Name : Bharya Rampal
Roll no : 102103409

Date:

Parameter Estimation Assignment

1. Normal distribution

Given : Mean $= \theta_1$

Variance $= \theta_2$

$$PDF = f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

joint density for $(X_1, X_2 \dots X_n) =$

$$J(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log

$$\log_e [J(\theta_1, \theta_2)] = \log_e \left[(2\pi\theta_2)^{-n/2} \cdot e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}} \right]$$

$$= -\frac{n}{2} \log_e (2\pi\theta_2)$$

$$- \frac{1}{2\theta_2} \Sigma (x_i - \theta_1)^2$$

Differentiate wrt θ_1

$$\frac{d \log_e J}{d\theta_1} = \frac{1}{\theta_2} \sum_{1}^{n} (x_i - \theta_1) = 0$$

PREMIER

$$\boxed{\theta_1 = \frac{\sum x_i}{n}}$$ (Sample Mean)

Differentiate w.r.t θ_2

$$\frac{d \log_e J}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum (x_i - \theta_1)^2 = 0$$

$$= \frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum (x_i - \theta_1)^2$$

$$\boxed{\theta_2 = \frac{1}{n} \sum (x_i - \theta_1)^2} = \text{Variance}$$

Maximum liklihood estimation of θ_1 is Sample Mean and θ_2 is Sample Variance

Date:

2. Binomial distribution (Given)

$m \to$ no. of trials

$\theta \to (0,1)$

$B(m, \theta)$

PDF of binomial distribution $= {}^{m}C_{x}\, p^{x} (1-p)^{m-x}$

Joint density distribution for 'm' trials,

$$J(\theta; x_1 \dots x_m) = \prod_{i=1}^{m} \left(P(x_i \mid m, p) \right)$$

$$J(\theta) = \prod_{i=1}^{n} \left({}^{n}C_{x} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i} \right)$$

Taking $\log_e$:

$$\ln [J(\theta)] = \sum_{i=1}^{n} \log \left({}^{n}C_{x_i} \right) + \sum_{i=1}^{n} x_i \log \theta$$

$$+ \sum_{i=1}^{n} (m - x_i) \log (1-\theta)$$

PREMIER

Date:

Differentiate wrt θ

$$\frac{d \log J(\theta)}{d(\theta)} = \frac{1}{\theta} \sum_{i=1}^{n} x_i + \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i)(-1) = 0$$

$$\Rightarrow \frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i)$$

$$\Rightarrow (1-\theta) \sum x_i = \theta \sum (m - x_i)$$

$$\Rightarrow \sum x_i = \theta \sum (m)$$

$$\boxed{\theta = \frac{\sum x_i}{m}} \rightarrow \text{(Mean)}$$

Maximum liklihood estimation of θ is Mean.

PREMIER