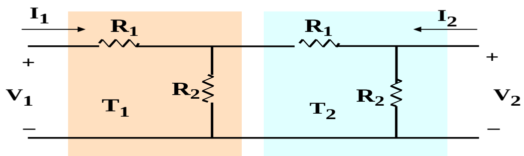
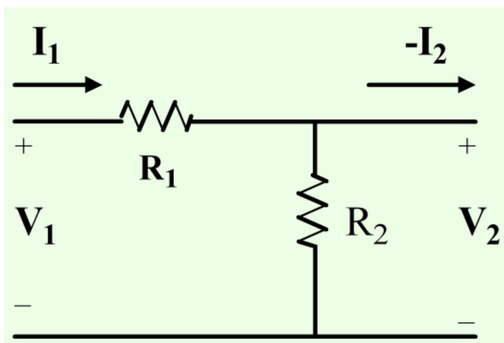
Q1.

Consider the following network:

Find $\frac{V_2}{V_1}$





We can write the following equations.

$$V_1 = (R_1 + R_2)I_1 + R_2I_2$$

$$V_2 = R_2I_1 + R_2I_2$$

It is not always possible to write 2 equations in terms of the V's and I's Of the parameter set.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{R_1 + R_2}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} \frac{R_1 + R_2}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Interconnection Of Two Port Networks

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & R_1 \\ \dfrac{1}{R_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

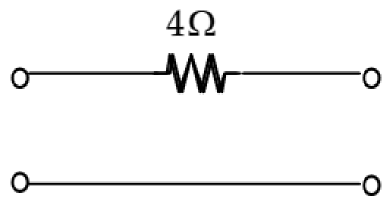
Multiply out the first row:

$$V_1 = \left[\left[\left(\frac{R_1 + R_2}{R_2} \right)^2 + \frac{R_1}{R_2} \right] V_2 + \left[\left(\frac{R_1 + R_2}{R_2} \right) R_1 + R_1 \right] (-I_2) \right]$$

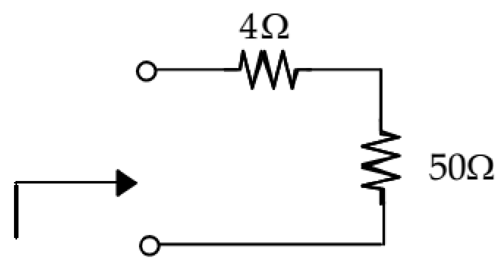
Set $I_2 =$ 0 (as in the diagram)

$$\frac{V_2}{V_1} = \frac{R_2^{\,2}}{R_1^{\,2} + 3R_1R_2 \;\; R_2^{\,2}}$$

Can be verified directly by solving the circuit



Given $Z_0 = 50\Omega$, what is S_{11}?



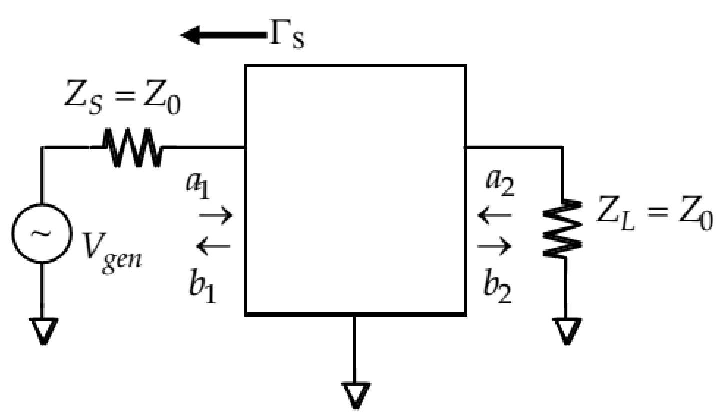
$$Z_{in}\big|_{Z_L=Z_0} = 54\Omega$$

$$S_{11} = \frac{54-50}{54+50} = \frac{4}{104}$$

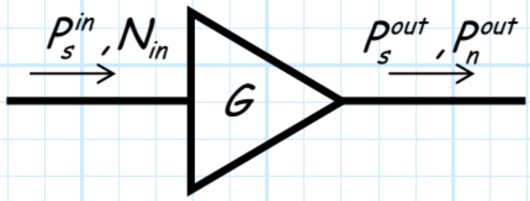
Similar arguments give $S_{22} = \dfrac{4}{104}$.

