Optimization Of Urban Emergency Supplies Distribution Paths for Epidemic Outbreaks Report

Bhavya Contractor (200001018)
Priyansh Jaseja (200001063)

December 2022

1 Introduction

In the aftermath of an epidemic breakout, hospitals located across a city need a lot of emergency supplies, such as oxygen, disposable protective gear, and food. The solution to the general Vehicle Routing Problem(VRP) is no longer relevant due to the features of frequent logistical operations and a significant quantity of information, short transportation distance, high timeliness requirements, multiple forms of transportation, and tiny batches. It has become a pressing issue for humanitarian logistics to figure out how to effectively distribute urban emergency supplies during an epidemic outbreak while also ensuring the equitable distribution and prompt delivery of such commodities. In contrast to other VRP variations that attempt to reduce distribution costs, the distribution of emergency supplies under the effect of epidemics must also take the prudence of emergency supplies distribution and the timeliness of transportation into account. The best distribution route for emergency supplies must be chosen based on the demand-urgency of various hospitals for various emergency supplies in order to allocate supplies properly under multi-dimensional restrictions, such as time windows and heterogeneous vehicles.

2 Problem Statement

The problem is to find an optimal solution in which we can efficiently distribute urban emergency supplies in an epidemic outbreak and find the fair distribution and timely delivery of such emergency supplies to the hospitals, also accounting for the urgent need of these supplies, proper utilization of each vehicle and lowering the transportation cost.

3 Problem Formulation

In this problem we have certain emergency supplies which are required by different hospitals. We have a set of vehicles of varying capacity which will transport these supplies to the hospitals.Our task is to find a route for all the vehicles carrying emergency supplies simultaneously, maximizing the objective function and satisfying a given set of constraints.
Objective function comprises of three components:

1. Vehicle Utilization(τ): Each vehicle should be utilized to its maximum capacity. This factor is formulated as the ratio of weight(or volume) of a vehicle used to its maximum loading weight(or volume).
Vehicle Utilization(τ) = $\frac{Weight\ \ (or\ \ Volume)\ \ utilized}{Maximum\ \ loading\ \ Weight(or\ \ Volume)}$

2. Vehicle Transportation Cost $(F_a)-$ Overall vehicle transportation cost should be minimized. This formulated as
Vehicle Transportation Cost $(F_a) = Freight\ \ per\ \ unit\ \ time\ * \ Total\ \ time\ \ taken\ \ by\ \ all\ \ vehicles.$

3. Demand Urgency (γ) - This factor is to maximize the demand urgency value of emergency materials delivered before the latest delivery time required by the hospital, indicating that the higher the demand urgency, the higher the priority of distribution

So, our final objective function becomes

$$max f = \frac{(Vehicle\ \ Utilization) * (Demand\ \ Urgency\ \ factor)}{Vehicle\ \ Transportation\ \ Cost}$$

3.1 Vehicle Utilization (τ)

Vehicle utilization rate mainly considers the two aspects of load and volume utilization rates. The larger of the two is taken as the evaluation standard.

$$\eta_a = (\frac{\sum_{j \in J} W_{ji}^a}{W_{max}}, \frac{\sum_{j \in J} V_{ji}^a}{V_{max}}), a \in A$$

where A is the set of all vehicles and a is one of the vehicles in the set. W_{max} is the maximum loading weight of vehicle and V_{max} is maximum loading volume of vehicle.

$$W_{ji}^a = \sum_{i \in I} W_{ji} * \psi_{ji}^a, \forall a \in A, \forall j \in J$$

$$V_{ji}^a = \sum_{i \in I} V_{ji} * \psi_{ji}^a, \forall a \in A, \forall j \in J$$

$$\psi_{ji}^a \in \{0, 1\}, \forall a \in A, \forall j \in J, i \in I$$

I is the set of different types of emergency supplies and J is the set of all hospitals. ψ_{ji}^a is a 0-1 variable, which indicates whether vehicle a distributes type i emergency supplies required by hospital j. W_{ji}^{a} is the Weight of type i emergency supplies required by hospital j. V_{ji}^{a} is the Volume of type i emergency supplies required by hospital j.

