Chapter-5 (Measures of dispersion)

Ex 1 Marks of 10 Students in Mathematics & Statistics are given below.

Maths	25	40	30	35	21	45	23	33	10	29
Statics	30	39	23	42	20	40	25	30	18	19

Find the range & Cofficients in both Subjects.

Sol^n: For Maths

$L = 45$, $S = 10$

Range $= 45 - 10 = 35$

Coff of Range $= \frac{L-S}{L+S} = \frac{45-10}{45+10} = \frac{\cancel{35}^{7}}{\cancel{55}_{11}}$

$= \frac{7}{11} = 0.636$

For Static

$L = 42$

$S = 18$

Range $= L - S = 42 - 18 = 24$

Coff of Range $= \frac{L-S}{L+S} = \frac{42-18}{42+18}$

$= \frac{\cancel{24}^{2}}{\cancel{60}_{5}} = \frac{2}{5} = 0.4$ Ans.

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Ex3 Find the range of the following distribution of ~~falies~~ following data:

Mid Value	5	10	15	20	25	30	35
Frequency	7	5	8	12	8	9	8

Sol^n

Mid Value	Class	Frequency
5	2.5 - 7.5	5
10	7.5 - 12.5	7
15	12.5 - 17.5	8
20	17.5 - 22.5	12
25	22.5 - 27.5	8
30	27.5 - 32.5	9
35	32.5 - 37.5	8

$$\text{Range} = L - S$$
$$= 37.5 - 2.5$$
$$= 35$$

$$\text{Coff of Range} = \frac{L-S}{L+S} = \frac{\cancel{35}\ 7}{\cancel{40}\ 8} = \frac{7}{8} = 0.875$$

Ans

Ex4 Find range & Cofficient of range from the following distribution:

Class	6-10	11-15	16-20	21-25	26-30	31-35	36-
Frequency	7	8	15	35	18	7	5

Sol^n →

Inclusive Class	Continuous Series	Frequency
6 - 10	5.5 - 10.5	7
11 - 15	10.5 - 15.5	8
16 - 20	15.5 - 20.5	15
21 - 25	20.5 - 25.5	35
26 - 30	25.5 - 30.5	18
31 - 35	30.5 - 35.5	7
36 - 40	35.5 - 40.5	5

$$\text{Range} = L - S$$
$$= 40.5 - 5.5$$
$$= 35$$

$$\text{Coff of Range} = \frac{L - S}{L + S} = \frac{40.5 - 5.5}{40.5 + 5.5}$$

$$= \frac{35}{46} = 0.76 \text{ Ans.}$$

Ex5 The Smallest Value in a Set of observations in 20 With a range of 25. Find the largest observation and the Coefficient of range

Sol^n Given, $S = 20$
$R = 25$

We Know

$$R = L - S$$
$$25 = L - 20$$
$$L = 25 + 20$$
$$L = 45$$

So largest observation is 45

Now Coff of Range $= \frac{L-S}{L+S}$

$= \frac{45-20}{45+20} \Rightarrow \frac{\cancel{25}\,5}{\cancel{65}\,13} = \frac{5}{13} = 0.37$ Ans

Ex6 From the following information of Wages of 15 Workers, find interquartile range, quartile deviation & Co-efficient of QD.

S. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Wages	520	550	440	580	450	620	470	680	400	490	420	480	440	480	500

Solⁿ Arrange the Given Wages in Assending order

400, 420, 440, 450, 470, 480, 480, 480, 500, 520, 550, 580, 620, 680

Here $N = 15$

So $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$Q_1 = \frac{(15+1)^{th}}{4}$ observation

$Q_1 = \frac{\cancel{16}\,4}{\cancel{4}} = 4^{th}$ observation

$Q_1 = 440$

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Now $Q_3 = \frac{3(N+1)^{th}}{4}$ observation

$Q_3 = \frac{3(16)}{4} = 3(4) = 12^{th}$ observation

$Q_3 = 550$

Interquartile Range $= Q_3 - Q_1$
$= 550 - 440$
$= 110$

Quartile Divation $\rightarrow \frac{Q_3 - Q_1}{2} \Rightarrow \frac{\cancel{110}^{55}}{\cancel{2}}$

Coff. of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{110}{990} = \frac{1}{9}$

0.11 Ans.

Ex 7 Scores are:

CI	f
45-49	2
40-44	3
35-39	2
30-34	6
25-29	8
20-24	8
15-19	5
10-14	7
5-9	9

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Sol 3.

CI	Centrid C.I	f	CF
5-9	4.5-9.5	9	9
10-14	9.5-14.5	7	16
15-19	14.5-19.5	5	21
20-24	19.5-24.5	8	29
25-29	24.5-29.5	8	37
30-34	29.5-34.5	6	43
35-39	34.5-39.5	2	45
40-44	39.5-44.5	3	48
45-49	44.5-49.5	2	50

for Q_1 $\left(\frac{N+1}{4}\right)^{th}$ observation

$$\frac{50+1}{4} = 12.75$$

So Q_1 Class 9.5 - 14.5

$F=7,\ C=9,\ l_1 = 9.5,\ l_2 = 14.5$

$$\text{So } Q_1 = l_1 + \frac{\frac{N}{2} - C}{f} \cdot (l_2 - l_1)$$

$$= 9.5 + \frac{\frac{50}{4} - 9}{7} (5)$$

$$= 9.5 + \frac{12.5 - 9}{7} \times 5$$

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$= 9.5 + \frac{3.5 \times 5}{7}$

$= 9.5 + 2.5 = 12$

for $Q_3 = \frac{3(N+1)}{4} = \frac{3(50+1)}{4}$

$= 38.25^{th}$ observation.