<u>Find S_{21}</u>

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}$$



Say the **power** of this input signal is P_s^{in}. The output of the amplifier will therefore include **both** a signal with power P_s^{out}, and noise with power P_n^{out}:



where:

$$P_s^{out} = G\, P_s^{in}$$

and:

$$\begin{aligned} P_n^{out} &= N_{in} + G\, k\, T_e\, B \\ &= G\, k\left(T_{in} + T_e\right) B \end{aligned}$$

In order to accurately demodulate the signal, it is important that signal power be **large** in comparison to the noise power. Thus, a fundamental and important measure in radio systems is the **Signal-to-Noise Ratio** (SNR):

$$\boxed{SNR \doteq \frac{P_s}{P_n}}$$

The **larger** the SNR, the **better**!

At the **output** of the amplifier, the SNR is:

$$
\begin{aligned}
SNR_{out} &= \frac{P_s^{out}}{P_n^{out}} \\
&= \frac{G\,P_s^{in}}{G\,k\left(T_{in}+T_e\right)B} \\
&= \frac{P_s^{in}}{k\left(T_{in}+T_e\right)B}
\end{aligned}
$$

Moreover, we can define an **input noise power** as the total noise power across the **bandwidth of the amplifier**:

$$
P_n^{in} = N_{in}\,B = k\,T_{in}\,B
$$

And thus the **input** SNR as:

$$
SNR_{in} = \frac{P_s^{in}}{P_n^{in}} = \frac{P_s^{in}}{k\,T_{in}B}
$$

Now, let's take the **ratio** of the input SNR to the output SNR:

$$
\begin{aligned}
\frac{SNR_{in}}{SNR_{out}} &= \frac{P_s^{in}}{k\,T_{in}B}\left(\frac{k\left(T_{in}+T_e\right)B}{P_s^{in}}\right) \\
&= \frac{T_{in}+T_e}{T_{in}} \\
&= 1+\frac{T_e}{T_{in}}
\end{aligned}
$$

Thus, the **Noise Figure** (F) of a device is defined as:

$$F \doteq \left.\frac{SNR_{in}}{SNR_{out}}\right|_{T_{in}=290K^\circ}$$

$$= \left.\left(1+\frac{T_e}{T_{in}}\right)\right|_{T_{in}=290K^\circ}$$

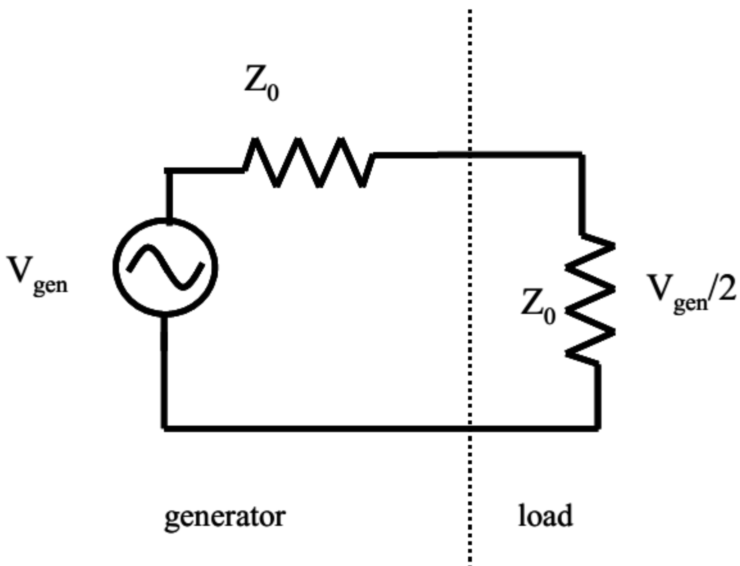
$$= 1+\frac{T_e}{290K^\circ}$$

3a)

P_{AVS} = <u>max power output from a source with impedance Z_s that can be absorbed into a load.</u>

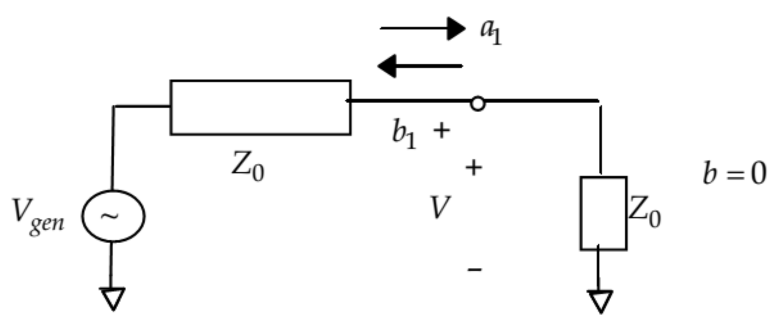
let $Z_S = Z_0$, $Z_L = Z_S^* = Z_0$ (in this case)

because maximum power transfer occurs when we have a conjugate match



$$P_{load} = P_{AVS} = \frac{1}{8}\frac{V_{gen}^2}{Z_0}$$

Or, in terms of a and b:



$$a_1 = \frac{V^+}{\sqrt{Z_0}} \quad and \ b_1 = 0; \qquad V^+ = V_{gen}\left(\frac{Z_0}{Z_0 + Z_0}\right) = \frac{Vgen}{2} \quad and \ \ V^- = 0$$