$$\tau = \sum_{a \in A} \eta_a$$

3.2 Vehicle Transportation Cost (F_a)

Vehicle Transportation cost can be formulated as

$$F_a = Q * \frac{L_{0j}}{V} * \sigma_{0j}^{a} + \sum_{j,k \in J} Q * \frac{L_{0j}}{V} * \sigma_{0j}^{a}, \forall a \in A, j \in J$$

$$\sigma_{0j}^{a} \in \{0, 1\}, \forall j \in J, \forall a \in A$$

$$\sigma_{jk}^{a} \in \{0, 1\}, \forall j, k \in J$$

Q is the freight per unit time for transporting emergency materials. L_{0j} is the distance from the emergency material distribution center to each hospital and L_{jk} is the distance from hospital j to hospital i. V is the average speed of each vehicle. σ_{0j}^{a} is a 0–1 variable, which indicates whether vehicle a will directly transport the emergency materials required by hospital j from emergency material distribution center 0 to the hospital j. σ_{jk}^{a} is a 0–1 variable, which indicates whether vehicle a will directly transport the emergency materials required by hospital k from hospital j to the hospital k.

3.3 Demand Urgency factor (γ)

Demand Urgency factor can be formulated as

$$\gamma = \sum_{j \in J} \sum_{i \in I} (\gamma_{ji} * \epsilon_{ji})$$

γ_{ji} is the relative demand urgency of hospital j for material i.

$$\epsilon_{ji} \in \{0, 1\}, \forall j \in J, \forall i \in I$$

ϵ_{ji} is a 0–1 variable, which indicates whether the emergency materials i are delivered before the latest delivery time required by hospital j.

3.4 Objective function and Constraints

The objective function is formulated as

$$max f = \frac{\tau * \gamma}{\sum_{a \in A} F_a}$$

The constraints are

$$\sum_{j \in J} W_j^a \leq W_{max}, \forall a \in A$$

$$\sum_{j \in J} V_j^a \leq V_{max}, \forall a \in A$$

$$\sum_{j \in J} \lambda_j^a \leq 5, \forall a \in A$$

$$\sum_{a \in A} \beta_a \leq N_{max}$$

$$\sum_{j \in J} \psi_{ji}^a = H_{SUM}, \forall i \in I, \forall a \in A$$

$$\lambda_j^a \in \{0,1\}, \forall j \in J. \forall a \in A$$

$$\beta_a \in \{0,1\}, \forall a \in A$$

The inequality in λ indicates that a standard van can traverse up yo five hospitals in one operation. The inequality in β is the maximum number limit of vehicles. The inequality in ψ indicates that various materials required by each hospital must be delivered to each hospital without omission.

4 Solution Scheme

In order to solve the formulated problem, we use Multiverse Optimizer algorithm based on Differential Evolution (DE-IMVO). The algorithm is as follows

Step 1: Parameter initialization

Parameters include the universe dimension D, the number of universes NP , the number of iterations G, the crossover probability CR, the difference coefficient F, the upper limit of the search space x_{max}, the lower limit of the search space x_{min}, the vehicle speed V , and the unit time freight Q.

Step 2: Multi-verse initialization

Initialize the loop according to the total number of orders N_0, and generate random integers between 1 and N_v (Number of vehicles) for each order. After the loop is completed, calculate the order owned by each vehicle and determine all routes for the delivery order. Calculate the delivery path with the least time, and judge whether the path meets the constraints. If it meets the constraints, add it to the multi-verse. If it does not, repeat this step until the multi-verse has NP universes that meet the constraints. Let the i^{th} universe in k^{th} iteration be denoted by X_i^k.

Step 3:Calculate and standardize the expansion degree of the universe.

