

Science Lab Report

Lab partners (name, student #): <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div> <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div> <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div> <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div>	degree course, semester: <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div> <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div> <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div> <div style="border-bottom: 1px solid black; height: 1.2em; margin-bottom: 10px;"></div>
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Performed the experiment on:
Name of the tutor:
Accepted:

No	Yes
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Submission of the report:	Checked on:
Submission of first revision:	Checked on:
Submission of second revision:	Checked on:

Title:

Kinematics

[illegible]

I) Theory

The purpose of this experiment is to record the position-time curve and the velocity-time curve for the two fundamental types of movement, the movement with constant velocity and the movement with constant acceleration. In addition, a velocity-position plot should be recorded for the two types of movement. The recorded curves are then used to determine the velocity and the acceleration in the two cases. The curves are described by the following formulas:

	constant velocity	constant acceleration
position-time relation	$x(t) = v_0 t + x_0$	$x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0$
velocity-time relation	$v(t) = v_0$	$v(t) = a_0 t + v_0$
velocity-position relation	$v(x) = v_0$	$v(x) = \sqrt{v_0^2 + 2a_0(x - x_0)}$

II) Setup

The experimental setup consists of a track with a cart. The cart is started with a starter system or accelerated with a pulling weight or by inclining the track. Light barriers are positioned at four positions and with a flag attached to the cart one can measure the arrival time of the cart or the blackout time of the light barrier.

