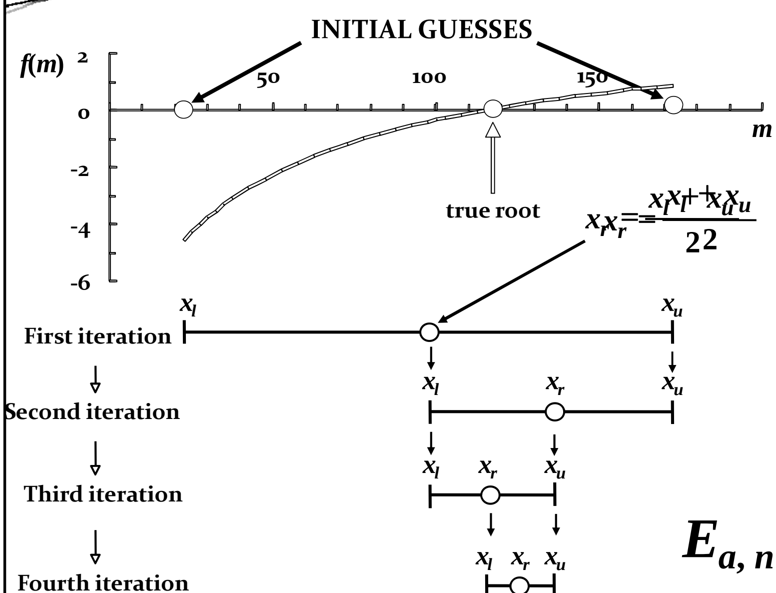
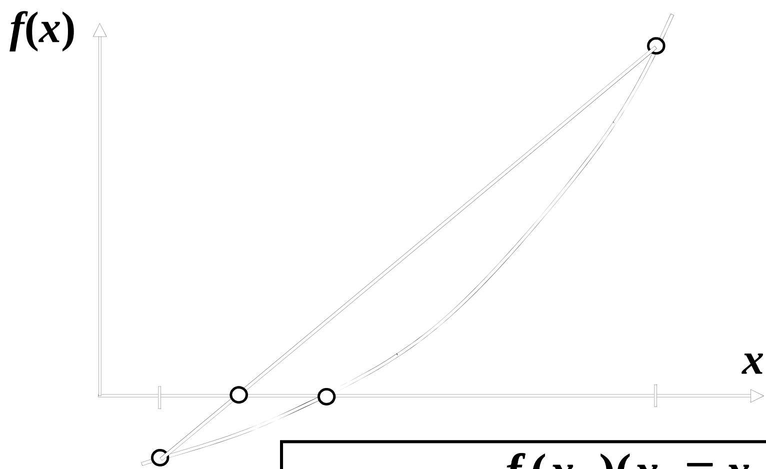
Lecture 06: Root of equations – SUMMARY

