Problem 1:

$$A = \begin{vmatrix} 10 & -3 \\ 4 & 2 \end{vmatrix} \qquad B = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} \qquad v = \begin{vmatrix} 1 \\ 2 \end{vmatrix} \qquad w = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

a) v'w

For calculating v'w we calculate v' first

$$v' = \begin{vmatrix} 1 & 2 \end{vmatrix}$$

$$v'w = \begin{vmatrix} 1 & 2 \end{vmatrix} X \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1*1+2*1 \end{vmatrix}$$

$$= \begin{vmatrix} 3 \end{vmatrix}$$

b) v.w'

For calculating vw' we calculate w' first

$$w' = \begin{vmatrix} 1 & 1 \end{vmatrix}$$

$$vw' = \begin{vmatrix} 1 \\ 2 \end{vmatrix} X \begin{vmatrix} 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1*1 & 1*1 \\ 2*1 & 2*1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$$

c) Av

$$Av = \begin{vmatrix} 10 & -3 \\ 4 & 2 \end{vmatrix} X \begin{vmatrix} 1 \\ 2 \end{vmatrix}$$

$$= \begin{vmatrix} 10* +(-3*2) \\ 4*1 + 2*2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 \\ 8 \end{vmatrix}$$

d) A'v

For calculating A'v we calculate A' first

$$A' = \begin{vmatrix} 10 & 4 \\ -3 & 2 \end{vmatrix}$$

$$A'v = \begin{vmatrix} 10 & 4 \\ -3 & 2 \end{vmatrix} X \begin{vmatrix} 1 \\ 2 \end{vmatrix}$$

$$= \begin{vmatrix} 10*1 + 4*2 \\ (-3)*1 + 2*2 \end{vmatrix}$$

$$= \begin{vmatrix} 18 \\ 1 \end{vmatrix}$$

e) AB

$$AB = \begin{vmatrix} 10 & -3 \\ 4 & 2 \end{vmatrix} X \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 10*1 + (-3*-1) & 10*0+(-3*2) \\ 4*1 + 2*(-1) & 4*0 + 2*2 \end{vmatrix}$$

$$= \begin{vmatrix} 13 & -6 \\ 2 & 4 \end{vmatrix}$$

f) BA

$$BA = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} X \begin{vmatrix} 10 & -3 \\ 4 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1*10 + 0*4 & 1*(-3) + 0*2 \\ (-1)*10 + 2*4 & (-1)*(-3) + 2*2 \end{vmatrix}$$

$$= \begin{vmatrix} 10 & -3 \\ -2 & 7 \end{vmatrix}$$

g) A^2

$$A^2 = A \quad X \quad A$$

$$= \begin{vmatrix} 10 & -3 \\ 4 & 2 \end{vmatrix} X \begin{vmatrix} 10 & -3 \\ 4 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 10*10 +(-3)*4 & 10*(-3) + (-3)*2 \\ 4*10 + 2*4 & 4*(-3) + 2*2 \end{vmatrix}$$

$$= \begin{vmatrix} 88 & -36 \\ 48 & -8 \end{vmatrix}$$

h) Vector y for which By = w

Since w is a 2x1 matrice the vector y should be 2x1 matrice

Lets assume

$$y = \begin{vmatrix} y1 \\ y2 \end{vmatrix}$$

$$By = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} X \begin{vmatrix} y1 \\ y2 \end{vmatrix} = w = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1*y1 + 0*y2 \\ (-1)*y1 + 2*y2 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

$$= \begin{vmatrix} y1 \\ 2y2-y1 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

Solving this gives us $y = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$