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Measures of Central tendency

In Statics, Various measures of Central tendency or averages are Studied. Three important Measures of Central tendency are :-

1. Airthmetic Mean (A.M) or Mean
2. Median
3. Mode

Airthmetic Mean → Airthmetic Mean, also Called average in general Sense, is the Most popular and widely used Measure of Central tendency.
Airthmetic Mean is of two types :-

1. Simple Arithmetic Mean
2. Weighted Airthmetic Mean

Simple Arithmetic Mean is are of two Method →
1. Direct Method
2. Short-Cut Method

Direct Method $= \frac{1}{N} \Sigma X$

Short-Cut Method $= A + \frac{\Sigma d}{N}$

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Ex 1: Calculate the Arithmetic mean of the following Marks in Statics obtained by ten Students in a Class.

Students No	1	2	3	4	5	6	7	8	9	10
Marks	15	20	25	19	12	11	13	17	18	20

Solⁿ →

Student No.	Marks (X)
1	15
2	20
3	25
4	19
5	12
6	11
7	13
8	17
9	18
10	20
$N=10$	$\Sigma X = 170$

Using Direct Method →

$$\bar{X} = \frac{1}{N} \Sigma X$$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{170}{10} = 17 \text{ Marks}$$ Ans

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Ex2: Use Short-Cut Method to Calculate arithmetic mean for the data in Statitics

Student No.	1	2	3	4	5	6	7	8	9	10
Marks(s)	15	20	25	19	12	11	13	17	18	20

Sol^n

Student No.	Mark(s)	Deviation $(X-A)$
1	11	-3
2	12	$+2$
3	13	$+7$
4	15	$+1$
5	17	-6
6	18(A)	-7
7	19	-5
8	20	-1
9	20	0
10	25	$+2$
$N=10$		$\Sigma d=-10$

$$\bar{X} = A + \frac{\Sigma}{N}$$

$$= 18 + \left(\frac{-10}{10}\right)$$

$$= 18 - 1$$

$$= 17 \text{ Marks}$$ Ans

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Ex 3: Show that the arithmetic mean of the series.

$1, 2, 2^2, 2^3, 2^4, \ldots 2^n$ is $\frac{(2^{n+1}-1)}{(n+1)}$.

Sol^n →

S No.	X
1	1
2	2
3	2^2
4	2^3
5	2^4
n	2^n
$N = n+1$	$\Sigma X = (2^{n+1}-1)$

$$\bar{X} = \frac{1}{N} \cdot \Sigma X$$

$$= \frac{\Sigma X}{N}$$

$$= \frac{(2^{n+1}-1)}{(n+1)}$$ Ans.

Ex 4: The mean of n numbers of a series is $\bar{X}$ and the sum of first $(n-1)$ numbers is λ. Find the value of the last number.

Sol^n → Let n number is $x_1, x_2, x_3 \ldots x_{n-1}, x_n$

So $\bar{x} = \dfrac{x_1 + x_2 + x_3 - - - - - - x_{n-1} + x_n}{n}$

$x_1 + x_2 + x_3 + - - - - - - x_{n-1} + x_n = n\bar{x}$

also given

$$x_1 + x_2 + x_3 + - - - - x_{n-1} = \lambda$$

So from ①.

$$\lambda + x_n = n\bar{x}$$

$$x_n = n\bar{x} - \lambda$$

So, last number is $(n\bar{x} - \lambda)$ Ans

Ex5: Following table gives the wages paid to 125 workers in a factory. Calculate the Arithmetic Mean of the wages:

Wage (₹)	200	210	220	230	240	250	260
No. of Workers	5	15	32	42	15	12	4

Solⁿ →.

Wages (X)	No. of Workers (f)	fx
200	5	1000
210	15	3150
220	32	7040
230	42	9660
240	15	3600
250	12	3000
260	4	1040
Total	$N = \Sigma f = 125$	$fx = 28490$

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$$\bar{X} = \frac{\Sigma fx}{N} = \frac{28490}{125} = ₹227.92$$

Ex6: What is the arithmetic mean of the following?

Variate	0	1	2	---- n
Frequency	${}^nC_0$	${}^nC_1$	${}^nC_2$	---- ${}^nC_n$

Solⁿ →.

x	f	fx
0	${}^nC_0$	0
1	${}^nC_1$	$1 \cdot {}^nC_1$
2	${}^nC_2$	$2 \cdot {}^nC_2$
3	${}^nC_3$	$3 \cdot {}^nC_3$
4	${}^nC_4$	$4 \cdot {}^nC_4$
5	${}^nC_5$	$5 \cdot {}^nC_5$
n	${}^nC_n$	$n \cdot {}^nC_n$

We know that A.M Given by

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\Rightarrow \frac{0 + 1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + \cdots n \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n}$$

$$= \frac{n + \cancel{2} \cdot \frac{n(n-1)}{\cancel{2}\angle 1} + \cancel{3} \cdot \frac{n(n-1)(n-2)}{\cancel{3}\angle 2} + \cdots n \cdot}{2^n}$$

So $\bar{x} = \dfrac{\dfrac{n[1+(n-1)+(n-1)(n-2)+ \cdots +1]}{\lfloor 2}}{2^n}$

$= \dfrac{n({}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-2}C_2 + \cdots {}^{n-1}C_n)}{2^n}$

$= \dfrac{n \cdot 2^{n-1}}{2^n} = \dfrac{n}{2}$

So $A.M = \dfrac{n}{2}$ Ans.

Ex7 If the arithmetic average of data given below be 165 Rupees, find the missing term.

Monthly Wages	100	150	200		300	500
Number of Labours	30	20	15	10	4	1

Sol^n → Let missing term is x_4 then

x	f	fx
100	30	3000
150	20	3000
200	15	3000
x_4	10	$10x_4$
300	4	1200
500	1	500
	80	$10700 + 10x_4$

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given $\bar{x} = 165$

$\Sigma f = 80$

$\Sigma f = 10700 + 10x_4$

So

$$165 = \frac{10700 + 10x_4}{80}$$

$$13200 = 10700 + 10x_4$$

$$10x_4 = 13200 - 10700$$

$$10x_4 = 2500$$

$$x_4 = 250 \quad \text{Ans.}$$

Ex 9: Use Step-deviation Method for Calculating arithmetic Mean for the data in following table gives the Wages paid to 125 workers in a factory

Wages	200	210	220	230	240	250	260
No. of Workers	5	15	32	42	15	12	4

Sol^n →

x	f	A	$d' = \frac{x-A}{h}$	fd'
200	5		-3	-15
210	15		-2	-30
220	32	230	-1	-32
230	42		0	0
240	15		1	15
250	12		2	24
260	4		3	12
	125			-26

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$$\bar{X} = A + \frac{\Sigma f d'}{\Sigma f} \times h$$

$$= 230 + \frac{-26}{125} \times 10$$

$$= 230 - 2.08$$

$$= 227.92 \text{ Ans.}$$

Ex10: Use direct Method to Calculate Arithmetic Mean from the following data

Class	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	8	10	12	20	11	4	5

Sol^n →.

Class	frequency	Mid-Value(x)	fx
20-25	8	22.5	180
25-30	10	27.5	275
30-35	12	32.5	390
35-40	20	37.5	750
40-45	11	42.5	467.5
45-50	4	47.5	190
50-55	5	52.5	262.5
	70		2515

$$AM = \frac{\Sigma fx}{\Sigma f} = \frac{2515}{70} = 35.93 \text{ Ans}$$

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Ex 11: Use Short-Cut Method for Calculating Mean for the data given by:

Class	20-25	25-30	30-35	35-40	40-45	45-50	50-50
Frequency	8	10	12	20	11	4	5

Solⁿ

Class	frequency	Mid-value	$d = x - A$	fd
20-25	8	22.5	-15	-120
25-30	10	27.5	-10	-100
30-35	12	32.5	-5	-60
35-40	20	37.5 (A)	0	0
40-45	11	42.5	5	55
45-50	4	47.5	10	40
50-55	5	52.5	15	75
	70			-110

So by Short-Cut Method.

$$AM = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 37.5 + \frac{-110}{70}$$

$$= 37.5 - 1.57$$

$$= 35.93 \text{ Ans.}$$

Ex 12: Obtain Aritmetic mean for the data in above example using Step-deviation Method.

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Class	frequency	Mid Value (X)	$d' = \frac{X-A}{h}$	fd'
20-25	8	22.5	-3	-24
25-30	10	27.5	-2	-20
30-35	12	32.5	-1	-12
35-40	20	37.5 (A)	0	0
40-45	11	42.5	1	11
45-50	4	47.5	2	8
50-55	5	52.5	3	15
	70			-22

So by Step divation Method.

$$AM = A + \frac{\Sigma fd}{\Sigma f} \times h$$

$$= 37.5 + \frac{-22}{70} \times 5$$

$$= 37.5 - 1.57$$

$$= 35.93 \text{ Ans}$$

Ex13: Find the average Marks of the Students from the following data:

Marks Below	10	20	30	40	50	60	70	80
No. of Stu	15	35	60	84	96	127	198	250

Marks (Below)	No. of Student	Class	f	M.V	d = x-A	fd
10	15	0-10	15	5	-30	-450
20	35	10-20	20	15	-20	-400
30	60	20-30	25	25	-10	-250
40	84	30-40	24	35(A)	0	0
50	96	40-50	12	45	10	120
60	127	50-60	31	55	20	620
70	198	60-70	71	65	30	2130
80	250	70-80	52	75	40	2080
			250			3850

$$\bar{X} = A + \frac{\Sigma fd}{N}$$

$$= 35 + \frac{3850}{250}$$

$$= 35 + 15.4$$

$$\bar{X} = 50.4 \text{ Ans.}$$

Ex 14: The following data related to the Marks of 70 Students in Statistics. Find the Mean:

Marks (More than)	20	30	40	50	60	70
No. of Student	70	63	40	30	18	7

Sol^n:

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Marks (Mark Class)	No. of Student	Class	f	M.V(X)	d	fd
20	70	20-30	7	25	-20	-140
30	63	30-40	23	35	-10	-230
40	40	40-50	10	45(A)	0	0
50	30	50-60	12	55	10	120
60	18	60-70	11	65	20	220
70	7	70-80	7	75	30	210
			70			180

$$\bar{X} = A + \frac{\Sigma fd}{N}$$

$$= 45 + \frac{180}{70}$$

$$= 45 + 2.57$$

$$= 47.57 \text{ Ans.}$$

Ex15 Calculate A.M from the following distribution:

Classes	10-19	20-29	30-39	40-49	50-59	60-69
frequency	5	12	17	20	18	8

Sol^n:

Classes	frequency	M.V(X)	d = X-A	fd
10-19	5	14.5	-20	-100
20-29	12	24.5	-10	-120
30-39	17	34.5(A)	0	0
40-49	20	44.5	10	200
50-59	18	54.5	20	360
60-69	8	64.5	30	240
	80			580

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$$\bar{X} = A + \frac{\Sigma fd}{N}$$

$$\bar{X} = 34.5 + \frac{58\cancel{0}}{8\cancel{0}}$$

$$\bar{X} = 34.5 + 7.25$$
$$\bar{X} = 41.75 \text{ Ans}$$

Ex 17: The mean wage of 100 workers in a factory running two shifts of 60 & 40 workers is 38. The mean wage of 60 workers in the morning shift is 40 ₹. Find the mean wage of 40 workers in the evening shift.