Calculate the expansion degree of each universe according to the objective function value (fitness function value, fit). In this model, the universe with the largest expansion represents the optimal solution, so the universe that does not meet the constraints is assigned negative infinity. Use the normalization formula to normalize the expansion of the universe:

$$Sfit(X_i^k) = \frac{fit(X_i^k) - min(fit)}{max(fit) - min(fit)}$$

Step 4: Update the global extreme value and save the universe with the largest expansion as X_{best}^k

Step 5: The universe exchanges matter through black holes and white holes

A random number r_i^1 between 0 and 1 is generated for each universe X_i^k. If $r_i^1 < Sfit(X_i^K)$, the universe X_j^k that generates the white hole is selected according to the roulette method. The universe with the black hole sends matter to the universe that created the white hole, and the black hole and white hole dimensions change. The process is denoted as

$$X_i^k = \begin{cases} X_j^k \text{ if } r_i^1 < Sfit(X_i^k) \\ X_i^k \text{ if } r_i^1 \geq Sfit(X_i^k) \end{cases}$$

Step 6: Verse differential

If $r_i^1 < Sfit(X_i^k)$, a random number r_i^2 between 0 and 1 is generated for the universe. If $r_i^2 < CR$, then a random number r_i^3 is generated, and the optimal universe sends matter to the universe through the wormhole, as shown

$$X_i^k = \begin{cases} \lfloor X_{best}^k + F * (X_j^k - X_k^k) \rfloor \text{ if } r_i^2 < CR \;\&\; r_i^3 < 0.5 \\ \lfloor X_{best}^k - F * (X_j^k - X_k^k) \rfloor \text{ if } r_i^2 < CR \;\&\; r_i^3 \geq 0.5 \\ X_i^k \text{ if } r_i^2 > CR \end{cases}$$

Step 7: Random search based on a chaotic tent map

If $r_i^2 > CR$, perform a chaos search on the universe as follows:

a. Generate the random number r_i^4 in the interval $[(x_{max} - x_{min}), 2(x_{max} - x_{min})]$ and calculate the minimum search step l_0, as shown

$$l_0 = \frac{x_{max} - x_{min}}{r_i^4}$$

b. Generate a random number r_i^5 in the interval [0.5, 0.5], where the search strategy for universe chaos is shown

$$X_i^{k'} = \lfloor X_i^k + 0.5 * r_i^5 * l_s * (x_{max} - x_{min}) + x_{min} \rfloor$$

c. Update the search step size, as shown

$$l_{s+1} = \begin{cases} 2 * l_s \text{ if } x_{min} < l_s < \frac{x_{max} - x_{min}}{2} \\ 2 * l_s - 1 \text{ if } \frac{x_{max} - x_{min}}{2} \leq l_s < x_{max} \end{cases}$$

d. Calculate the expansion degree of the universe and perform individual updates, as shown

$$X_i^k = \begin{cases} X_i^{k'} \text{ if } fit(X_i^{k'}) > fit(X_i^k) \\ X_i^k \text{ if } fit(X_i^{k'}) \leq fit(X_i^k) \end{cases}$$

Step 8: If the maximum number of iterations is reached, exit the optimization and output the result, otherwise return to Step 3.

5 Implementation of Algorithm

5.1 Algorithm Parameters

Shijiazhuang is a city in northern China with a population of 10 million. The data used in the simulation is taken between January and February 2021, during the epidemic prevention and control period in Shijiazhuang, to ensure that the city's emergency supplies can be in place when necessary. The parameters of the DE-IMVO algorithm are set as follows:

The universe dimension $D = 33$, the number of universes $NP = 20$, the number of iterations $G = 500$, the crossover probability $CR = 0.2$, the difference coefficient F $= 0.5$, the upper limit of the search space $x_{max} = 15$, the lower limit of the search space $x_{min} = 1$, the vehicle speed $V = 50$, and the unit time freight $Q = 58.5$. Among them, each hospital requires emergency supplies to arrive at 8:30 a.m. every day. The loading time of each vehicle is 6:30 a.m. every day, and the loading time of each delivery vehicle is one hour.

