1.

(a) $\hat{\beta}_1$ means that for every unit increase in x_1 (students expected score in exam), students actual score increases by 0.469.

(b) $\beta_2 = 3.369$ $\quad \alpha = 1 - 95\% = 1 - 0.95 = 0.05 \quad \therefore \alpha/2 = 0.025$

$SE(\beta_2) = 0.456 \quad n = 64, \; p = 3 \quad \therefore t_{n-p-1,\alpha/2} = t_{64-3-1,0.025} = t_{60,0.025}$

$= 2$

$$C.I. = [\beta_2 \pm t_{n-p-1,\alpha/2} \cdot SE(\beta_2)]$$

$$= [3.369 - 2 \cdot 0.456 \, , \; 3.369 + 2 \cdot 0.456] = [3.369 - 0.912 \, , \; 3.369 + 0.912]$$

$$= [2.457, 4.281]$$

(c) $H_0: \beta_3 = 0 \quad H_a: \beta_3 \neq 0$

$$t = \frac{\beta_3 - 0}{SE(\beta_3)} = \frac{3.054}{1.457} = 2.096$$

$n = 64, \; p = 3, \; \alpha = 0.05 \quad \therefore t_{n-p-1,\alpha/2} = t_{60,0.025} = 2$

Here $t > t_{n-p-1,\alpha/2}$ $\therefore$ We can reject Null Hypothesis.

(d) $F = \dfrac{TSS - RSS}{RSS} \cdot \dfrac{(n-p-1)}{p} = \left(\dfrac{TSS}{RSS} - 1\right) \cdot \dfrac{(n-p-1)}{p}$ $\quad \left(\dfrac{1}{1-0.686} - 1\right)\left(\dfrac{60}{3}\right)$

$= \dfrac{1}{0.314} - 1 \times 20$

$R^2 = 1 - \dfrac{RSS}{TSS} \rightarrow \dfrac{RSS}{TSS} = 1 - R^2 \quad \therefore \dfrac{TSS}{RSS} = \dfrac{1}{1-R^2}$ using this

$= 43.694$

In order to reject NULL HYPOTHESIS, $F > F_{3,60,0.01}$ should be true.

$F_{3,60,0.01} = 4.1259$. Here $F > F_{3,60,0.01}$ so null hypothesis is rejected.

(e) $\hat{y} = 2.178 + 0.469 \times 80 + 3.369 \times 8 + 3.054 \times 3 = 2.178 + 37.52 + 26.952 + 9.162$

$= 75.812$

2.

Leave Out Index	Y	$\hat{Y}$ (Predicted)
1	+	+
2	−	+
3	−	+
4	*	+
5	−	+
6	+	+
7	+	+
8	*	+
9	+	+

Here every ~~s~~ leave one out sample is assigned to '+' class.

Hence out of 9 samples, 5 ~~&~~ are misclassified

∴ Misclassification error = $\frac{5}{9}$ = $\underline{\underline{55.556\%}}$

3.

(a) For class $k=1$

$$k^* = \arg\max\left[\pi_1 \cdot f_1(x)\right] = \arg\max\left[\pi_1 \cdot \frac{1}{2} e^{-x/2}\right]$$

$$= \arg\max\left[\log\left[\pi_1 \cdot 1/2\, e^{-x/2}\right]\right]$$

$$= \arg\max\left[\log \pi_1 + \log\left[1/2\; e^{-x/2}\right]\right]$$

$$= \arg\max\left[\log \pi_1 + \log[1/2] + \log[e^{-x/2}]\right]$$

$$= \arg\max\left(\log \pi_1 + -x/2\right)$$

$$\therefore\ \delta_1(x) = \log \pi_1 - x/2$$

For class $k=2$

$$k^* = \arg\max\left[\pi_2 \cdot f_2(x)\right] = \arg\max\left[\pi_2 \cdot \frac{1}{4} x \cdot e^{-x/2}\right]$$

