Unit 1: Linear Relationships and Equations

Topic 6: Equation of a Line

Concept Summary

A **linear equation** represents all points (x, y) that form a straight line on the coordinate plane. There are several ways to write the equation of a line, depending on what information is given.

1. Slope–Intercept Form

$$y = mx + b$$

where:

- m is the slope (rate of change)
- b is the y-intercept (the point where the line crosses the y-axis)

Example: $y = 2x + 3$ has slope 2 and y-intercept 3.

2. Point–Slope Form

$$y - y_1 = m(x - x_1)$$

Used when you know the slope m and one point (x_1, y_1) on the line. This form is especially useful for building an equation quickly from limited data.

Example: The line through $(2, 5)$ with slope 3 is

$$y - 5 = 3(x - 2)$$

which can be rewritten as $y = 3x - 1$.

3. Standard Form

$$Ax + By = C$$

where A, B, and C are integers, and A is usually positive. This form is often used to find intercepts easily:

$$x\text{-intercept: set } y = 0 \quad \text{and} \quad y\text{-intercept: set } x = 0$$

Example 1: Writing an Equation from a Graph

A line passes through $(0, 2)$ and has slope 4.

$$y = 4x + 2$$

Slope $= 4$, y-intercept $= 2$.

Example 2: Writing an Equation from Two Points

Find the equation of the line through $(1, 3)$ and $(5, 11)$.

Step 1: Find the slope.

$$m = \frac{11 - 3}{5 - 1} = \frac{8}{4} = 2$$

Step 2: Use point–slope form with $(1, 3)$**.**

$$y - 3 = 2(x - 1)$$

Step 3: Simplify to slope–intercept form.

$$y = 2x + 1$$

Final Answer: $\boxed{y = 2x + 1}$

Key Takeaways

- Use $y = mx + b$ when slope and intercept are known.
- Use $y - y_1 = m(x - x_1)$ when you know a slope and one point.
- Use $Ax + By = C$ for standard form or when working with intercepts.
- All forms describe the same line — they are just written differently.

Practice Questions: Equation of a Line

Part A: Slope–Intercept Form

1. Identify the slope and y-intercept of $y = 2x + 5$.
2. Identify the slope and y-intercept of $y = -3x + 1$.
3. Write the equation of a line with slope 4 and y-intercept -2.
4. Write the equation of a line with slope -1 and y-intercept 6.
5. Find the slope and y-intercept of $y = \frac{1}{2}x - 4$.

Part B: Point–Slope Form

6. Write an equation of a line with slope 3 passing through $(2, 5)$.
7. Write an equation of a line with slope -2 passing through $(4, -1)$.
8. Write an equation of a line with slope $\frac{1}{2}$ passing through $(0, 8)$.
9. Find the equation of a line through $(1, 2)$ and $(3, 8)$.
10. Find the equation of a line through $(-2, 5)$ and $(4, -7)$.

Part C: Standard Form and Conversion

11. Write $y = 3x - 6$ in standard form $Ax + By = C$.
12. Write $2x - y = 4$ in slope–intercept form.
13. Write $3x + 4y = 12$ in slope–intercept form.
14. Write the equation of a line with slope $-\frac{3}{2}$ and y-intercept 4 in standard form.
15. Find the x- and y-intercepts of $2x + 3y = 12$.

Part D: Parallel and Perpendicular Lines

16. Write the equation of a line parallel to $y = 2x + 3$ passing through $(1, 5)$.
17. Write the equation of a line perpendicular to $y = -\frac{1}{3}x + 2$ passing through $(3, 4)$.
18. Are the lines $y = 4x + 1$ and $4x - y = 3$ parallel, perpendicular, or neither?

19. Determine whether the lines $2x + y = 8$ and $x - 2y = 5$ are parallel, perpendicular, or neither.

20. Write the equation of the line perpendicular to $y = \frac{1}{2}x + 7$ that passes through $(2, 3)$.

Part E: SAT-Style Word and Context Problems

21. A taxi ride costs \$4 plus \$2 per mile. Write the linear equation that models the total cost C for m miles.

22. A company's revenue is \$500 when 10 units are sold and \$800 when 25 units are sold. Write the equation for revenue R as a function of units x.