BISECTION



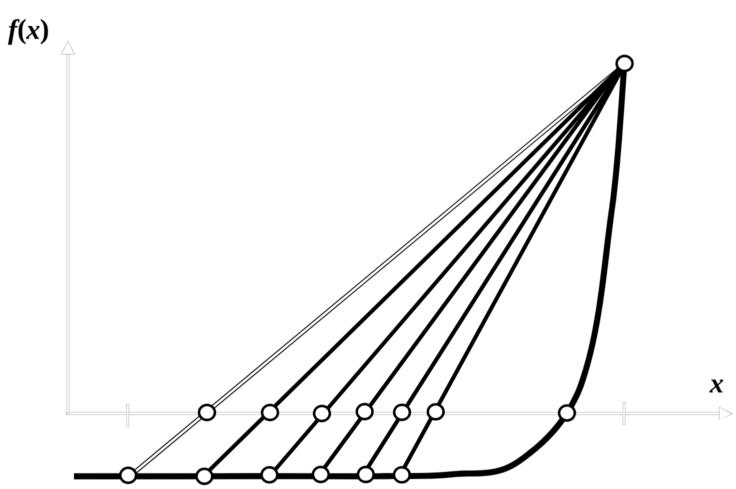
$$E_{a,n} = \frac{\Delta x_0}{2^n}$$

FALSE POSITION

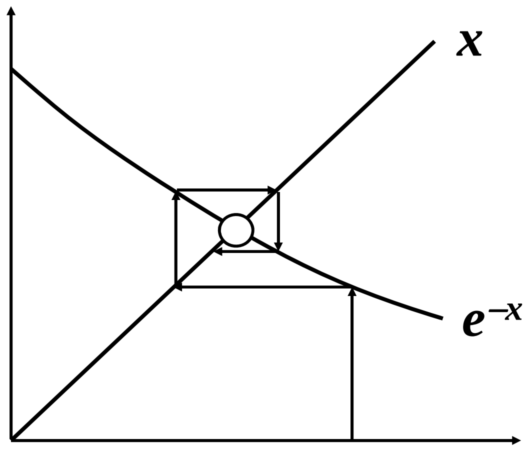


$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

PITFALLS OF FALSE POSITION



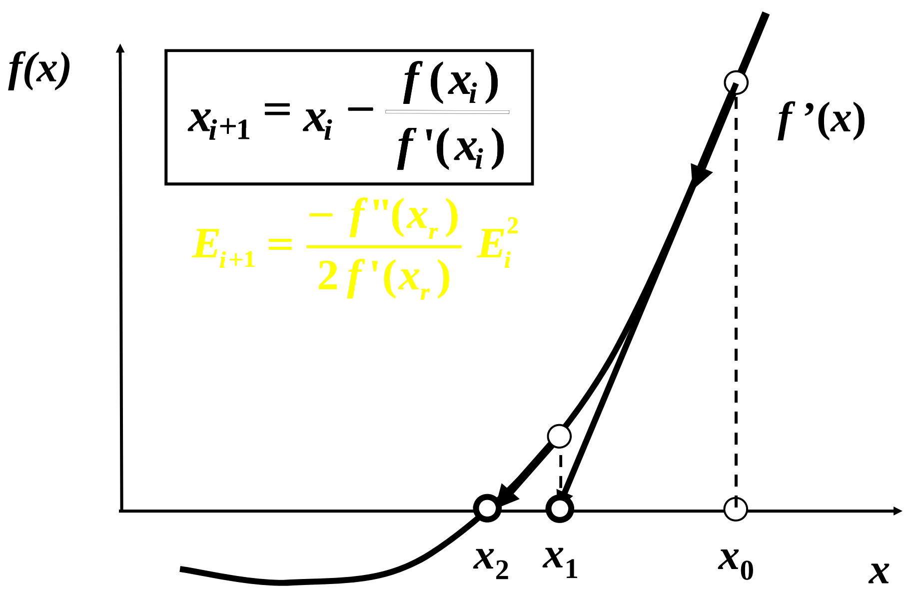
FIXED-POINT ITERATION



$$\boxed{x_{i+1} = g(x_i)}$$

$$E_{i+1} = g'(\xi)\, E_i$$

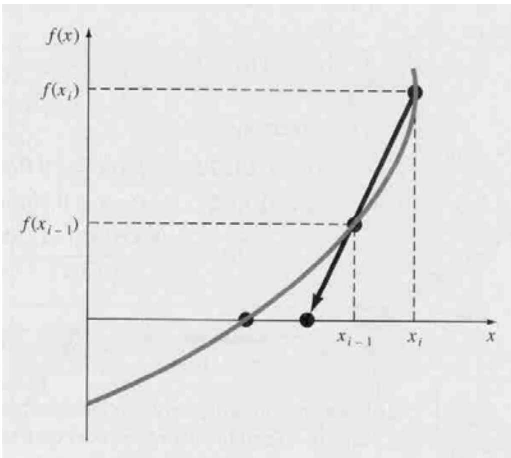
NEWTON-RAPHSON



$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

$$E_{i+1} = \frac{-f''(x_r)}{2f'(x_r)} E_i^2$$

SECANT METHOD

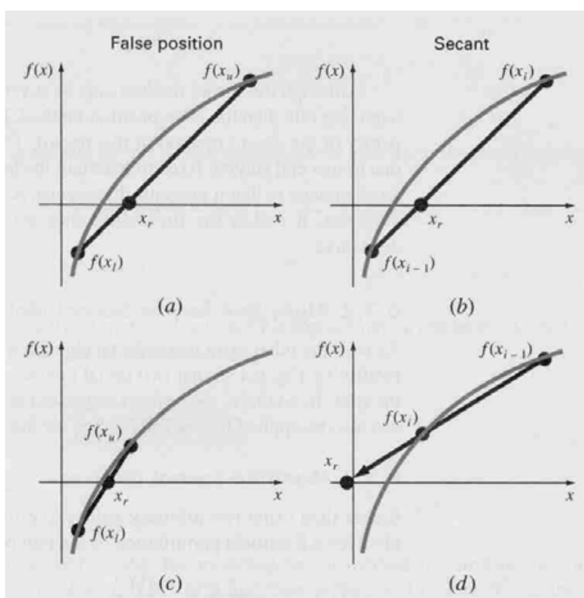


$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x_i) \cong \frac{f(x_{i-1}) - f(x_i)}{x_{i-1} - x_i}$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}}$$

