Math Internal Question Bank Solution -

Section - A] (Quiz Questions)

1.) True
2.) False
3.) $A \cap B = \{3\}$
4.) 6, 3
5.) $A \cup A = A$ and $A \cap A = A$
6.) $(a, b) \in R \Rightarrow (b, a) \in R$

OR

$aRb \Rightarrow bRa \ \forall a, b \in A$

7.) reflexive, symmetric and transitive.

8.) Constant function.

9.) unique least upper bound (sup) and a unique greatest lower bound (inf).

10.) complemented.

11.) ~~False~~ True

12.) ~~False~~ subset

13.) False

14.) $2\sqrt{26}$

15.) Saddle point

16.) $(1, 1, 2)$

17.) Normal / Perpendicular

18.) $rt - s^2 > 0$ and $r > 0$

19.) Skew lines or non-intersecting lines

20.) $(1, 0, 1)$

21.) reflexive, antisymmetric and transitive

22.) $x^2 + y^2 + z^2 + 6x - 8y - 10z + 1 = 0$

23.) $(0, 1, 0)$

~~24.) $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = r^2$~~

25.) $\cos\theta = \dfrac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}$

26.) $y = 0$

27.) $\dfrac{x-3}{l} = \dfrac{y+4}{m} = \dfrac{z-5}{n} = r$

28.) 1

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29.) 3

30.) 18

24.) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

Short Question :-

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Q-1 $u = \sin^{-1}\left[\frac{x^2+y^2}{x+y}\right]$

Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

$\Rightarrow$ We have, $u = \sin^{-1}\left[\frac{x^2+y^2}{x+y}\right]$

$$\sin u = \left[\frac{x^2+y^2}{x+y}\right] = f(x,y)$$

$n = 2-1 \Rightarrow 1$

This $f(x,y)$ is homogeneous of degree 1 then by Euler theorem :-

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 1 \cdot f$$

$$\left[\Rightarrow x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \sin u\right]$$

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = \sin u$$

$$x\cos u\frac{\partial u}{\partial u} + y\cos u\frac{\partial u}{\partial y} = \sin u$$

$$\cos u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = \sin u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

Q-2. Intersection point of the plane :

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda \text{ (say)}$$

$x - 2 = 3\lambda$, $x = 3\lambda + 2$ —— (1)

$y + 1 = 4\lambda$, $y = 4\lambda - 1$ —— (2)

$z - 2 = 12\lambda$, $z = 12\lambda + 2$ —— (3)

Now,

Eqn of plane,

$$x - y + z = 5$$

Put the values of x, y & z from eqn (1), (2) & (3)

$$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 5$$

$$11\lambda + 5 = 5$$

$$11\lambda = 0$$

$$\boxed{\lambda = 0}$$

Put the value of λ in eqn (1), (2) & (3)

$$\boxed{x = 2}, \boxed{y = -1}, \boxed{z = 2}$$

Now,

The distance b/w (0, 0, 0) & (x, y, z)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$D = \sqrt{(2)^2 + (-1)^2 + (2)^2}$$

$$D = \sqrt{4 + 1 + 4}$$

$$\boxed{D = \sqrt{9}}$$

Q-3 $x - y$ is divisible by 5

(i) Reflexive : If $(x, x) \in R$

$x - y$ is divisible by 5

$\Rightarrow (x, x)$ $[x - x = 0]$

0 is divisible by 5 :

Hence, R is Reflexive relation.

ii) Symmetric : If $(x, y) \in R$, then $(y, x) \in R$

$(x, y) \in R$

$(x - y)$ is divisible by 5

$x - y = 5$

$-x + y = -5$

$y - x = 5$

$(y - x)$ is also divisible by 5

Hence, R is also a symmetric relation.

(iii) Transitive : If $(x, y) \in R$ & $(y, z) \in R$

then $(x, z) \in R$

$x - y = 5k_1$ — ①

$y - z = 5k_2$ — ②

Adding both eqn

$x - z = 5(k_1 + k_2)$

$x - z = 5C$

(x, z) is also $\in R$

Hence R is Transitive also.