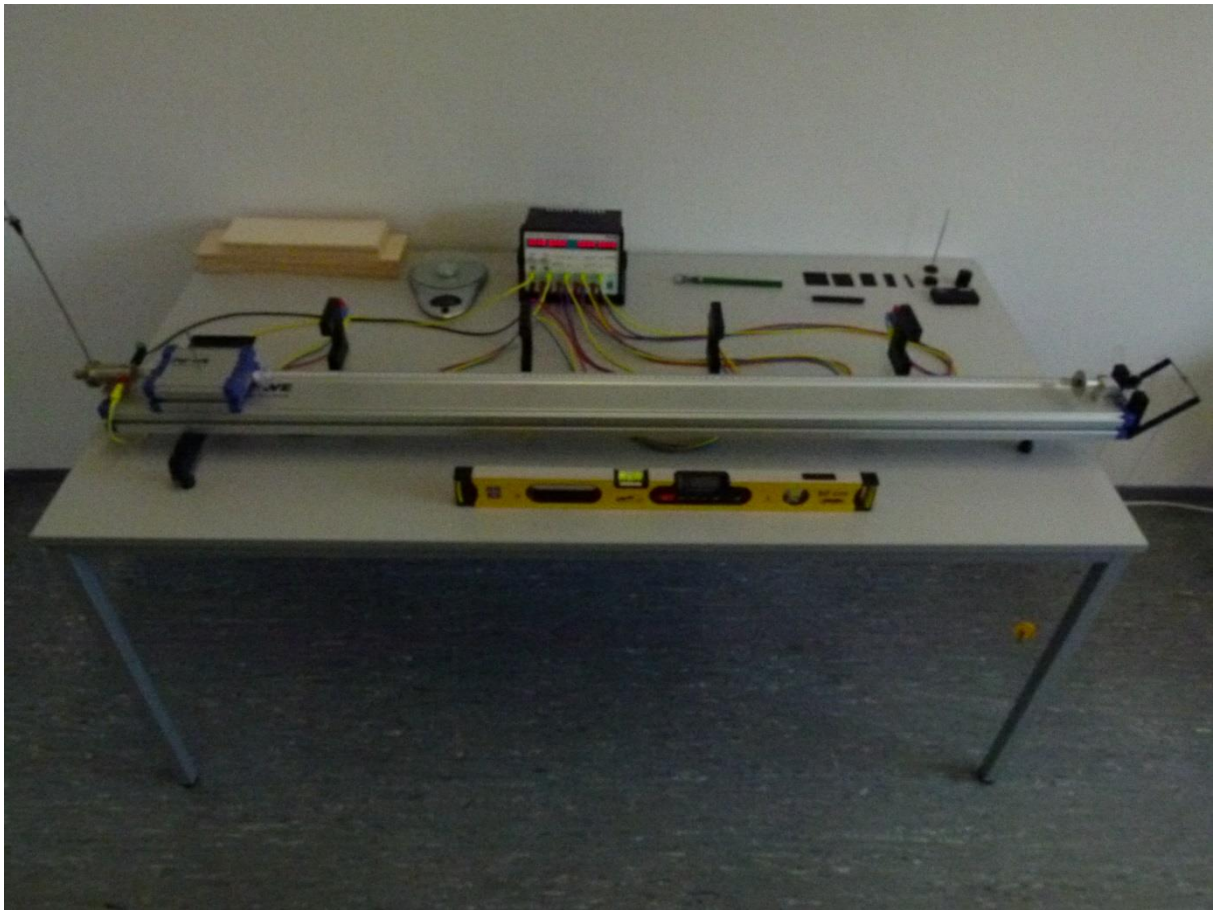


Figure 1: The experimental setup

III) Evaluation

1a) *Position-time plot for constant velocity*

Step 1: The data

The light barriers were fixed at the positions $x_1 = 30$ cm, $x_2 = 60$ cm, $x_3 = 90$ cm and $x_4 = 120$ cm with an uncertainty of $u(x_i) = \pm 0.1$ cm. The following tables contain the $N=3$ measured arrival times and their mean and uncertainty calculated with the formula:

$$\bar{t} \pm u(\bar{t}) = \frac{1}{N} \sum_{i=1}^N t_i \pm t_{(N-1,95\%)} \frac{1}{\sqrt{N}} \sqrt{\frac{1}{N-1} \sum_{i=1}^N (t_i - \bar{t})^2}, \quad (1)$$

using the student-t-factor $t_{(2,95\%)} = 4.303$.

stage 1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.372	1.393	2.371	3.324
2	0.375	1.402	2.388	3.350
3	0.379	1.421	2.422	3.398
mean \bar{t} in s	0.375	1.405	2.393	3.357
$u(\bar{t})$ in s	0.009	0.036	0.065	0.093

stage 3				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.178	0.671	1.152	1.634
2	0.180	0.678	1.163	1.649
3	0.179	0.671	1.151	1.632
mean \bar{t} in s	0.179	0.673	1.155	1.638
$u(\bar{t})$ in s	0.003	0.010	0.017	0.023

Table 1: The arrival times for the case of constant velocity with their mean values and uncertainties.

Step 2: The plots

The position-time plots are obtained by plotting the positions against the corresponding mean values of the arrival times. For the case of constant velocity position and time are related by the linear position-time law

$$x(t) = v_0 t + x_0, \quad (2)$$

i.e., a linear regression is performed for the data points of stage 1 and the data points of stage 3.

The slope of the regression line corresponds then to the velocity v_0 and the intercept of the regression line corresponds to the front edge x_0 of the cart at its start position.