So Q_3, Class in $29.5 - 34.5$

$F = 6$, $C = 37$, $l_1 = 29.5$, $l_2 = 34.5$, $N = 50$

$Q_3 = l_1 + \frac{\frac{3N}{4} - C}{f} (l_2 - l_1)$

$= 29.5 + \frac{\frac{3 \times 50}{4} - 37}{6} \times 5$

$= 29.5 + \frac{5 \times 5}{6}$

$= 29.5 + 0.417$

$Q_3 = 29.917$

So $QD = \frac{Q_3 - Q_1}{2} = \frac{29.917 - 12}{2}$

$= \frac{17.917}{2} = 8.9585$ Ans.

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Ex8 Calculate QD and its Co efficient from the following distribution:

Weekly Income	58	59	60	61	62	63	64	65	66
No. of Workers	2	3	6	15	10	5	4	3	1

Sol^n

x	f	c.f
58	2	2
59	3	5
60	6	11
61	15	26
62	10	36
63	5	41
64	4	45
65	3	48
66	1	49

50

For $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$= \frac{49+1}{4}$ $(12.5)^{th}$ observation

$= 61$

$Q_3 = \frac{3(N+1)^{th}}{4}$ observation

$= \frac{3(49+1)^{th}}{4}$ observation

$= 37.5^{th}$ observation

$= 63$

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$$QD = \frac{Q_3 - Q_1}{2} = \frac{63-61}{2} = \frac{2}{2} = 1 \text{ Ans}$$

$$\text{Coeff of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{63-61}{63+61}$$

$$= \frac{2}{124} = 0.016 \text{ Ans}$$

Ex 9 The following is the age distribution of 799 workers

Age-Group	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of Workers	50	70	100	180	150	120	70	59

Solⁿ

Class	f	C.F
20-25	50	50
25-30	70	120
30-35	100	220
35-40	180	400
40-45	150	550
45-50	120	670
50-55	70	740
55-60	59	799

For $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$$\frac{799+1}{4} = \frac{800}{4} = 200^{th} \text{ observation}$$

So, Q_1 Class $30-35$

$F = 100$, $C = 120$, $N = 799$, $l_1 = 30$, $l_2 = 35$

So $$Q_1 = l_1 + \frac{\frac{N}{4} - C}{f}(l_2 - l_1)$$

$$= 30 + \frac{\frac{799}{4} - 120}{100} \times 5$$

$$= 30 + \frac{199.75 - 120}{20}$$

$$= 30 + \frac{79.75}{20} \Rightarrow 30 + 3.9875$$

$$= 33.9875$$

& Q_3 $\frac{3(N+1)}{4}^{th}$ observation

$$\frac{3(799+1)}{4}^{th} \text{ observation}$$

600th observation

So Q_3 Class is $45-50$

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$F = 120,\ C = 550,\ l_1 = 45,\ l_2 = 50,\ N = 799$

$$Q_3 = l_1 + \frac{\frac{3N}{4} - C}{f}(l_2 - l_1)$$

$$= 45 + \frac{\frac{3 \times 799}{4} - 550}{120} \times 5$$

$$= 45 + \frac{49.25 \times 5}{120}$$

$$= 45 + 2.05 \Rightarrow 47.05$$

$$\text{So } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{47.05 - 33.9875}{47.05 + 33.9875}$$

$$= \frac{13.0625}{81.6375} = 0.16 \text{ Ans}$$

Example 10 Compute mean deviation and its Coefficient from the following data relating to the marks obtained by a batch of 11 students in a Class test:

Marks	10	70	50	53	20	95	55	42	60	48	80

Sol^n →

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$
10		-43	43
70		17	17
50		-3	3
53		0	0
20	53	-33	33
95		42	42
55		2	2
42		-11	11
60		7	7
48		-5	5
80		27	27
			190

Now $\bar{x} = \frac{\Sigma X}{N} = \frac{10+70+50+53+20+95+55+42+60+48+80}{11}$

$\frac{583}{11} = 53$

So $MD = \frac{\Sigma \lvert x-\bar{x}\rvert}{N} = \frac{190}{11} = 17.27$

& Coff of $MD = \frac{MD}{Mean} = \frac{17.27}{53} = 0.326$ Ans.

Ex 11 Calculate mean deviation from Median for the following data. Also Compute the Cofficent of M.D

Size	2	4	6	8	10	12	14	16
Frequency	2	2	4	5	3	2	1	1

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Solⁿ

X	f	C.F	Md	x−Md	|x−Md|	f|x−Md|
2	2	2		−6	6	12
4	2	4		−4	4	8
6	4	8	8	−2	2	8
8	5	13		0	0	0
10	3	16		2	2	6
12	2	18		4	4	8
14	1	19		6	6	6
16	1	20		8	8	8
						56

For Median $= \left(\frac{N+1}{2}\right)^{th}$ Observation

$\left(\frac{20+1}{2}\right)^{th}$ observation $= 10.5^{th}$ observation

Now $MD = \frac{\Sigma f\,|X - Md|}{\Sigma f} = \frac{56}{20} = 2.8$ Ans.