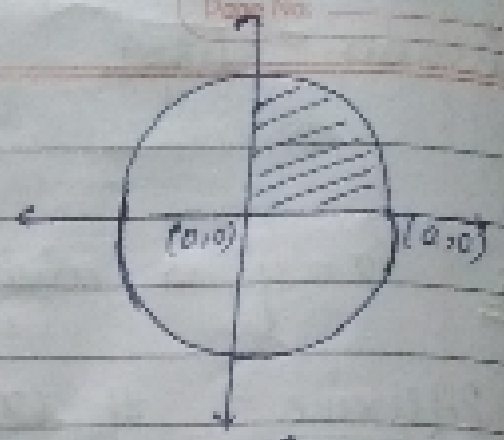
from (i), (ii) & (iii), we get to know that R is an equivalence relation.

Q-4. Given eqn of circle

$x^2 + y^2 = a^2$

$y^2 = a^2 - x^2$

$y = \sqrt{a^2 - x^2}$



$\therefore$ Area of circle = 4 × Area of first quadrant

$$\Rightarrow 4\int_0^a y\,dx$$

$$\Rightarrow 4\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$\Rightarrow 4\times\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$\Rightarrow 4\times\left[\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a} - 0\right]$$

$$\Rightarrow 4\times\left[\frac{a^2}{2}\sin^{-1}\left(\sin\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \cancel{4}\times\frac{a^2}{\cancel{2}}\times\frac{\pi}{\cancel{2}}$$

$$\Rightarrow \boxed{\pi a^2}$$

[Short Q-4]

Theorem-3 : Let $f: X \to Y$ and $g: Y \to Z$ be one-one onto mapping. Then the composite function $gof: X \to Z$ will be one-one onto $(gof)^{-1} = f^{-1}og^{-1}$.

Proof : gof one-one

Since f and g are one-one mapping

Then f^{-1} and g^{-1} are also one-one mapping.

Let $x_1, x_2 \in X$ & $y_1, y_2 \in Y$

$f(x_1) = y_1$, $f(x_2) = y_2$

$(gof) = y\ x_1 = (gof)\ x_2$

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$g\{f(x_1)\} = g\{f(x_2)\}$

$g(y_1) = g(y_2)$ ($\because$ g is one-one)

$y_1 = y_2$

$f(x_1) = f(x_2)$ ($\because$ f is one-one)

$x_1 = x_2$

So gof is also one-one.

gof onto : let $z \in Z$

g is onto, then there exist an element $y \in Y$.

$g(y) = z$

and f is onto then there exist an element $x \in X$.

$\because$ $gof(x) = g\{f(x)\} = g(y) = z$

So gof is also onto.

Given that

$f: X \to Y$ then $f(x) = y$ $\forall$ $x \in X$, $y \in Y$

$f: Y \to Z$ then $g(y) = z$ $\forall$ $y \in Y$, $z \in Z$

and $gof: X \to Z$ then $(gof)^{-1} x = z$ $\forall$ $x \in X$, $z \in Z$

$f^{-1}(y) = x$, $g^{-1}(z) = y$, $(gof)^{-1} z = x$

To prove $(gof)^{-1} = f^{-1} o g^{-1}$

R.H.S $\Rightarrow$ $(f^{-1} o g^{-1}) z = f^{-1}\{g^{-1}(z)\}$

$= f^{-1}(y) = x$

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$\Rightarrow (g \circ f)^{-1} z = (f^{-1} \circ g^{-1}) z$

$\Rightarrow (g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Theorem 21: *Dual of a complemented lattice is complemented.*

Proof: Let (L, R) be a complemented lattice with 0 and 1 as least and greatest elements. Let $(L, \overline{R})$ be the dual of (L, R). Then 1 and 0 are the least and greatest elements of $(L, \overline{R})$.