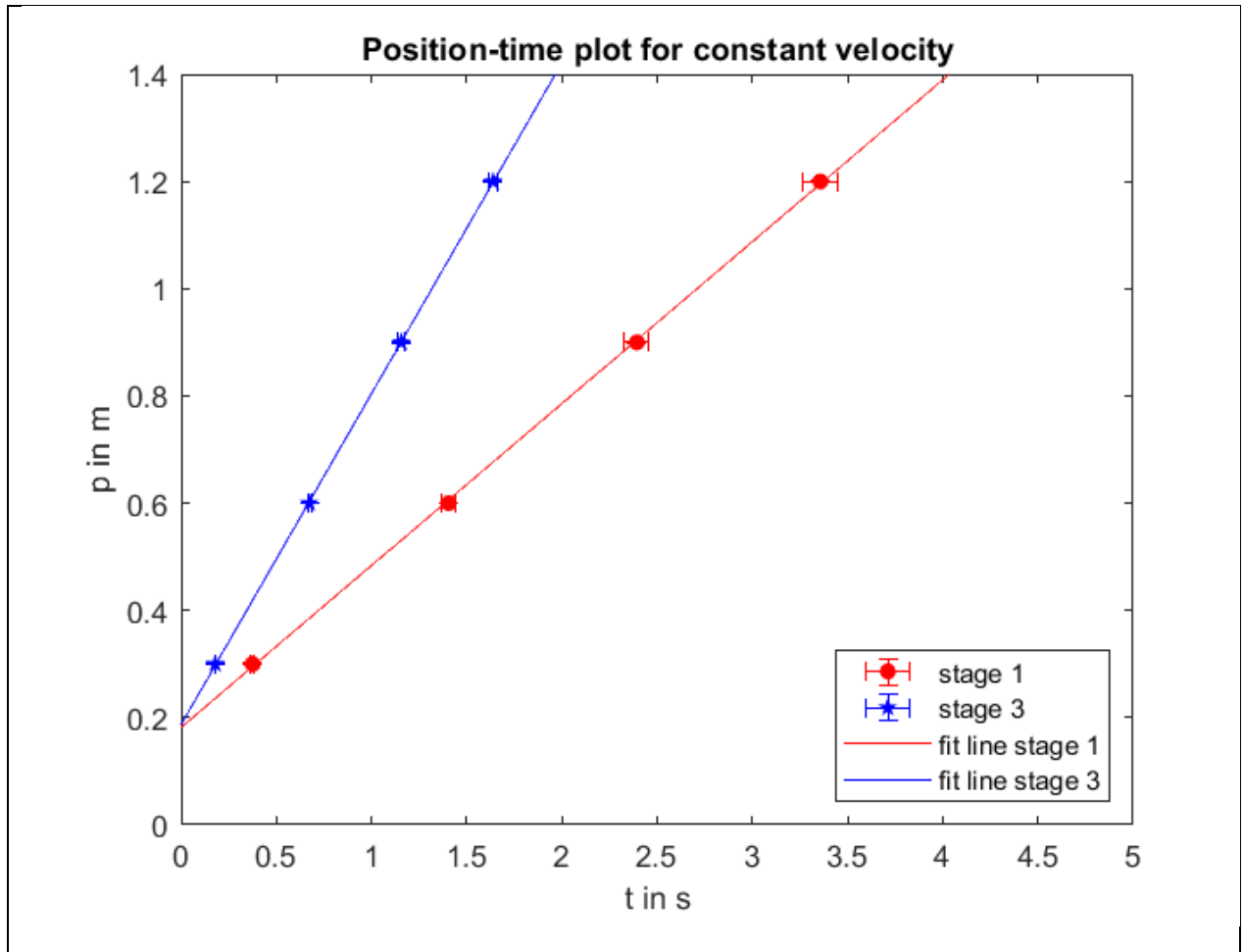


Figure 2: The position-time plots for constant velocity.

The regression lines are given by:

$$\text{stage 1: } x_1(t) = (a_1 \pm sa_1)t + (b_1 \pm sb_1) = (0.3019 \pm 0.016)t + (0.1815 \pm 0.010), \quad (3)$$

$$\text{stage 3: } x_3(t) = (a_3 \pm sa_3)t + (b_3 \pm sb_3) = (0.6173 \pm 0.032)t + (0.1874 \pm 0.010). \quad (4)$$

Step 3: The results and answer to the question

The measurement values for the velocity follow from the slope of the regression lines:

$$\text{velocity in stage 1: } v_0 \pm u(v_0) = (0.3019 \pm 0.016) \text{ m/s}, \quad (5)$$

$$\text{velocity in stage 3: } v_0 \pm u(v_0) = (0.6173 \pm 0.032) \text{ m/s}. \quad (6)$$

The positions of the front edge of the cart follows from the intercept of the regression line:

$$\text{front edge position in stage 1: } x_0 \pm u(x_0) = (0.1815 \pm 0.010) \text{ m}, \quad (7)$$

$$\text{front edge position in stage 3: } x_0 \pm u(x_0) = (0.1874 \pm 0.010) \text{ m}. \quad (8)$$

1b) Velocity-time plot and velocity-position plot for constant velocity

Step 1: The data

The light barriers are now measuring the blackout times t_{bl} . Mean and uncertainty of the blackout times are calculated as in 1a). The velocity at the corresponding light barrier is calculated with the length $l \pm u(l) = (10 \pm 0.1) \text{ cm}$ of the flag:

$$v \pm u(v) = \frac{l}{t_{bl}} \pm \sqrt{\left(\frac{1}{t_{bl}}\right)^2 (u(l))^2 + \left(\frac{-l}{t_{bl}^2}\right)^2 (u(t_{bl}))^2}. \quad (9)$$

