R.H.L

$$f(a+h) = \lim_{h \to 0} f(a+h)$$

$$\Rightarrow \lim_{h \to 0} a - \frac{(a^2 \cancel{+ah})}{(a+h)}$$

$$\Rightarrow \lim_{h \to 0} a - \frac{a^2}{(a+h)}$$

Put the lt

$$a - \frac{a^2}{a} \Rightarrow \frac{a^2 - a^2}{a} \Rightarrow 0$$

Hence, L.H.L = R.H.L

then its continuous.

(6)

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of the matrix is 2.

Sec B (1) $5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7$, Exp^n along first row

-430, 21, 451, 396, 55 ; 231, 36, 195, 55

$$|\Delta| = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix} \Rightarrow 5[48+2] + 7[-36+3] + 1[12+24]$$

$$\Rightarrow 5 \times 50 - 231 + 36$$

$$\Rightarrow 250 - 195 \Rightarrow 55$$

$$|\Delta_1| = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix} \Rightarrow 11(48+2) + 7(-90+7) + 1(30+56)$$

$$\Rightarrow 550 - 581 + 86$$

$$\Rightarrow -581 + 636 \Rightarrow 55$$

$$|\Delta_2| = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix} \Rightarrow 5(-90+7) - 11(-36+3) + 1(42-45)$$

$$\Rightarrow -415 + 360$$

$$\Rightarrow -55$$

$$|\Delta_3| = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix} \Rightarrow 5(-56-30) + 7(42-45) + 11(12+24)$$

$$\Rightarrow 5 \times (-86) - 21 + 396$$

$$\Rightarrow -430 - 21 + 396 \Rightarrow -55$$

$x = \frac{\Delta_1}{\Delta} = \frac{55}{55} = 1$, $y = \frac{\Delta_2}{\Delta} = \frac{-55}{55} \Rightarrow -1$, $z = \frac{\Delta_3}{\Delta} = \frac{-55}{55} \Rightarrow -1$

(2) Let $f(x) = \sin x$, $x \in R$

$f'(x) = \cos x$, $f''(x) = -\sin x$, $f'''(x) = -\cos x$

In general form ~~$\sin^n x$~~ $f^n(x) = \sin\left(x + \frac{n\pi}{2}\right)$

Put $x = 0$ in above eq^n.

$f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, $f'''(0) = -1$

$f^n(0) = \sin\left(\frac{n\pi}{2}\right)$

(4.) $y = (\sin x)^x$

Taking log both sides

$\log y = x \cdot \log(\sin x)$

Diff w.r.t. to x

$$\frac{1}{y}\frac{dy}{dx} = \cancel{\log(\sin x)}\ x \frac{d}{dx}[\log(\sin x)] + \log \sin x \frac{d}{dx}(x)$$

$$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{\sin x} \times \cos x + \log \sin x\ (1)$$

$$\frac{dy}{dx} = y\left[x\cot x + \log(\sin x)\right]$$

$$\frac{dy}{dx} = (\sin x)^x \left[x\cot x + \log(\sin x)\right]$$

(5.) $$f(x) = \begin{cases} \frac{x^2}{a} - a & , \text{ if } x < a \\ 0 & , \text{ if } x = a \\ a - \frac{a^2}{x} & , \text{ if } x > a \end{cases}$$

at $x = a$

L.H.L

$$f(a-h) = \lim_{h \to 0} f(a-h)$$

$$\Rightarrow \lim_{h \to 0} \frac{(a-h)^2}{a} - a$$

$$\Rightarrow \lim_{h \to 0} \frac{a^2 + h^2 - 2ah - a}{a}$$

Put the lt

$$\Rightarrow \frac{a^2 + 0 - 0 - a^2}{a}$$

$$\Rightarrow \frac{a^2 - a^2}{a} = 0$$

(8) $$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$$

Multiply & divide by a^2-b^2 in denominator

$$\Rightarrow \frac{1}{a^2-b^2}\int_0^\infty \frac{(a^2-b^2)\,dx}{(x^2+a^2)(x^2+b^2)}$$

Now, Add & Sub x^2 in numerator.