Let a be any element of L. Since (L, R) is complemented, there exists $a' \in L$ such that

$$a \wedge a' = 0 \text{ and } a \vee a' = 1 \text{ in } (L, R)$$

$$0 = \inf\{a, a'\} \text{ and } 1 = \sup\{a, a'\} \text{ in } (L, R)$$

i.e.,

$\Rightarrow$ $0 \, R \, a, 0 \, R \, a'$ and $a \, R \, 1, a' \, R \, 1$

$\Rightarrow$ $a \, \overline{R} \, 0, a' \, \overline{R} \, 0$ and $1 \, \overline{R} \, a, 1 \, \overline{R} \, a'$

$\Rightarrow$ 0 is an upper bound of $\{a, a'\}$ in $\{L, \overline{R}\}$ and 1 is a lower bound of $\{a, a'\}$ in $\{L, \overline{R}\}$.

Let k be any upper bound of $\{a, a'\}$ in $\{L, \overline{R}\}$. Then $a \, \overline{R} \, k$ and $a' \, \overline{R} \, k$

$\Rightarrow$ $k \, R \, a$ and $k \, R \, a'$

$\Rightarrow$ $k \, R \, 0$ as 0 is the inf. of $\{a, a'\}$ in (L, R)

$\Rightarrow$ $0 \, \overline{R} \, k$.

This shows that 0 is the least upper bound of $[a, a']$ in $\{L, \overline{R}\}$. Therefore $a \vee a' = 0$ in (L, R).

Similarly, we can show that $a \wedge a' = 1$ in $(L, \overline{R})$. Thus, a is the complement of a' in $(L, \overline{R})$.

Hence $(L, \overline{R})$ is complemented.

Long Questions :-

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Q-1. (a) $x = r\cos\theta$
$y = r\sin\theta$

$\frac{\partial r}{\partial x} = -\sin\theta$ | $\frac{\partial r}{\partial y} = \cos\theta$

$\Rightarrow \left[\frac{\partial r}{\partial x}\right]^2 + \left[\frac{\partial r}{\partial y}\right]^2 = 1$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$
$1 = 1$

(b) $u = xy(a-x-y)$

$u = axy - x^2y - xy^2$

Firstly diff. w.r. to x [Treating y as constant]

$\frac{\partial u}{\partial x} = ay - 2xy - y^2$

$r \Rightarrow \frac{\partial^2 u}{\partial x^2} = -2y$

Secondly, diff. w.r. to y [Treating x as constant]

$\frac{\partial u}{\partial y} = ax - x^2 - 2xy$

$t \Rightarrow \frac{\partial^2 u}{\partial y^2} = -2x$

$$s \Rightarrow \frac{\partial}{\partial x}\left[\frac{\partial u}{\partial y}\right] \Rightarrow \frac{\partial}{\partial x}\left[ax - x^2 - 2xy\right]$$

$$s \Rightarrow a - 2x - 2y$$

Put $\frac{\partial u}{\partial x} = 0$ & $\frac{\partial u}{\partial y} = 0$ then,

$$\Rightarrow \frac{\partial u}{\partial x} = 0$$

$$ay - 2xy - y^2 = 0$$

$$y\left[a - 2x - y\right] = 0$$

$$a - 2x - y = 0$$

$$\Rightarrow \frac{\partial u}{\partial y} = 0$$

$$ax - 2xy - x^2 = 0$$

$$x\left[a - 2y - x\right] = 0$$

$$a - 2y - x = 0$$

$$-x = 2y - a$$

$$x = a - 2y$$

Put value of x in eqn

$$\Rightarrow a - 2a + 4y - y = 0$$

$$\Rightarrow -a + 3y = 0$$

$\Rightarrow \quad 3y = a$

$\Rightarrow \quad \boxed{y = a/3}$

put value of in eq^n

$$x = a - \frac{2a}{3}$$

$$x = \frac{3a - 2a}{3}, \quad \boxed{x = \frac{a}{3}}$$

Stationary point $\boxed{\frac{a}{3}, \frac{a}{3}}$

$\Rightarrow \quad r = -2 \times \frac{a}{3} \Rightarrow \frac{-2a}{3}$

$\Rightarrow \quad t = -2 \times \frac{a}{3} \Rightarrow \frac{-2a}{3}$

$\Rightarrow \quad s = a - \frac{2a}{3} - \frac{2a}{3}$

$\Rightarrow \quad \frac{3a - 4a}{3} \Rightarrow \frac{-a}{3}$

if $rt - s^2$ is greater than 0 then we check the fun^n ~~is~~ r if r is less than 0 then is maximum & if its greater than 0 then the fun^n is minimum.

