340: Theory of Computation (Homework Assignment 2)

Due Date: 6th September, 2019 (in class)

Total Number of Pages: 1

Total Points 40

Question 1. (5 points) Give a regular expression for the following language.

$$B = \{x \in \{0,1\}^* \mid x \text{ does not contain the substring } 101\}$$

Question 2. (5 points) Prove that $\{0^i 1^j \mid \gcd(i, j) = 1\}$ is not regular?

Question 3. (6 points) Let $A \subseteq \mathbb{N}$ be a subset of natural numbers. A is said to be *ultimately periodic* if there exists numbers $p > 0$ and $n \geq 0$, such that for all $m \geq n$, $m \in A$ if and only if $m + p \in A$. In other words, after a certain point (the number n) the numbers in the set A occur in a fixed regular interval of length p .

Consider $L \subseteq \{a\}^*$. Prove that L is regular if and only if the set $\{m \mid a^m \in L\}$ is ultimately periodic.

(Hint: Think how will the DFA of a unary regular language look like.)

Question 4. (12 points) Give CFGs for the following languages

- (a) $L_1 = \{a^i b^j c^k d^l \mid i, j, k, l \geq 1, i = l, j = k\}$
- (b) $L_2 = \{a^n b^m \mid n, m \geq 0, n \neq m\}$
- (c) $L_3 = \{a^i b^j c^k \mid i, j, k \geq 0, i > j \text{ or } j > k\}$

Question 5. Consider the following CFG G over the set of terminals $T = \{+, *, 0, 1, (,)\}$

$$S \rightarrow S + S \mid S * S \mid (S) \mid 0 \mid 1$$

- (a) (2 points) Give a string of length 5 that is ambiguous with respect to G .
- (b) (4 points) Give two parse trees for the string in part (a) with respect to G .
- (c) (6 points) Give an unambiguous CFG for the language generated by the above grammar that gives proper precedence to the operators (i.e. highest precedence to brackets followed by the $*$ operator and then the $+$ operator).