& Coeff of $M.D = \frac{MD}{Median} = \frac{2.8}{8} = 0.35$ Ans.

Ex 12: Compute the mean deviation (M.D) from the following data.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	50	84	32	10	6

Solⁿ →

Class	f	X	fx	$\bar{x}$	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
0-20	5	10	50		41.07	205.35
20-40	50	30	1500		21.07	1053.5
40-60	54	50	4200	51.07	1.07	89.88
60-80	32	70	2240		18.93	605.76
80-100	10	90	900		38.93	389.3
100-120	6	110	660		58.93	353.58
	187		9550			2697.37

So $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{9550}{187} = 51.07$

Now $MD = \frac{\Sigma f\lvert x-\bar{x}\rvert}{\Sigma f} = \frac{2697.37}{187} = 14.42$

& Coeff. of $MD = \frac{M.D}{Mean} = \frac{14.42}{51.07} = 0.28$ Ans.

Ex 13 Calculate SD from the following set of observation:

X: 10, 11, 17, 25, 7, 13, 21, 10, 12, 14

Sol^n:

We use direct Method in this

X: 10, 11, 17, 25, 7, 13, 21, 10, 12, 14

X	$x-\bar{x}$	$(x-\bar{x})^2$
10	-4	16
11	-3	9
17	3	9
25	11	121
7	-7	49
13	-1	1
21	7	99
10	-4	16
12	-2	4
14	0	0
140		274

Here $\bar{x} = \frac{\Sigma X}{N} = \frac{140}{10} = 14$

$$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}}$$

$$= \sqrt{\frac{274}{10}} = \sqrt{27.4}$$

$$= 5.23$$

So S.D = 5.23 Ans.

Ex 14 Find SD from the data in above example

Solⁿ $SD = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$

$$= \sqrt{\frac{284}{10} - \left(\frac{70}{10}\right)^2}$$

$$= \sqrt{28.4 - 1}$$

$$= \sqrt{27.4} = 5.23$$

SD = 5.23 Ans.

Ex15: Use direct Method to Calculate S.D of the following discrete frequency distribution:

Size (x)	4	5	6	7	8	9	10
Frequency	6	12	15	28	20	14	5

Sol:

X	f	fx	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
4	6	24	-3.06	9.3636	56.1816
5	12	60	-2.06	4.2436	50.9232
6	15	90	-1.06	1.1236	16.8540
7	28	196	0.06	0.0036	0.1008
8	20	160	0.94	0.8836	17.6720
9	14	126	1.94	3.7636	52.6904
10	5	50	2.94	8.6436	43.2180
	100	706			237.6400

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{706}{100} = 7.06$$

$$SD = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}} = \sqrt{\frac{237.64}{100}}$$

SD $\sqrt{2.3764} = 1.54$ Ans.

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Ex 16: Use Short-Cut Method to find the S.D of the data in above example

Sol^n

$$SD = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{238}{100} - \left(\frac{6}{100}\right)^2}$$

$$= \sqrt{2.38 - (0.06)^2}$$

$$= \sqrt{2.38 - 0.0036}$$

$$= \sqrt{2.3764}$$

$$= 1.5416 \text{ Ans.}$$

Ex 17 Use Short-Cut and Step-deviation Method for Calculating S.D of the following distribution.

Age Groups (Years)	25-30	30-35	35-40	40-45	45-50	50-55
No. of Workers	10	12	25	40	10	3

Sol^n:

Class	M.V	f	A	$d' = \frac{x-A}{h}$	fd'	d'^2	fd'^2
25-30	27.5	10		-2	-20	4	40
30-35	32.5	12		-1	-12	1	12
35-40	37.5	25	37.5	0	0	0	0
40-45	42.5	40		1	40	1	40
45-50	47.5	10		2	20	4	40
50-55	52.5	3		3	9	9	27
		100			37		159

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$$SD = h\sqrt{\frac{\Sigma f d'^2}{N} - \left(\frac{\Sigma f d'}{N}\right)^2}$$

$$5\sqrt{\frac{159}{100} - \left(\frac{37}{100}\right)^2}$$

$$= 5\sqrt{1.59 - (0.37)^2}$$
$$= 5\sqrt{1.4531}$$
$$= 5 \times 1.2054$$
$$\Rightarrow SD \Rightarrow 6.027$$

& Variance = 36.3247 Ans

Ex21 The mean of 100 items is 50 and their S.D is 4. Find the Sum and the Sum of square of the items.

