Unit 1: Linear Relationships and Equations

Topic 2: Solving Linear Equations

Concept Summary

A **linear equation** is an equation where the variable has an exponent of 1. When written in one variable, it has the general form:

$$ax + b = c$$

where a, b, and c are constants.

The goal in solving a linear equation is to **isolate the variable** on one side of the equation. We do this by using the properties of equality:

- You can add or subtract the same number from both sides.
- You can multiply or divide both sides by the same nonzero number.

The solution is the value of the variable that makes the equation true.

Core Skills

- Simplify each side of the equation before solving.
- Use inverse operations to isolate the variable.
- Check the solution by substituting it back into the original equation.

Example 1: One-Step Equation

Solve for x:

$$x - 7 = 12$$

Step 1: Add 7 to both sides.

$$x - 7 + 7 = 12 + 7$$

$$x = 19$$

Check: Substitute $x = 19$ into the equation.

$$19 - 7 = 12 \quad \checkmark$$

Final Answer: $\boxed{x = 19}$

Example 2: Two-Step Equation

Solve for x:

$$4x - 5 = 11$$

Step 1: Add 5 to both sides.

$$4x = 16$$

Step 2: Divide both sides by 4.

$$x = \frac{16}{4}$$

$$\boxed{x = 4}$$

Check: Substitute $x = 4$ into the equation.

$$4(4) - 5 = 16 - 5 = 11 \quad \checkmark$$

Key Takeaways

- Perform the same operation on both sides of the equation.
- Simplify carefully before isolating the variable.
- Always verify your solution.

Practice Questions: Solving Linear Equations

Part A: One-Step Equations

1. Solve for x: $x + 9 = 14$
2. Solve for x: $x - 5 = -2$
3. Solve for x: $4x = 20$
4. Solve for x: $\frac{x}{3} = 7$
5. Solve for x: $-8x = 32$

Part B: Two-Step Equations

6. Solve for x: $3x + 4 = 13$
7. Solve for x: $5x - 6 = 19$
8. Solve for x: $\frac{x}{2} + 7 = 12$
9. Solve for x: $8x - 9 = 23$
10. Solve for x: $4x + 5 = -11$

Part C: Equations with Variables on Both Sides

11. Solve for x: $2x + 7 = 3x - 5$
12. Solve for x: $5x - 8 = 2x + 7$
13. Solve for x: $9x + 4 = 4x + 19$
14. Solve for x: $6x - 10 = 2x + 14$
15. Solve for x: $3x + 9 = -x + 25$

Part D: Equations with Parentheses and Fractions

16. Solve for x: $3(x - 2) = 9$
17. Solve for x: $2(x + 4) = 10$
18. Solve for x: $5(x - 1) = 3x + 9$

19. Solve for x: $\dfrac{2x-3}{5} = 7$

20. Solve for x: $\dfrac{x+2}{4} = \dfrac{x-1}{2}$

Part E: Word Problems

21. The sum of a number and 8 is equal to 15. What is the number?

22. A number decreased by 6 equals twice that number minus 12. Find the number.

23. Five more than three times a number is equal to 20. What is the number?

24. When 4 is subtracted from half a number, the result is 6. What is the number?

25. The perimeter P of a rectangle is given by $P = 2L + 2W$. If $P = 30$ and $W = 5$, find the length L.

Answer Key and Solutions: Solving Linear Equations

Part A Solutions: One-Step Equations

1. $x + 9 = 14 \Rightarrow x = 14 - 9 = \boxed{5}$
2. $x - 5 = -2 \Rightarrow x = -2 + 5 = \boxed{3}$
3. $4x = 20 \Rightarrow x = \dfrac{20}{4} = \boxed{5}$
4. $\dfrac{x}{3} = 7 \Rightarrow x = 7 \cdot 3 = \boxed{21}$
5. $-8x = 32 \Rightarrow x = \dfrac{32}{-8} = \boxed{-4}$

Part B Solutions: Two-Step Equations

6. $3x + 4 = 13 \Rightarrow 3x = 9 \Rightarrow x = \boxed{3}$
7. $5x - 6 = 19 \Rightarrow 5x = 25 \Rightarrow x = \boxed{5}$
8. $\dfrac{x}{2} + 7 = 12 \Rightarrow \dfrac{x}{2} = 5 \Rightarrow x = \boxed{10}$
9. $8x - 9 = 23 \Rightarrow 8x = 32 \Rightarrow x = \boxed{4}$
10. $4x + 5 = -11 \Rightarrow 4x = -16 \Rightarrow x = \boxed{-4}$

Part C Solutions: Variables on Both Sides

11. $2x + 7 = 3x - 5 \Rightarrow 7 = x - 5 \Rightarrow x = \boxed{12}$
12. $5x - 8 = 2x + 7 \Rightarrow 3x = 15 \Rightarrow x = \boxed{5}$
13. $9x + 4 = 4x + 19 \Rightarrow 5x = 15 \Rightarrow x = \boxed{3}$
14. $6x - 10 = 2x + 14 \Rightarrow 4x = 24 \Rightarrow x = \boxed{6}$
15. $3x + 9 = -x + 25 \Rightarrow 4x = 16 \Rightarrow x = \boxed{4}$

Part D Solutions: Parentheses and Fractions

16. $3(x-2) = 9 \Rightarrow 3x - 6 = 9 \Rightarrow 3x = 15 \Rightarrow x = \boxed{5}$

17. $2(x+4) = 10 \Rightarrow 2x + 8 = 10 \Rightarrow 2x = 2 \Rightarrow x = \boxed{1}$

18. $5(x-1) = 3x + 9 \Rightarrow 5x - 5 = 3x + 9 \Rightarrow 2x = 14 \Rightarrow x = \boxed{7}$

19. $\dfrac{2x-3}{5} = 7 \Rightarrow 2x - 3 = 35 \Rightarrow 2x = 38 \Rightarrow x = \boxed{19}$

20. $\dfrac{x+2}{4} = \dfrac{x-1}{2} \Rightarrow 2(x+2) = 4(x-1) \Rightarrow 2x + 4 = 4x - 4 \Rightarrow 8 = 2x \Rightarrow x = \boxed{4}$

Part E Solutions: SAT-Style Word Problems

21. Let the number be n. $n + 8 = 15 \Rightarrow n = \boxed{7}$

22. Let the number be n. $n - 6 = 2n - 12 \Rightarrow 6 = n \Rightarrow \boxed{6}$

23. Let the number be n. $3n + 5 = 20 \Rightarrow 3n = 15 \Rightarrow n = \boxed{5}$

24. Let the number be n. $\dfrac{n}{2} - 4 = 6 \Rightarrow \dfrac{n}{2} = 10 \Rightarrow n = \boxed{20}$

25. $P = 2L + 2W$. With $P = 30$ and $W = 5$: $30 = 2L + 10 \Rightarrow 20 = 2L \Rightarrow L = \boxed{10}$