Solⁿ → $\bar{x}_{12} = 38$, $N_1 = 60$, $N_2 = 40$, $\bar{x}_1 = 40$
$\bar{x}_2 = ?$

We know combined mean is given by

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$38 = \frac{60 \times 40 + 40 \times \bar{x}_2}{60 + 40}$$

$$3800 = 2400 + 40 \bar{x}_2$$
$$40 \bar{x}_2 = 3800 - 2400$$
$$\bar{x}_2 = \frac{1400}{40}$$
$$\bar{x}_2 = 35 \text{ Ans}$$

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Ex18: Find the Combined mean from the following data:

Groups	1	2	3
Numbers	200	250	300
Mean	25	20	15

Sol^n→ Given

$N_1 = 200, \; N_2 = 250, \; N_3 = 300$

$N = N_1 + N_2 + N_3 = 200 + 250 + 300$

$N = 750$

$\overline{X}_{123} = ?$

We know Combined Mean is given by

$$\overline{X}_{123} = \frac{N_1 \overline{x}_1 + N_2 \overline{x}_2 + N_3 \overline{x}_3}{N_1 + N_2 + N_3}$$

$$= \frac{200 \times 25 + 250 \times 20 + 300 \times 15}{750}$$

$$= \frac{5000 + 5000 + 4500}{750}$$

$$= \frac{14500}{750}$$

$\overline{x}_{123} = 19.33$ Ans

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Ex 20: The weight of 150 students in a certain class is 60 kilograms. The mean weight of boys in the class is 70 kilograms & that of the girls is 55 kilograms find the number of boys and girls in the class.

Sol^n → Let N_1 boys & N_2 Girls in the Class

then $N_1 + N_2 = 150$ — ①

& given

$$\overline{N_{12}} = 60$$
$$\overline{x_1} = 75$$
$$\overline{x_2} = 55$$

We know Combined Mean is given by

$$\overline{x_{12}} = \frac{N_1 \overline{x_1} + N_2 \overline{x_2}}{N_1 + N_2}$$

$$60 = \frac{70 N_1 + 55 N_2}{150}$$

$$9000 = 70 N_1 + 55 N_2$$

$$14 N_1 + 11 N_2 = 1800 \quad \text{— ②.}$$
$$\underline{11 N_1 + 11 N_2 = 1650}$$

$$3 N_1 = 150$$
$$N_1 = 50$$

from ①.

$$50 + N_2 = 150$$
$$N_2 = 100 \quad \text{Ans.}$$

Ex21: A person makes a trip which entire travelling 900 Kms by train at an average speed of 60 K/h, 300 Kms by bus at an average speed 25 K/h, 400 Kms by plane at 350 K/h and finally 15 Kms by taxi at 25 K/h. What is the average speed of the entire distance.

Sol⇒

Journey by	Distance (Kms)	Average Speed (K/h)	Time Taken (Hrs)
Train	900	60	15
Bus	300	25	12
Plane	400	350	$8/7$
Taxi	15	25	$3/5$
Total	1615		$\frac{1006}{35}$

Now Average Speed $= \frac{\text{Total distance}}{\text{Total Time}}$

$= \frac{1615}{\frac{1006}{35}} = \frac{1615 \times 35}{1006} = \frac{56525}{1006}$

$= 56.19$ Km/h Ans.

Ex22: A person travels n equal distances with velocities $V_1, V_2 --- V_n$. Show that his average velocity $V_1, V_2, V_3 ----V_n$. Show that his average velocity cannot exceed $\frac{(V_1 + V_2 + --- V_n)}{n}$ when it when it will be equal to this value?

Sol⇒ Total distance = $n \cdot$ equal distance

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Velocity per distance be $V_1, V_2, V_3 ---- V_3$

Per Units distance time $= \frac{\text{Speed}}{\text{distance}}$

$= \frac{V_1 + V_2 + ---V_n}{n}$

The Max Speed $= \frac{V_1 + V_2 + ---V_n}{n} \leq$ Total time

Maximum Average Speed $= \frac{V_1 + V_2 + ---V_n}{n}$ Ans

Ex 23: In a Class of 50 Students, 10 have failed and their average of Marks is 2.5 The Total of Marks Secured by the entire class was 285. Find the average Marks of those Who have passed.

Sol^n → Total Students = 50
Fail Students = 10
Passed Students = 50 − 10 = 40

Total Marks = 285
Total Marks of fail students = 10 × 2.5
= 25

Total Marks of passed Students = 285 − 25
= 260

So, average Marks of Passed Students
$= \frac{\Sigma X}{N} = \frac{260}{40} = 6.5$ Ans

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Ex24: In the examination, the average grade of all Students in Class A ~~and B be N_1 and N_2 respectively.~~ ~~the~~ is 68.4 and that for Students in Class B is 71.2 if there average of both the Class Combined is 70, find the Number of Students in Class A to the Number in Class B.

Sol^n → Given $\bar{x}_1 = 68.4$, $\bar{x}_2 = 71.2$
$\bar{x}_{12} = 70$

Let Number of Students in Class A be N_1 & Number of Student in Class B be N_2 then We know.

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$70 = \frac{68.4 N_1 + 71.2 N_2}{N_1 + N_2}$$

$$70 N_1 + 70 N_2 = 68.4 N_1 + 71.2 N_2$$

$$70 N_1 - 68.4 N_1 = 71.2 N_2 - 70 N_2$$
$$1.6 N_1 = 1.2 N_2$$

$$\frac{N_1}{N_2} = \frac{1.2}{1.6} = \frac{3}{4}$$

$N_1 : N_2 = 3 : 4$ Ans

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Ex25 : The Average Marks of 80 students were found to be 40. Later, it was discovered that a Score of 54 was misread as 84. Find the Correct Mean of the 80 Students

Sol^n → Given, $N = 80$
Incorrect Mean $= 40$
Correct Value $= 54$
Incorrect Value $= 84$

We know $\bar{x} = \frac{\Sigma X}{N}$

Incorrect $\Sigma X = \bar{X} \cdot N$
$= 40 \times 80$
$= 3200$

Correct $\Sigma X = 3200 - 84 + 54$
$= 3170$

So Correct Mean $= \frac{\text{Correct } \Sigma X}{N}$

$= \frac{3170}{80}$

$= 39.625$ Ans.

Ex26: Mean of 100 items is found to be 30. if at the time collection, two items are wrongly taken as 32 & 12 instead of 23 and 11, find the Correct Mean.

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Sol^n: $N = 100$, $\bar{x} = 40$
Correct item = 23 & 11
incorrect item = 32 & 12

$$\bar{X} = \frac{\Sigma X}{N}$$

Now $\Sigma X = \bar{X} \cdot A$
$= 30 \times 100$
$= 3000$

So, Correct $\Sigma X = 3000 - 32 - 12 + 23 + 11$
$= 2990$

So, Correct Mean $= \frac{\text{Correct } \Sigma X}{N}$

$= \frac{2990}{100} = 29.9$ Ans

Ex28: The average Monthly wages of all workers in a factory is 444 ₹. if the average wages paid to Male and female Workers are 480 ₹ and 360 ₹ respectively, find the persentage of Male and female workers employed by the factory.

Sol^n: Let No. of Male Workers = N_1
Let No. of Female Workers = N_2

$\bar{X}_{12} = 444$, $\bar{x}_1 = 480$, $\bar{x}_2 = 360$

We Know $\bar{x}_{12} = \dfrac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$

$$444 = \frac{480 N_1 + 360 N_2}{N_1 + N_2}$$

$$444 N_1 + 444 N_2 = 480 N_1 + 360 N_2$$
$$444 N_2 - 360 N_2 = 480 N_1 - 444 N_1$$
$$84 N_2 = 36 N_1$$
$$\frac{N_1}{N_2} = \frac{\cancel{84}\,7}{\cancel{36}\,3} = \frac{7}{3}$$

$$N_1 = \frac{7}{3} N_2 \quad \& \quad N_2 = \frac{3}{7} N_1$$

Now male % $= \dfrac{N_1}{N_1 + N_2} \times 100$

$$= \frac{N_1}{N_1 + \frac{3}{7} N_1} \times 100$$

$$= \frac{\cancel{N_1}}{\frac{10}{7} \times \cancel{N_1}} \times 100$$

$$= \frac{7}{\cancel{10}} \times 10\cancel{0}$$

$$= 70$$

So % of Male Workers = 70 %
& % of Female Workers = 100 − 70
= 30 % Ans.

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Ex29: Average rainfall of a City from Monday to Saturday is 0.3 inch. Due to heavy rainfall on Sunday, the average rainfall increased to 0.5 inches. What was the rainfall on Sunday?

Sol^n → Let rainfall in Sunday to x & given average Rainfall Monday to Saturday is 0.3 inch. So

$$0.3 = \frac{\text{Monday to Saturday}}{6}$$

Monday to Saturday total Rainfall

$= 0.3 \times 6$

$= 1.8$ inch

Now average Rainfall from Monday to Saturday $= 0.5$ inch

$$\Rightarrow 0.5 = \frac{1.8 + x}{7}$$

$$3.5 = 1.8 + x$$

$$x = 3.5 - 1.8$$

$$x = 1.7 \text{ inch}$$ Ans

Ex31: The daily wages of there person are 6, 9 and 13.5 ₹. Find the geometric mean

Sol^n → Given $x_1 = 6$, $x_2 = 9$, $x_3 = 13.5$

We know

$$G.M = (x_1 \cdot x_2 \cdot x_3)^{1/3}$$

$$= (6 \times 9 \times 13.5)^{1/3}$$

$$= (3 \times 9 \times 27)^{1/3}$$

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$= (3\times3\times3 \times 3\times3\times3)^{1/3}$

$= 3\times3$

$\Rightarrow GM = 9$ Ans.

Ex32: Calculate the geometric Mean of the following 2574, 475, 75, 5, 0.8, 0.08, 0.005, 0.0009.

Sol^n:

Value (x)	Logarithm ($\log x$)
2574	3.4106
475	2.6767
75	1.8751
5	0.6999
0.8	$\bar{1}.9031$
0.08	$\bar{2}.9031$
0.005	$\bar{3}.6990$
0.0009	$\bar{4}.9545$
$N=8$	Total $= 2.1208$

So, $N=8$, $\Sigma \log x = 2.1208$

$$G.M = \text{Antilog}\left(\frac{\Sigma \log x}{N}\right)$$

$$= \text{Antilog}\left(\frac{2.1208}{8}\right)$$

$= \text{Antilog}(0.2651)$

$= 1.841$

$GM = 1.841$ Ans

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Ex 33: Find the geometric Mean for the following data.

Class	100-200	200-300	300-400	400-500	500-600
Frequency	15	18	30	20	17

Sol^n:

Class	frequency	Mid Value	log x	f log x
100-200	15	150	2.1761	32.6415
200-300	18	250	2.3979	43.1622
300-400	30	350	2.5441	76.3230
400-500	20	450	2.6532	53.0640
500-600	17	550	2.7407	46.5919
	100			251.7826

We know $G.M = \text{Antilog}\left(\frac{\Sigma f \log x_i}{\Sigma f}\right)$

$= \text{Antilog}\left(\frac{251.7826}{100}\right) = \text{Antilog}(2.517826)$

$= (329.5) = GM$ Ans.

