CODE: *APR4_APT_TEST_IITM_2025* Test Duration: 60 *minutes*

Read the questions well ! Think and Answer ! Each question carries 10 marks

Dear Friend,

- You are writing this exam as a part of the admission process to a research program at IIT Madras.
- We wish you all the best to become one among the most successful researchers impacting positively the world that we live in.
- History has repeatedly proved that every successful researcher whose contribution is remembered and used for long has always been honest.
- Here is an opportunity to demonstrate your honesty.
- This is a simple exam, which will help us understand your strengths and weaknesses in analytical thinking, that will help IITM as a system to guide you better in the event of you getting selected.
- We are NOT proctoring this exam with a firm belief that you will maintain the necessary code of honest conduct and shall upload your answer script within the prescribed time.

Instructions

1. Answer in Plain paper, scan and send as a single PDF.
2. Do not forget to write your Name and Application/Registration number on your Answer sheet.
3. This exam is to enable us know more about your approach to problem solving which is extremely important for a research based program.
4. Note that significant portion of the marks shall be awarded for intermediate steps.
5. For every question attempted please give detailed explanation on how you arrive at the answers.

I Answer each of the following questions with detailed justification

1. A 2− digit palindrome number x divides a 3−digit palindrome number y exactly.

   How many such combinations are there including the example $(x, y) \equiv (33, 858)$?

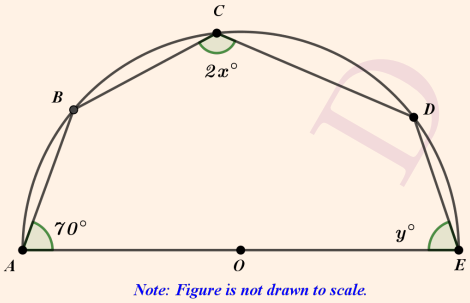
   Give reasons.

2. The function f has the property that for each real number x,

   $$f(x) + f(x-1) = x^2$$

   If $f(20) = 25$, then find the value of $f(25)$. Give reasons.

3. Here you observe a semicircle with diameter AE and centre O.



   B, C, D are points on the arc of the semicircle such that $\angle BAO = 70°, \angle BCD = 2x°; \angle DEO = y°$.

   If $\angle AOB + \angle DOE = x°$, then find the value of y. Give reasons.

4. The product of five different natural numbers is 7560. Their sum is 33.

   What is the sum of all possible medians of the natural numbers? Give reasons.

5. The number 2025 is expressed in the form

   $$2025 = \frac{a_1! \times a_3!}{a_2! \times a_4!} \text{ where } a_1 > a_2 > a_3 > a_4 > 0$$

   are natural numbers.

   Find the smallest possible value of $a_1 + a_2 + a_3 + a_4$. Give reasons.

   Note: $n!$ is termed as factorial function of natural number n, defined as

   the product of the first n natural numbers.

6. Y is a year in this 21^{st} century $(2001 - 2100)$, having the following properties:

   - Y is the sum of 31 consecutive positive integers.
   - Y is the product of three prime numbers.

   Find the value of Y. Give reasons.