So,

$$\boxed{P_{load} = P_{AVS} = \frac{1}{2}a_1 a_1^* = \frac{V_{gen}^2}{8Z_0}}$$

Actual Load Power

$$\mathrm{P_{Load}} = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2 = \frac{1}{2}\mathrm{Re}\left[I_1 V_1^*\right]$$

or

$$\boxed{P_{Load} = P_{AVS}(1-|S_{11}|^2)}$$

Reflected Power $\qquad b_1 = a_1 S_{11}$

$$P_R = \frac{1}{2}|b_1|^2 = \frac{1}{2}|a_1|^2|S_{11}|^2 = P_{AVS}|S_{11}|^2$$

$$|S_{11}|^2 = \frac{\text{Power reflected from input}}{\text{Power incident on input}} = \frac{|b_1|^2}{|a_1|^2}$$

$$|S_{22}|^2 = \frac{\text{Power reflected from network output}}{\text{Power incident on output}} = \frac{|b_2|^2}{|a_2|^2}$$

Similarly,

$$\frac{1}{2}|a_2|^2 \quad = \quad \text{Power incident on output}$$

$$= \quad \text{Reflected power from load}$$

$$\frac{1}{2}|b_1|^2 \quad = \quad \text{Power reflected from input port}$$

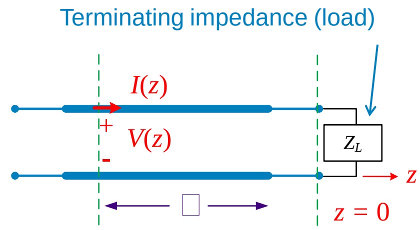
$$\frac{1}{2}|b_2|^2 \quad = \quad \text{Power incident on load from the network}$$

Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Ampl. of voltage wave propagating in positive z direction at $z = 0$.

Ampl. of voltage wave propagating in negative z direction at $z = 0$.



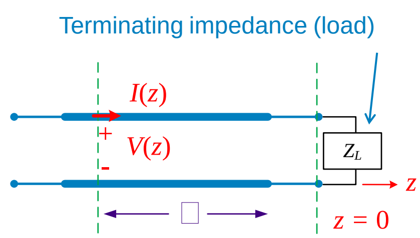
Where do we assign $z = 0$?

The usual choice is at the load.

Note: The length l measures distance from the load: $l = -z$

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What if we know

V^+ and V^- @ $z = -l$

Can we use $z = -l$ as a reference plane?

$$V_0^+ = V^+(0) = V^+(-l)e^{-\gamma l}$$

$$V^-(-l) = V^-(0)e^{-\gamma l}$$

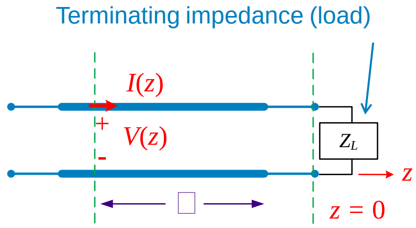
$$\Rightarrow V_0^- = V^-(0) = V^-(-l)e^{\gamma l}$$

Hence

$$V(z) = V^+(-l)e^{-\gamma(z+l)} + V^-(-l)e^{\gamma(z+l)}$$

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What if we know

V^+ and V^- @ $z = -\ell$

Can we use $z = -\ell$ as a reference plane?

$$V_0^+ = V^+(0) = V^+(-\ell)e^{-\gamma \ell}$$

$$V^-(-\ell) = V^-(0)e^{-\gamma \ell}$$

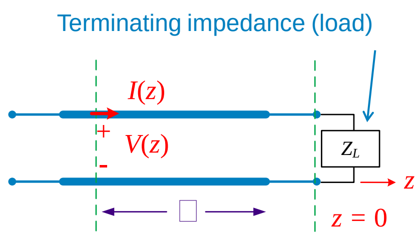
$$\Rightarrow V_0^- = V^-(0) = V^-(-\ell)e^{\gamma \ell}$$

Hence

$$V(z) = V^+(-\ell)e^{-\gamma(z+\ell)} + V^-(-\ell)e^{\gamma(z+\ell)}$$

Terminated Transmission Line (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$



What is $V(-\ell)$?

$$V(-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell}$$

propagating forwards

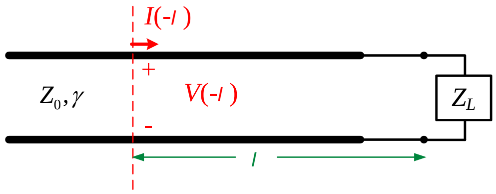
propagating backwards

The current at $z = -\ell$ is then

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{\gamma \ell} - \frac{V_0^-}{Z_0} e^{-\gamma \ell}$$

$\ell \equiv$ distance away from load

Terminated Transmission Line (cont.)