Ex 35: Find the harmonic mean of 12, 8, 6 and 24.

Sol^n: $x_1 = 12,\ x_2 = 8,\ x_3 = 6,\ x_4 = 24$

We know H.M is given by

$$\frac{1}{H} = \frac{1}{N}\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right)$$

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$$= \frac{1}{4}\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4}\right.$$

$$= \frac{1}{4}(0.833 + 0.1250 + 0.1167 + 0.0417)$$

$$= \frac{1}{4}(0.4167) = 0.104175$$

So $H = \frac{1}{0.104175}$

$H = 9.5992$ Ans.

Ex 36: Find the Harmonic Mean for the following table of Marks obtained in Class test:

Marks obtained	11	12	13	14	15
No. of Students	3	7	8	5	2

Soln →

x	f	$\frac{1}{x}$	$f \cdot \frac{1}{x}$
11	3	0.0909	0.2727
12	7	0.0833	0.5831
13	8	0.0769	0.6152
14	5	0.0714	0.3570
15	2	0.667	0.1334
	25		1.9614

Now H.M is given by

$$\frac{1}{H} = \frac{\Sigma f \cdot \frac{1}{x}}{\Sigma f} = \frac{1.9614}{25}$$

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$= 0.078456$

$$H = \frac{1}{0.078456} = 12.746 \text{ Ans.}$$

Ex.37: A Variate takes Values $a, ar, ar^2 \cdots ar^{n-1}$ each with frequency one. Show that the

$$A.M = \frac{a(1-r^n)}{n(1-r)}$$

Sol^n: Given $x_1 = a$, $x_2 = ar$, $x_3 = ar^2 \cdots x_n = ar^{n-1}$

Now $AM = \frac{x_1 + x_2 + x_3 + \cdots x_n}{n}$

$$A = \frac{a + ar + ar^2 + \cdots ar^{n-1}}{n}$$

$$AM = \frac{a(1-r^n)}{n(1-r)}$$

$= [a \cdot ar \cdot ar^2 \cdots ar^{n-1}]^{1/n}$

$G = a^n \cdot r^{1+2+3+\cdots n-1}]^{1/n}$

Put formula.

$= \frac{n-1}{2}[2a - ((n-1)-1)d]$

$= a^n \cdot r^{\frac{n-1}{2}[2\times 1 + (n-1-1)]}]^{1/n}$

$= [a^n \cdot r^{\frac{n-1 \cdot n}{2}}]^{1/n}$

$G = a \cdot r^{\frac{n-1}{2}}$ Ans.

$$H.M = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} - - - \frac{1}{x_n}}$$

$$= \frac{n}{\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + - - - - \frac{1}{ar^{n-1}}}$$

$$= \frac{n}{\frac{\frac{1}{a}\left[\frac{1}{r}\right]^{n-1}}{\frac{1}{r} - 1}}$$

$$H = \frac{n}{\frac{\frac{1}{a}\left(\frac{1-r^n}{r^n}\right)}{\frac{1-r}{r}}}$$

$$= \frac{n}{\frac{-r^n}{ar^n} \times \frac{r}{1-r}}$$

$$H = \frac{n a r^{n-1}(1-r}{1-r^n}$$ Ans.

Ex 38: Find the average rate of

(a) Motion in the case of a person who rides the first mile at 10 m.p.h, the next mile at 8 m.p.h and the third mile at 6 m.p.h

(b) Increase in population which in the first decade has increase 20% in the next 25% and in the third 44%.

Sol 3: $x_1 = 10$, $x_2 = 8$, $x_3 = 6$ & $n = 3$

So average speed $= \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$

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$$= \frac{3}{0.1 + 0.125 + 0.1667}$$

$$= \frac{3}{0.3917}$$

Average Speed = 7.6589 m.p.h.

(b) $x_1 = 20$, $x_2 = 25$, $x_3 = 44$ & $n = 3$

So G.M $(x_1 \cdot x_2 \cdot x_3)^{1/n}$

$$G.M = (20 \times 25 \times 44)^{1/3}$$

Taking log on both side

$$\log G = (\log 20 \times \log 25 \times \log 44)^{1/3}$$

$$\log G = \frac{1}{3}(1.3010 + 1.3979 + 1.6435)$$

$$\log G = \frac{1}{3} \times 4.3424$$

$$\log G = \frac{4.3424}{3}$$

$$\log G = 1.4475$$

$$G = \log 28.02$$

$$G = 28.02$$ Ans.

Ex39: A Motor When traveling from rest travels the first twentieth of a Mile at 6 Miles per hours and the next three twentieth of the mile at 8, 12, 24 m.p.h respectively. But its average speed over the first one-fifth of a Mile is not 12.5 m.p.h but 9.6 m.p.h. Explain the apparent paradox.

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Let A&H be A.M & H.M of speeds

$$A.M = \frac{6+8+12+24}{4} = \frac{50}{4} = 12.5$$

$$\&M = \frac{4}{\frac{1}{6}+\frac{1}{8}+\frac{1}{12}+\frac{1}{24}}$$

$$= \frac{4 \times 24}{4+3+2+1} = \frac{4 \times 24}{10}$$

$$= 9.6$$

Now average speed $= \frac{\text{Total distance}}{\text{Total Time}}$

$$= \frac{20+20+20+20}{20/6 + 20/8 + 20/12 + 20/24} = \frac{80}{\frac{20(4+3+2+1)}{24}}$$

$$= \frac{\cancel{80}^{4} \times 24}{\cancel{20} \times 10} = \frac{96}{10} = 9.6 \text{ HM}$$

Then the average speed is given by H.M But Now A.M Ans.

Ex 41: If the price of a commodity doubles is period of 4 years from 1997 to 2001. What is the average persentage increase per year?

Sol:→ Let $P = P_0$, $A = 2P_0$, $n = 4$

We know $A = P\left(1 + \frac{r}{100}\right)^n$

$$2P_0 = P_0\left(1 + \frac{r}{100}\right)^4$$

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$\frac{r}{100} + 1 = 2^{\frac{1}{4}}$

$= r = 100(2^{\frac{1}{4}} - 1)$

let $2^{\frac{1}{4}} = x$

$= \log x = \frac{1}{4} \log 2$

$= \frac{1}{4} \times 0.3010$

$\log x = 0.07525$

$x = \text{antilog}(0.07525)$

$x = 1.190$

So from 1 $r = 100(1.190 - 1)$

$r = 19\%$ Ans.

Ex 42: The geometric Mean of a Number is x & that of b numbers it is y, obtain the G.M of those $a+b$ numbers Combined.

Sol:→ Given,

GM of a Number $= x$

$(\text{Product of a Number})^{\frac{1}{a}} = x$

Product of a Number $= x^a$

& GM of b number $= y$

$(\text{Product of } b \text{ number})^{\frac{1}{b}} = y$

Product of b Number $= y^b$

New product of $(a+b)$ number $= x^a \cdot y^b$

So G.M of $(a+b)$ number $= (\text{Product of } (a+b) \text{ number})^{\frac{1}{a+b}}$

$= (x^a \cdot y^b)^{\frac{1}{a+b}}$

$= x^{\frac{a}{a+b}} \cdot y^{\frac{b}{a+b}}$ Ans.

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Ex 43: Find the geometric Mean of the series $1, 2, 4, 8, 16, \ldots 2^n$

Sol^n →

$1, 2, 4, 8, 16 \ldots 2^n$

$2^0, 2^1, 2^2, 2^3, 2^4 \ldots 2^n$

Then Number of term $= n+1$

So $G.M = (\text{Product of Number})^{\frac{1}{n+1}}$

$= (2^0 \cdot 2^1 \cdot 2^2 \cdot 2^3 \cdot 2^4 \ldots 2^n)^{\frac{1}{n+1}}$

$= 2^{0+1+2+3+4 \ldots n)\frac{1}{n+1}}$

$= [2^{\frac{1}{2}n(n+1)}]^{\frac{1}{n+1}}$

$= 2^{\frac{n(n+1)}{2(n+1)}}$

$G.M = 2^{\frac{n}{2}}$ Ans.

Ex 44. The rise in price of a Certain Commodity was 5% in 1994, 8% in 1995 and 77% in 1996. It is said that the average price rise between 1994 was 26% and net was 30%.

Sol^n → Let price in 1993 be $= 100$

So price in 1994 $= 105$

price in 1995 $= 108$

price in 1996 $= 177$

Now average price $= G.M$ of $105, 108, 177$

$G = (105 \times 108 \times 177)^{\frac{1}{3}}$

$\log G = \frac{1}{3}(\log 105 + \log 108 + \log 177)$

$= \frac{1}{3}[2.0212 + 2.0334 + 2.2480$

$= \frac{1}{3}[6.3026]$

$\Rightarrow \log G = 2.1009$

$G = \text{Antilog}(2.1009)$

$= 125.09 = 126$

So Average price rise $= 126 - 100 = 26\%$.

Ans

Ex 46 The following data relate to the height of nine students in a class. Compute the Median height.

S. No	1	2	3	4	5	6	7	8	9
Heights	153	142	151	144	149	146	147	150	154

Sol^n → First Arrange in Assending order.

142, 144, 146, 147, 149, 150, 151, 153, 154,

Here $N = 9$

So, Median $= \left(\frac{N+1}{2}\right)^{th}$ observation

$= \left(\frac{9+1}{2}\right)^{th}$ observation

$= \frac{10}{2} = 5^{th}$ observation

So Median is 149 Ans

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Ex 47: Calculate Median from the following data:

S. No.	1	2	3	4	5	6	7	8	9	10
Marks	17	32	35	33	15	21	41	32	11	18

Determine the Median.

Sol^n → First arrange in assending order.

11, 15, 17, 18, 21, 32, 33, 35, 41

Here $N = 10$

So, Median $= \left(\frac{N+1}{2}\right)^{th}$ observation

$= \left(\frac{10+1}{2}\right)^{th}$ observation

$= \frac{11}{2} = 5.5^{th}$ observation

Median $= \frac{\text{Value of } 5^{th} + \text{Value of } 6^{th}}{2} = \frac{21+32}{2} = \frac{53}{2}$

$= 26.5$ Ans

Ex 48: The following table represent the Marks obtained by a batch of 20 Students in Certain Class tests in Statistics & Accountancy:

Serial No	1	2	3	4	5	6	7	8	9	10
Marks (Statistics)	53	54	52	32	30	60	47	46	35	28
Marks (Accountancy)	58	55	25	32	56	85	44	80	33	72

Serial No	11	12	13	14	15	16	17	18	19	20
Statistics	25	42	33	48	72	51	45	33	65	29
Accountancy	10	42	15	46	50	64	39	38	30	36

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Sol ⇒. Here $N=20$

For Statics →

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{21}{2}\right)^{th} \text{ observation} = 10.5^{th} \text{ observation}$$

$$\text{Median} = \frac{\text{Value of } 10th + \text{Value of } 11}{2} = \frac{45+46}{2} = 45.5$$

For Account

$$\text{Median} = \frac{10th \text{ Value} + 11 \text{ Value}}{2} = \frac{39+42}{2} = 40.5$$

Since Median of Statics > Median of Accountency
= Knowledge is higher in Statics Ans.

