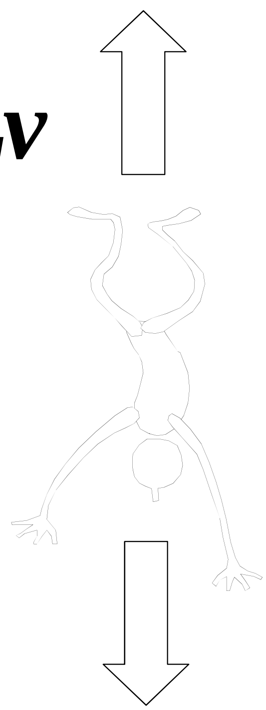
LECTURE 01: MODELING AND COMPUTING

Reading: Chapter 1

(Modified from Dr. Chapra's lecture)

Free Falling Parachutist Problem

$$F_U = -c_d v$$



Free-body Diagram

$$F_D = mg$$

$$F_{\text{net}} = mg - c_d v$$

FORCE BALANCE

Newton's Second Law:

$$F = m\,a$$

$$a = \frac{F}{m} = \frac{m\,g - c_d\,v}{m}$$

$$a = g - \frac{c_d\,v}{m}$$

FORCE BALANCE

$$a = g - \frac{c_d v}{m}$$

$$a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{c_d v}{m}$$

"MATHEMATICAL MODEL" OF THE PARACHUTIST

$$\frac{dv}{dt} = g - \frac{c_d\, v}{m}$$

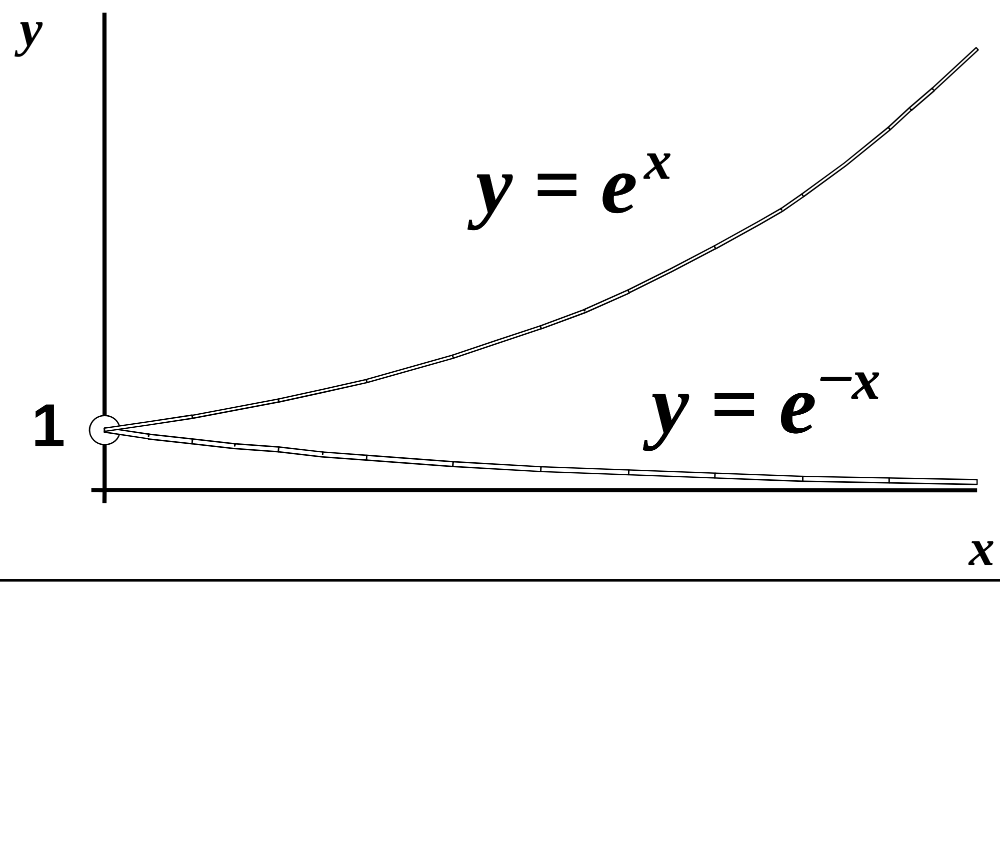
If $v = 0$ at $t = 0$, calculus can be used to solve it for:

$$v = \frac{gm}{c_d}\left(1 - e^{-\frac{c_d}{m}t}\right)$$

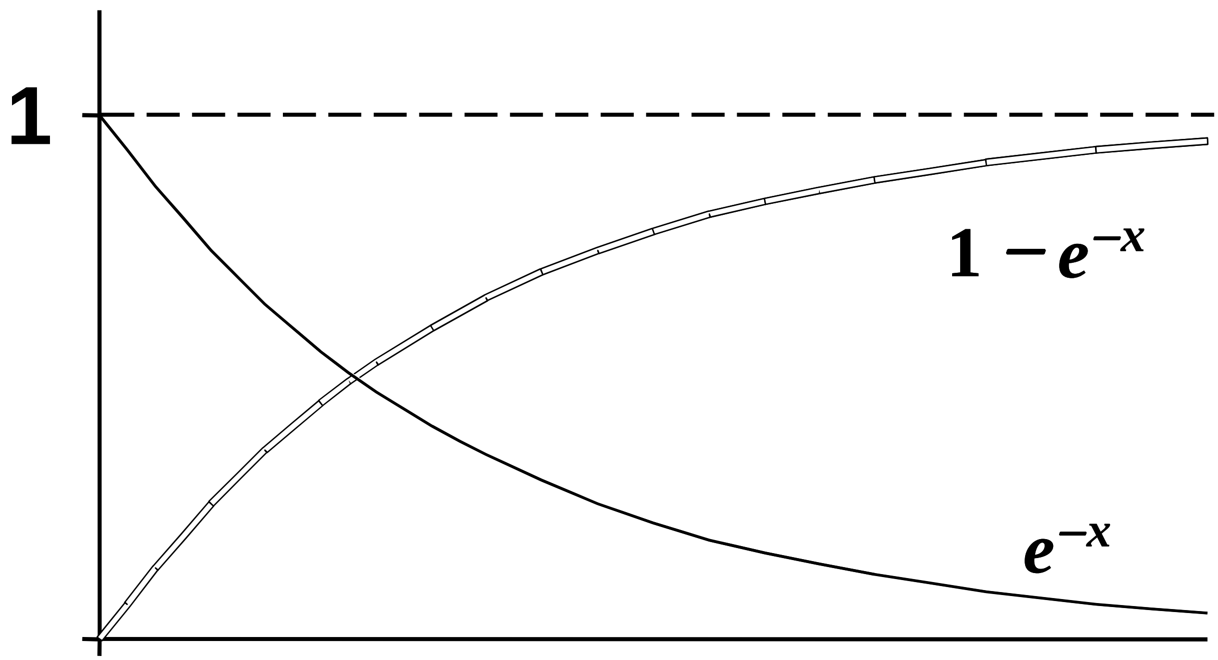
EXPONENTIAL FUNCTION

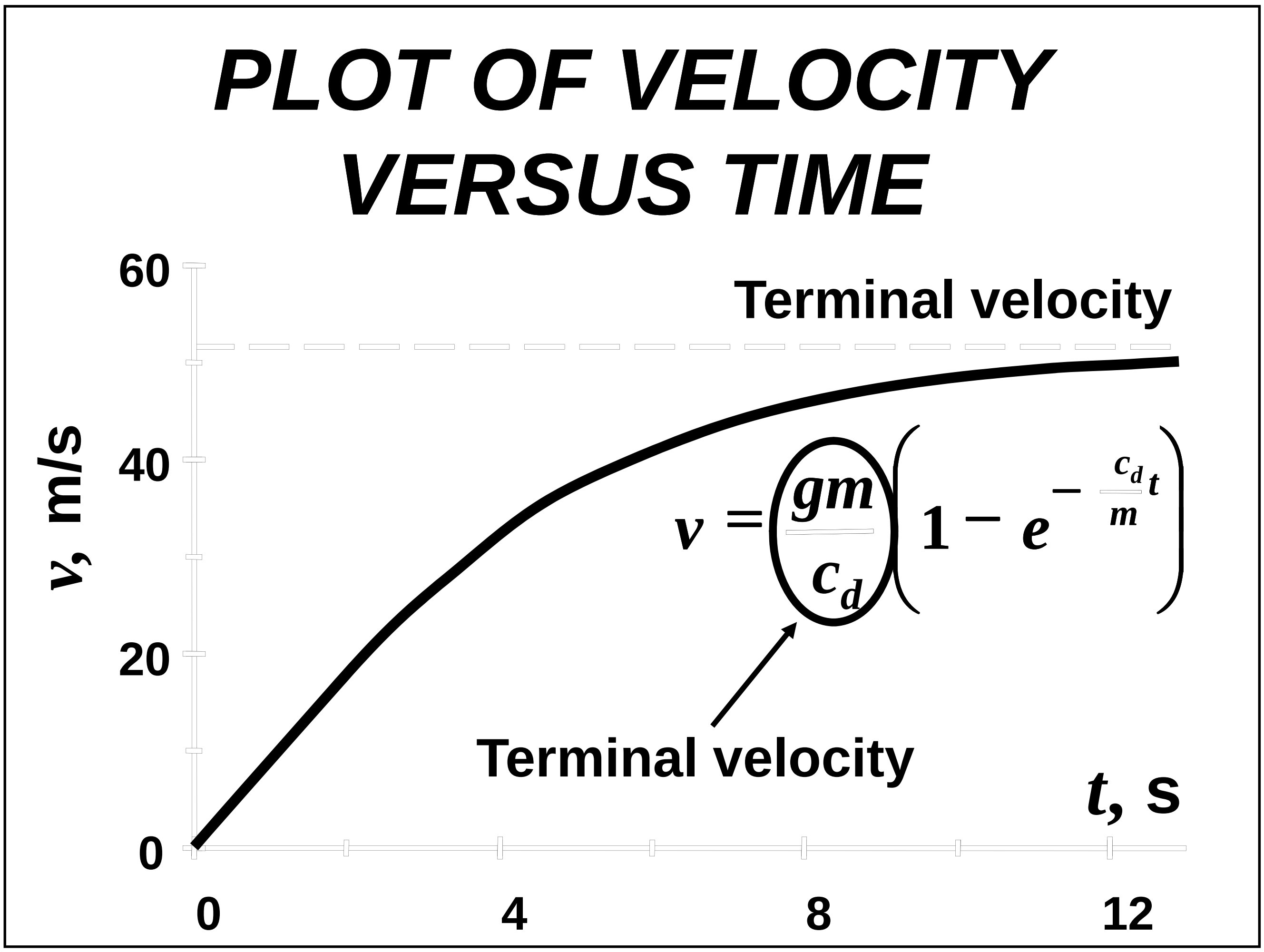
$$y = e^{x}$$

2.7182818

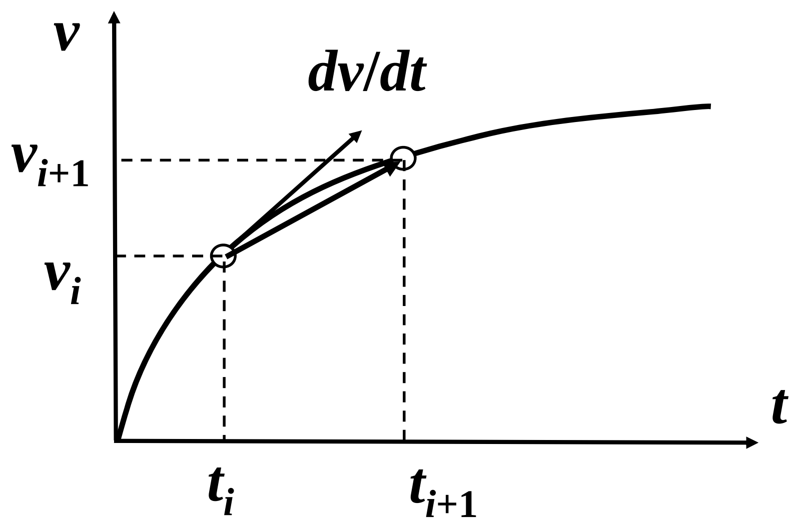


$$1 - e^{-x}$$




PLOT OF VELOCITY VERSUS TIME
60
40
20
0
v, m/s
Terminal velocity
$v = \frac{gm}{c_d}\left(1 - e^{-\frac{c_d}{m}t}\right)$
Terminal velocity
t, s
0
4
8
12

HOW DO YOU SOLVE IT WITH A COMPUTER???



$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

NUMERICAL APPROXIMATION OF THE DERIVATIVE

$$\frac{dv}{dt} = g - \frac{c_d}{m} v$$

$$\Downarrow$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c_d}{m} v(t_i)$$