Sol^n Given, $N = 100$, $\bar{x} = 50$, $SD = 4$

We know,

$$\bar{x} = \frac{\Sigma X}{N} = \frac{\Sigma X}{100}$$

$$\Sigma X = 50 \times 100$$
$$\Sigma X = 5000$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$$

$$\sigma^2 = \frac{\Sigma X^2}{N} - (\bar{x})^2$$

$$(4)^2 = \frac{\Sigma x^2}{100} - (50)^2$$

$$\frac{\Sigma x^2}{100} = 16 + 2500$$

$$\frac{\Sigma x^2}{100} = 2516$$

$$\Sigma x^2 = 2516 \times 100$$

$$\Sigma x^2 = 251600 \quad \text{Ans.}$$

Ex 22 The Sum of Squares of 20 observations was worked out as 5100. But while calculating it. an observation 31 was wrongly considered as 13. Find the Corrected Sum of Squares.

Solⁿ: Given $N = 20$, $\Sigma x^2 = 5100$

So Correct $\Sigma x^2 = 5100 - (13)^2 + (31)^2$

$= 5100 - 169 + 961$

$= 5892$ Ans.

Ex 23 The Sum of Squares of 100 observations was calculated as 7961. Later, it was found that two values, 53 and 42 were wrongly read as 35 and 24 at the time of calculation. Find the Corrected Sum of Squares.

Solⁿ Given, $N = 100$

$\Sigma x^2 = 7961$

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Wrong item be 35 & 24
Correct item be 53 & 42

So Correct $\Sigma X^2 = 7961 - (35^2 + 24^2) + (53^2 + 42^2)$
$= 7961 - (1225 + 576) + (2809 + 1764)$
$= 7961 - 1801 + 4573$
$= 10733$ Ans.

Ex25 The arithmetic mean and the SD of a Series of 20 items were Calculated as 20 Cm and 5 Cm respectively. But while Calculating them, an item 13 Was measured as 30. Find the Correct arithmetic Mean and Standard deviation.

Solⁿ Given, $N = 20$, $\bar{x} = 20$
$SD = 5$

Correct item = 13
incorrect observation = 30

$$\bar{x} = \frac{\Sigma X}{N}$$

$$20 = \frac{\Sigma X}{20}$$

$\Sigma X = 400$
Correct $\Sigma X = 400 - 30 + 13$
$= 413 - 30 = 383$

So Correct Mean $= \frac{\Sigma X}{N} = \frac{383}{20} = 19.15$

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Correct $\bar{x} = 19 \cdot 15$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - (20)^2}$$

$$25 = \frac{\Sigma X^2}{20} - 400$$

$$\frac{\Sigma X^2}{20} = 400 + 25$$
$$= 425$$

$$\Sigma X^2 = 425 \times 20$$
$$\Sigma X^2 = 8500$$

$$\text{Correct } \Sigma X^2 = 8500 - (30)^2 + (13)^2$$
$$= 8500 - 900 + 169$$
$$= 7769$$

$$\text{So Correct } SD = \sqrt{\frac{\text{Correct } \Sigma X^2}{N} - \left(\frac{\text{Correct } \Sigma X}{N}\right)^2}$$

$$= \sqrt{\frac{7769}{20} - \left(\frac{383}{20}\right)^2}$$

$$= \sqrt{388 \cdot 45 - 366 \cdot 7225}$$
$$= \sqrt{21 \cdot 7275}$$
$$\sigma = 4 \cdot 66 \text{ Ans.}$$

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Ex 27 A Sample of 35 Value has mean 80 & SD. 4. A Second Sample of 65 Values from the Same population has mean 70 and SD. 3. Find the Mean & Standard deviation of the combined Sample of 100 Values.

Sol^n:- $N_1 = 35$, $\bar{x}_1 = 80$, $\sigma_1 = 4$
$N_2 = 65$, $\bar{x}_2 = 70$, $\sigma_2 = 3$

Combined Mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$\bar{x}_{12} = \frac{35 \times 80 + 65 \times 70}{100} = \frac{2800 + 4550}{100}$$

$$\bar{x}_{12} = \frac{7350}{100} = 73.5$$

Now $D_1 = \bar{x}_1 - \bar{x}_{12} = 80 - 73.5 = 6.5$
$D_2 = \bar{x}_2 - \bar{x}_{12} = 70 - 73.5 = -3.5$

So, $D_1^2 = 42.25$, $D_2^2 = 12.25$

Now Combined SD

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{35(16+42.25)+65(9+12.25)}{100}}$$

$$= \sqrt{\frac{35 \times 58.25 + 65 \times 21.25}{100}}$$

$$= \sqrt{\frac{2038.75 + 1381.25}{100}}$$

$$= \sqrt{\frac{3420}{100}} = \sqrt{34.2} = 5.848 \text{ Ans}$$

So $\sigma = 5.848$ Ans.