Total volt. at distance l from the load

$$V(-l) = V_0^+ e^{\gamma l} + V_0^- e^{-\gamma l} = V_0^+ e^{\gamma l}\left(1 + \underbrace{\frac{V_0^-}{V_0^+}}_{\Gamma_L} e^{-2\gamma l}\right)$$

Ampl. of volt. wave prop. towards load, at the load position (z = 0).

Ampl. of volt. wave prop. away from load, at the load position (z = 0).

$\Gamma_L \equiv$ Load reflection coefficient

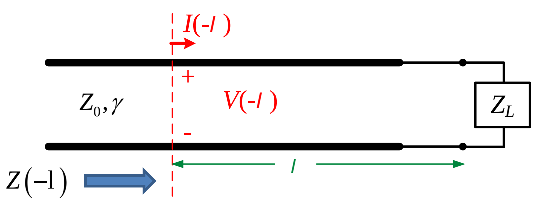
$\Gamma_l \equiv$ Reflection coefficient at z = - l

$$= V_0^+ e^{\gamma l}\left(1 + \Gamma_L e^{-2\gamma l}\right)$$

Similarly,

$$I(-l) = \frac{V_0^+}{Z_0} e^{\gamma l}\left(1 - \Gamma_L e^{-2\gamma l}\right)$$

Terminated Transmission Line (cont.)



$$V(-l) = V_0^+ e^{\gamma l}\left(1 + \Gamma_L e^{-2\gamma l}\right)$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{\gamma l}\left(1 - \Gamma_L e^{-2\gamma l}\right)$$

$$Z(-l) = \frac{V(-l)}{I(-l)} = Z_0\left(\frac{1 + \Gamma_L e^{-2\gamma l}}{1 - \Gamma_L e^{-2\gamma l}}\right)$$

Input impedance seen "looking" towards load at z = -l.

Terminated Transmission Line (cont.)

At the load ($l = 0$):

$$Z(0) = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \equiv Z_L \qquad \boxed{\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Recall $\quad Z(-l) = Z_0\left(\dfrac{1+\Gamma_L e^{-2\gamma l}}{1-\Gamma_L e^{-2\gamma l}}\right)$

Thus,

$$\boxed{Z(-l) = Z_0\left(\frac{1+\left(\dfrac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma l}}{1-\left(\dfrac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma l}}\right)}$$

Terminated Transmission Line (cont.)

Simplifying, we have

$$\begin{aligned} Z(-l) &= Z_0\left(\frac{1+\left(\dfrac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma l}}{1-\left(\dfrac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma l}}\right) = Z_0\left(\frac{(Z_L+Z_0)+(Z_L-Z_0)e^{-2\gamma l}}{(Z_L+Z_0)-(Z_L-Z_0)e^{-2\gamma l}}\right) \\ &= Z_0\left(\frac{(Z_L+Z_0)e^{+\gamma l}+(Z_L-Z_0)e^{-\gamma l}}{(Z_L+Z_0)e^{+\gamma l}-(Z_L-Z_0)e^{-\gamma l}}\right) \\ &= Z_0\left(\frac{Z_L\cosh(\gamma l)+Z_0\sinh(\gamma l)}{Z_0\cosh(\gamma l)+Z_L\sinh(\gamma l)}\right) \end{aligned}$$

Hence, we have

$$\boxed{Z(-l) = Z_0\left(\frac{Z_L + Z_0\tanh(\gamma l)}{Z_0 + Z_L\tanh(\gamma l)}\right)}$$

Terminated Lossless Transmission Line

$$\gamma = \cancel{\alpha} + j\beta = j\beta$$

$$V(-l) = V_0^+ e^{j\beta l}\left(1 + \Gamma_L e^{-2j\beta l}\right)$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\beta l}\left(1 - \Gamma_L e^{-2j\beta l}\right)$$

$$Z(-l) = Z_0\left(\frac{1 + \Gamma_L e^{-2j\beta l}}{1 - \Gamma_L e^{-2j\beta l}}\right)$$

$$\boxed{Z(-l) = Z_0\left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}\right)}$$

