CODE: $APR3\_APT\_TEST\_IITM\_2025$ Test Duration: $60 \; minutes$

Read the questions well ! Think and Answer ! Each question carries 10 marks

Dear Friend,

- You are writing this exam as a part of the admission process to a research program at IIT Madras.

- We wish you all the best to become one among the most successful researchers impacting positively the world that we live in.

- History has repeatedly proved that every successful researcher whose contribution is remembered and used for long has always been honest.

- Here is an opportunity to demonstrate your honesty.

- This is a simple exam, which will help us understand your strengths and weaknesses in analytical thinking, that will help IITM as a system to guide you better in the event of you getting selected.

- We are NOT proctoring this exam with a firm belief that you will maintain the necessary code of honest conduct and shall upload your answer script within the prescribed time.

Instructions

1. Answer in Plain paper, scan and send as a single PDF.

2. Do not forget to write your Name and Application/Registration number on your Answer sheet.

3. This exam is to enable us know more about your approach to problem solving which is extremely important for a research based program.

4. Note that significant portion of the marks shall be awarded for intermediate steps.

5. For every question attempted please give detailed explanation on how you arrive at the answers.

# I   Answer each of the following questions with detailed justification

1. 4 vertices among the 9 vertices of a regular nonagon should be selected such that the

   selected 4 form the vertices of a trapezium. How many such possibilities are there?

   Give reasons.

2. x, y are real numbers such that they satisfy the system of equations:

$$\begin{cases} x^2 + y^2 = 6y - 4x + 12 \\ \\ 4y = x^2 + 4x + 12 \end{cases}$$

   Find the number of such pairs (x, y). Give reasons.

3. OAB is a quadrant with $\angle AOB = 90°$. D, F are points on arc AB

   such that $AD = DF = FB$ as shown.
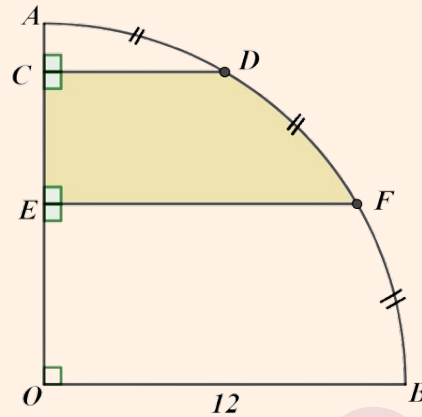
Note: Figure not drawn to scale

   C, E are points on OA such that $DC \perp AC$ and $FE \perp AE$, as shown.

   If $OB = 12 \ unit$, then find the area of the shaded region. Give reasons.

4. a, b, c, d, e, f are six positive integers of which only one of them, is composite.

   The integers satisfy the system of equations:

$$a + b = 75$$
$$b + c = 120$$
$$c + d = 148$$
$$d + e = 194$$
$$e + f = 100$$

   Find the difference between the largest prime number and the composite number.

   Give reasons.

5. The sequence of words $< a_n >$ is defined as follows:

   $a_1 = N; a_2 = O$; and for $n \geq 3$,

   we have a_n is a_{n-1} followed by the reverse of a_{n-2}.

   $\therefore a_3 = ON; a_4 = ONO; a_5 = ONONO; a_6 = ONONOONO$ and so on...

   How many of the first 1000 terms are palindromes? Give reasons.

   Note:a_1, a_2, a_4, a_5 are palindrome words (may not be meaningful!)

6. A finite Arithmetic Progression $(A.P.)$ has 10 distinct positive integer terms in it.

   If the sum of all the terms of the $A.P.$ is 2025, How many such $A.P.'s$ are there? Give reasons.