Ex. 28 Find the mean & Standard deviation of the two groups taken together:

Group	Number	Mean	SD
A	113	160	22
B	120	150	20

Sol^n $N_1 = 113$, $\bar{x}_1 = 160$, $\sigma_1 = 22$
$N_2 = 120$, $\bar{x}_2 = 150$, $\sigma_2 = 20$

Combined Mean :-

$$\bar{x}_{12} = \frac{N\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

$$\bar{x}_{12} = \frac{113 \times 160 + 120 \times 150}{113 + 120}$$

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$$\bar{x}_{12} = \frac{18080 + 18000}{233}$$

$$\bar{x}_{12} = \frac{36080}{233} = 154.85$$

$$D_1 = \bar{x}_1 - \bar{x}_{12} = 160 - 154.85 = 5.515$$
$$D_2 = \bar{x}_2 - \bar{x}_{12} = 150 - 154.85 = -4.85$$

$$D_1^2 = 26.5225 \quad \& \quad D_2^2 = 23.5225$$

Combined SD

$$\sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{113(484 + 26.5225) + 120(400 + 23.5225)}{113 + 120}}$$

$$= \sqrt{\frac{113 \times 510.5225 + 120 \times 423.5225}{233}}$$

$$= \sqrt{\frac{576.8904 + 50822.70}{233}}$$

$$= \sqrt{465.72}$$

$$= \sigma_{12} = 21.58 \text{ Ans}$$

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Ex 30 The mean of the two samples of sizes 50 and 100 respectively is 54.1 and 50.3 and standard deviations are 8 and 7. Obtain the mean and standard deviation of combined sample of size 150.

Sol^n Given $N_1 = 50$, $\sigma_1 = 8$, $\bar{x}_1 = 54.1$
$N_2 = 100$, $\sigma_2 = 7$, $\bar{x}_2 = 50.3$

Combined Mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{50 \times 54.1 + 100 \times 50.3}{50 + 100}$$

$$= \frac{2705 + 5030}{150} \Rightarrow \frac{7735}{150}$$

$$\bar{x}_{12} = 51.57 \quad \text{Ans.}$$

$$D_1 = \bar{x}_1 - \bar{x}_{12} = 54.1 - 51.57 = 2.53$$
$$D_2 = \bar{x}_2 - \bar{x}_{12} = 50.3 - 51.57 = -1.27$$

So $D_1^2 = 6.4009$, $D_2^2 = 1.6129$

So Combined SD

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{50(64+6.4009)+100(49+1.6129)}{150}}$$

$$= \sqrt{\frac{3520.045+5061.29}{150}}$$

$$= \sqrt{57.2089} = 7.564 \text{ Ans}$$

Ex32 A Sample of 5 items was taken from the output of a factory. The length was weight of 5 items are given below:

Length	5	6	7	9	12
Weight	13	15	18	19	20

Sol^n →

Length			Width		
x	$dx=(x-A)$	d^2x	y	$dy=(x-A)$	d^2y
5	−2	4	13	−5	25
6	−1	1	15	−3	9
7	0	0	18	0	0
9	2	4	19	1	1
12	5	25	20	2	4
N=5	4	34	N=5	−5	39

$$\bar{x} = A + \frac{\Sigma dx}{N}$$

$$= 7 + \frac{4}{5}$$

$$= 7 + 0.8$$

$$= 7.8$$

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$$\sigma_x = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma dx}{N}\right)^2}$$

$$= \sqrt{\frac{34}{5} - \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{6.8 - 0.64}$$

$$= \sqrt{6.16}$$

$$\sigma_x = 2.48 \text{ inches}$$

So $CV = \frac{\sigma x}{x} \times 100$

$$= \frac{2.48}{7.8} \times 100$$

$$= \frac{248}{7.8} = 31.79\ \%$$

& $\bar{Y} = A + \frac{\Sigma dy}{N}$

$$\bar{Y} = 18 - \frac{5}{5} = 18 - 1 = 17$$

$$\sigma_y = \sqrt{\frac{\Sigma dy^2}{N} - \left(\frac{\Sigma dy}{N}\right)^2}$$

$$= \sqrt{\frac{39}{5} - \left(\frac{-5}{5}\right)^2}$$

$$= \sqrt{7.8 - 1} = \sqrt{6.8}$$

$$\sigma_y = 2.6$$

So $C.V = \frac{\sigma_y}{y} \times 100$

$= \frac{260 \times 100}{17} = \frac{260}{17} = 15.29$

Since Coeff of Variation of length is Greater than Coeff of Variation of Weight.
So length is more variation then weight Ans.

Ex 33 Below are the Numbers of runs Scored by two batsmen is eight Matches.

Batsman A	27	16	39	45	101	80	40	52
Batsman B	0	100	80	5	60	40	10	121

Sol^n →

X	$d_x = (x-45)$	d^2x	Y	$d_y = (Y-5)$	d^2y
27	-18	324	0	-5	25
16	-29	841	100	95	9025
39	-6	36	80	75	5625
45	0	0	5	10	0
101	56	3136	60	55	3025
80	35	1225	40	35	1225
40	-5	25	10	5	25
52	7	49	121	116	13456
N=8	40	5636	N=8	376	32406

So $\bar{x} = A + \frac{\Sigma d_x}{N}$

$$= 45 + \frac{40}{8}$$

$$\bar{x} = 45 + 5 = 50$$

& $$\bar{y} = A + \frac{\Sigma dy}{N}$$

$$= 5 + \frac{376}{8}$$

$$\bar{y} = 5 + 47 = 52$$

$$\sigma_x = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{5636}{8} - \left(\frac{40}{8}\right)^2}$$

$$= \sqrt{704.5 - 25} = \sqrt{679.5} = 26.06$$

$$CV = \frac{\sigma_x}{x} \times 100$$

$$= \frac{26.06}{50} \times 10$$

$$C.V = \frac{2606}{50} = 52.12$$

$$\sigma_y = \sqrt{\frac{\Sigma dy^2}{N} - \left(\frac{\Sigma dy}{N}\right)^2}$$

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$$= \sqrt{\frac{32406}{8} - \left(\frac{376}{8}\right)^2}$$

$$= \sqrt{4050.75 - 2209}$$

$$= \sqrt{1841.75}$$

$$= 42.92$$

$$\& \; CV = \frac{\sigma_y}{y} \times 100$$

$$= \frac{42.92}{52} \times 100$$

$$CV = 82.53$$

From Mean, Batsman B is Better in Scoring & by comparing C.V, Coff of Variation of Batsman A is less than the Coff of Variation of Batsman B, So Batsman A is more Consistant than B Ans.