Note: $\tanh(\gamma l) = \tanh(j\beta l) = j\tan(\beta l)$

Impedance is periodic with period $\lambda_g/2$

tan repeats when

$$\beta l = \pi$$

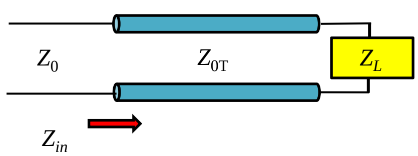
$$\frac{2\pi}{\lambda_g} l = \pi$$

$$\Rightarrow l = \lambda_g / 2$$

Ans to 4b)

Quarter-Wave Transformer

$$Z_{in} = Z_{0T}\left(\frac{Z_L + jZ_{0T}\tan\beta l}{Z_{0T} + jZ_L \tan\beta l}\right)$$



$$\beta l = \beta\frac{\lambda_g}{4} = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Rightarrow Z_{in} = Z_{0T}\left(\frac{jZ_{0T}}{jZ_L}\right)$$

so

$$\boxed{Z_{in} = \frac{Z_{0T}^2}{Z_L}}$$

$$\Gamma_{in} = 0 \quad \Rightarrow \quad Z_{in} = Z_0$$

$$\Rightarrow \quad Z_0 = \frac{Z_{0T}^2}{Z_L}$$

This requires Z_L to be real.

Hence

$$\boxed{Z_{0T} = \left[Z_0 Z_L\right]^{1/2}}$$

Consider the standing wave patterns as in Figure 11.23(a). From this, we observe that

$$\frac{\lambda}{2} = 19 - 11 = 8 \text{ cm} \qquad \text{or} \qquad \lambda = 16 \text{ cm}$$

$$f = \frac{u}{\lambda} = \frac{3 \times 10^8}{16 \times 10^{-2}} = 1.875 \text{ GHz}$$

Electrically speaking, the load can be located at 16 cm or 24 cm. If we assume that the load is at 24 cm, the load is at a distance ℓ from V_{min}, where

$$\ell = 24 - 19 = 5 \text{ cm} = \frac{5}{16}\lambda = 0.3125\,\lambda$$

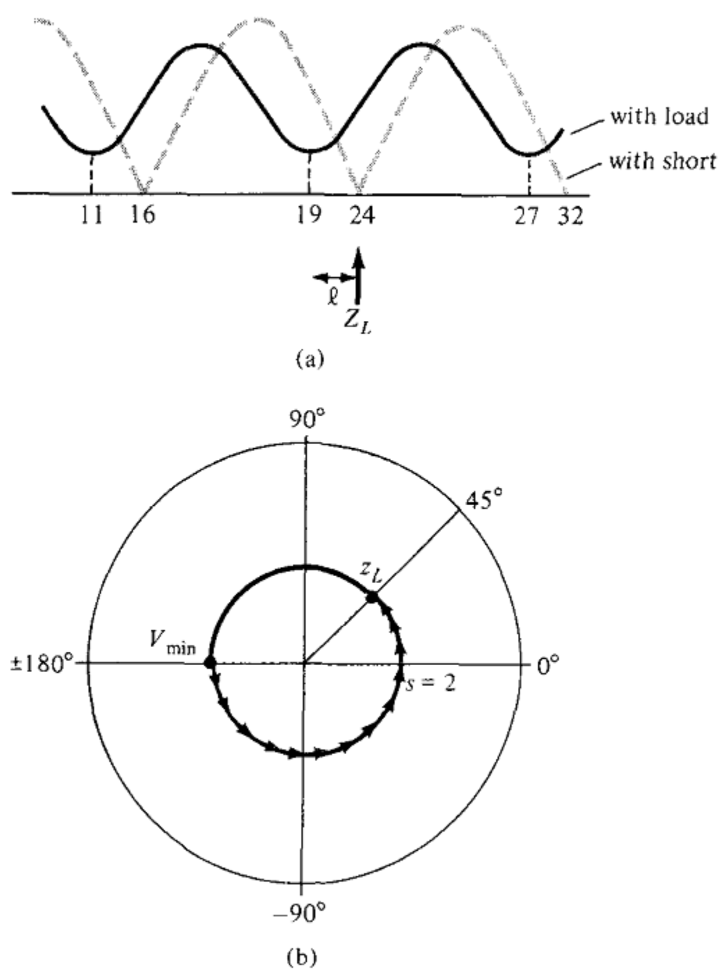


Figure 11.23 Determining Z_L using the slotted line: **(a)** wave pattern, **(b)** Smith chart for Example 11.6.

