Unit 1: Linear Relationships and Equations

Topic 4: Linear Inequalities (Including Compound and Absolute Value)

Concept Summary

An **inequality** shows a relationship where two expressions are not necessarily equal. Instead, one side is greater or less than the other.

$a < b$ means a is less than b

$a > b$ means a is greater than b

$a \leq b$ means a is less than or equal to b

$a \geq b$ means a is greater than or equal to b

Solving inequalities is very similar to solving equations — the same operations can be performed on both sides. However, there is one key rule to remember:

When you multiply or divide by a negative number, you must flip the inequality sign.

Compound Inequalities

Sometimes two inequalities are joined by the words **and** or **or**.

- "**and**" means both conditions must be true — the overlap of the solution sets.
- "**or**" means either condition can be true — the combined solution set.

Example of an "and" compound inequality:

$$-2 < x \leq 5$$

This means x is greater than -2 *and* less than or equal to 5.

Absolute Value Equations and Inequalities

The **absolute value** of a number represents its distance from 0 on the number line. Distance is always positive.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

- If $|x| = a$, then $x = a$ or $x = -a$.
- If $|x| < a$, then $-a < x < a$.
- If $|x| > a$, then $x < -a$ or $x > a$.

Core Skills

- Apply the same operations to both sides of an inequality.
- Reverse the inequality symbol when multiplying or dividing by a negative.
- Solve and represent compound inequalities.
- Split absolute value inequalities into two separate linear inequalities.

Example 1: Solving a Simple Inequality

Solve for x:

$$2x - 5 < 7$$

Step 1: Add 5 to both sides.

$$2x < 12$$

Step 2: Divide by 2.

$$x < 6$$

Final Answer: $\boxed{x < 6}$

Graph: Shade all numbers less than 6 on the number line (open circle at 6).

Example 2: Absolute Value Inequality

Solve for x:

$$|x - 3| \leq 5$$

Step 1: Write as a compound inequality.

$$-5 \leq x - 3 \leq 5$$

Step 2: Add 3 to all sides.

$$-2 \leq x \leq 8$$

Final Answer: $\boxed{-2 \leq x \leq 8}$
Interpretation: All values of x that are within 5 units of 3 satisfy the inequality.

Key Takeaways

- Solving inequalities follows the same steps as solving equations, except when multiplying or dividing by a negative number - always flip the sign.
- Compound inequalities combine solution sets with "and" or "or".
- Absolute value inequalities describe distances from a central point on the number line.

Practice Questions: Linear Inequalities (Including Compound and Absolute Value)

Part A: Basic Inequalities

1. Solve for x: $x + 5 < 9$
2. Solve for x: $2x - 3 \geq 7$
3. Solve for x: $5x + 4 < 19$
4. Solve for x: $4x - 8 > 0$
5. Solve for x: $-3x + 2 \leq 11$

Part B: Inequalities with Negative Coefficients

6. Solve for x: $-2x + 5 > 9$
7. Solve for x: $-4x - 8 < 0$
8. Solve for x: $-3x + 7 \leq 1$
9. Solve for x: $-6x > 12$
10. Solve for x: $-5x + 4 \geq 9$

Part C: Compound Inequalities

11. Solve for x: $2 < x + 5 \leq 9$
12. Solve for x: $-3 \leq 2x - 1 < 5$
13. Solve for x: $x - 4 > 2$ or $x + 1 < 0$
14. Solve for x: $3x + 2 \leq 8$ and $x - 1 > 0$
15. Solve for x: $x + 2 > 6$ or $x - 3 < -2$

Part D: Absolute Value Equations and Inequalities

16. Solve for x: $|x| = 4$

17. Solve for x: $|x - 3| = 7$

18. Solve for x: $|x + 2| < 5$

19. Solve for x: $|2x - 4| \leq 6$

20. Solve for x: $|3x + 1| > 7$

Part E: SAT-Style Word and Application Problems

21. A phone plan costs \$20 per month plus \$0.10 per text message. If a customer's bill must stay under \$50, write and solve an inequality for the number of messages t.