$$\Rightarrow \frac{1}{a^2-b^2}\int_0^\infty \frac{(x^2+a^2-x^2-b^2)\,dx}{(x^2+a^2)(x^2+b^2)}$$

$$\Rightarrow \frac{1}{a^2-b^2}\left\{\int_0^\infty \frac{\cancel{(x^2+a^2)}}{\cancel{(x^2+a^2)}(x^2+b^2)}\,dx - \int_0^\infty \frac{\cancel{(x^2+b^2)}\,dx}{(x^2+a^2)\cancel{(x^2+b^2)}}\right\}$$

$$\Rightarrow \frac{1}{a^2-b^2}\left\{\int_0^\infty \frac{dx}{(x^2+b^2)} - \int_0^\infty \frac{dx}{(x^2+a^2)}\right\}$$

$$\Rightarrow \frac{1}{a^2-b^2}\left\{\left[\frac{1}{b}\tan^{-1}\frac{x}{b} - \frac{1}{a}\tan^{-1}\frac{x}{a}\right]_0^\infty\right\}$$

Put the lt

$$\rightarrow \frac{1}{a^2-b^2}\left[\left\{\frac{1}{b}\tan^{-1}\frac{\infty}{b} - \frac{1}{b}\tan^{-1}\frac{0}{b}\right\} - \left\{\frac{1}{a}\tan^{-1}\frac{\infty}{a} - \frac{1}{a}\tan^{-1}\frac{0}{a}\right\}\right]$$

$$\Rightarrow \frac{1}{a^2-b^2}\left[\frac{1}{b}\tan^{-1}\infty - \frac{1}{b}\tan^{-1}0 - \frac{1}{a}\tan^{-1}\infty + \frac{1}{a}\tan^{-1}0\right]$$

$$\Rightarrow \frac{1}{a^2-b^2}\left[\frac{1}{b}\tan^{-1}\left(\tan\frac{\pi}{2}\right) - \frac{1}{a}\tan^{-1}\tan\left(\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \frac{1}{a^2-b^2}\left[\frac{\pi}{2b} - \frac{\pi}{2a}\right] \Rightarrow \frac{1}{2(a^2-b^2)}\left[\frac{\pi}{b} - \frac{\pi}{a}\right]$$

Sec C ① $y = \sin(m\sin^{-1}x)$

Diff w.r. to x

$y_1 = \cos(m\sin^{-1}x) \cdot m \cdot \frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\, y_1 = m\cos(m\sin^{-1}x)$

Squaring both sides

$(1-x^2)(y_1)^2 = m^2[\cos(m\sin^{-1}x)]^2$

Diff w.r. to x

$(1-x^2)(2y_1)(y_2) + (y_1)^2(-2x) \Rightarrow m^2[2\cos(m\sin^{-1}x)]$

$[-\sin(m\sin^{-1}x)] \cdot \left[m \times \frac{1}{\sqrt{(1-x^2)}}\right]$

$\Rightarrow 2y_1[(1-x^2)y_2 - xy_1] \Rightarrow -2m^2\left[\frac{m\cos(m\sin^{-1}x)}{\sqrt{1-x^2}}\right][\sin(m\sin^{-1}x)]$

$\Rightarrow 2y_1[(1-x^2)y_2 - xy_1] \Rightarrow -2m^2(y_1)(y)$

$\Rightarrow (1-x^2)y_2 - xy_1 \Rightarrow \frac{(-m^2)(\cancel{2y_1})(y)}{(\cancel{2y_1})}$

$\Rightarrow (1-x^2)y_2 - xy_1 = -m^2y$

Section

$\sin x = 1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!} - \cdots$

(5) (a) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\, dx$

$\boxed{\cos 2x = 2\cos^2 x - 1}$

$\Rightarrow \int \frac{2\cos^2 x - \cancel{1} - 2\cos^2\alpha + \cancel{1}}{\cos x - \cos\alpha}\, dx$

$\Rightarrow \int \frac{2\cos^2 x - 2\cos^2\alpha}{\cos x - \cos\alpha}\, dx$

$\Rightarrow 2\int \frac{\cancel{(\cos x - \cos\alpha)}(\cos x + \cos\alpha)}{\cancel{(\cos x - \cos\alpha)}}\, dx$