Ex 49: Find the Median Size of the following Series:

Size	4	5	6	7	8	9	10
Frequency	6	12	15	28	20	14	5

Sol ⇒

x	f	cf
6	6	6
12	12	18
15	15	33
28	28	61
20	20	81
14	14	95
5	5	100

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Here $N = 100$

Median $= \left(\frac{N+1}{2}\right)^{th}$ observation

$= \left(\frac{100+1}{2}\right)^{th}$ observation $= 50.5^{th}$ observation

$=$ 50.5 lies in 61 frequency

So, Median is 7 Ans.

Ex 50: From the following table find the value of Median:

Measurements	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-
Frequency	7	10	13	26	35	22	11	6

Sol^n →

Measurement	Frequency	Class-Boundries	C.f
11-15	7	10.5 - 15.5	7
16-20	10	15.5 - 20.5	17
21-25	13	20.5 - 25.5	30
26-30	26	25.5 - 30.5	56
31-35	35	30.5 - 35.5	91
36-40	22	35.5 - 40.5	113
41-45	11	40.5 - 45.5	124
46-50	6	45.5 - 50.5	130

Here $N = 130$, So $\frac{N}{2} = \frac{130}{2} = 65$

Which lies is 91, So Median Class is 30.5-35

So here $l_1 = 30.5$, $l_2 = 35.5$, $f = 35$, $c = 56$

Median → $l_1 + \frac{\frac{N}{2} - c}{f} (l_2 - l_1)$

$$= 30.5 + \frac{\frac{130}{2} - 56}{35} (35.5 - 30.5)$$

$$= 30.5 + \frac{65 - 56}{35} (5)$$

$$= 30.5 + \frac{9}{7} = 30.5 + 1.29 = 31.79$$

Median = 31.79 Ans.

Ex 51: Calculate the Median for the following:

Marks Below	10	20	30	40	50	60	70	80
No. of Students	15	35	60	84	96	127	198	250

Sol^n:

Class	f	C.f
0 - 10	15	15
10 - 20	20	35
20 - 30	25	60
30 - 40	24	84
40 - 50	12	96
50 - 60	31	127
60 - 70	71	198
70 - 80	52	250
	N = 250	

Here $N = 250$, So $\frac{N}{2} = 125$

So this lies in 127 so Median Class is 50-60

Here. $l_1 = 50$, $l_2 = 60$, $C = 96$, $f = 31$

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$$\text{Median} = l_1 + \frac{\frac{N}{2} - C}{f}(l_2 - l_1)$$

$$= 50 + \frac{125 - 96}{31}(60 - 50)$$

$$= 50 + \frac{29}{31} \times 10$$

$$= 50 + 0.935 \times 10$$

$$= 50 + 9.35$$

Median = 59.35 Ans.

Ex 52: Following is the distribution of Marks of 50 students in a Class:

Marks (More than)	0	10	20	30	40	50
No. of students	50	46	40	20	10	3

Sol^n →

Class	f	Cf
0-10	4	4
10-20	6	10
20-30	20	30
30-40	10	40
40-50	7	47
50-60	3	50
	N=50	

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Here $N = 50$, so $\frac{N}{2} = 25$

$\therefore$ Median Class $20 - 30$

So $l_1 = 20$, $l_2 = 30$, $f = 20$, $C = 10$

$$\text{Median} = l_1 + \frac{\frac{N}{2} - C}{f}(l_2 - l_1)$$

$$= 20 + \frac{25-10}{20}(30-20)$$

$$= 20 + \frac{15}{2\cancel{0}} \times 1\cancel{0}$$

$$= 20 + 7.5$$

$$= 27.5 \text{ Ans}$$

Ex 55: Determine Mode from the following distribution:

x	10	12	14	16	18	20	22
f	4	6	10	11	21	10	5

Sol^n ⇒

~~Class~~ Here Max frequency is 21, So mode is above distribution is 18.

Ex 56 The following table gives the measurement of Collar size of 230 students in a University. Determine the Modal size of the Collar:

Collar-Size	32	33	34	35	36	37	38	39	40	41
No. of Students	7	14	30	28	35	34	16	14	36	16

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Sol 27

x	f	I	II	III	IV	V
32	7	21				
33	14		44	51		
34	30	58			72	
35	28		63			83
36	35	69		87		
37	34		50		85	
38	16	30				64
39	14		50	66		
40	36	52			66	
41	16					

x	32	33	34	35	36	37	38	39	40	41
f										
I					✓	✓				
II				✓	✓					
III				✓	✓	✓				
IV					✓	✓	✓			
V			✓	✓	✓					
	0	0	1	3	5	3	1	0	0	0

Since 36 Repeated highest Number of time
So

Mode = 36 Ans

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Ex57 Determine the Modal Value in the following Series.

Value	10	12	14	16	18	20	22	24	26	28	30	32
Frequency	7	15	21	38	34	34	11	19	10	38	5	2

Sol^n →

Value	f	I	II	III	IV	V
10	7	22				
12	15		36	43		
14	21	59			74	
16	38		72			93
18	34	68		106		
20	34		45		79	
22	11	30				64
24	19		29	40		
26	10	48			67	
28	38		43			53
30	5	7		45		
32	2					

X	10	12	14	16	18	20	22	24	26	28	30	32
f				✓						✓		
I					✓	✓						
II				✓	✓							
III				✓	✓	✓						
IV					✓	✓	✓					
V			✓		✓							
	0	0	1	3	5	3	1	0	0	0	0	0

Since 18 repeated Max of times So Mode of given distribution is 18 Ans.

Ex 58: Obtain Mode of the following distribution:

Classes	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	12	25	45	11	9

Solⁿ:- Here Max frequency = 45
So Mode Class is 40-50

$l_1 = 40,\ f_1 = 45,\ f_0 = 25,\ f_1 = 11,\ h = 10$

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_2} \times h$$

$$= 40 + \frac{45-25}{2\times 45 - 25 - 11} \times 10$$

$$= 40 + \frac{20}{90-36} \times 10$$

$$= 40 + \frac{200}{54}$$

$$= 40 + 3.7$$

$$= 43.7 \text{ Ans}$$

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Ex59: Locate Mode from the following data:

Mid-Value	30	40	50	60	70	80	90
Frequency	7	12	17	29	31	5	3

Sol^n:

Mid Value	Class	f	I	II	III	IV	V
30	25-35	7	19				
40	35-45	12		29			
50	45-55	17	46		36		
60	55-65	29		60		58	
70	65-75	31	36				77
80	75-85	5		8	65		
90	85-95	3				39	

Class	25-35	35-45	45-55	55-65	65-75	75-85	85-95
f					✓		
I			✓	✓			
II				✓	✓		
III				✓	✓	✓	
IV		✓	✓	✓			
V			✓	✓	✓		
	0	1	3	5	4	1	0

Since 55-65 repeated Max 5 times so Mode Class 55-65

So $l_1 = 55$, $f_1 = 29$, $f_0 = 17$, $f_2 = 31$, $h = 10$

So $\text{Mode} = l_1 + \frac{f_1 - f_2}{2f_1 - f_0 - f_2} \times h$

$= 55 + \frac{29 - 17}{2 \times 29 - 17 - 31} \times 10$

$= 55 + \frac{12 \times 10}{10}$

$= 67$

Since 67 not lies in 55 - 65
So 67 is not Mode.

So again
$\text{Mode} = l_1 + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \times h$

$= 55 + \frac{|29 - 17|}{|f_1 - f_0| + |f_1 - f_2|} \times h$

$= 55 + \frac{12}{|29 - 17| + |29 - 31|} \times 10$

$= 55 + \frac{120}{14}$

$= 55 + 8.57$

$= 63.57$ Ans

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Ex 60: Find the Mode for the following distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	20	22	24	6	2

Sol^n:

Class	f	I	II	III	IV	V
0-10	5	15				
10-20	10		30	35		
20-30	20	42			52	
30-40	22		46			66
40-50	24	30		52		
50-60	6		8		32	
60-70	2					

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
f					✓		
I			✓	✓			
II				✓	✓		
III				✓	✓	✓	
IV		✓	✓	✓			
V			✓	✓	✓		
	0	1	3	5	4	1	0

Since 5 Repeated Max times of Class 30-40
So Mode Class is 30-40

So $l_1 = 30$, $f_1 = 22$, $f_0 = 20$, $f_2 = 24$, $h = 10$

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{22-20}{44-20-24} \times 10$$

$$= 30 + \frac{20}{0}$$

$$= 30 + \infty = \infty$$

Which is Not possible So again :-

$$\text{Mode} = l_1 + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \times h$$

$$= 30 + \frac{|22-20|}{|22-20| + |22-24|} \times 10$$

$$= 30 + \frac{2}{2+2} \times 10$$

$$= 30 + \frac{\cancel{20}\,5}{\cancel{4}}$$

$$= 30 + 5$$

Mode = 35 Ans.

Ex 65: A distribution $x_1, x_2, \ldots, x_n$ Which frequencies $f_1, f_2, \ldots f_n$ is transformed into the distribution $x_1, x_2, \ldots x_n$ With the Same corresponding frequincies of the relation

$$x_i = ax_i + b, \quad i = 1, 2, 3 \ldots n$$

Where a & b are Constants. Show that the means mode and Median of the New distribution are

given in terms of those of the first distribution are given in terms of those of the first distribution by the Same transformation.

Sol^n:- Given $x_i = ax_i + b$

Let $\bar{x}$ be the mean of initial distribution & $\bar{x}$ be the mean of New distribution by Some transformation

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{\Sigma f_i (a x_i + b)}{\Sigma f_i}$$

$$= \frac{a \Sigma f_i x_i}{\Sigma f_i} + \frac{b \Sigma f_i}{\Sigma f_i}$$

$$\boxed{\bar{x} = a\bar{x} + b}$$ Mean

& Let Median of initial distribution is Md & New distribution is Md' then.

$$Md = L_0 + \frac{\frac{N}{2} - C}{f} \times h$$

So $L_0' = aL_0 + b$

So $Md' = L_0' + \frac{\frac{N}{2} - C}{f} \times ah$

$$= aL_0 + b + \frac{\frac{N}{2} - C}{f} \times ah$$

$$= a\left(L_0 + \frac{\frac{N}{2} - C}{f} \cdot h\right) + b$$

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$\boxed{M_d' = a M_d + b}$ Median

Let Mode of initial distribution is M_o & New distribution is M_o' then

$$M_o = L_o + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$x_1 = a x_1 + b$$

$$\text{So } M_o' = (a L_o + b) + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times ah$$

$$a\left(L_o + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h\right) + b$$

$\boxed{M_o' = a M_o + b}$ Mode Ans.

Ex 67: The Monthly incomes of 10 families are given by the following table:

A	B	C	D	E	F	G	H	I	J	Total
85	70	25	75	500	20	45	250	40	36	1136

Compute the airthmetic Mean, geometric Mean & Harmonic Mean of the above incomes. Which of the above three means represents the data Most Suitable? Give reasons.