22. The inequality $5x + 20 \leq 70$ represents a budget constraint. What is the greatest possible value of x?

23. The temperature T (in °F) must stay within 8 degrees of 72°F. Write an absolute value inequality that represents this situation.

24. The length L of a rod must be within 0.5 cm of 10 cm. Write and solve an inequality for L.

25. The profit P from selling x products is given by $P = 15x - 120$. The company wants at least \$60 profit. Write and solve an inequality for x.

Answer Key and Solutions: Linear Inequalities (Including Compound and Absolute Value)

Part A Solutions: Basic Inequalities

1. $x + 5 < 9 \Rightarrow x < \boxed{4}$
2. $2x - 3 \geq 7 \Rightarrow 2x \geq 10 \Rightarrow x \geq \boxed{5}$
3. $5x + 4 < 19 \Rightarrow 5x < 15 \Rightarrow x < \boxed{3}$
4. $4x - 8 > 0 \Rightarrow 4x > 8 \Rightarrow x > \boxed{2}$
5. $-3x + 2 \leq 11 \Rightarrow -3x \leq 9 \Rightarrow x \geq \boxed{-3}$ (flip sign when dividing by -3)

Part B Solutions: Inequalities with Negative Coefficients

6. $-2x + 5 > 9 \Rightarrow -2x > 4 \Rightarrow x < \boxed{-2}$
7. $-4x - 8 < 0 \Rightarrow -4x < 8 \Rightarrow x > \boxed{-2}$
8. $-3x + 7 \leq 1 \Rightarrow -3x \leq -6 \Rightarrow x \geq \boxed{2}$
9. $-6x > 12 \Rightarrow x < \boxed{-2}$
10. $-5x + 4 \geq 9 \Rightarrow -5x \geq 5 \Rightarrow x \leq \boxed{-1}$

Part C Solutions: Compound Inequalities

11. $2 < x + 5 \leq 9 \Rightarrow -3 < x \leq \boxed{4}$
12. $-3 \leq 2x - 1 < 5 \Rightarrow -2 \leq 2x < 6 \Rightarrow -1 \leq x < \boxed{3}$
13. $x - 4 > 2$ or $x + 1 < 0 \Rightarrow x > 6$ or $x < \boxed{-1}$
14. $3x + 2 \leq 8$ and $x - 1 > 0 \Rightarrow x \leq 2$ and $x > 1$. Combine: $\boxed{1 < x \leq 2}$
15. $x + 2 > 6$ or $x - 3 < -2 \Rightarrow x > 4$ or $x < \boxed{1}$

Part D Solutions: Absolute Value Equations and Inequalities

16. $|x| = 4 \Rightarrow x = \boxed{4}$ or $x = \boxed{-4}$

17. $|x - 3| = 7 \Rightarrow x - 3 = 7$ or $x - 3 = -7 \Rightarrow x = \boxed{10}$ or $x = \boxed{-4}$

18. $|x + 2| < 5 \Rightarrow -5 < x + 2 < 5 \Rightarrow \boxed{-7 < x < 3}$

19. $|2x - 4| \leq 6 \Rightarrow -6 \leq 2x - 4 \leq 6 \Rightarrow -2 \leq 2x \leq 10 \Rightarrow \boxed{-1 \leq x \leq 5}$

20. $|3x + 1| > 7 \Rightarrow 3x + 1 > 7$ or $3x + 1 < -7 \Rightarrow x > 2$ or $x < \boxed{-\frac{8}{3}}$

Part E Solutions: SAT-Style Word and Application Problems

21. Cost model: $20 + 0.10t < 50 \Rightarrow 0.10t < 30 \Rightarrow t < \boxed{300}$. For whole texts, $t \leq 299$.

22. $5x + 20 \leq 70 \Rightarrow 5x \leq 50 \Rightarrow x \leq \boxed{10}$. Greatest possible value is 10.

23. Temperatures within 8 of 72: $\boxed{|T - 72| \leq 8}$.

24. Length within 0.5 of 10: $|L - 10| \leq 0.5 \Rightarrow \boxed{9.5 \leq L \leq 10.5}$.

25. Profit at least 60: $15x - 120 \geq 60 \Rightarrow 15x \geq 180 \Rightarrow x \geq \boxed{12}$.