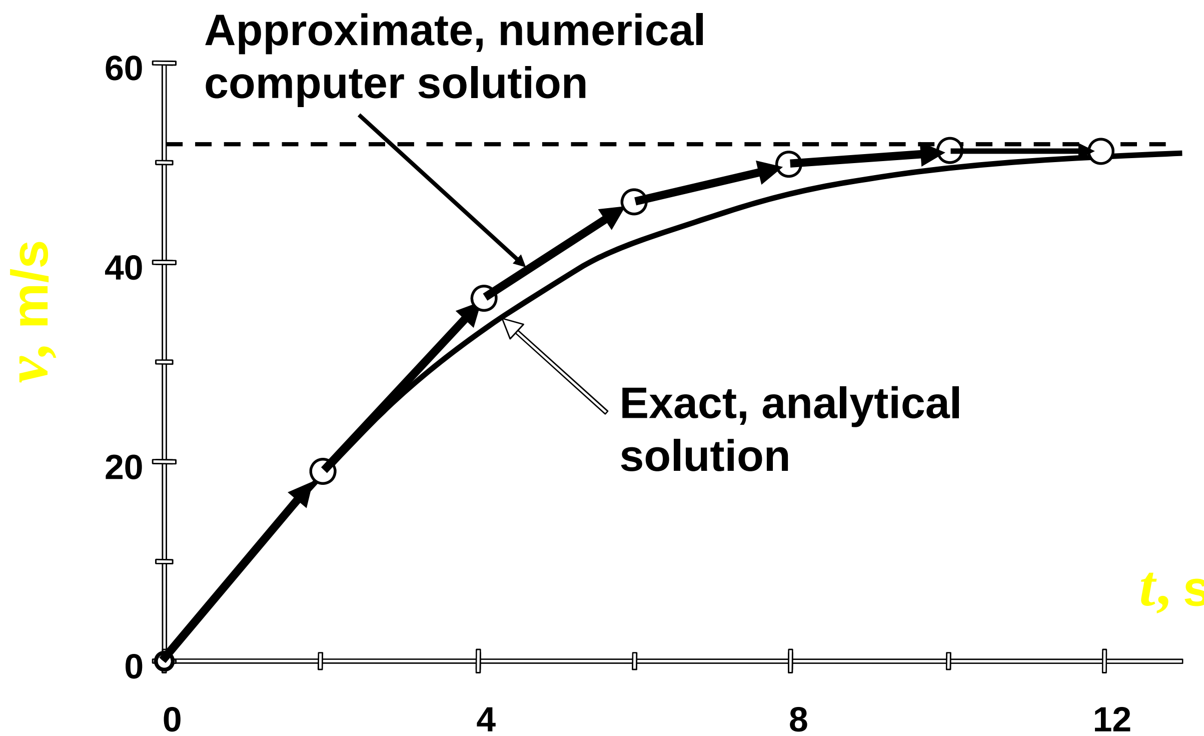
EULER'S METHOD

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c_d}{m} v(t_i) \right] (t_{i+1} - t_i)$$

$$v_{i+1} = v_i + \frac{dv_i}{dt} \Delta t$$

New value = old value + slope × step size

EULER'S METHOD



HOW DO WE MAKE IT MORE ACCURATE???

Take smaller steps

$\Downarrow$

More computational effort

$\Downarrow$

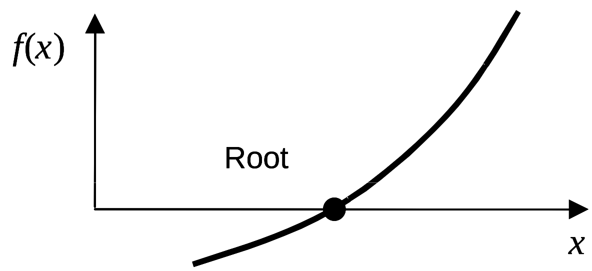
The computer doesn’t care!!!

HOW DO WE IMPLEMENT ON COMPUTER

MATLAB

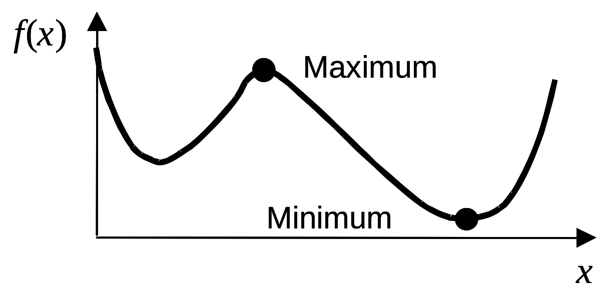
(a) *Part 2*: Roots of equations

Solve $f(x) = 0$ for x



(b) *Part 3*: Optimization

Determine x that gives optimum $f(x)$

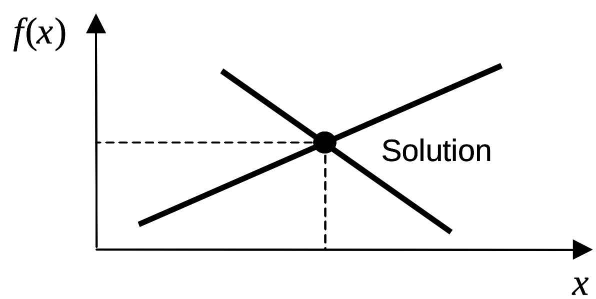


(c) *Part 4*: Linear algebraic equations

Given the a's and the b's, solve

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

Solve for the x's



(*d*) *Part 5*: Curve fitting