$\Rightarrow \quad rt - s^2 \Rightarrow \left(\frac{-2a}{3}\right) \times \left(\frac{-2a}{3}\right) - \left(\frac{-a}{3}\right)^2$

$\Rightarrow \quad \frac{4a^2}{9} - \frac{a^2}{9} \Rightarrow \frac{\cancel{3}a^2}{\cancel{9}\,3}$

$\Rightarrow \quad \frac{a^2}{3} > 0$

$r = -\frac{2a}{3} < 0$

The funⁿ is maxima.

Q.2

(a) Eqⁿ of a plane :

$$Ax + By + Cz + D = 0 \quad \text{—— (1)}$$

The points are : $[0, -1, -1]$, $[4, 5, 1]$, $[3, 9, 4]$

The eqⁿ (1) passes through these points :

$$-B - C + D = 0 \quad \text{—— (2)}$$

$$4A + 5B + C + D = 0 \quad \text{—— (3)}$$

$$3A + 9B + 4C + D = 0 \quad \text{—— (4)}$$

Solving (2), (3) & (4) :

$-B - C + D = 0$

$-C = -D + B$

$-C = -(D - B)$

$C = D - B$

$C = D + \frac{7D}{4}$

$$\boxed{C = \frac{11D}{4}}$$

$4A + 5B + C + D = 0$

$4A + 5B + D - B + D = 0$

$4A + 4B + 2D = 0$

$2A + 2B + D = 0$

$2A = -2B - D$

$A = \frac{-2B - D}{2}$

$A = \frac{14D - 4B}{8}$

$A = \frac{10D}{8}$

$$\boxed{A = \frac{5D}{4}}$$

$3A + 9B + 4C + D = 0$

$\frac{-6B - 3D}{2} + 9B + 4D - 4B + D = 0$

$-6B - 3D + 18B + 8D - 8B + 2D = 0$

$4B + 7D = 0$

$4B = -7D$

$$\boxed{B = \frac{-7D}{4}}$$

$\Rightarrow \frac{5Dx}{4} - \frac{7Dy}{4} + \frac{11Dz}{4} + D = 0$

$\Rightarrow \frac{D}{4}[5x - 7y + 11z + 4] = 0$

$\Rightarrow 5x - 7y + 11z + 4 = 0$

$[-4, 4, 4]$ these has to be satisfy the eqn of plane passes through the given points

$\Rightarrow -20 - 28 + 44 + 4 = 0$

$\Rightarrow -48 + 48 = 0$

$\Rightarrow 0 = 0$

(b) Let the given eqn of the sphere be

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ — (1)

If the sphere (1) passes through these points $(1, -3, 4)$, $(1, -3, 0)$, $(1, -5, 2)$, these must satisfy the eqn (1). Therefore,

$\Rightarrow 1 + 9 + 16 + 2u - 6v + 8w + d = 0$

$\Rightarrow 26 + 2u - 6v + 8w + d = 0$ — (2)

$\Rightarrow 1 + 9 + 2u - 6v + d = 0$

$\Rightarrow 10 + 2u - 6v + d = 0$ — (3)

$\Rightarrow 1 + 25 + 4 + 2u - 10v + 4w + d = 0$

$\Rightarrow 30 + 2u - 10v + 4w + d = 0$ — (4)