Sol^n → Here $N = 10$

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Arithmetic Mean

$$A = \frac{\Sigma X_i}{n} = \frac{1136}{10} = \boxed{113.6}$$ Arithmetic Mean

Geometric Mean

$$G = (85 \times 70 \times 15 \times 75 \times 500 \times 20 \times 40 \times 250 \times 40 \times 36)^{1/10}$$

$$\log G = \frac{1}{10}[\log 85 + \log 70 + \log 15 + \log 75 + \log 500 + \log 20 + \log 40 + \log 250 + \log 40 + \log 36]$$

$$= \frac{1}{10}[1.9294 + 1.8451 + 1.1761 + 1.8751 + 2.6980 + 1.3010 + 1.6532 + 2.3979 + 1.6021 + 1.5563]$$

$$= \frac{1}{10} \times 18.0342 = 1.80342 = \log 63.59$$

$$\boxed{\text{So } G = 63.59 \text{ ₹}}$$ Geometric Mean

$$\text{Harmonic Mean} = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} - - - - - - - - - - - \frac{1}{x_{10}}}$$

$$= \frac{10}{\frac{1}{85} + \frac{1}{70} + \frac{1}{15} + \frac{1}{75} + \frac{1}{500} + \frac{1}{20} + \frac{1}{75} + \frac{1}{250} + \frac{1}{40} + \frac{1}{56}}$$

$$= \frac{10}{0.01176 + 0.0143 + 0.0667 + 0.0133 + 0.002 + 0.05 + 0.0222 + 0.004 + 0.025 + 0.0278}$$

$$= \frac{10}{0.23706} = \boxed{42.18}$$ Harmonic Mean

Ans

The Geometric Mean Representation the data Most Suitable. Since out of 10 families, 8 families has their income less than A.M & H.M is represent the lower income Group only Ans.

Ex 68: The Expenditure of 100 families are given by

Expenses in Ruppies	0-10	10-20	20-30	30-40	40-50
No. of families	14	2	27	?	15

The Mode for the distribution is 24. Find the Missing frequencies.

Sol^n →

Class	f	Cf
0-10	14	14
10-20	f_1	$14 + f_1$
20-30	27	$41 + f_1$
30-40	f_2	$41 + f_1 + f_2$
40-50	15	$56 + f_1 + f_2$

$N = 100 = 56 + f_1 + f_2$

$56 + f_1 + f_2 = 100$

$f_1 + f_2 = 100 - 56$

$f_1 + f_2 = 44$ — (1)

& Mode = 24

Mode Class is 20-30

$l_1 = 20$, $f_1 = 27$, f_0, $f_2 = f_2$

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$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$24 = 20 + \frac{27 - f_1}{54 - f_1 - f_2} \times 10$$

$$24 = \frac{27 - f_1}{54 - f_1 - f_2} \times 10$$

$$108 - 2f_1 - 2f_2 = 135 - 5f_1$$
$$5f_1 - 2f_1 - 2f_2 = 135 - 108$$

$$3f_1 - 2f_2 = 27$$
$$f_1 + f_2 = 44 \quad \times 2$$
$$5f_1 = 27 + 88$$

$$5f_1 = 115$$
$$f_1 = 23$$

from ① $23 + f_2 = 44$
$$f_2 = 44 - 23$$
$$f_2 = 21$$

So $f_1 = 23$, & $f_2 = 21$ Ans

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Ex 75: The Mean of the following frequency distribution is 62.8 and the Sum of all frequencies is 50. Compute the Missing frequency f_1 & f_2

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	5	f_1	10	f_2	7	8	50

Sol^n:

Given $\bar{x} = 62.8$

$\Sigma f = 50$

Class	x	f	fx
0-20	10	5	50
20-40	30	f_1	$30f_1$
40-60	50	10	500
60-80	70	f_2	$70f_2$
80-100	90	7	630
100-120	110	8	880
		$30+f_1+f_2$	$2060+30f_1+70f_2$

Since $\Sigma f = 50$

$30 + f_1 + f_2 = 50$

$f_1 + f_2 = 20$ — (1)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$62.8 = \frac{2060 + 30f_1 + 70f_2}{50}$$

$$3140 = 2060 + 30f_1 + 70f_2$$

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$$3f_1 + 7f_2 = 108$$
$$-\ 3f_1 \pm 3f_2 = 60$$

$$4f_2 = 48$$
$$f_2 = 12$$

From (1) :-

$$f_1 + 12 = 20$$
$$f_1 = 8$$

So $f_1 = 8$ & $f_2 = 12$ Ans

Chapter-5 (Measures of dispersion)

Ex 1 Marks of 10 Students in Mathematics & Statistics are given below.

Maths	25	40	30	35	21	45	23	33	10	29
Statics	30	39	23	42	20	40	25	30	18	19

Find the range & Cofficients in both Subjects.

Sol^n: For Maths

$L = 45$, $S = 10$

Range $= 45 - 10 = 35$

Coff of Range $= \frac{L-S}{L+S} = \frac{45-10}{45+10} = \frac{\cancel{35}^{7}}{\cancel{55}_{11}}$

$= \frac{7}{11} = 0.636$

For Static

$L = 42$

$S = 18$

Range $= L - S = 42 - 18 = 24$

Coff of Range $= \frac{L-S}{L+S} = \frac{42-18}{42+18}$

$= \frac{\cancel{24}^{2}}{\cancel{60}_{5}} = \frac{2}{5} = 0.4$ Ans.

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Ex3 Find the range of the following distribution of ~~falies~~ following data:

Mid Value	5	10	15	20	25	30	35
Frequency	7	5	8	12	8	9	8

Sol^n

Mid Value	Class	Frequency
5	2.5 - 7.5	5
10	7.5 - 12.5	7
15	12.5 - 17.5	8
20	17.5 - 22.5	12
25	22.5 - 27.5	8
30	27.5 - 32.5	9
35	32.5 - 37.5	8

$$\text{Range} = L - S$$
$$= 37.5 - 2.5$$
$$= 35$$

$$\text{Coff of Range} = \frac{L-S}{L+S} = \frac{\cancel{35}\ 7}{\cancel{40}\ 8} = \frac{7}{8} = 0.875$$

Ans

Ex4 Find range & Cofficient of range from the following distribution:

Class	6-10	11-15	16-20	21-25	26-30	31-35	36-
Frequency	7	8	15	35	18	7	5

Sol^n →

Inclusive Class	Continuous Series	Frequency
6 - 10	5.5 - 10.5	7
11 - 15	10.5 - 15.5	8
16 - 20	15.5 - 20.5	15
21 - 25	20.5 - 25.5	35
26 - 30	25.5 - 30.5	18
31 - 35	30.5 - 35.5	7
36 - 40	35.5 - 40.5	5

$$\text{Range} = L - S$$
$$= 40.5 - 5.5$$
$$= 35$$

$$\text{Coff of Range} = \frac{L - S}{L + S} = \frac{40.5 - 5.5}{40.5 + 5.5}$$

$$= \frac{35}{46} = 0.76 \text{ Ans.}$$

Ex5 The Smallest Value in a Set of observations in 20 With a range of 25. Find the largest observation and the Coefficient of range

Sol^n Given, $S = 20$
$R = 25$

We Know

$$R = L - S$$
$$25 = L - 20$$
$$L = 25 + 20$$
$$L = 45$$

So largest observation is 45

Now Coff of Range $= \frac{L-S}{L+S}$

$= \frac{45-20}{45+20} \Rightarrow \frac{\cancel{25}\,5}{\cancel{65}\,13} = \frac{5}{13} = 0.37$ Ans

Ex6 From the following information of Wages of 15 Workers, find interquartile range, quartile deviation & Co-efficient of QD.

S. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Wages	520	550	440	580	450	620	470	680	400	490	420	480	440	480	500

Solⁿ Arrange the Given Wages in Assending order

400, 420, 440, 450, 470, 480, 480, 480, 500, 520, 550, 580, 620, 680

Here $N = 15$

So $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$Q_1 = \frac{(15+1)^{th}}{4}$ observation

$Q_1 = \frac{\cancel{16}\,4}{\cancel{4}} = 4^{th}$ observation

$Q_1 = 440$

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Now $Q_3 = \frac{3(N+1)^{th}}{4}$ observation

$Q_3 = \frac{3(16)}{4} = 3(4) = 12^{th}$ observation

$Q_3 = 550$

Interquartile Range $= Q_3 - Q_1$
$= 550 - 440$
$= 110$

Quartile Divation $\rightarrow \frac{Q_3 - Q_1}{2} \Rightarrow \frac{\cancel{110}^{55}}{\cancel{2}}$

Coff. of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{110}{990} = \frac{1}{9}$

0.11 Ans.

Ex 7 Scores are:

CI	f
45-49	2
40-44	3
35-39	2
30-34	6
25-29	8
20-24	8
15-19	5
10-14	7
5-9	9

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Sol 3.

CI	Centrid C.I	f	CF
5-9	4.5-9.5	9	9
10-14	9.5-14.5	7	16
15-19	14.5-19.5	5	21
20-24	19.5-24.5	8	29
25-29	24.5-29.5	8	37
30-34	29.5-34.5	6	43
35-39	34.5-39.5	2	45
40-44	39.5-44.5	3	48
45-49	44.5-49.5	2	50

for Q_1 $\left(\frac{N+1}{4}\right)^{th}$ observation

$$\frac{50+1}{4} = 12.75$$

So Q_1 Class 9.5 - 14.5

$F=7,\ C=9,\ l_1 = 9.5,\ l_2 = 14.5$

$$\text{So } Q_1 = l_1 + \frac{\frac{N}{2} - C}{f} \cdot (l_2 - l_1)$$

$$= 9.5 + \frac{\frac{50}{4} - 9}{7} (5)$$

$$= 9.5 + \frac{12.5 - 9}{7} \times 5$$

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$= 9.5 + \frac{3.5 \times 5}{7}$

$= 9.5 + 2.5 = 12$

for $Q_3 = \frac{3(N+1)}{4} = \frac{3(50+1)}{4}$

$= 38.25^{th}$ observation.

So Q_3, Class in $29.5 - 34.5$

$F = 6$, $C = 37$, $l_1 = 29.5$, $l_2 = 34.5$, $N = 50$

$Q_3 = l_1 + \frac{\frac{3N}{4} - C}{f} (l_2 - l_1)$

$= 29.5 + \frac{\frac{3 \times 50}{4} - 37}{6} \times 5$

$= 29.5 + \frac{5 \times 5}{6}$

$= 29.5 + 0.417$

$Q_3 = 29.917$

So $QD = \frac{Q_3 - Q_1}{2} = \frac{29.917 - 12}{2}$

$= \frac{17.917}{2} = 8.9585$ Ans.