stage 1				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1	0.336	0.332	0.317	0.307
2	0.335	0.330	0.315	0.306
3	0.331	0.326	0.312	0.303
mean $\underline{t_{bl}}$ in s	0.334	0.329	0.315	0.305
$u(\underline{t_{bl}})$ in s	0.007	0.008	0.006	0.005
velocity v in m/s	29.940	30.395	31.746	32.787
$u(v)$ in m/s	0.695	0.799	0.683	0.630

stage 3				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1	0.162	0.163	0.160	0.159
2	0.161	0.161	0.159	0.158
3	0.164	0.165	0.162	0.161
mean $\underline{t_{bl}}$ in s	0.162	0.163	0.160	0.159
$u(\underline{t_{bl}})$ in s	0.004	0.005	0.004	0.004
velocity v in m/s	61.728	61.350	62.5	62.893
$u(v)$ in m/s	1.644	1.979	1.683	1.703

Table 2: The blackout times for the case of constant velocity together with the velocities and their uncertainties.

Step 2: The plots

The velocity-time plots are obtained by plotting the velocities of table 2 against the mean arrival times at the corresponding position given in table 1. For the case of constant velocity and time are related by the linear velocity-time law

$$v(t) = v_0, \quad (10)$$

i.e., a linear regression is performed for the data points of stage 1 and the data points of stage 3.

The slope of the regression line should then be close to zero and the intercept of the regression line corresponds to the initial velocity v_0 of the cart.

