

Problem 1. Solution

	Time Period	1 Sec	1 min	1 hour	1 month	1yr	1 century
f(n)	microsec (t)	1.00E+06	6.00E+07	3.60E+09	2.59E+12	3.15E+13	3.15E+15
lg n	$n = 2^{(t)}$	$2^{1.00E+06}$	$2^{6.00E+07}$	$2^{3.60E+09}$	$2^{2.59E+12}$	$2^{3.15E+13}$	$2^{3.15E+15}$
\sqrt{n}	$n = (t)^2$	1.00E+12	3.60E+15	1.30E+19	6.72E+24	9.95E+26	9.95E+30
n	$n = (t)$	1.00E+06	6.00E+07	3.60E+09	2.59E+12	3.15E+13	3.15E+15
n lg n	$n = e^{(W(\ln(2)t))}$	5.15E+04	2.37E+06	1.15E+08	6.37E+10	7.12E+11	6.19E+13
n^2	$n = \sqrt{t}$	1000	7745	60000	1609968	5615692	56156922
n^3	$n = \sqrt[3]{t}$	100	391	1532	13736	31593	146645
2^n	$\lg(t)$	19	25	31	41	44	51
n!		9	11	12	15	16	17

Solving algorithm

Since the time required to solve for n data points is f(n) microsecs

Therefore number of data points that can be solved in t microsec = $f^{-1}(t)$ rounded down to whole number

Table shows Possible data set of size n that can be solved for the given Time

n lg n can be solved using Lamber W Function

converting log to the base 2 to natural log

$$n (\ln(n)) / \ln(2)$$

$$n \ln(n) = \ln(2) \cdot t$$

$$n = e^{(W(\ln(2) \cdot t))}$$

$$W_0(x) = \ln x - \ln \ln x + O(1)$$

$$W_0 = \ln(.6931472 \cdot t) - \ln \ln(.6931472 \cdot t) + O(1)$$

n! can be solved using an iterative method with below steps

$$n=1$$

$$t$$

```
while(t/n >= (n+1)){
    t=t/n;
    n++
}
```

Problem 2. Solution

	A	B	O	o	Ω	ω	Θ
a	lgkn	n^E	Yes	Yes	Yes	Yes	Yes
b	n^k	c^n	Yes	Yes	No	No	No
c	\sqrt{n}	$n^{\sin n}$	No	No	No	No	No
d	2^n	$2^{n/2}$	No	No	Yes	Yes	No
e	$n \lg c$	$c \lg n$	No	No	Yes	Yes	No
f	$\lg(n!)$	$n \lg(n)$	Yes	No	Yes	No	Yes

Function Definitions

Function (A) belongs to the set of Big O of function B if $A \leq C \cdot B$ for some value of C

Function (A) belongs to the set of Small o of function B if $A < C \cdot B$ for all values of C

Function (A) belongs to the set of Ω function B if $A \geq B$ for some value of C

Function (A) belongs to the set of ω function B if $A > C \cdot B$ for all values of C

Function (A) belongs to the set of Θ function B if $C_1 \cdot B \leq A \leq C_2 \cdot B$

a $\lg kn$ n^E

For $k = 1$ and $E = 2$; $A < B$ hence A is Big (0) of B

For $k = 1$ and $E = .1$; $A > B$ hence A is Big (Omega) of B

Since the curve of the function is highly dependent on the K and e terms

The function will behave differently based on K and E hence all are true

b n^k c^n

n^k is smaller than c^n hence it's a Big O of c^n

This will stand true for all values of C after a certain n hence its also small O of C^n

c \sqrt{n} $n^{\sin n}$

$\sin n$ is a cyclic function (-1 to 1) so function A is not a part of any function

as B will fluctuate between 1 and n

d 2^n $2^{n/2}$

2^n will always be bigger than $2^{n/2}$ hence its Big Omega of B

Its true for all values of C so its also w of B

e $n \lg c$ $c \lg n$

$\lg c^n$ $\lg n^c$

since c^n will always be bigger than n^c it's a Big Omega of B

it will be true for all values of C its also w of the B

1.f

A	B
$\lg(n!)$	$n \lg(n)$

$\lg(n!) = \log(n) + \log(n-1) + \dots + \log(1)$

$n \lg(n) = \log(n) + \log(n) + \dots + \log(n)$

so $B > A$

but since its a log function it grows much slower and if we use

a very large constant $B < A$ hence it can be big omega, big theta and big O

Since its dependent on C its neither small O small w