$$= \arg\max\left[\log\left[\pi_2 \cdot \frac{x}{4} e^{-x/2}\right]\right]$$

$$= \arg\max\left[\log \pi_2 + \log\left[x/4\; e^{-x/2}\right]\right]$$

$$= \arg\max\left[\log \pi_2 + \log(x/4) - x/2\right]$$

$$= \arg\max\left[\log \pi_2 + \log(x) - \cancel{\log 4} - x/2\right]$$

$$\therefore\ \delta(x) = \log \pi_2 + \log(x) - x/2$$

(b) For a point x on decision boundary, $f_1(x) = f_2(x)$

$$\therefore\ \frac{1}{2} \cdot \cancel{e^{-x/2}} = \frac{1}{4} \cdot x \cdot \cancel{e^{-x/2}}$$

$$\frac{x}{4} = \frac{1}{2} \qquad 2x = 4 \qquad \underline{x = 2} \text{ is the decision boundary.}$$

All points $x > 2$ will be assigned to class 2 & $x < 2$ will be assigned to class 1.

$$f_1(3) = \frac{1}{2} e^{-3/2} = \frac{1}{2 \cdot e^{1.5}} = 0.111 \qquad f_2(3) = \frac{1}{4} \cdot 3 \cdot e^{-3/2} = 0.167$$

$$P(Y=1 \mid X=3) \propto \pi_1 f_1(x) = 0.5 \times 0.111 = 0.0555$$

$$P(Y=2 \mid X=3) \propto \pi_2 f_2(x) = 0.5 \times 0.167 = 0.0835$$

$\therefore$ $x=3$ will be assigned to class $k=2$ as $0.0835 > 0.0555$

4.

(a) $\alpha = 0.05$ $\therefore \alpha/2 = 0.025$

For $\beta_{11} \rightarrow Z = \frac{-2}{S1}$ For β_{11} to be significant, $Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$

$\therefore Z_{0.025} = 1.960$ $\therefore \frac{+2}{S1} < +1.960$ or $\frac{-2}{S1} > 1.960$

$= \frac{2}{1.960} < S1$

$= 1.02 < S1$

$\therefore \boxed{S1 > 1.02}$

For $\beta_{21} \rightarrow Z = \frac{-1}{S_2}$ $\therefore \frac{-1}{S_2} < -1.960$ $S_2 > \frac{1}{1.960} =$ $\boxed{S_2 > 0.510}$

For $\beta_{31} \rightarrow Z = \frac{1.5}{S_3}$ $\therefore \frac{1.5}{S_2} > 1.960$ $= \frac{1.5}{1.960} > S_3$ $= 0.765 > S_3$ $\therefore \boxed{S_3 < 0.765}$

For $\beta_{22} \rightarrow Z = \frac{-2.5}{S_4}$ $\therefore \frac{+2.5}{S_4} < +1.960$ $\frac{2.5}{1.96} < S_4$ $\therefore \boxed{S_4 > 1.275}$

For $\beta_{33} \rightarrow Z = \frac{2}{S5}$ $\therefore \frac{2}{S5} > 1.96$ $\therefore \frac{2}{1.96} > S5$ $1.02 > S5$ $\therefore \boxed{S_5 < 1.02}$

(b) $x^* = (0, 0, -1) \rightarrow x_1 = 0,\ x_2 = 0,\ x_3 = -1$

For $k = 1$ $\therefore P_k(x^*) = \frac{e^{1+0+0-1.5}}{e^{1+0+0-1.5} + e^{0+0+00} + e^{0+0+0+2}} = \frac{e^{-0.5}}{e^{-0.5} + 1 + e^{-2}} = \frac{0.606}{0.606 + 1 + 0.135}$

$= \frac{0.606}{1.741} = \underline{\underline{0.348}}$

For $k = 2$ $\therefore P_{(k)}(x^*) = \frac{e^0}{1.741} = \frac{1}{1.741} = \underline{\underline{0.574}}$