This corresponds to an angular movement of $0.3125 \times 720° = 225°$ on the $s = 2$ circle. By starting at the location of $V_{\min}$ and moving 225° toward the load (counterclockwise), we reach the location of z_L as illustrated in Figure 11.23(b). Thus

$$z_L = 1.4 + j0.75$$

and

$$Z_L = Z_o z_L = 50\,(1.4 + j0.75) = 70 + j37.5\ \Omega$$

Solution Question 5

Solution:

1. a) compute Γ_{in} and write it in terms of Γ_{short} for a lossy transmission-line

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0.915 e^{-j0.675} = \Gamma_{short} e^{-2\alpha\ell} e^{-j2\beta\ell}$$

knowing that $\Gamma_{short} = -1 = e^{j\pi}$, we can match magnitudes and phases

$$e^{-2\alpha\ell} = 0.915 \quad \Rightarrow \quad \alpha = \frac{1}{2(1.5)} \ln\left(\frac{1}{0.915}\right) = 0.0297 \text{ Np/m}$$

$$-2\beta\ell + \pi = -0.675 \quad \Rightarrow \quad \beta = \frac{0.675 + \pi}{2(1.5)} = 1.27 \text{ rad/m}$$

b) compute Γ_L from which you can calculate Γ_{in}

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.2 + j0.4$$

$$\Gamma_{in} = \Gamma_L e^{-2\alpha\ell} e^{-j2\beta\ell} = \frac{\Gamma_L}{\Gamma_{short}} \Gamma_{in}^{(a)} = (0.2 - j0.4)(0.915 e^{-j0.675}) = -0.0857 - j0.4$$

now convert this reflection coefficient to an impedance

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 62.2 - j59.8\ \Omega$$

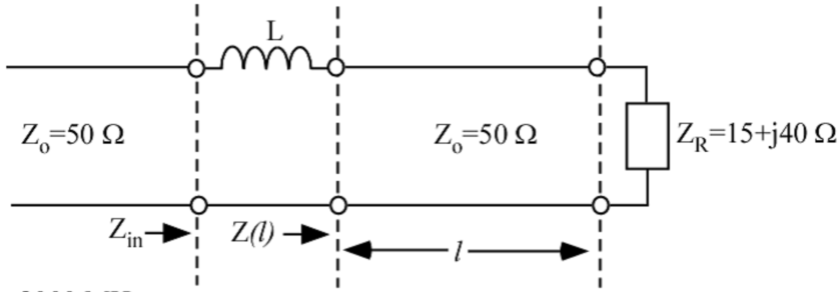
c) we apply the same equation as in part a) but with a new ℓ and using $\lambda = 2\pi/\beta = 4.94$ m

$$\Gamma_{in} = \Gamma_{short} e^{-j2\beta\ell} e^{-2\alpha\ell} = -e^{-j2(2\pi)(0.15)} e^{-2(0.0297)(0.15)(4.94)} = 0.296 + j0.910$$

and convert to

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 6.36 + j137\ \Omega$$

6a)



f_o=2000 MHz
v_o=0.3 m/ns

(a) Using the Smith chart, determine the SWR on the section of line of length l.

$$\lambda = \frac{v_o}{f_o} = \frac{0.3}{2\times 10^9} = 15cm$$

$$15 + j40\Omega \Rightarrow z_R = \frac{15+j40}{50} = 0.3 + j0.8$$

From Smith chart, VSWR=5.7

SWR = 5.7

(b) Using the Smith chart find two values for the length l such that $Z(l)$ is equal to $Z_o \pm jX$.