23. A line passes through the points $(0, 40)$ and $(5, 25)$. Write its equation and interpret the slope.

24. A plumber charges a \$60 service fee plus \$40 per hour. Write the cost equation.

25. The temperature is 70°F at noon and 58°F at 4 PM. Write the linear equation modeling temperature T as a function of time t (hours after noon).

Answer Key and Solutions: Equation of a Line

Part A Solutions: Slope–Intercept Form

1. $y = 2x + 5$: slope $m = \boxed{2}$, y-intercept $\boxed{5}$.
2. $y = -3x + 1$: $m = \boxed{-3}$, y-intercept $\boxed{1}$.
3. Slope 4, y-int -2: $\boxed{y = 4x - 2}$.
4. Slope -1, y-int 6: $\boxed{y = -x + 6}$.
5. $y = \frac{1}{2}x - 4$: $m = \boxed{\frac{1}{2}}$, y-intercept $\boxed{-4}$.

Part B Solutions: Point–Slope Form

6. Through $(2, 5)$, $m = 3$: $y - 5 = 3(x - 2) \Rightarrow \boxed{y = 3x - 1}$.
7. Through $(4, -1)$, $m = -2$: $y + 1 = -2(x - 4) \Rightarrow \boxed{y = -2x + 7}$.
8. Through $(0, 8)$, $m = \frac{1}{2}$: $y - 8 = \frac{1}{2}(x - 0) \Rightarrow \boxed{y = \frac{1}{2}x + 8}$.
9. $(1, 2)$, $(3, 8)$: $m = \frac{8-2}{3-1} = 3$. $y - 2 = 3(x - 1) \Rightarrow \boxed{y = 3x - 1}$.
10. $(-2, 5)$, $(4, -7)$: $m = \frac{-7-5}{4-(-2)} = -2$. $y - 5 = -2(x + 2) \Rightarrow \boxed{y = -2x + 1}$.

Part C Solutions: Standard Form and Conversion

11. $y = 3x - 6 \Rightarrow \boxed{3x - y = 6}$.
12. $2x - y = 4 \Rightarrow -y = -2x + 4 \Rightarrow \boxed{y = 2x - 4}$.
13. $3x + 4y = 12 \Rightarrow 4y = -3x + 12 \Rightarrow \boxed{y = -\frac{3}{4}x + 3}$.
14. $m = -\frac{3}{2}, b = 4 \Rightarrow y = -\frac{3}{2}x + 4 \Rightarrow 2y = -3x + 8 \Rightarrow \boxed{3x + 2y = 8}$.
15. $2x + 3y = 12$: x-int $(6, 0)$, y-int $(0, 4)$.

Part D Solutions: Parallel and Perpendicular Lines

16. Parallel slope 2: $y - 5 = 2(x - 1) \Rightarrow \boxed{y = 2x + 3}$.

17. Perpendicular to $-\frac{1}{3}$ has slope 3: $y - 4 = 3(x - 3) \Rightarrow \boxed{y = 3x - 5}$.

18. $y = 4x + 1$ and $4x - y = 3 \Rightarrow y = 4x - 3$: same slope $\Rightarrow \boxed{\textit{Parallel}}$.

19. $2x + y = 8 \Rightarrow m = -2$; $x - 2y = 5 \Rightarrow m = \frac{1}{2}$; product $-1 \Rightarrow \boxed{\textit{Perpendicular}}$.

20. Perpendicular to $m = \frac{1}{2}$ has slope -2: $y - 3 = -2(x - 2) \Rightarrow \boxed{y = -2x + 7}$.

Part E Solutions: SAT-Style Word and Context Problems

21. Cost model: $\boxed{C = 2m + 4}$.

22. Through $(10, 500)$, $(25, 800)$: $m = \frac{300}{15} = 20$. $500 = 20(10) + b \Rightarrow b = 300$. $\boxed{R = 20x + 300}$.

23. $(0, 40)$, $(5, 25)$: $m = \frac{25-40}{5-0} = -3$. $\boxed{y = -3x + 40}$. Slope means y decreases by 3 per 1 increase in x.

24. $\boxed{C = 40h + 60}$.

25. $t = 0 \Rightarrow 70$, $t = 4 \Rightarrow 58$: $m = \frac{58-70}{4} = -3$. $\boxed{T = -3t + 70}$.