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Ex8 Calculate QD and its Co efficient from the following distribution:

Weekly Income	58	59	60	61	62	63	64	65	66
No. of Workers	2	3	6	15	10	5	4	3	1

Sol^n

x	f	c.f
58	2	2
59	3	5
60	6	11
61	15	26
62	10	36
63	5	41
64	4	45
65	3	48
66	1	49

50

For $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$= \frac{49+1}{4}$ $(12.5)^{th}$ observation

$= 61$

$Q_3 = \frac{3(N+1)^{th}}{4}$ observation

$= \frac{3(49+1)^{th}}{4}$ observation

$= 37.5^{th}$ observation

$= 63$

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$$QD = \frac{Q_3 - Q_1}{2} = \frac{63-61}{2} = \frac{2}{2} = 1 \text{ Ans}$$

$$\text{Coeff of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{63-61}{63+61}$$

$$= \frac{2}{124} = 0.016 \text{ Ans}$$

Ex 9 The following is the age distribution of 799 workers

Age-Group	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of Workers	50	70	100	180	150	120	70	59

Solⁿ

Class	f	C.F
20-25	50	50
25-30	70	120
30-35	100	220
35-40	180	400
40-45	150	550
45-50	120	670
50-55	70	740
55-60	59	799

For $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ observation

$$\frac{799+1}{4} = \frac{800}{4} = 200^{th} \text{ observation}$$

So, Q_1 Class $30-35$

$F = 100$, $C = 120$, $N = 799$, $l_1 = 30$, $l_2 = 35$

So $$Q_1 = l_1 + \frac{\frac{N}{4} - C}{f}(l_2 - l_1)$$

$$= 30 + \frac{\frac{799}{4} - 120}{100} \times 5$$

$$= 30 + \frac{199.75 - 120}{20}$$

$$= 30 + \frac{79.75}{20} \Rightarrow 30 + 3.9875$$

$$= 33.9875$$

& Q_3 $\frac{3(N+1)}{4}^{th}$ observation

$$\frac{3(799+1)}{4}^{th} \text{ observation}$$

600th observation

So Q_3 Class is $45-50$

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$F = 120,\ C = 550,\ l_1 = 45,\ l_2 = 50,\ N = 799$

$$Q_3 = l_1 + \frac{\frac{3N}{4} - C}{f}(l_2 - l_1)$$

$$= 45 + \frac{\frac{3 \times 799}{4} - 550}{120} \times 5$$

$$= 45 + \frac{49.25 \times 5}{120}$$

$$= 45 + 2.05 \Rightarrow 47.05$$

$$\text{So } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{47.05 - 33.9875}{47.05 + 33.9875}$$

$$= \frac{13.0625}{81.6375} = 0.16 \text{ Ans}$$

Example 10 Compute mean deviation and its Coefficient from the following data relating to the marks obtained by a batch of 11 students in a Class test:

Marks	10	70	50	53	20	95	55	42	60	48	80

Sol^n →

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$
10		-43	43
70		17	17
50		-3	3
53		0	0
20	53	-33	33
95		42	42
55		2	2
42		-11	11
60		7	7
48		-5	5
80		27	27
			190

Now $\bar{x} = \frac{\Sigma X}{N} = \frac{10+70+50+53+20+95+55+42+60+48+80}{11}$

$\frac{583}{11} = 53$

So $MD = \frac{\Sigma \lvert x-\bar{x}\rvert}{N} = \frac{190}{11} = 17.27$

& Coff of $MD = \frac{MD}{Mean} = \frac{17.27}{53} = 0.326$ Ans.

Ex 11 Calculate mean deviation from Median for the following data. Also Compute the Cofficent of M.D

Size	2	4	6	8	10	12	14	16
Frequency	2	2	4	5	3	2	1	1

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Solⁿ

X	f	C.F	Md	x−Md	|x−Md|	f|x−Md|
2	2	2		−6	6	12
4	2	4		−4	4	8
6	4	8	8	−2	2	8
8	5	13		0	0	0
10	3	16		2	2	6
12	2	18		4	4	8
14	1	19		6	6	6
16	1	20		8	8	8
						56

For Median $= \left(\frac{N+1}{2}\right)^{th}$ Observation

$\left(\frac{20+1}{2}\right)^{th}$ observation $= 10.5^{th}$ observation

Now $MD = \frac{\Sigma f\,|X - Md|}{\Sigma f} = \frac{56}{20} = 2.8$ Ans.

& Coeff of $M.D = \frac{MD}{Median} = \frac{2.8}{8} = 0.35$ Ans.

Ex 12: Compute the mean deviation (M.D) from the following data.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	50	84	32	10	6

Solⁿ →

Class	f	X	fx	$\bar{x}$	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
0-20	5	10	50		41.07	205.35
20-40	50	30	1500		21.07	1053.5
40-60	54	50	4200	51.07	1.07	89.88
60-80	32	70	2240		18.93	605.76
80-100	10	90	900		38.93	389.3
100-120	6	110	660		58.93	353.58
	187		9550			2697.37

So $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{9550}{187} = 51.07$

Now $MD = \frac{\Sigma f\lvert x-\bar{x}\rvert}{\Sigma f} = \frac{2697.37}{187} = 14.42$

& Coeff. of $MD = \frac{M.D}{Mean} = \frac{14.42}{51.07} = 0.28$ Ans.

Ex 13 Calculate SD from the following set of observation:

X: 10, 11, 17, 25, 7, 13, 21, 10, 12, 14

Sol^n:

We use direct Method in this

X: 10, 11, 17, 25, 7, 13, 21, 10, 12, 14

X	$x-\bar{x}$	$(x-\bar{x})^2$
10	-4	16
11	-3	9
17	3	9
25	11	121
7	-7	49
13	-1	1
21	7	99
10	-4	16
12	-2	4
14	0	0
140		274

Here $\bar{x} = \frac{\Sigma X}{N} = \frac{140}{10} = 14$

$$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}}$$

$$= \sqrt{\frac{274}{10}} = \sqrt{27.4}$$

$$= 5.23$$

So S.D = 5.23 Ans.

Ex 14 Find SD from the data in above example

Solⁿ $SD = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$

$$= \sqrt{\frac{284}{10} - \left(\frac{70}{10}\right)^2}$$

$$= \sqrt{28.4 - 1}$$

$$= \sqrt{27.4} = 5.23$$

SD = 5.23 Ans.

Ex15: Use direct Method to Calculate S.D of the following discrete frequency distribution:

Size (x)	4	5	6	7	8	9	10
Frequency	6	12	15	28	20	14	5

Sol:

X	f	fx	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
4	6	24	-3.06	9.3636	56.1816
5	12	60	-2.06	4.2436	50.9232
6	15	90	-1.06	1.1236	16.8540
7	28	196	0.06	0.0036	0.1008
8	20	160	0.94	0.8836	17.6720
9	14	126	1.94	3.7636	52.6904
10	5	50	2.94	8.6436	43.2180
	100	706			237.6400

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{706}{100} = 7.06$$

$$SD = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}} = \sqrt{\frac{237.64}{100}}$$

SD $\sqrt{2.3764} = 1.54$ Ans.

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Ex 16: Use Short-Cut Method to find the S.D of the data in above example

Sol^n

$$SD = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{238}{100} - \left(\frac{6}{100}\right)^2}$$

$$= \sqrt{2.38 - (0.06)^2}$$

$$= \sqrt{2.38 - 0.0036}$$

$$= \sqrt{2.3764}$$

$$= 1.5416 \text{ Ans.}$$

Ex 17 Use Short-Cut and Step-deviation Method for Calculating S.D of the following distribution.

Age Groups (Years)	25-30	30-35	35-40	40-45	45-50	50-55
No. of Workers	10	12	25	40	10	3

Sol^n:

Class	M.V	f	A	$d' = \frac{x-A}{h}$	fd'	d'^2	fd'^2
25-30	27.5	10		-2	-20	4	40
30-35	32.5	12		-1	-12	1	12
35-40	37.5	25	37.5	0	0	0	0
40-45	42.5	40		1	40	1	40
45-50	47.5	10		2	20	4	40
50-55	52.5	3		3	9	9	27
		100			37		159

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$$SD = h\sqrt{\frac{\Sigma f d'^2}{N} - \left(\frac{\Sigma f d'}{N}\right)^2}$$

$$5\sqrt{\frac{159}{100} - \left(\frac{37}{100}\right)^2}$$

$$= 5\sqrt{1.59 - (0.37)^2}$$
$$= 5\sqrt{1.4531}$$
$$= 5 \times 1.2054$$
$$\Rightarrow SD \Rightarrow 6.027$$

& Variance = 36.3247 Ans

Ex21 The mean of 100 items is 50 and their S.D is 4. Find the Sum and the Sum of square of the items.

Sol^n Given, $N = 100$, $\bar{x} = 50$, $SD = 4$

We know,

$$\bar{x} = \frac{\Sigma X}{N} = \frac{\Sigma X}{100}$$

$$\Sigma X = 50 \times 100$$
$$\Sigma X = 5000$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$$

$$\sigma^2 = \frac{\Sigma X^2}{N} - (\bar{x})^2$$

$$(4)^2 = \frac{\Sigma x^2}{100} - (50)^2$$

$$\frac{\Sigma x^2}{100} = 16 + 2500$$

$$\frac{\Sigma x^2}{100} = 2516$$

$$\Sigma x^2 = 2516 \times 100$$

$$\Sigma x^2 = 251600 \quad \text{Ans.}$$

Ex 22 The Sum of Squares of 20 observations was worked out as 5100. But while calculating it. an observation 31 was wrongly considered as 13. Find the Corrected Sum of Squares.

Solⁿ: Given $N = 20$, $\Sigma x^2 = 5100$

So Correct $\Sigma x^2 = 5100 - (13)^2 + (31)^2$

$= 5100 - 169 + 961$

$= 5892$ Ans.

Ex 23 The Sum of Squares of 100 observations was calculated as 7961. Later, it was found that two values, 53 and 42 were wrongly read as 35 and 24 at the time of calculation. Find the Corrected Sum of Squares.

Solⁿ Given, $N = 100$

$\Sigma x^2 = 7961$

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Wrong item be 35 & 24
Correct item be 53 & 42

So Correct $\Sigma X^2 = 7961 - (35^2 + 24^2) + (53^2 + 42^2)$
$= 7961 - (1225 + 576) + (2809 + 1764)$
$= 7961 - 1801 + 4573$
$= 10733$ Ans.

Ex25 The arithmetic mean and the SD of a Series of 20 items were Calculated as 20 Cm and 5 Cm respectively. But while Calculating them, an item 13 Was measured as 30. Find the Correct arithmetic Mean and Standard deviation.

Solⁿ Given, $N = 20$, $\bar{x} = 20$
$SD = 5$

Correct item = 13
incorrect observation = 30

$$\bar{x} = \frac{\Sigma X}{N}$$

$$20 = \frac{\Sigma X}{20}$$

$\Sigma X = 400$
Correct $\Sigma X = 400 - 30 + 13$
$= 413 - 30 = 383$

So Correct Mean $= \frac{\Sigma X}{N} = \frac{383}{20} = 19.15$

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Correct $\bar{x} = 19 \cdot 15$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - (20)^2}$$

$$25 = \frac{\Sigma X^2}{20} - 400$$

$$\frac{\Sigma X^2}{20} = 400 + 25$$
$$= 425$$

$$\Sigma X^2 = 425 \times 20$$
$$\Sigma X^2 = 8500$$

$$\text{Correct } \Sigma X^2 = 8500 - (30)^2 + (13)^2$$
$$= 8500 - 900 + 169$$
$$= 7769$$

$$\text{So Correct } SD = \sqrt{\frac{\text{Correct } \Sigma X^2}{N} - \left(\frac{\text{Correct } \Sigma X}{N}\right)^2}$$

$$= \sqrt{\frac{7769}{20} - \left(\frac{383}{20}\right)^2}$$

$$= \sqrt{388 \cdot 45 - 366 \cdot 7225}$$
$$= \sqrt{21 \cdot 7275}$$
$$\sigma = 4 \cdot 66 \text{ Ans.}$$

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Ex 27 A Sample of 35 Value has mean 80 & SD. 4. A Second Sample of 65 Values from the Same population has mean 70 and SD. 3. Find the Mean & Standard deviation of the combined Sample of 100 Values.