l_1=0.186λ-0.112λ=0.074λ=0.074×15=1.11 cm

l_2=0.313λ-0.112λ=0.201λ=0.201×15=3.015 cm

$Z(l_1) = Z_o + jX$, l_1 =1.11cm

$Z(l_2) = Z_o - jX$, l_2 =3.015cm

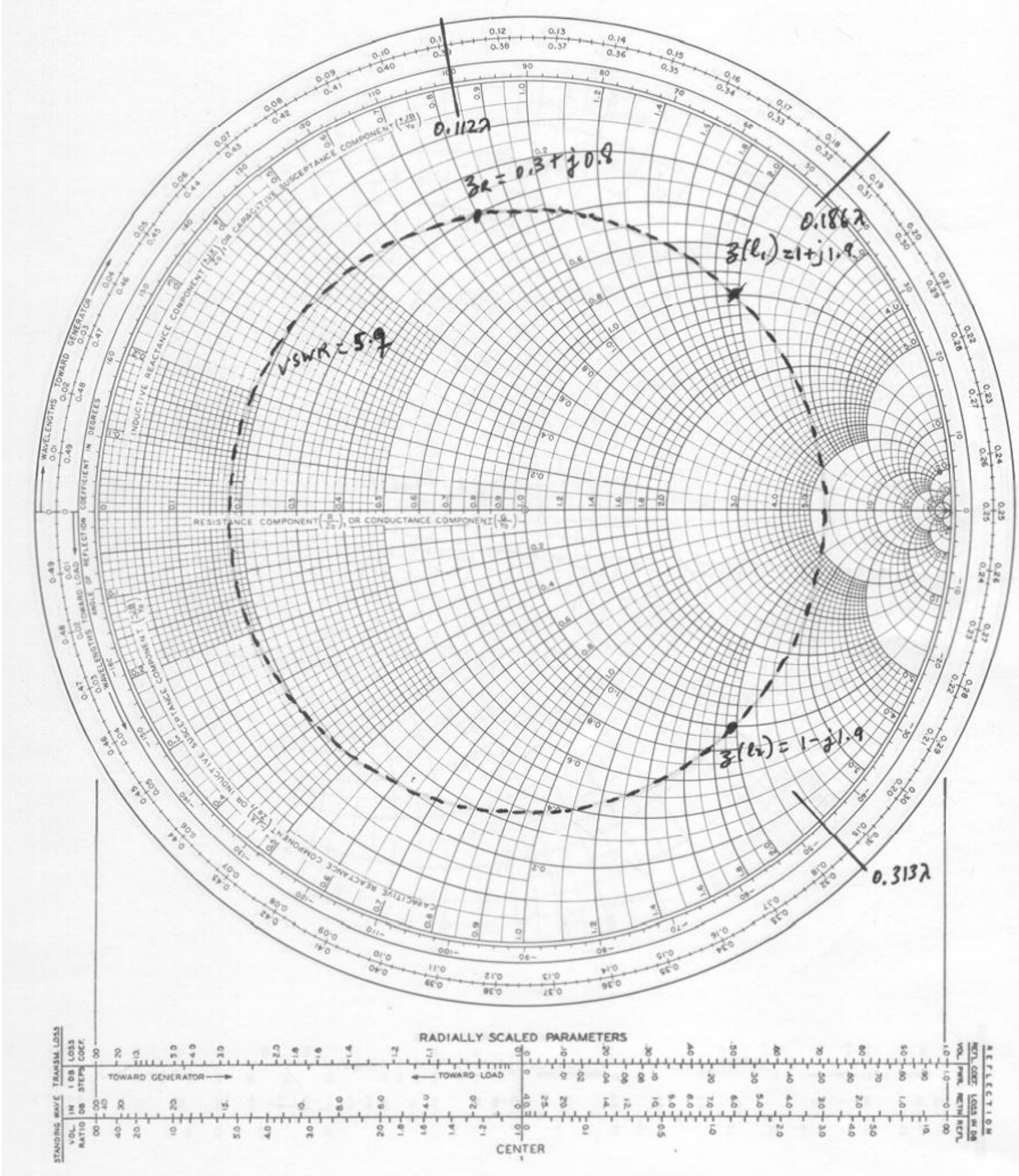
(c) Determine the value of series inductance L and the proper length of the transmission line section (l_1 or l_2) that insures $Z_{in} = Z_o$

Impedance is capacitive at l2 with z=1-j1.9 or Z=50+j95 Ω. Can be compensated with inductor such that

$$L = \frac{95}{2\pi\times 2\times 10^9} = 7.55\ nH$$

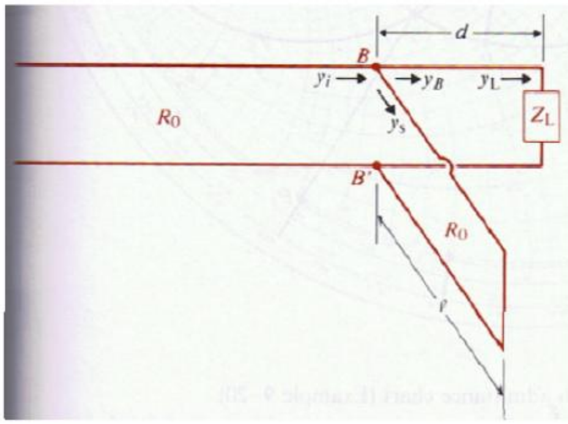
L = 7.55_ (nanoHenries)

