Unit 1: Linear Relationships and Equations

Topic 1: Variables, Expressions, and Equations

Concept Summary

Mathematical expressions and equations are the building blocks of algebra. **Variables** represent unknown values. We often use letters like x, y, or n to stand for a number that can change.

An **expression** is a combination of numbers, variables, and operations, but it does not include an equals sign.

$$3x + 5 \quad \text{or} \quad 2(a - 4)$$

An **equation** shows that two expressions are equal.

$$3x + 5 = 17$$

To **solve an equation** means to find the value of the variable that makes the equation true. The goal is to isolate the variable on one side of the equation using opposite operations.

Core Skills

- Simplify expressions by combining like terms and using the distributive property.
- Translate word phrases into algebraic expressions.
- Solve simple linear equations by isolating the variable.

Example 1: Simplifying an Expression

Simplify the expression:

$$3(2y - 4) + y$$

Step 1: Apply the distributive property.

$$3 \times 2y = 6y, \quad 3 \times (-4) = -12$$

So the expression becomes:

$$6y - 12 + y$$

Step 2: Combine like terms.

$$6y + y = 7y$$

Final Answer:

$$\boxed{7y - 12}$$

Example 2: Solving a Simple Equation

Solve for x:

$$2x + 5 = 19$$

Step 1: Subtract 5 from both sides.

$$2x + 5 - 5 = 19 - 5$$

$$2x = 14$$

Step 2: Divide both sides by 2.

$$x = \frac{14}{2}$$

$$\boxed{x = 7}$$

Check: Substitute $x = 7$ back into the original equation:

$$2(7) + 5 = 19 \quad \checkmark$$

Key Takeaways

- Expressions do *not* have an equals sign; equations do.
- Always perform the same operation on both sides of an equation to keep it balanced.
- Checking your solution helps confirm accuracy.

Practice Questions

Part A: Simplifying Expressions

1. Simplify: $4x + 3x - 5$
2. Simplify: $5(2y - 1) + 3y$
3. Simplify: $2(a + 3) - 4(a - 1)$
4. Simplify: $7m - 2(3m - 5)$
5. Simplify: $3x + 2y - (4x - 3y)$

Part B: Evaluating Expressions

6. If $x = 4$, evaluate $3x + 5$.
7. If $a = 2$ and $b = -3$, evaluate $2a - 3b$.
8. If $p = -1$, evaluate $4p^2 - 3p + 2$.
9. If $x = 5$ and $y = -2$, evaluate $2x - 3y$.
10. If $m = 6$, evaluate $\dfrac{2m - 4}{m}$.

Part C: Solving Linear Equations

11. Solve for x: $x + 7 = 12$
12. Solve for x: $4x - 5 = 11$
13. Solve for y: $3y + 2 = 17$
14. Solve for x: $2x - 3 = 9$
15. Solve for x: $5x + 8 = 23$

Part D: Multi-Step and Fractional Equations

16. Solve for x: $3(x - 2) = 9$
17. Solve for x: $2(x + 4) = 5x - 6$

18. Solve for x: $\dfrac{2x-3}{5} = 3$

19. Solve for x: $\dfrac{x+2}{4} = \dfrac{x-1}{2}$

20. Solve for x: $7x - 4 = 2x + 11$

Answer Key and Solutions

Part A Solutions

1. $4x + 3x - 5 = 7x - 5$
2. $5(2y - 1) + 3y = 10y - 5 + 3y = 13y - 5$
3. $2(a + 3) - 4(a - 1) = 2a + 6 - 4a + 4 = -2a + 10$
4. $7m - 2(3m - 5) = 7m - 6m + 10 = m + 10$
5. $3x + 2y - (4x - 3y) = 3x + 2y - 4x + 3y = -x + 5y$

Part B Solutions

6. $3(4) + 5 = 12 + 5 = 17$
7. $2(2) - 3(-3) = 4 + 9 = 13$
8. $4(-1)^2 - 3(-1) + 2 = 4 + 3 + 2 = 9$
9. $2(5) - 3(-2) = 10 + 6 = 16$
10. $\dfrac{2(6) - 4}{6} = \dfrac{8}{6} = \dfrac{4}{3}$

Part C Solutions

11. $x + 7 = 12 \Rightarrow x = 5$
12. $4x - 5 = 11 \Rightarrow 4x = 16 \Rightarrow x = 4$
13. $3y + 2 = 17 \Rightarrow 3y = 15 \Rightarrow y = 5$
14. $2x - 3 = 9 \Rightarrow 2x = 12 \Rightarrow x = 6$
15. $5x + 8 = 23 \Rightarrow 5x = 15 \Rightarrow x = 3$

Part D Solutions

16. $3(x-2) = 9 \Rightarrow 3x - 6 = 9 \Rightarrow 3x = 15 \Rightarrow x = 5$

17. $2(x+4) = 5x - 6 \Rightarrow 2x + 8 = 5x - 6 \Rightarrow 14 = 3x \Rightarrow x = \dfrac{14}{3}$

18. $\dfrac{2x-3}{5} = 3 \Rightarrow 2x - 3 = 15 \Rightarrow 2x = 18 \Rightarrow x = 9$

19. $\dfrac{x+2}{4} = \dfrac{x-1}{2} \Rightarrow 2(x+2) = 4(x-1) \Rightarrow 2x + 4 = 4x - 4 \Rightarrow 8 = 2x \Rightarrow x = 4$

20. $7x - 4 = 2x + 11 \Rightarrow 5x = 15 \Rightarrow x = 3$