Sol^n:- $N_1 = 35$, $\bar{x}_1 = 80$, $\sigma_1 = 4$
$N_2 = 65$, $\bar{x}_2 = 70$, $\sigma_2 = 3$

Combined Mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$\bar{x}_{12} = \frac{35 \times 80 + 65 \times 70}{100} = \frac{2800 + 4550}{100}$$

$$\bar{x}_{12} = \frac{7350}{100} = 73.5$$

Now $D_1 = \bar{x}_1 - \bar{x}_{12} = 80 - 73.5 = 6.5$
$D_2 = \bar{x}_2 - \bar{x}_{12} = 70 - 73.5 = -3.5$

So, $D_1^2 = 42.25$, $D_2^2 = 12.25$

Now Combined SD

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{35(16+42.25)+65(9+12.25)}{100}}$$

$$= \sqrt{\frac{35 \times 58.25 + 65 \times 21.25}{100}}$$

$$= \sqrt{\frac{2038.75 + 1381.25}{100}}$$

$$= \sqrt{\frac{3420}{100}} = \sqrt{34.2} = 5.848 \text{ Ans}$$

So $\sigma = 5.848$ Ans.

Ex. 28 Find the mean & Standard deviation of the two groups taken together:

Group	Number	Mean	SD
A	113	160	22
B	120	150	20

Sol^n $N_1 = 113$, $\bar{x}_1 = 160$, $\sigma_1 = 22$
$N_2 = 120$, $\bar{x}_2 = 150$, $\sigma_2 = 20$

Combined Mean :-

$$\bar{x}_{12} = \frac{N\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

$$\bar{x}_{12} = \frac{113 \times 160 + 120 \times 150}{113 + 120}$$

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$$\bar{x}_{12} = \frac{18080 + 18000}{233}$$

$$\bar{x}_{12} = \frac{36080}{233} = 154.85$$

$$D_1 = \bar{x}_1 - \bar{x}_{12} = 160 - 154.85 = 5.515$$
$$D_2 = \bar{x}_2 - \bar{x}_{12} = 150 - 154.85 = -4.85$$

$$D_1^2 = 26.5225 \quad \& \quad D_2^2 = 23.5225$$

Combined SD

$$\sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{113(484 + 26.5225) + 120(400 + 23.5225)}{113 + 120}}$$

$$= \sqrt{\frac{113 \times 510.5225 + 120 \times 423.5225}{233}}$$

$$= \sqrt{\frac{576.8904 + 50822.70}{233}}$$

$$= \sqrt{465.72}$$

$$= \sigma_{12} = 21.58 \text{ Ans}$$

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Ex 30 The mean of the two samples of sizes 50 and 100 respectively is 54.1 and 50.3 and standard deviations are 8 and 7. Obtain the mean and standard deviation of combined sample of size 150.

Sol^n Given $N_1 = 50$, $\sigma_1 = 8$, $\bar{x}_1 = 54.1$
$N_2 = 100$, $\sigma_2 = 7$, $\bar{x}_2 = 50.3$

Combined Mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{50 \times 54.1 + 100 \times 50.3}{50 + 100}$$

$$= \frac{2705 + 5030}{150} \Rightarrow \frac{7735}{150}$$

$$\bar{x}_{12} = 51.57 \quad \text{Ans.}$$

$$D_1 = \bar{x}_1 - \bar{x}_{12} = 54.1 - 51.57 = 2.53$$
$$D_2 = \bar{x}_2 - \bar{x}_{12} = 50.3 - 51.57 = -1.27$$

So $D_1^2 = 6.4009$, $D_2^2 = 1.6129$

So Combined SD

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{50(64+6.4009)+100(49+1.6129)}{150}}$$

$$= \sqrt{\frac{3520.045+5061.29}{150}}$$

$$= \sqrt{57.2089} = 7.564 \text{ Ans}$$

Ex32 A Sample of 5 items was taken from the output of a factory. The length was weight of 5 items are given below:

Length	5	6	7	9	12
Weight	13	15	18	19	20

Sol^n →

Length			Width		
x	$dx=(x-A)$	d^2x	y	$dy=(x-A)$	d^2y
5	−2	4	13	−5	25
6	−1	1	15	−3	9
7	0	0	18	0	0
9	2	4	19	1	1
12	5	25	20	2	4
N=5	4	34	N=5	−5	39

$$\bar{x} = A + \frac{\Sigma dx}{N}$$

$$= 7 + \frac{4}{5}$$

$$= 7 + 0.8$$

$$= 7.8$$

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$$\sigma_x = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma dx}{N}\right)^2}$$

$$= \sqrt{\frac{34}{5} - \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{6.8 - 0.64}$$

$$= \sqrt{6.16}$$

$$\sigma_x = 2.48 \text{ inches}$$

So $CV = \frac{\sigma x}{x} \times 100$

$$= \frac{2.48}{7.8} \times 100$$

$$= \frac{248}{7.8} = 31.79\ \%$$

& $\bar{Y} = A + \frac{\Sigma dy}{N}$

$$\bar{Y} = 18 - \frac{5}{5} = 18 - 1 = 17$$

$$\sigma_y = \sqrt{\frac{\Sigma dy^2}{N} - \left(\frac{\Sigma dy}{N}\right)^2}$$

$$= \sqrt{\frac{39}{5} - \left(\frac{-5}{5}\right)^2}$$

$$= \sqrt{7.8 - 1} = \sqrt{6.8}$$

$$\sigma_y = 2.6$$

So $C.V = \frac{\sigma_y}{y} \times 100$

$= \frac{260 \times 100}{17} = \frac{260}{17} = 15.29$

Since Coeff of Variation of length is Greater than Coeff of Variation of Weight.
So length is more variation then weight Ans.

Ex 33 Below are the Numbers of runs Scored by two batsmen is eight Matches.

Batsman A	27	16	39	45	101	80	40	52
Batsman B	0	100	80	5	60	40	10	121

Sol^n →

X	$d_x = (x-45)$	d^2x	Y	$d_y = (Y-5)$	d^2y
27	-18	324	0	-5	25
16	-29	841	100	95	9025
39	-6	36	80	75	5625
45	0	0	5	10	0
101	56	3136	60	55	3025
80	35	1225	40	35	1225
40	-5	25	10	5	25
52	7	49	121	116	13456
N=8	40	5636	N=8	376	32406

So $\bar{x} = A + \frac{\Sigma d_x}{N}$

$$= 45 + \frac{40}{8}$$

$$\bar{x} = 45 + 5 = 50$$

& $$\bar{y} = A + \frac{\Sigma dy}{N}$$

$$= 5 + \frac{376}{8}$$

$$\bar{y} = 5 + 47 = 52$$

$$\sigma_x = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{5636}{8} - \left(\frac{40}{8}\right)^2}$$

$$= \sqrt{704.5 - 25} = \sqrt{679.5} = 26.06$$

$$CV = \frac{\sigma_x}{x} \times 100$$

$$= \frac{26.06}{50} \times 10$$

$$C.V = \frac{2606}{50} = 52.12$$

$$\sigma_y = \sqrt{\frac{\Sigma dy^2}{N} - \left(\frac{\Sigma dy}{N}\right)^2}$$

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$$= \sqrt{\frac{32406}{8} - \left(\frac{376}{8}\right)^2}$$

$$= \sqrt{4050.75 - 2209}$$

$$= \sqrt{1841.75}$$

$$= 42.92$$

$$\& \; CV = \frac{\sigma_y}{y} \times 100$$

$$= \frac{42.92}{52} \times 100$$

$$CV = 82.53$$

From Mean, Batsman B is Better in Scoring & by comparing C.V, Coff of Variation of Batsman A is less than the Coff of Variation of Batsman B, So Batsman A is more Consistant than B Ans.

Ex 37 In the factories A and B the average Weekly wages and Standard deviation are as follow:

Follow	Average Weekly Wages	S.D. of Wages	No. of Workers
A	460	50	100
B	490	40	80

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Sol^n: Given,

$\sigma_x = 50, \quad \sigma_y = 40$

$N_1 = 100, \quad N_2 = 80$

$\bar{x}_1 = 460, \quad \bar{y} = 490$

(i) Total Weekly Wages paid by factory A

$A = \Sigma X = N_1, \bar{X}_1 = 100 \times 460 = 46,000$

Total Weekly Wages Paid by factory B

$B = \Sigma X = N_2 \bar{X}_2 = 80 \times 490 = 39200$

So factory A Paid larger Amount as Weekly Wages.

(ii) For factory A

$C.V = \frac{\sigma_1}{\bar{x}_1} \times 100 = \frac{50}{460} \times 100 = 10.87$

For Factory B

$C.V = \frac{\sigma_2}{\bar{y}} \times 100 = \frac{40}{490} \times 100 = 8.16$

Since Coff of Variation of factor A is greater than B. So factory A is more Variable than B.

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(ii) Now

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{100 \times 460 + 80 \times 490}{100 + 80}$$

$$= \frac{46000 + 39200}{180} = \frac{85200}{180}$$

$$= 473.33 \text{ Ans.}$$

Now $D_1 = \bar{x}_1 - \bar{x}_{12}$

$= 460 - 473.33$
$= -13.33$

$D_2 = \bar{x}_2 - \bar{x}_{12}$
$= 490 - 473.33$
$= 16.67$

$D_1^2 = 177.69$
$D_1^2 = 277.89$

Now $\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$

$$= \sqrt{\frac{100(2500 + 177.69) + 80(1600 + 277.89}{100 + 80}}$$

$$= \sqrt{\frac{267769 + 80 \times 1877.89}{180}}$$

$$= \sqrt{2322.22} = \sqrt{72} = 48.18 \text{ Ans.}$$

Ex 39 The following distribution shows the life of bulbs manufactured by two companies X and Y:

Use of Bulbs	Company (X)	Company (Y)
100-200	10	10
200-300	15	40
300-400	25	25
400-500	40	15
500-600	10	10

Sol 39.

			Company (X)			Company Y		
Class	x	$d = x - A$	f	fd	fd^2	f	fd	fd
100-200	150	-200	10	-2000	400000	10	-2000	400
200-300	250	-100	15	-1500	150000	40	-4000	400
300-400	350	0	25	0	0	25	0	0
400-500	450	100	40	4000	400000	15	1500	1500 00
500-600	550	200	10	2000	400000	10	2000	400 00

For Company X:-

$$\bar{x}_1 = A + \frac{\Sigma fd}{N}$$

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$$= 350 + \frac{2500}{100} = 375$$

$$\sigma_1 = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{1350000}{100} - \left(\frac{2500}{100}\right)^2}$$

$$= \sqrt{13500 - 625}$$

$$= \sqrt{12875}$$

$$\sigma_1 = 113.47$$

So Coeff of Variation $= \frac{\sigma_1}{\bar{x}_1} \times 100$

$$= \frac{113.47 \times 100}{375}$$

$$= 30.26$$

For Company Y:-

$$\bar{x}_2 = A + \frac{\Sigma fd}{N}$$

$$= 350 - \frac{2500}{100} \rightarrow \bar{x}_2 = 325$$

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& $\sigma_2 = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= \sqrt{\frac{1350000}{100} - \left(\frac{-2500}{100}\right)^2}$

$= \sqrt{13500 - 625} = \sqrt{12875} = 113.47$

So, $C_1 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$= \frac{113.47 \times 100}{325}$

$CV = 34.91.$

Since $\bar{x}_1 = \bar{x}_2$ So bulbs of Company has higher average life.

