

A-3

Average runtime complexity of the non random pivot version of quick sort.

→ Average case running time is much closer to the best case.

→ If suppose the partitioning algorithm produce a 9 to 1 proportional split the recurrence will be

→ $O(n)$ is time complexity for partitioning step

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + O(n)$$

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expand

$$T\left(\frac{9n}{10}\right) = T\left(\frac{9n}{100}\right) + T\left(\frac{81n}{100}\right) + \left(\frac{81n}{1000}\right)$$

$$+ \left(\frac{729n}{1000}\right) + \dots$$

let establish general pattern

$$n \left(\frac{9}{10}\right)^K \approx 1 \quad \left(\because K \approx \log_{10/9} n\right)$$

→ The work done at each level is $O(n)$ and there are approx $\log_{10/9} n$ level

$$T(n) = O(n) \cdot K$$

$$= O(n) \cdot \log_{10/9} n$$

$$\boxed{T(n) = O(n \log n)}$$