$\Rightarrow 2\int \cos x\, dx + 2\cos\alpha \int dx$

$\Rightarrow 2\sin x + 2x\cos\alpha + C$

(b) $\int \tan^4 x\, dx$

$\boxed{\tan^n x \Rightarrow \frac{\tan^{n-1}}{n-1} x - \int \tan^{n-2} x\, dx}$

$\tan^4 x = \frac{\tan^{4-1} x}{4-1} - \int \tan^{4-2} x\, dx$

$\Rightarrow \frac{\tan^3 x}{3} - \int \tan^2 x\, dx$

$\Rightarrow \frac{\tan^3 x}{3} - \left[\frac{\tan^{2-1} x}{2-1} - \int \tan^{2-2} x\, dx\right]$

$\Rightarrow \dfrac{\tan^3 x}{3} - \left[\tan x - \int \tan^0 x \, dx\right]$

$\Rightarrow \dfrac{\tan^3 x}{3} - \tan x + x$

(6) $y = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x}}}}}$

$y = \sqrt{x+y}$

Squaring both sides

$y^2 = x+y$

Diff w.r to x

$2y \dfrac{dy}{dx} = 1 + \dfrac{dy}{dx}$

$2y \dfrac{dy}{dx} - \dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx}[2y-1] = 1$

$$\boxed{\dfrac{dy}{dx} = \dfrac{1}{2y-1}}$$

Section-C

Page No.

Date / /

(2) (a) We have $f(x) = (x^2+2x-3).e^x$. The fun^n (x^2+2x-3) and e^x are polynomial and exponential functions, therefore continuous. Also their product is cont^n i.e $f(x) = (x^2+2x-3)e^x$ is coi in $[-3, 1]$.

(b) $f(-3) = f(1) = 0$

(c) $f'(x) = (2x+2)e^x + e^x(x^2+2x-3)$

$\Rightarrow e^x[2x+2+x^2+2x-3]$

$\Rightarrow e^x[x^2+4x-1]$

which exists in the interval $]-3, 1[$.

Thus, the fun^n satisfies all the three conditions of Rolle's theorem, therefore there exists a point c.

$\Rightarrow f'(c) = e^c[c^2+4c-1]$

$c^2+4c-1 = 0$

$\frac{4 \pm \sqrt{16-4}}{2} = 0$

$\frac{4 \pm 2\sqrt{3}}{2}$

$2 \pm \sqrt{3} = 0$

$(2+\sqrt{3}), (2-\sqrt{3})$

not yet.

$2\sqrt{3}$

$2+\sqrt{3}$

2.00
0.58
1.42

(4.) sinx in power of $\left(x-\frac{\pi}{2}\right)$

$$f(x) = \sin x$$

$$f(x) = f\left[\frac{\pi}{2} + (x) - \frac{\pi}{2}\right] = f(a+h)$$

where $a = \frac{\pi}{2}$ and $h = x - \frac{\pi}{2}$

$$f(x) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \frac{h^4}{4!} f''''(a) + \cdots$$

Put all the value -

$$\sin x = f\left(\frac{\pi}{2}\right) + (x-\pi/2) f'\left(\frac{\pi}{2}\right) + \frac{(x-\pi/2)^2}{2!} f''\left(\frac{\pi}{2}\right) + \frac{(x-\pi/2)^3}{3!} f'''\left(\frac{\pi}{2}\right) + \frac{(x-\pi/2)^4}{4!} f''''\left(\frac{\pi}{2}\right) + \cdots \quad (1)$$

$f(x) = \sin x$	$f(\pi/2) = 1$
$f'(x) = \cos x$	$f'(\pi/2) = 0$
$f''(x) = -\sin x$	$f''(\pi/2) = -1$
$f'''(x) = -\cos x$	$f'''(\pi/2) = 0$
$f''''(x) = \sin x$	$f''''(\pi/2) = 1$

Put these values in eqn (1)

$$\sin x = 1 + (x-\pi/2)\,0 + \frac{(x-\pi/2)^2}{2!}(-1) + \frac{(x-\pi/2)^3}{3!}(0) + \frac{(x-\pi/2)^4}{4!}(1)$$