

Unit 1: Linear Relationships and Equations

Topic 3: Multi-Step and Fractional Equations

Concept Summary

Many SAT algebra questions involve equations that take more than one step to solve. These are called **multi-step equations**. They may include parentheses, fractions, or variables on both sides.

The goal is still the same: **isolate the variable**. To do that, we simplify both sides of the equation by:

1. Using the distributive property to remove parentheses.
2. Combining like terms on each side.
3. Using addition, subtraction, multiplication, or division to isolate the variable.

When fractions appear, it is often easiest to eliminate them first by multiplying both sides by a common denominator.

Core Skills

- Use the distributive property to expand expressions.
- Combine like terms carefully.
- Multiply through by the least common denominator (LCD) to remove fractions.
- Check your final answer by substituting it back into the equation.

Example 1: Multi-Step Equation

Solve for x :

$$3(x - 2) = 9$$

Step 1: Distribute.

$$3x - 6 = 9$$

Step 2: Add 6 to both sides.

$$3x = 15$$

Step 3: Divide by 3.

$$x = 5$$

Check:

$$3(5 - 2) = 9 \Rightarrow 3(3) = 9 \quad \checkmark$$

$$\boxed{x = 5}$$

Example 2: Fractional Equation

Solve for x :

$$\frac{2x - 3}{5} = 3$$

Step 1: Multiply both sides by 5 to remove the denominator.

$$2x - 3 = 15$$

Step 2: Add 3 to both sides.

$$2x = 18$$

Step 3: Divide by 2.

$$x = 9$$

Check:

$$\frac{2(9) - 3}{5} = \frac{18 - 3}{5} = \frac{15}{5} = 3 \quad \checkmark$$

$$\boxed{x = 9}$$

Key Takeaways

- Simplify both sides before isolating the variable.
- Use the distributive property correctly when parentheses are involved.
- When fractions are present, multiply by the LCD to clear denominators.
- Always check your answer to avoid sign or arithmetic errors.

Practice Questions: Multi-Step and Fractional Equations

Part A: Multi-Step Equations

1. Solve for x : $2(x + 3) = 14$
2. Solve for x : $4x - 7 = 13$
3. Solve for x : $5x + 6 = 3x + 16$
4. Solve for x : $3(x - 4) + 5 = 14$
5. Solve for x : $6x - 3 = 2x + 13$

Part B: Parentheses and Distribution

6. Solve for x : $2(3x - 5) = 14$
7. Solve for x : $5(x + 2) - 3 = 17$
8. Solve for x : $4(x - 3) + 2x = 18$
9. Solve for x : $3(x + 1) = 2(x + 5)$
10. Solve for x : $7x - 4(2x - 3) = 10$

Part C: Fractional Equations (One Denominator)

11. Solve for x : $\frac{x - 4}{3} = 2$
12. Solve for x : $\frac{2x + 5}{4} = 7$
13. Solve for x : $\frac{3x - 2}{5} = 4$
14. Solve for x : $\frac{5x - 1}{2} = 9$
15. Solve for x : $\frac{x + 3}{6} = 2$

Part D: Fractional Equations (Two Denominators)

16. Solve for x : $\frac{x+1}{4} = \frac{x-3}{2}$
17. Solve for x : $\frac{3x-5}{6} = \frac{x+1}{3}$
18. Solve for x : $\frac{2x-1}{5} = \frac{x+4}{10}$
19. Solve for x : $\frac{x-2}{3} = \frac{2x+4}{6}$
20. Solve for x : $\frac{5x+2}{8} = \frac{x+6}{4}$

Part E: SAT-Style Word and Context Problems

21. Twice a number minus 6 equals 14. What is the number?
22. When 3 is added to half a number, the result is 11. Find the number.
23. The sum of 4 times a number and 7 is equal to 23. Find the number.
24. A plumber charges a \$50 service fee plus \$30 per hour. If a customer paid \$200, how many hours did the plumber work?
25. A taxi ride costs \$2.50 plus \$0.75 per mile. If the total cost was \$11.75, how many miles was the ride?