Ex 37 In the factories A and B the average Weekly wages and Standard deviation are as follow:

Follow	Average Weekly Wages	S.D. of Wages	No. of Workers
A	460	50	100
B	490	40	80

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Sol^n: Given,

$\sigma_x = 50, \quad \sigma_y = 40$

$N_1 = 100, \quad N_2 = 80$

$\bar{x}_1 = 460, \quad \bar{y} = 490$

(i) Total Weekly Wages paid by factory A

$A = \Sigma X = N_1, \bar{X}_1 = 100 \times 460 = 46,000$

Total Weekly Wages Paid by factory B

$B = \Sigma X = N_2 \bar{X}_2 = 80 \times 490 = 39200$

So factory A Paid larger Amount as Weekly Wages.

(ii) For factory A

$C.V = \frac{\sigma_1}{\bar{x}_1} \times 100 = \frac{50}{460} \times 100 = 10.87$

For Factory B

$C.V = \frac{\sigma_2}{\bar{y}} \times 100 = \frac{40}{490} \times 100 = 8.16$

Since Coff of Variation of factor A is greater than B. So factory A is more Variable than B.

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(ii) Now

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{100 \times 460 + 80 \times 490}{100 + 80}$$

$$= \frac{46000 + 39200}{180} = \frac{85200}{180}$$

$$= 473.33 \text{ Ans.}$$

Now $D_1 = \bar{x}_1 - \bar{x}_{12}$

$= 460 - 473.33$
$= -13.33$

$D_2 = \bar{x}_2 - \bar{x}_{12}$
$= 490 - 473.33$
$= 16.67$

$D_1^2 = 177.69$
$D_1^2 = 277.89$

Now $\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$

$$= \sqrt{\frac{100(2500 + 177.69) + 80(1600 + 277.89}{100 + 80}}$$

$$= \sqrt{\frac{267769 + 80 \times 1877.89}{180}}$$

$$= \sqrt{2322.22} = \sqrt{72} = 48.18 \text{ Ans.}$$

Ex 39 The following distribution shows the life of bulbs manufactured by two companies X and Y:

Use of Bulbs	Company (X)	Company (Y)
100-200	10	10
200-300	15	40
300-400	25	25
400-500	40	15
500-600	10	10

Sol 39.

			Company (X)			Company Y		
Class	x	$d = x - A$	f	fd	fd^2	f	fd	fd
100-200	150	-200	10	-2000	400000	10	-2000	400
200-300	250	-100	15	-1500	150000	40	-4000	400
300-400	350	0	25	0	0	25	0	0
400-500	450	100	40	4000	400000	15	1500	1500 00
500-600	550	200	10	2000	400000	10	2000	400 00

For Company X:-

$$\bar{x}_1 = A + \frac{\Sigma fd}{N}$$

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$$= 350 + \frac{2500}{100} = 375$$

$$\sigma_1 = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{1350000}{100} - \left(\frac{2500}{100}\right)^2}$$

$$= \sqrt{13500 - 625}$$

$$= \sqrt{12875}$$

$$\sigma_1 = 113.47$$

So Coeff of Variation $= \frac{\sigma_1}{\bar{x}_1} \times 100$

$$= \frac{113.47 \times 100}{375}$$

$$= 30.26$$

For Company Y:-

$$\bar{x}_2 = A + \frac{\Sigma fd}{N}$$

$$= 350 - \frac{2500}{100} \rightarrow \bar{x}_2 = 325$$

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& $\sigma_2 = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= \sqrt{\frac{1350000}{100} - \left(\frac{-2500}{100}\right)^2}$

$= \sqrt{13500 - 625} = \sqrt{12875} = 113.47$

So, $C_1 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$= \frac{113.47 \times 100}{325}$

$CV = 34.91.$

Since $\bar{x}_1 = \bar{x}_2$ So bulbs of Company has higher average life.

& C.V of Company X is less than C.V of Company Y So Company X is more Consistant than Y Ans.

Ex 40: From the prices of Shares of X and Y given below, State which Share is more Stable in Value.