& C.V of Company X is less than C.V of Company Y So Company X is more Consistant than Y Ans.

Ex 40: From the prices of Shares of X and Y given below, State which Share is more Stable in Value.

X	41	44	43	48	45	46	49	50	42	40
Y	91	93	96	92	90	97	99	94	98	95

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Sol 3)

x	y	$d_x = x - A$	dx^2	$d_y = x - 90$	dy^2
41	91	−4	16	+1	1
44	93	−1	1	3	9
43	96	−2	4	6	36
48	92	3	9	2	4
45	90	0	0	0	0
46	97	1	1	7	49
49	99	4	16	9	81
50	94	5	25	4	16
52	98	−3	9	8	64
40	95	−5	25	5	25
488	945	−2	106	45	285

Now $\bar{x} = A + \frac{\Sigma d}{N}$

$45 + \left(\frac{-2}{10}\right)$

$45 - 0.2 = 44.8$

$\sigma x = \sqrt{\frac{\Sigma dx^2}{N} - \left(\frac{\Sigma dx}{N}\right)^2}$

$= \sqrt{\frac{106}{10} - \left(\frac{-2}{10}\right)^2}$

$= \sqrt{10.6 - 0.04}$

$\sigma x = \sqrt{10.56} = 3.24$

$\& \bar{y} = A + \frac{\Sigma d}{N}$

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$= 90 + \frac{45}{10} \Rightarrow 90 + 4.5 = 94.5$

$\sigma_y = \sqrt{\frac{\Sigma dy^2}{N} - \left(\frac{\Sigma dy}{N}\right)^2}$

$= \sqrt{\frac{285}{10} - \left(\frac{45}{10}\right)^2}$

$= \sqrt{28.5 - 20.25}$

$= \sqrt{8.25} \Rightarrow \sigma_y = 2.87$

Now

CV for $x = \frac{\sigma_x}{\bar{x}} \times 100$

$= \frac{3.25}{44.8} \times 100 \Rightarrow \frac{328}{448} = 7.25$

CV for $y = \frac{\sigma_y}{\bar{y}} \times 100$

$= \frac{2.87}{94.5} \times 100 \Rightarrow \frac{287}{94.5} = 3.03$

Since C.V of Y is less than C.V of X
So Y is more Stable than x Ans.

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Ex 4.1 (i) Coefficient of Variation of two Series are 50 % and 69 %. Their Standard diaviation are 21.2 & 15.6 respectively. What are their arithmetic means?

(ii) Mean of 100 items is 58 and their Standard deviation is 4 find the Sum of Squares of all items.

Sol^n:- Given

$$CV \text{ of } x = 50$$
$$CV \text{ of } y = 69$$
$$\sigma x = 21.2$$
$$\sigma y = 15.6$$

We know $C.V \text{ of } x = \frac{\sigma x}{x} \times 100$

$$\bar{x} = \frac{\sigma x}{CV \text{ of } x} \times 100$$

$$= \frac{21.2}{\cancel{50}} \times \cancel{100}^{2} \Rightarrow \bar{x} = 42.4 \text{ Ans}$$

$$CV \text{ of } y = \frac{\sigma y}{\bar{y}} \times 100$$

$$\bar{y} = \frac{\sigma y}{CV \text{ of } y} \times 100$$

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$$= \frac{15.6 \times 100}{69} \Rightarrow \frac{1560}{69}$$

$$\bar{y} = 22.6$$ Ans.

iii) $N = 100, \ \bar{x} = 58, \ \sigma = 4$

We know $\bar{x} = \frac{\Sigma X}{N}$

$$58 = \frac{\Sigma X}{100} \Rightarrow \Sigma X = 5800$$

& $\sigma = \sqrt{\frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$

$$4 = \sqrt{\frac{\Sigma X^2}{100} - (58)^2}$$

$$16 = \frac{\Sigma X^2}{100} - 3364$$

$$\Sigma X^2 = 100 \times (16 + 3364)$$
$$\Sigma X^2 = 100 \times 3380$$
$$\Sigma X^2 = 338000$$ Ans.

Ex42 Mean and Standard deviation of the following Continuous Series are 31 and 15.94 respectively.

The distribution after step deviation:

d	-3	-2	-1	0	1	2	3
f	10	15	25	25	10	11	5

Sol^n ⇒ $\bar{x} = 31$, $\sigma = 15.94$

d	f	fd	fd^2
-3	10	-30	90
-2	15	-30	60
-1	25	-25	25
0	25	0	0
1	10	10	10
2	11	22	44
3	5	15	45
	101	-38	274

Now $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times i$

$\Rightarrow 15.94 = \sqrt{\frac{274}{101} - \left(\frac{-38}{101}\right)^2} \times i$

$15.94 = \sqrt{2.7129 - 0.1416} \times i$

$15.94 = 1.6035 \times i$

$i = \frac{15.94}{1.6035} \Rightarrow 9.94$

$i = 10$ (approx)

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& $\bar{x} = A + \frac{\Sigma fd}{N} \times i$

$31 = A + \frac{(-38)}{101} \times 10$

$31 = A - \frac{380}{101}$

$A = 31 + 3.76$

$A = 34.76 \Rightarrow 35$ (approx)

$A = 35$

So Class interval be

0-10, 10-20, 20-30, 30-40, 40-50, 50-60, 60-70

Ans.

Ex 43: A Collar manufacturer is considering the production of new style of collar to attract young man. The following statistics of neck circumference are available based upon measurement of a typical group of college students:

Mid Values in In	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5
Number of Students	4	19	30	63	66	28	18	1	1

Sol^n

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x	f	d	fd	d^2	fd^2
12.5	4	-2	-8	4	16
13.0	19	-1.5	-28.5	3.25	42.75
13.5	30	-1	-30	1	30
14.0	63	-0.5	-31.5	0.25	15.75
14.5	66	0	0	0	0
15.0	29	0.5	14.5	0.25	7.25
15.5	18	1	18	1	18
16.0	1	1.5	1.5	2.25	2.25
16.5	1	2	2	4	4
	231		-62		136

So $\bar{x} = A + \frac{\Sigma fd}{N}$

$= 14.5 + \frac{(-62)}{231}$

$= 14.5 - 2.68$

$\bar{x} = 14.232$

$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= \sqrt{\frac{136}{231} - \left(\frac{-62}{231}\right)^2}$

$= \sqrt{0.5887 - 0.0720}$

$\sigma = 0.7188$

Using the critrion $\bar{x} \pm 3\sigma$

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Smallest size of Collar $= \bar{x} - 3\sigma + \frac{3}{4}$

$= 14.232 - 3(0.7188) + 0.75$
$= 14.232 - 2.1564 + 0.75$
$= 12.8256$

Largest size of Collar $= \bar{x} + 3\sigma - \frac{3}{4}$

$= 14.232 + 3(0.07188) + 0.75$
$= 14.232 - 2.1564 + 0.75$
$= 17.1384$ Ans

Ex 44 Following is the data related to two factories:

No. of Workers	200	250
Average wage per hour	₹15.00	₹12.00
Variance	16	9

(i) Which factory pays larger amount as total wages per hour.
(ii) Which factory is more variable?
(iii) Find Combined S.D

Sol^n: (i) For Factory A

$\bar{x}_A = \frac{\Sigma x_A}{N} \rightarrow 15 = \frac{\Sigma x_A}{200}$

$\Sigma x_A = 3000$

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For factory B

$$\bar{x}_B = \frac{\Sigma X_B}{N} \Rightarrow 12 = \frac{\Sigma X_B}{250}$$

$$\Sigma X_B = 3000$$

So Both factory pay equal amount Ans

Now $\sigma_A^2 = 16$

$\sigma_A = 4$

$$C.V \text{ of } A = \frac{\sigma_A}{x_A} \times 100$$

$$= \frac{4}{15} \times 100 = \frac{400}{15} = 26.67$$

& $\sigma_B^2 = 9$

$\sigma_B = 3$

$$C.V \text{ of } B = \frac{\sigma_B}{X_B} \times 100$$

$$= \frac{3}{12} \times 100 \Rightarrow \frac{300}{12} = 25$$

Since C.V of Factory A is greater than C.V of Factory B

So Factory A is more variable than B

Now $\bar{x}_1 = 15, \quad \bar{x}_2 = 12, \quad \sigma_1 = 4, \quad \sigma_2 = 3$

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{200 \times 15 + 250 \times 12}{200 + 250}$$

$$= \frac{3000 + 3000}{450} = \frac{6000}{450} = 13.3$$

So $D_1 = \bar{x}_1 - \bar{x}_{12} = 15 - 13.3 = 1.7$

$D_2 = \bar{x}_2 - \bar{x}_{12} = 12 - 13.3 = -1.3$

So $D_1^2 = 2.89$ $D_2^2 = 1.69$

So Combined S.D

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + D_1^2) + N_2(\sigma_2^2 + D_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{200(16 + 2.89) + 250(9 + 1.69)}{200 + 250}}$$

$$= \sqrt{\frac{200 \times 18.89 + 250 \times 10.69}{450}}$$

$$= \sqrt{\frac{3778 + 2672.5}{450}}$$

$$= \sqrt{14.3344} \Rightarrow 3.786 \text{ Ans}$$

Ex 45 Calculate Karl Pearson's Coefficient of Skewness from the following data:

Marks Above	0	10	20	30	40	50	60	70	80
No. of Students	150	140	100	80	80	70	30	14	0

Sol^n:

Class	f	x	d	fd	d^2	fd^2	C.F
0-10	10	5	-30	-300	900	9000	10
10-20	40	15	-20	-800	400	16000	50
20-30	20	25	-10	-200	100	2000	70
30-40	0	35	0	0	0	0	70
40-50	10	45	10	100	100	1000	80
50-60	40	55	20	800	400	16000	120
60-70	16	65	30	480	900	14400	136
70-80	14	75	40	560	1600	22400	150
	150			640		80800	

$$\bar{x} = A + \frac{\Sigma fd}{N}$$

$$= 35 + \frac{640}{150}$$

$$= 35 + 4.27$$
$$= 39.27$$

$$SD = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{80800}{150} - \left(\frac{640}{150}\right)^2}$$

$= \sqrt{538.67 - 18.20}$

$\sigma = 22.81$

For Median

Median Class $= \frac{150}{2} = 75^{th}$ observation

So Median Class $= 40 - 50$

$f = 10,\ C = 70,\ l_1 = 40,\ l_2 = 50$

So $Md = l_1 + \frac{\frac{N}{2} - C}{f}(l_2 - l_1)$

$= 40 + \frac{75 - 70}{10} \times 10$

$= 40 + \frac{5\cancel{0}}{1\cancel{0}}$

$= 40 + 5$

$Md = 45$

So Carl person's Coeff of (Skewness) =

$\frac{3(\bar{x} - Md)}{\sigma}$

$S_k = \frac{3(39.27 - 45)}{22.81}$

$= \frac{3 \times (-5.73)}{22.81}$

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$S_K = -0.75$ Ans.