Also the centre $(-u, -v, -w)$ of the sphere (1) lies on plane $(x+y+z)=1$

$$\Rightarrow -u - v - w = 1 \quad \text{——— (5)}$$

~~From~~ solve eqn (2), (3) & (4)

$$10 + 2u - 6v + d = 0$$
$$2u - 6v = -d - 10$$
$$2u = -d - 10 + 6v$$
$$u \Rightarrow \frac{-d - 10 + 6v}{2}$$

$$u = \frac{-d - 10 + 18}{2}$$

$$\boxed{u = \frac{-d + 8}{2}}$$

$$26 + 2u - 6v + 8w + d = 0$$
$$26 - \cancel{d} - 10 + \cancel{6v} - \cancel{6v} + 8w + \cancel{d} = 0$$
$$16 + 8w = 0$$
$$\cancel{8}w = -\cancel{16}^{2}$$
$$\boxed{w = -2}$$

$$30 + 2u - 10v + 4w + d = 0$$
$$30 - \cancel{d} - 10 + 6v - 10v - \cancel{8} + \cancel{d} = 0$$
$$12 - 4v = 0$$
$$12 = 4v$$
$$\boxed{v = 3}$$

From eqn (2), (3) & (4)

$$u = \left(\frac{-d + 8}{2}\right), \quad v = 3, \quad w = -2$$

Substituting these values of u, v, w in (5) we get,

$$-\left[\frac{-d + 8}{2}\right] - 3 + 2 = 0$$

$$d - 8 - 6 + 4 = 0$$

$$d - 14 + 4 = 0$$

$$\boxed{d = 10}$$

Put the value of d in $u = \left(\frac{d+8}{10 \times 2}\right)$

$$u = \frac{-10+8}{2} = \frac{-2}{2}$$

$$\boxed{u = -1}$$

Put these values of u, v, w & d in eq(1), the required eqn of the sphere is:

$$x^2 + y^2 + z^2 - 2x + 6y - 4z + 10 = 0$$

Q-3. We have,

$$l + m + n = 0 \quad \text{--- (1)}$$

$$l^2 + m^2 + n^2 = 0 \quad \text{--- (2)}$$

From eqn (1)

$$n = -(l+m) \quad \text{--- (3)}$$

Put eqn (3) in eqn (2)

$$\left[\begin{aligned} \cancel{l^2} + m^2 - \cancel{l^2} + m^2 + 2lm &= 0 \\ 2m^2 + 2lm &= 0 \end{aligned}\right] \times$$

$$l^2 + m^2 - (l+m)^2 = 0$$

$$l^2 + m^2 - [l^2 + m^2 + 2lm] = 0$$

$$\cancel{l^2} + \cancel{m^2} - \cancel{l^2} - \cancel{m^2} - 2lm = 0$$

$$-2lm = 0$$

$$\text{or} \quad lm = 0$$

$$l = 0 \quad \text{or} \quad m = 0$$

When $l = 0$, we have from eqn (1)

$$l + m + n = 0$$
$$0 + m + n = 0$$
$$\boxed{m = -n}$$

When $m = 0$, we have from eqn (1)

$$l + m + n = 0$$
$$l + 0 + n = 0$$
$$\boxed{l = -n}$$

$\because$ the direction ratios of two lines are $[0, -n, n]$ & $[-n, 0, n]$ or $(0, -1, 1)$ & $(-1, 0, 1)$

If the θ acute angle b/w two lines, then

$$\cos\theta = \frac{|a_1b_1 + a_2b_2 + a_3b_3|}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$\cos\theta = \frac{|0 + 0 + 1|}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

$$\boxed{\theta = \frac{\pi}{3}}$$

Q-4: (a) $n(H) = 35$, $n(H \cap E) = 25$, $n(H \cup E) = 50$
$n(U) = 50$

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$
$$n(E) = n(H \cup E) + n(H \cap E) - n(H)$$
$$= 50 + 25 - 35$$
$$= 75 - 35$$
$$\boxed{n(E) = 40}$$

$n(E-H) \Rightarrow n(E) - n(E \cap H)$

$\Rightarrow 40 - 25$

$\boxed{n(E-H) \Rightarrow 15}$

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40 people speaks english & 15 people speak only english not Hindi.