IMPEDANCE OR ADMITTANCE COORDINATES



Solution Question 6d

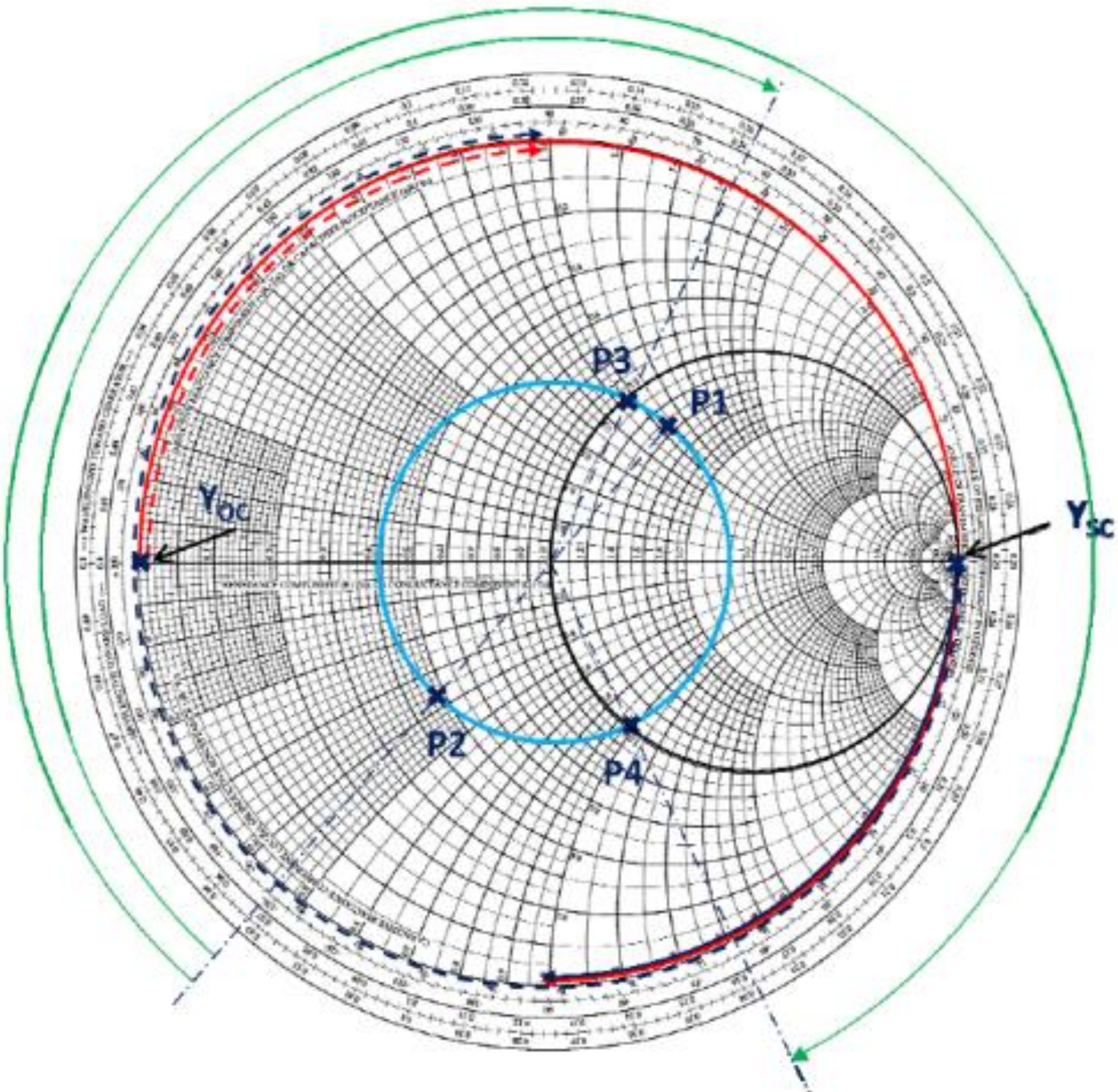
Solution:



$z_L = \frac{Z_L}{Z_0} = \frac{100+j80}{75} = 1.33 + j1.07.$

- Enter z_L (P1) on the Smith chart.
- Draw the SWR circle for the P1 (light blue).
- Transform into admittance y_L (P2).
- Travel towards generator until the intersection of the SWR circle and the $g = 1$ circle.
- Intersections:
  At P3: $y_{B1} = 1 + j0.98 \Rightarrow y_{s1} = -j0.98;\ d_1 = (0.161 - 0.432 + 0.5)\lambda = 0.229\lambda$
  At P4: $y_{B2} = 1 - j0.98 \Rightarrow y_{s2} = +j0.98;\ d_2 = (0.339 - 0.432 + 0.5)\lambda = 0.407\lambda$

b) Using a short-circuited stub:
Match from P3: $l_{1,SC} = (0.377 - 0.25)\lambda = 0.127\lambda$; (dark blue arrow)
Match from P4: $l_{2,OC} = (0.123 - 0.25 + 0.5)\lambda = 0.373\lambda$; (dark blue dashed arrow)


P3
P1
Y_{OC}
Y_{SC}
P2
P4