List of hospitals:

Symbols and their representations	
G1	Changan District (The Fourth Hospital of Hebei Medical University and Hebei Cancer Hospital)
G2	Qiaoxi District (The Third hospital of Hebei Medical University)
G3	Xinhua District (The Second Hospital of Hebei Medical University)
G4	Yuhua District (The First hospital of Hebei Medical University)
G5	Jingxing Mining Area (Jingxing Mining District Hospital)
G6	Gaocheng District (Gaocheng People's Hospital)
G7	Luquan District (Luquan People's Hospital)
G8	Luancheng District (Luancheng People's Hospital)
G9	High-Tech Zone (The Fourth Hospital of Shijiazhuang)
G10	Pingshan County (Pingshan Zhongshan Hospital)
G11	Zhengding County (Zhengding County People's Hospital)
G12	Distribution center (Zhonghua N Ave, Xinhua District, Shijiazhuang, Hebei.)

Distance matrix (unit: km).

\	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
G1	0	6.4	4.6	7.8	50	32	20	30	11	47	14	7.4
G2	6.4	0	5.2	10	44	37	19	28	14	46	19	6.5
G3	4.6	5.2	0	11	47	39	17	36	17	44	16	4.5
G4	7.8	10	11	0	49	32	27	18	8.1	55	23	13.9
G5	50	44	47	49	0	86	32	66	57	38	61	49
G6	32	37	39	32	86	0	55	35	23	82	41	36
G7	20	19	17	27	32	55	0	44	36	30	31	17.7
G8	30	28	36	18	66	35	44	0	22	73	37	34
G9	11	14	17	8.1	57	23	36	22	0	55	20	20.8
G10	47	46	44	55	38	82	30	73	55	0	52	38
G11	14	19	16	23	61	41	31	37	20	52	0	13.2
G12	7.4	6.5	4.5	13.9	49	36	17.7	34	20.8	38	13.2	0

Models and specifications of vehicles carried by carriers.

Vehicle model	Symbol	I-11–1	I-11–2	I-11-3
Standard van-4.2 m	$n = 1$	6	15	5
Standard van-5.2 m	$n = 2$	10	17	5
Standard van-6.8 m	$n = 3$	13	30	5

Emergency supplies parameters.

Emergency supplies	Symbol	Ratio of weight to volume
Oxygen (40 liter can)	$i = 1$	1.7 m^3/Ton
Disposable protective clothing	$i = 2$	12.66 m^3/Ton
Food	$i = 3$	3 m^3/Ton

Demand urgency values for emergency supplies needed by hospitals in each jurisdiction.

Hospital	Emergency supplies		
	$i = 1$	$i = 2$	$i = 3$
G1	0.5522	0.4291	0.5249
G2	0.4482	0.5062	0.6896
G3	0.4086	0.4509	0.4768
G4	0.3147	0.3954	0.3882
G5	0.0000	0.0000	0.0000
G6	1.0000	1.0000	1.0000
G7	0.2284	0.1169	0.1066
G8	0.1478	0.2059	0.1856
G9	0.1130	0.1858	0.1456
G10	0.1668	0.3042	0.1823
G11	0.2149	0.2677	0.2824

Gap rate and demand for emergency supplies in each hospital.

I-14-1	I-14-2	I-14-3	I-14-4	I-14-1	I-14-2	I-14-3	I-14-4
G1	$i = 1$	45%	3	G7	$i = 1$	40%	2.4
	$i = 2$	20%	0.8		$i = 2$	20%	0.8
	$i = 3$	15%	1		$i = 3$	5%	0.5
G2	$i = 1$	20%	1.5	G8	$i = 1$	30%	1.3
	$i = 2$	40%	1		$i = 2$	50%	1
	$i = 3$	36%	2		$i = 3$	20%	0.8
G3	$i = 1$	18%	1	G9	$i = 1$	20%	1
	$i = 2$	35%	0.8		$i = 2$	45%	1.2
	$i = 3$	15%	1.3		$i = 3$	15%	1
G4	$i = 1$	25%	2	G10	$i = 1$	15%	0.6
	$i = 2$	50%	1		$i = 2$	55%	1.2
	$i = 3$	20%	1.5		$i = 3$	10%	0.6
G5	$i = 1$	15%	0.8	G11	$i = 1$	20%	1
	$i = 2$	25%	0.5		$i = 2$	40%	1.5
	$i = 3$	10%	1		$i = 3$	18%	1
G6	$i = 1$	80%	5				
	$i = 2$	75%	1.8				
	$i = 3$	30%	1.5				