Q.4. (b) $f(x) = \log\left(\frac{1+x}{1-x}\right)$,

Show $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

$\Rightarrow f(x) = \log\left(\frac{1+x}{1-x}\right)$, $f(y) = \log\left(\frac{1+y}{1-y}\right)$

$\Rightarrow$ Taking L.H.S : $f(x) + f(y)$

$$\Rightarrow \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$$

$$\Rightarrow \log\left[\left(\frac{1+x}{1-x}\right) \times \left(\frac{1+y}{1-y}\right)\right]$$

$$\Rightarrow \log\left[\frac{1+y+x+xy}{1-y-x+xy}\right] \quad \text{——— (1)}$$

Now, Taking R.H.S : $f\left(\frac{x+y}{1+xy}\right)$

$$\Rightarrow \log\left[\frac{1+\left(\frac{x+y}{1+xy}\right)}{1-\left(\frac{x+y}{1+xy}\right)}\right]$$

$$\Rightarrow \log\left[\frac{\frac{1+xy+x+y}{1+xy}}{\frac{1+xy-x-y}{1+xy}}\right]$$

$$\Rightarrow \log\left[\frac{1+x+y+xy}{1-x-y+xy}\right] \quad \text{--- (2)}$$

By eqn (1) & (2), we get

$$f(x)+f(y) = f\left(\frac{x+y}{1+xy}\right)$$

Q-5. (a) $\int_0^a \int_0^{\sqrt{a^2-b^2}} x^2 y \, dx \, dy$

Diff. w.r. to y

$$\int_0^a x^2 \left[\int_0^{\sqrt{a^2-b^2}} y \, dy \right] dx$$

$$\Rightarrow \int_0^a x^2 \left[\frac{y^2}{2} \right]_0^{\sqrt{a^2-b^2}} dx$$

$$\Rightarrow \frac{1}{2} \int_0^a x^2 \left[a^2 - b^2 \right] dx$$

$$\Rightarrow \frac{a^2+b^2}{2} \int_0^a x^2 \, dx$$

$$\Rightarrow \frac{a^2+b^2}{2} \left[\frac{x^3}{3} \right]_0^a$$

$$\Rightarrow \frac{a^2+b^2}{6} \left[a^3 \right]$$

$$\Rightarrow \boxed{\frac{a^3 (a^2+b^2)}{6}}$$

(b) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$ is $\frac{1}{48}$.

Diff w.r.to z.

$$\Rightarrow \int_0^1 x \int_0^{\sqrt{1-x^2}} y \left[\int_0^{\sqrt{1-x^2-y^2}} z \, dz \right] dy \, dx$$

$$\Rightarrow \int_0^1 x \int_0^{\sqrt{1-x^2}} y \left[\frac{z^2}{2} \right]_0^{\sqrt{1-x^2-y^2}} dy \, dx$$

$$\Rightarrow \frac{1}{2} \int_0^1 x \left[\int_0^{\sqrt{1-x^2}} y - yx^2 - y^3 \, dy \right] dx$$

$$\Rightarrow \frac{1}{2} \int_0^1 x \left[\frac{y^2}{2} - \frac{y^2x^2}{2} - \frac{y^4}{4} \right]_0^{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{1}{8} \int_0^1 x \left[2y^2 - 2y^2x^2 - y^4 \right]_0^{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{1}{8} \int_0^1 x \left[2 - 2x^2 - 2x^2(1-x^2) - (1-x^2)^2 \right]$$

$$\Rightarrow \frac{1}{8} \int_0^1 x \left[2 - 2x^2 - 2x^2 + 2x^4 - 1 - x^4 + 2x^2 \right]$$

$$\Rightarrow \frac{1}{8}\int_0^1 x\left(1-2x^2+x^4\right)$$

$$\Rightarrow \frac{1}{8}\left[\frac{x^2}{2}-\frac{\cancel{2}x^4}{\cancel{4}2}+\frac{x^6}{6}\right]_0^1$$

$$\Rightarrow \frac{1}{8}\left[\frac{\cancel{1}}{2}-\frac{\cancel{1}}{\cancel{2}}+\frac{1}{6}\right]$$

$$\boxed{\Rightarrow \frac{1}{48}}$$