For $k = 3$, $P_{(k)}(x^*) = \frac{e^{-2}}{1.741} = \frac{0.135}{1.741} = \underline{0.077} \approx [1 - 0.348 - 0.574]$

Hence $x^*(0, 0, -1)$ will be assigned to class $K = 2$

4

(c) Decision boundary b/w class 1,2

Let $\beta_{01} + \beta_{11}X_1 + \beta_{21}X_2 + \beta_{31}X_3 = x_1$

$\beta_{02} + \beta_{12}X_1 + \beta_{22}X_2 + \beta_{32}X_3 = x_2$

$\beta_{03} + \beta_{13}X_1 + \beta_{23}X_2 + \beta_{33}X_3 = x_3$.

For classes 1,2 , $p(x) = \dfrac{e^{x_1}}{e^{x_1} + e^{x_2}}$ $\quad 1 - p(x) = \dfrac{e^{x_2}}{e^{x_1} + e^{x_2}}$

$$\frac{p x}{1-p(x)} = \frac{\dfrac{e^{x_1}}{\cancel{e^{x_1} + e^{x_2}}}}{\dfrac{e^{x_2}}{\cancel{e^{x_1} + e^{x_2}}}} = 0.5 \qquad \log\left[\frac{e^{x_1}}{e^{x_2}}\right] = 0$$

$x_1 - x_2 = 0$

Substituting β's in x_1 & x_2

$= 1 - 2X_1 - 2X_2 + 1.5X_3 - [0 + 0 - 2.5X_2 + 0] = 0$

$= \boxed{1 - 2X_1 + 0.5X_2 + 1.5X_3 = 0}$

For classes 2,3 $\quad p(x) = \dfrac{e^{x_2}}{e^{x_2} + e^{x_3}}$ $\quad 1 - p(x) = \dfrac{e^{x_3}}{e^{x_2} + e^{x_3}}$

$\therefore \log\left[\dfrac{e^{x_2}}{e^{x_3}}\right] = 0$ [USING SAME STEPS AS ABOVE]

$x_2 - x_3 = 0 \quad \therefore \quad 0 + 0 - 2.5X_2 + 0 - [0 + 0 + 0 + 2X_3] = 0$

$\therefore \boxed{-2.5X_2 - 2X_3 = 0}$

4

(c) For class 1,3

Using same steps used before

$$\log\left[\frac{e^{x_1}}{e^{x_3}}\right] = 0$$

$$x_1 - x_3 = 0$$

$$1 - 2x_1 - 2x_2 + 1.5x_3 - [0 + 0 + 0 + 2x_3] = 0$$

$$= \boxed{1 - 2x_1 - 2x_2 - 0.5x_3 = 0.}$$

Decision boundaries were calculated by assuming a binary logistic classifier b/w 2 classes and setting

$$\frac{p(x)}{1-p(x)} = 0.5$$

5.

(a) β_2^2 - Valid Shrinkage penalty as it will shrink the values of X_2 as it does in the ridge shrinkage penalty and will not have effects on other parameters.

(b) $\beta_1^5 + \beta_2^5 + \beta_3^5 + \beta_4^5$ - Valid shrinkage penalty as $\sum_{i=1}^{5} \beta_i^5$ will penalize all the coefficient and even though the features X_1 and X_3 have a linear relationship, the values of β's will be adjusted to incorporate the correlation.

(c) $|\beta_1| + \beta_2^2 + |\beta_3| + \beta_4^6$ → Invalid shrinkage penalty as model will give different feature selection as X_1 & X_2 are highly correlated. L_1 norm cannot incorporate correlation b/w features.

(d) $\beta_1^2 + |\beta_2| + \beta_3^6 + |\beta_4|$ ⇒ Valid shrinkage penalty as it does ridge regression for correlated features (X_1 & X_3) and uses L_1 norm (lasso) for other features.

(e) $\sqrt{|\beta_1|} + \beta_3^2$ → Valid shrinkage penalty as the correlation can be incorporated by β_3 which uses L_2 norm.