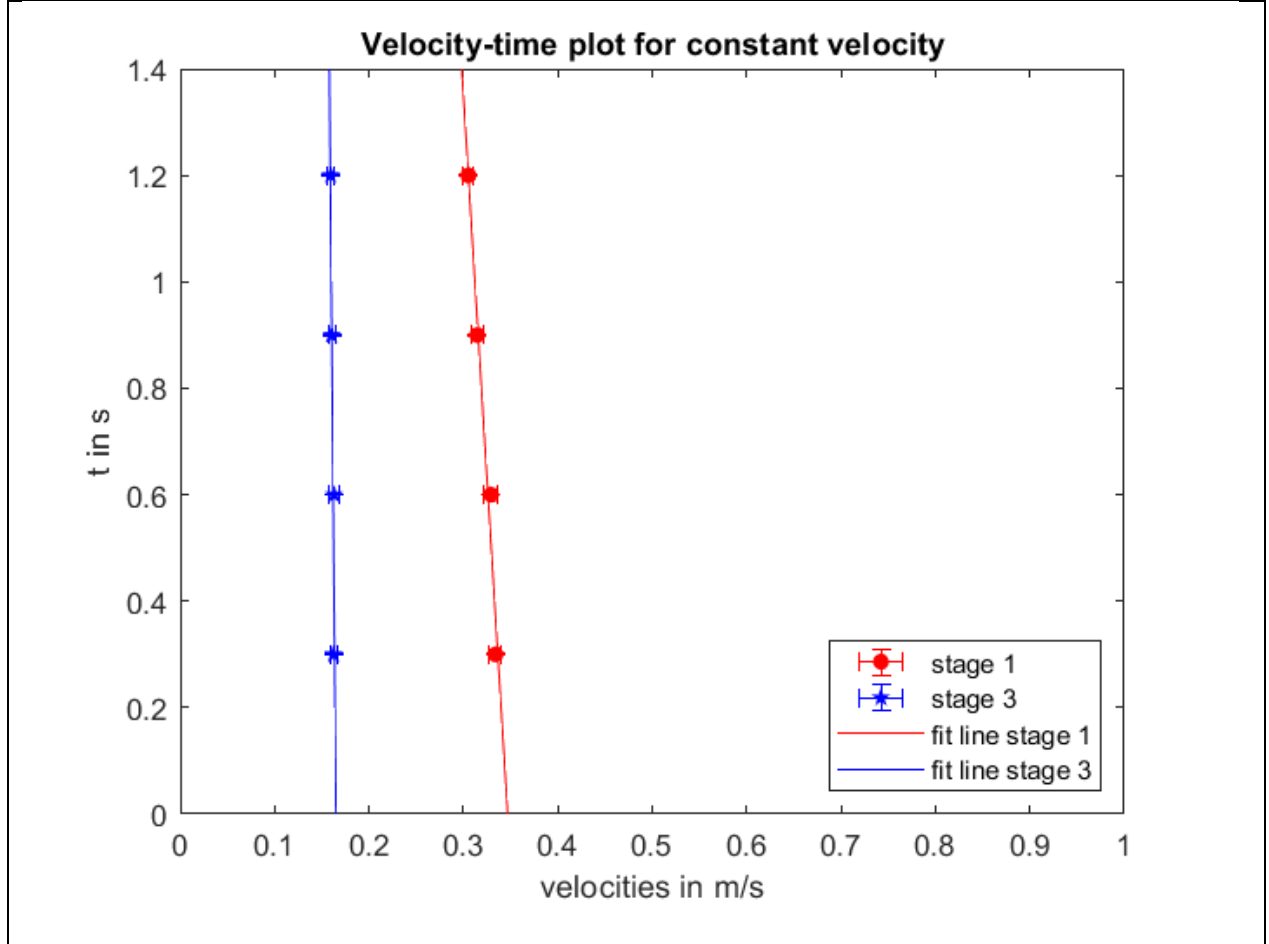


Figure 3: The velocity-time plots for constant velocity.

The regression lines are given by:

$$\text{stage 1: } v_1(t) = (a_1 \pm sa_1)t + (b_1 \pm sb_1) = (28.8200 \pm 1.5168)t + (9.9970 \pm 0.5262), \quad (11)$$

$$\text{stage 3: } v_3(t) = (a_3 \pm sa_3)t + (b_3 \pm sb_3) = (200 \pm 10.5263)t + (33 \pm 1.7368). \quad (12)$$

The velocity-position plot is obtained, by plotting the velocities from table 2 against the corresponding positions of the light barriers. For the case of constant velocity, the velocity and the position are connected by the linear velocity-position relation

$$v(x) = v_0, \quad (13)$$

i.e. for the data points of stage 1 and stage 3 one calculates a regression line. The slope of the regression line should be approximately zero and the intercept corresponds again to the initial velocity v_0 of the cart.

The regression lines of the velocity-position plot are given by:

$$\text{stage 1: } v_1(t) = (a_1 \pm sa_1)x + (b_1 \pm sb_1) = (0 \pm)t + (0.346 \pm), \quad (14)$$

$$\text{stage 3: } v_3(t) = (a_3 \pm sa_3)x + (b_3 \pm sb_3) = (0 \pm)t + (0.164 \pm). \quad (15)$$