X	41	44	43	48	45	46	49	50	42	40
Y	91	93	96	92	90	97	99	94	98	95

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Sol 3)

x	y	$d_x = x - A$	dx^2	$d_y = x - 90$	dy^2
41	91	−4	16	+1	1
44	93	−1	1	3	9
43	96	−2	4	6	36
48	92	3	9	2	4
45	90	0	0	0	0
46	97	1	1	7	49
49	99	4	16	9	81
50	94	5	25	4	16
52	98	−3	9	8	64
40	95	−5	25	5	25
488	945	−2	106	45	285

Now $\bar{x} = A + \frac{\Sigma d}{N}$

$45 + \left(\frac{-2}{10}\right)$

$45 - 0.2 = 44.8$

$\sigma x = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma dx}{N}\right)^2}$

$= \sqrt{\frac{106}{10} - \left(\frac{-2}{10}\right)^2}$

$= \sqrt{10.6 - 0.04}$

$\sigma x = \sqrt{10.56} = 3.24$

$\& \bar{y} = A + \frac{\Sigma d}{N}$

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$= 90 + \frac{45}{10} \Rightarrow 90 + 4.5 = 94.5$

$\sigma_y = \sqrt{\frac{\Sigma dy^2}{N} - \left(\frac{\Sigma dy}{N}\right)^2}$

$= \sqrt{\frac{285}{10} - \left(\frac{45}{10}\right)^2}$

$= \sqrt{28.5 - 20.25}$

$= \sqrt{8.25} \Rightarrow \sigma_y = 2.87$

Now

CV for $x = \frac{\sigma_x}{\bar{x}} \times 100$

$= \frac{3.25}{44.8} \times 100 \Rightarrow \frac{328}{448} = 7.25$

CV for $y = \frac{\sigma_y}{\bar{y}} \times 100$

$= \frac{2.87}{94.5} \times 100 \Rightarrow \frac{287}{94.5} = 3.03$

Since C.V of Y is less than C.V of X
So Y is more Stable than x Ans.

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Ex 4.1 (i) Coefficient of Variation of two Series are 50 % and 69 %. Their Standard diaviation are 21.2 & 15.6 respectively. What are their arithmetic means?

(ii) Mean of 100 items is 58 and their Standard deviation is 4 find the Sum of Squares of all items.

Sol^n:- Given

$$CV \text{ of } x = 50$$
$$CV \text{ of } y = 69$$
$$\sigma x = 21.2$$
$$\sigma y = 15.6$$

We know $C.V \text{ of } x = \frac{\sigma x}{x} \times 100$

$$\bar{x} = \frac{\sigma x}{CV \text{ of } x} \times 100$$

$$= \frac{21.2}{\cancel{50}} \times \cancel{100}^{2} \Rightarrow \bar{x} = 42.4 \text{ Ans}$$

$$CV \text{ of } y = \frac{\sigma y}{\bar{y}} \times 100$$

$$\bar{y} = \frac{\sigma y}{CV \text{ of } y} \times 100$$

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$$= \frac{15.6 \times 100}{69} \Rightarrow \frac{1560}{69}$$

$$\bar{y} = 22.6$$ Ans.

iii) $N = 100, \ \bar{x} = 58, \ \sigma = 4$

We know $\bar{x} = \frac{\Sigma X}{N}$

$$58 = \frac{\Sigma X}{100} \Rightarrow \Sigma X = 5800$$

& $\sigma = \sqrt{\frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$

$$4 = \sqrt{\frac{\Sigma X^2}{100} - (58)^2}$$

$$16 = \frac{\Sigma X^2}{100} - 3364$$

$$\Sigma X^2 = 100 \times (16 + 3364)$$
$$\Sigma X^2 = 100 \times 3380$$
$$\Sigma X^2 = 338000$$ Ans.

Ex42 Mean and Standard deviation of the following Continuous Series are 31 and 15.94 respectively.

The distribution after step deviation:

d	-3	-2	-1	0	1	2	3
f	10	15	25	25	10	11	5

Sol^n ⇒ $\bar{x} = 31$, $\sigma = 15.94$

d	f	fd	fd^2
-3	10	-30	90
-2	15	-30	60
-1	25	-25	25
0	25	0	0
1	10	10	10
2	11	22	44
3	5	15	45
	101	-38	274

Now $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times i$

$\Rightarrow 15.94 = \sqrt{\frac{274}{101} - \left(\frac{-38}{101}\right)^2} \times i$

$15.94 = \sqrt{2.7129 - 0.1416} \times i$

$15.94 = 1.6035 \times i$

$i = \frac{15.94}{1.6035} \Rightarrow 9.94$

$i = 10$ (approx)

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& $\bar{x} = A + \frac{\Sigma fd}{N} \times i$

$31 = A + \frac{(-38)}{101} \times 10$

$31 = A - \frac{380}{101}$

$A = 31 + 3.76$

$A = 34.76 \Rightarrow 35$ (approx)

$A = 35$

So Class interval be

0-10, 10-20, 20-30, 30-40, 40-50, 50-60, 60-70

Ans.