FALSE POSITION VS SECANT



MODIFIED SECANT METHOD

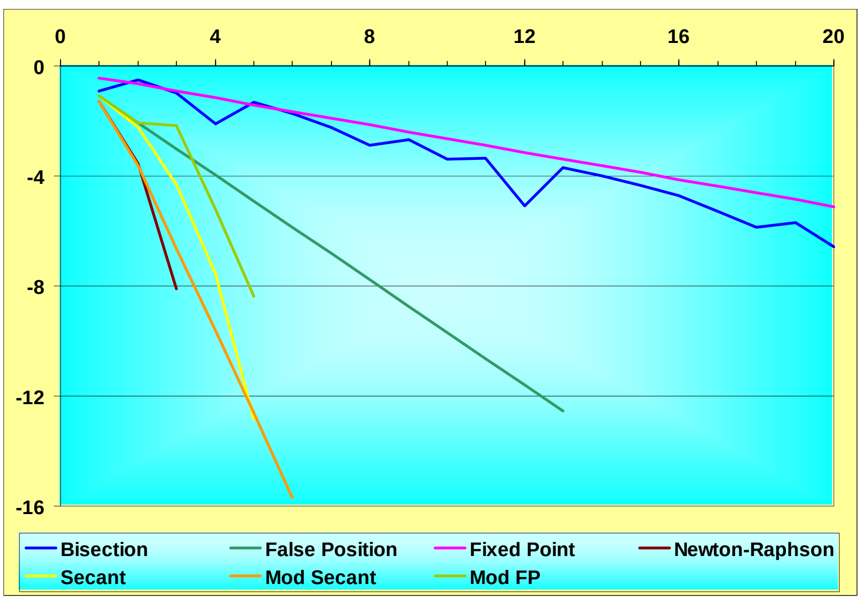
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x_i) \cong \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i}$$

$$\boxed{x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)}}$$

Generally faster than secant

CONVERGENCE SPEEDS



ROOT COMPARISON

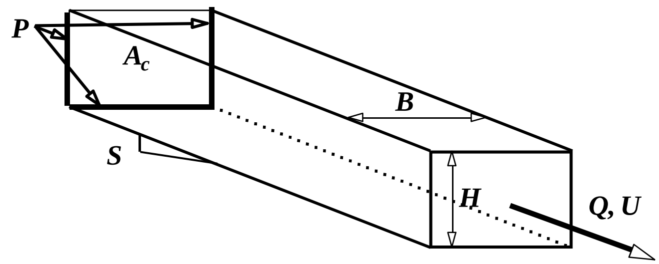
TABLE PT2.3 Comparison of the characteristics of alternative methods for finding roots of algebraic and transcendental equations. The comparisons are based on general experience and do not account for the behavior of specific functions.

Method	Initial Guesses	Convergence Rate	Stability	Accuracy	Breadth of Application	Programming Effort	Comments
Direct	—	—	—	—	Limited		
Graphical	—	—	—	Poor	Real roots	—	May take more time than the numerical method
Bisection	2	Slow	Always	Good	Real roots	Easy	
False-position	2	Slow/medium	Always	Good	Real roots	Easy	
Modified FP	2	Medium	Always	Good	Real roots	Easy	
Fixed-point iteration	1	Slow	Possibly divergent	Good	General	Easy	
Newton-Raphson	1	Fast	Possibly divergent	Good	General	Easy	Requires evaluation of $f'(x)$
Modified Newton-Raphson	1	Fast for multiple roots; medium for single	Possibly divergent	Good	General	Easy	Requires evaluation of $f''(x)$ and $f'(x)$
Secant	2	Medium to fast	Possibly divergent	Good	General	Easy	Initial guesses do not have to bracket the root
Modified secant	1	Medium to fast	Possibly divergent	Good	General	Easy	
Müller	2	Medium to fast	Possibly divergent	Good	Polynomials	Moderate	
Bairstow	2	Fast	Possibly divergent	Good	Polynomials	Moderate	

MATLAB ROOT SOLVING FUNCTIONS

- fzero
- roots

ENGINEERING APPLICATION: MANNING EQUATION



Continuity equation: (mass balance)

$$Q = UBH$$

Manning equation: (momentum balance)

$$U = \frac{1}{n}\left(\frac{BH}{B+2H}\right)^{2/3} S^{1/2}$$

MANNING AS ROOTS

$$U = \frac{1}{n}\left(\frac{BH}{B+2H}\right)^{2/3} S^{1/2}$$

$$Q = UBH$$

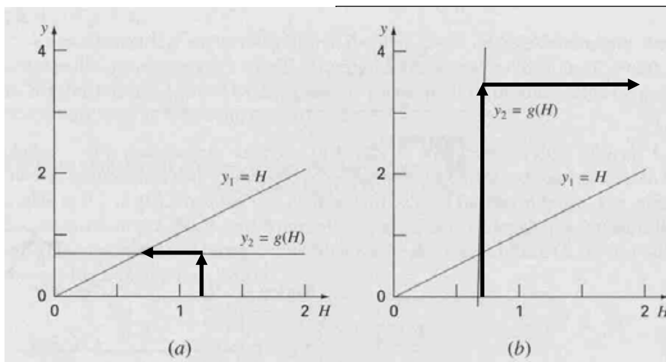
$$Q = \frac{1}{n}\,\frac{(BH)^{5/3}}{(B+2H)^{2/3}}\,S^{1/2}$$

$$f(H) = \frac{1}{n}\,\frac{(BH)^{5/3}}{(B+2H)^{2/3}}\,S^{1/2} - Q$$

HOW ABOUT FIXED POINT?

$$\frac{1}{n}\frac{(BH)^{5/3}}{(B+2H)^{2/3}}S^{1/2} - Q = 0$$

$$H = \frac{(Qn)^{3/5}(B+2H)^{2/5}}{B\,S^{3/10}} \qquad H = \frac{1}{2}\left[\frac{S^3(BH)^{5/2}}{(Qn)^{3/2}} - B\right]$$



CASE STUDY