Answer Key and Solutions: Multi-Step and Fractional Equations

Part A Solutions: Multi-Step Equations

1. $2(x + 3) = 14 \Rightarrow x + 3 = 7 \Rightarrow x = \boxed{4}$

2. $4x - 7 = 13 \Rightarrow 4x = 20 \Rightarrow x = \boxed{5}$

3. $5x + 6 = 3x + 16 \Rightarrow 2x = 10 \Rightarrow x = \boxed{5}$

4. $3(x - 4) + 5 = 14 \Rightarrow 3x - 12 + 5 = 14 \Rightarrow 3x - 7 = 14 \Rightarrow 3x = 21 \Rightarrow x = \boxed{7}$

5. $6x - 3 = 2x + 13 \Rightarrow 4x = 16 \Rightarrow x = \boxed{4}$

Part B Solutions: Parentheses and Distribution

6. $2(3x - 5) = 14 \Rightarrow 6x - 10 = 14 \Rightarrow 6x = 24 \Rightarrow x = \boxed{4}$

7. $5(x + 2) - 3 = 17 \Rightarrow 5x + 10 - 3 = 17 \Rightarrow 5x + 7 = 17 \Rightarrow 5x = 10 \Rightarrow x = \boxed{2}$

8. $4(x - 3) + 2x = 18 \Rightarrow 4x - 12 + 2x = 18 \Rightarrow 6x = 30 \Rightarrow x = \boxed{5}$

9. $3(x + 1) = 2(x + 5) \Rightarrow 3x + 3 = 2x + 10 \Rightarrow x = \boxed{7}$

10. $7x - 4(2x - 3) = 10 \Rightarrow 7x - 8x + 12 = 10 \Rightarrow -x + 12 = 10 \Rightarrow -x = -2 \Rightarrow x = \boxed{2}$

Part C Solutions: Fractional Equations (One Denominator)

11. $\frac{x - 4}{3} = 2 \Rightarrow x - 4 = 6 \Rightarrow x = \boxed{10}$

12. $\frac{2x + 5}{4} = 7 \Rightarrow 2x + 5 = 28 \Rightarrow 2x = 23 \Rightarrow x = \boxed{\frac{23}{2}}$

13. $\frac{3x - 2}{5} = 4 \Rightarrow 3x - 2 = 20 \Rightarrow 3x = 22 \Rightarrow x = \boxed{\frac{22}{3}}$

14. $\frac{5x - 1}{2} = 9 \Rightarrow 5x - 1 = 18 \Rightarrow 5x = 19 \Rightarrow x = \boxed{\frac{19}{5}}$

15. $\frac{x + 3}{6} = 2 \Rightarrow x + 3 = 12 \Rightarrow x = \boxed{9}$

Part D Solutions: Fractional Equations (Two Denominators)

$$16. \frac{x+1}{4} = \frac{x-3}{2} \Rightarrow 2(x+1) = 4(x-3) \Rightarrow 2x+2 = 4x-12 \Rightarrow 14 = 2x \Rightarrow x = \boxed{7}$$

$$17. \frac{3x-5}{6} = \frac{x+1}{3} \Rightarrow 3(3x-5) = 6(x+1) \Rightarrow 9x-15 = 6x+6 \Rightarrow 3x = 21 \Rightarrow x = \boxed{7}$$

$$18. \frac{2x-1}{5} = \frac{x+4}{10} \Rightarrow 10(2x-1) = 5(x+4) \Rightarrow 20x-10 = 5x+20 \Rightarrow 15x = 30 \Rightarrow x = \boxed{2}$$

$$19. \frac{x-2}{3} = \frac{2x+4}{6}. \text{ Reduce right side: } \frac{2x+4}{6} = \frac{x+2}{3}. \text{ Then } \frac{x-2}{3} = \frac{x+2}{3} \Rightarrow x-2 = x+2, \text{ which is impossible. No solution.}$$

$$20. \frac{5x+2}{8} = \frac{x+6}{4} \Rightarrow 4(5x+2) = 8(x+6) \Rightarrow 20x+8 = 8x+48 \Rightarrow 12x = 40 \Rightarrow x = \boxed{\frac{10}{3}}$$

Part E Solutions: SAT-Style Word and Context Problems

$$21. \text{ Let the number be } n. 2n - 6 = 14 \Rightarrow 2n = 20 \Rightarrow n = \boxed{10}$$

$$22. \text{ Let the number be } n. \frac{n}{2} + 3 = 11 \Rightarrow \frac{n}{2} = 8 \Rightarrow n = \boxed{16}$$

$$23. \text{ Let the number be } n. 4n + 7 = 23 \Rightarrow 4n = 16 \Rightarrow n = \boxed{4}$$

$$24. \text{ Cost model: } C = 50 + 30h. \text{ Given } C = 200: 50 + 30h = 200 \Rightarrow 30h = 150 \Rightarrow h = \boxed{5}$$

$$25. \text{ Cost model: } C = 2.50 + 0.75m. \text{ Given } C = 11.75: 0.75m = 11.75 - 2.50 = 9.25 \Rightarrow m = \frac{9.25}{0.75} = \frac{37}{3} \Rightarrow \boxed{\frac{37}{3} \text{ miles}}$$