CS 314 Fall 2018
Homework Assignment 2
Answers

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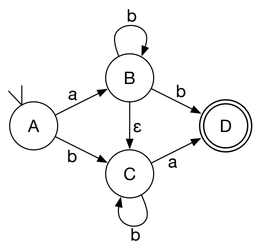
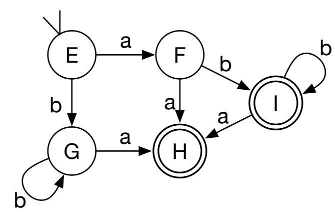
1. Convert this NFA to a DFA. Clearly indicate the set of NFA states that each state in the DFA corresponds to.





Where $E = \{A\}$, $F = \{B, C\}$, $G = \{C\}$, $H = \{D\}$ and $I = \{B, C, D\}$.

2. Reduce the following lambda terms to normal form, or argue that no normal form exists.

   (a) $(\lambda a.\ \lambda b.\ a)\ x\ y\ z$
   $x\ z$

   (b) $\lambda x.\ (\lambda y.\ y)\ (\lambda z.\ z)\ x$
   $\lambda x.\ x$

   (c) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (y\ z)\ x$
   $y\ z\ x\ x$

   (d) $(\lambda a.\ (\lambda b.\ a\ (b\ b))\ (\lambda c.\ a\ (c\ c)))\ s$
   No normal form.

The term β-reduces to $(\lambda b.\ s\ (b\ b))(\lambda c.\ s\ (c\ c))$ and then to $s\ ((\lambda c.\ s\ (c\ c))\ (\lambda c.\ s\ (c\ c)))$, which contains the previous term as a subterm. This means that further β-reduction will only create additional opportunities for β-reduction, and therefore we will not converge to a normal form.

(e) $(\lambda a.\ \lambda b.\ a)\ (\lambda c.\ c)\ x\ y\ z$

$y\ z$

3. Assume the following definitions:

$$\mathsf{TRUE} = \lambda t.\ \lambda f.\ t \tag{1}$$
$$\mathsf{FALSE} = \lambda t.\ \lambda f.\ f \tag{2}$$

Using these definitions, we can define terms such as NOT that behave in the expected way, e.g.,

$$\mathsf{NOT} = \lambda b.\ \lambda t.\ \lambda f.\ b\ f\ t \tag{3}$$

Note that $\mathsf{NOT}\ \mathsf{TRUE} = \mathsf{FALSE}$.

Define the following terms and briefly explain how they work.

(a) AND

$\mathsf{AND} = \lambda a.\ \lambda b.\ a\ b\ \mathsf{FALSE}$

We determine $\mathsf{AND}\ a\ b$ based on a: If a is FALSE, the result must be FALSE. Otherwise, it will be TRUE exactly when b is TRUE.

(b) OR

$\mathsf{OR} = \lambda a.\ \lambda b.\ a\ \mathsf{TRUE}\ b$

We determine $\mathsf{OR}\ a\ b$ based on a: If a is TRUE, the result must be TRUE. Otherwise, it will be TRUE exactly when b is TRUE.

(c) XOR

$\mathsf{XOR} = \lambda a.\ \lambda b.\ a\ (\mathsf{NOT}\ b)\ b$

We determine $\mathsf{OR}\ a\ b$ based on a: If a is TRUE, the result will be TRUE exactly when b is FALSE. Otherwise, it will be TRUE exactly when b is TRUE.