Ex 43: A Collar manufacturer is considering the production of new style of collar to attract young man. The following statistics of neck circumference are available based upon measurement of a typical group of college students:

Mid Values in In	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5
Number of Students	4	19	30	63	66	28	18	1	1

Sol^n

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x	f	d	fd	d^2	fd^2
12.5	4	-2	-8	4	16
13.0	19	-1.5	-28.5	3.25	42.75
13.5	30	-1	-30	1	30
14.0	63	-0.5	-31.5	0.25	15.75
14.5	66	0	0	0	0
15.0	29	0.5	14.5	0.25	7.25
15.5	18	1	18	1	18
16.0	1	1.5	1.5	2.25	2.25
16.5	1	2	2	4	4
	231		-62		136

So $\bar{x} = A + \frac{\Sigma fd}{N}$

$= 14.5 + \frac{(-62)}{231}$

$= 14.5 - 2.68$

$\bar{x} = 14.232$

$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= \sqrt{\frac{136}{231} - \left(\frac{-62}{231}\right)^2}$

$= \sqrt{0.5887 - 0.0720}$

$\sigma = 0.7188$

Using the critrion $\bar{x} \pm 3\sigma$

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Smallest size of Collar $= \bar{x} - 3\sigma + \frac{3}{4}$

$= 14.232 - 3(0.7188) + 0.75$
$= 14.232 - 2.1564 + 0.75$
$= 12.8256$

Largest size of Collar $= \bar{x} + 3\sigma - \frac{3}{4}$

$= 14.232 + 3(0.07188) + 0.75$
$= 14.232 - 2.1564 + 0.75$
$= 17.1384$ Ans

Ex 44 Following is the data related to two factories:

No. of Workers	200	250
Average wage per hour	₹15.00	₹12.00
Variance	16	9

(i) Which factory pays larger amount as total wages per hour.
(ii) Which factory is more variable?
(iii) Find Combined S.D

Sol^n: (i) For Factory A

$\bar{x}_A = \frac{\Sigma x_A}{N} \rightarrow 15 = \frac{\Sigma x_A}{200}$

$\Sigma x_A = 3000$

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For factory B

$$\bar{x}_B = \frac{\Sigma X_B}{N} \Rightarrow 12 = \frac{\Sigma X_B}{250}$$

$$\Sigma X_B = 3000$$

So Both factory pay equal amount Ans

Now $\sigma_A^2 = 16$

$\sigma_A = 4$

$$C.V \text{ of } A = \frac{\sigma_A}{x_A} \times 100$$

$$= \frac{4}{15} \times 100 = \frac{400}{15} = 26.67$$

& $\sigma_B^2 = 9$

$\sigma_B = 3$

$$C.V \text{ of } B = \frac{\sigma_B}{X_B} \times 100$$

$$= \frac{3}{12} \times 100 \Rightarrow \frac{300}{12} = 25$$

Since C.V of Factory A is greater than C.V of Factory B

So Factory A is more variable than B

Now $\bar{x}_1 = 15, \quad \bar{x}_2 = 12, \quad \sigma_1 = 4, \quad \sigma_2 = 3$

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{200 \times 15 + 250 \times 12}{200 + 250}$$

$$= \frac{3000 + 3000}{450} = \frac{6000}{450} = 13.3$$

So $D_1 = \bar{x}_1 - \bar{x}_{12} = 15 - 13.3 = 1.7$

$D_2 = \bar{x}_2 - \bar{x}_{12} = 12 - 13.3 = -1.3$

So $D_1^2 = 2.89$ $D_2^2 = 1.69$

So Combined S.D

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{200(16 + 2.89) + 250(9 + 1.69)}{200 + 250}}$$

$$= \sqrt{\frac{200 \times 18.89 + 250 \times 10.69}{450}}$$

$$= \sqrt{\frac{3778 + 2672.5}{450}}$$

$$= \sqrt{14.3344} \Rightarrow 3.786 \text{ Ans}$$

Ex 45 Calculate Karl Pearson's Coefficient of Skewness from the following data:

Marks Above	0	10	20	30	40	50	60	70	80
No. of Students	150	140	100	80	80	70	30	14	0

Sol^n:

Class	f	x	d	fd	d^2	fd^2	C.F
0-10	10	5	-30	-300	900	9000	10
10-20	40	15	-20	-800	400	16000	50
20-30	20	25	-10	-200	100	2000	70
30-40	0	35	0	0	0	0	70
40-50	10	45	10	100	100	1000	80
50-60	40	55	20	800	400	16000	120
60-70	16	65	30	480	900	14400	136
70-80	14	75	40	560	1600	22400	150
	150			640		80800	

$$\bar{x} = A + \frac{\Sigma fd}{N}$$

$$= 35 + \frac{640}{150}$$

$$= 35 + 4.27$$
$$= 39.27$$

$$SD = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{80800}{150} - \left(\frac{640}{150}\right)^2}$$

$= \sqrt{538.67 - 18.20}$

$\sigma = 22.81$

For Median

Median Class $= \frac{150}{2} = 75^{th}$ observation

So Median Class $= 40 - 50$

$f = 10,\ C = 70,\ l_1 = 40,\ l_2 = 50$

So $Md = l_1 + \frac{\frac{N}{2} - C}{f}(l_2 - l_1)$

$= 40 + \frac{75 - 70}{10} \times 10$

$= 40 + \frac{5\cancel{0}}{1\cancel{0}}$

$= 40 + 5$

$Md = 45$

So Carl person's Coeff of (Skewness) =

$\frac{3(\bar{x} - Md)}{\sigma}$

$S_k = \frac{3(39.27 - 45)}{22.81}$

$= \frac{3 \times (-5.73)}{22.81}$

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$S_K = -0.75$ Ans.