In above table, I-14-1 denotes Hospital, I-14-2 denotes Types of Emergency Supplies, I-14-3 denotes Gap Degree and I-14-4 denotes Demand Quantity in Tons

5.2 Code

Refer to Github. Readme describes structure of code and how to run code.

5.3 Results

With initial parameters listed above, when DE-IMVO was implemented, then after 64 iterations, the result obtained is

The function value obtained is

$$0.09782475043146364$$

The total Transportation cost is

$$594.7109999999999$$

The value of Vehicle Utilization factor is

$$5.161282051282051$$

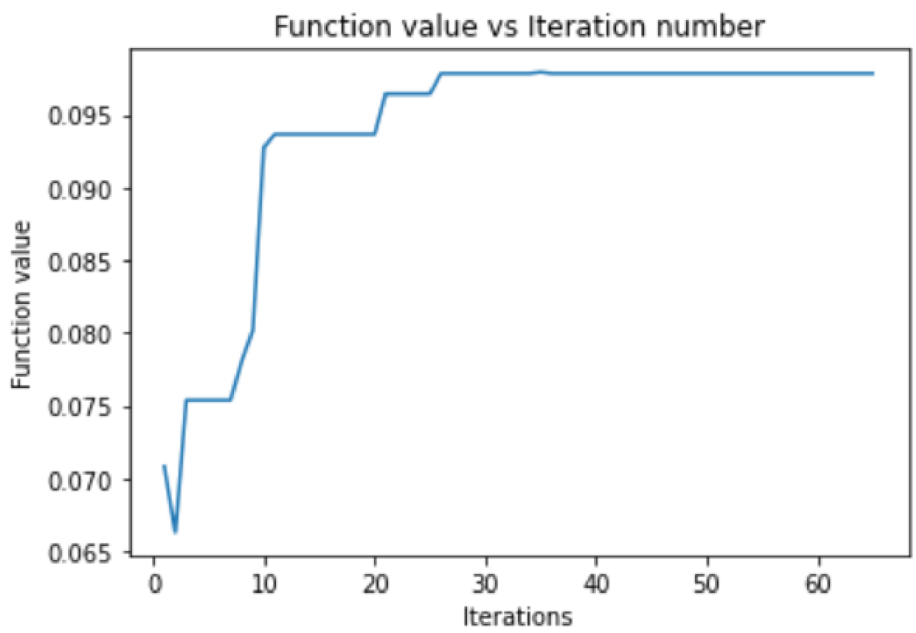
The value of Demand Urgency factor is

11.271900000000002

The route followed by different vehicles is -

Vehicle	Route	Distance
1	G1(i1)→G4(i3)→G8(i1)→G8(i3)	34.5
2	G1(i1)→G1(i3)	7.4
3	G4(i1)→G6(i1)	45.9
4	G7(i1)	17.7
5	G2(i1)	4.5
6	G7(i3)→G5(i1)→G5(i2)→G5(i3)	49.7
7	G9(i1)→G9(i2)→G6(i1)	43.8
8	G10(i2)	38
9	G3(i3)→G11(i3)→G10(i1)	72.5
10	G6(i3)	36
11	G11(i2)→G9(i3)	33.2
12	G7(i2)→G10(i3)	47.7
13	G1(i2)→G4(i2)	15.2
14	G2(i2)→G8(i2)	34.5
15	G2(i3)→G3(i2)→G11(i1)	27.7

Graph of Function Value at different Iteration number -



Note that this is a Genetic Algorithm. You may obtain different solutions every time you run the program. The above result is best among multiple results obtained.