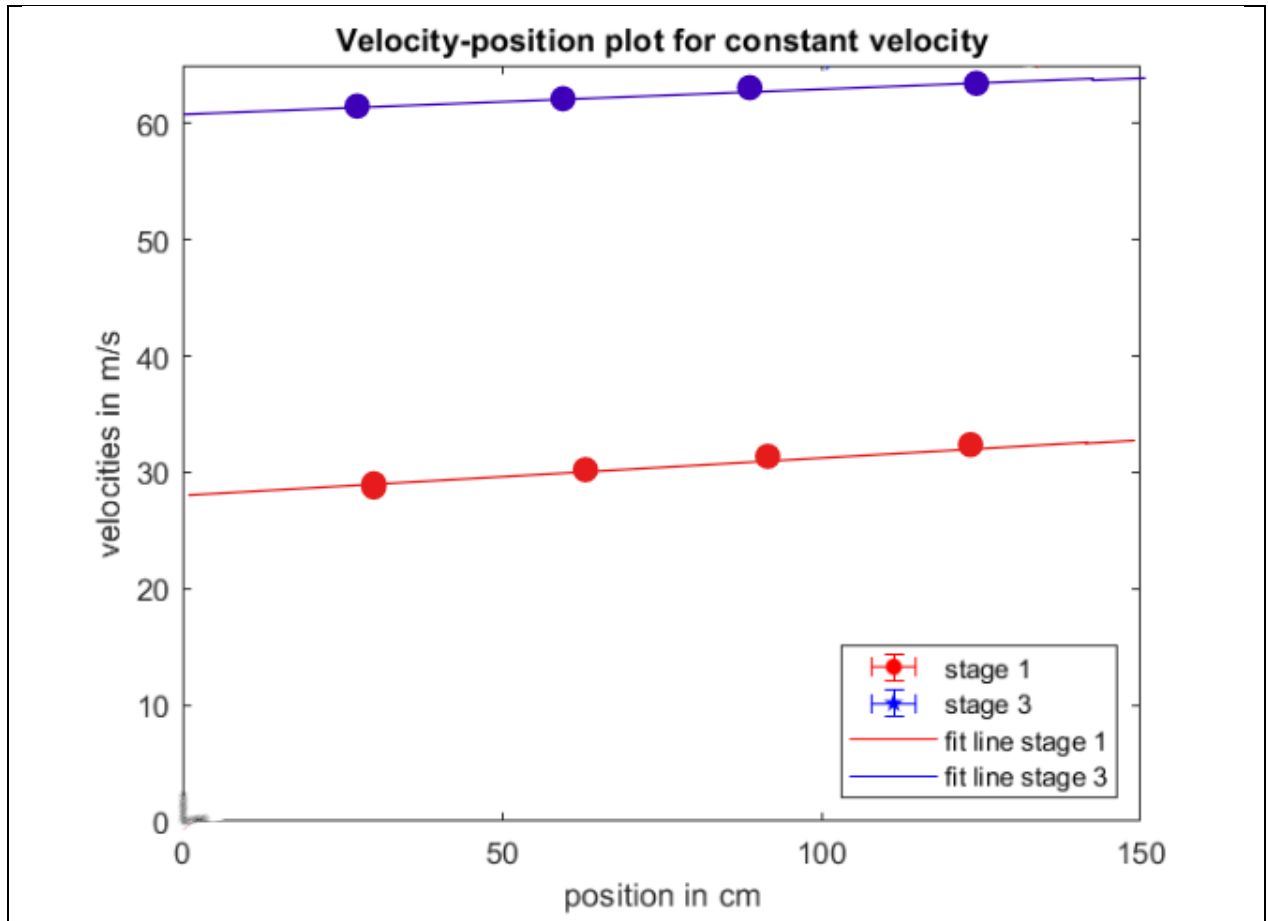


Figure 4: The velocity-position plot for constant velocity.

Step 3: The results and answers to the question

For the determination of the acceleration and the initial velocity one uses the velocity-time plot. The slope in the velocity-time plot should be close to zero in the case of constant velocity, but there is a small negative acceleration appears due to friction. It is given by the slope of the regression line:

$$\text{acceleration in stage 1: } a_0 \pm u(a_0) = (0 \pm 0.1) \text{ m/s}^2 , \quad (16)$$

$$\text{acceleration in stage 3: } a_0 \pm u(a_0) = (0 \pm 0.1) \text{ m/s}^2 . \quad (17)$$

The measurement value for the initial velocity of the cart follows from the intercept of the regression line in the velocity-time plot:

$$\text{initial velocity in stage 1: } v_0 \pm u(v_0) = (0.346 \pm 0.018) \text{ m/s} , \quad (18)$$

$$\text{initial velocity in stage 3: } v_0 \pm u(v_0) = (0.164 \pm 0.09) \text{ m/s} . \quad (19)$$

2a) Position-time plot for constant acceleration

Step 1: The data

The mean values and uncertainties are calculated just as in 1a).

20 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.932	1.774	2.324	2.770
2	0.927	1.768	2.316	2.760
3	0.924	1.752	2.298	2.742
mean \bar{t} in s	0.928	1.765	2.313	2.757
$u(\bar{t})$ in s	0.003	0.007	0.008	0.009

40 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.694	1.329	1.741	2.076
2	0.706	1.341	1.754	2.088
3	0.705	1.339	1.751	2.086
mean \bar{t} in s	0.702	1.336	1.749	2.083
$u(\bar{t})$ in s	0.004	0.004	0.004	0.004

Table 3: The arrival times for the case of constant acceleration with their mean values and uncertainties.

Step 2: The plots

The position-time plots are obtained by plotting the positions against the corresponding mean values of the arrival times. In the performed experiment the initial velocity was $v_0 = 0 \frac{m}{s}$. In this case the position-time relation for constant acceleration reduces to:

$$x(t) = \frac{1}{2} a_0 t^2 + x_0, \quad (20)$$

where x_0 is again the start position. This means that the data points have to be fitted by a parabola without linear term.

The fit parameter of the quadratic part corresponds then to one half of the acceleration and the intercept of the parabola corresponds again to the front edge x_0 of the cart at its start position.

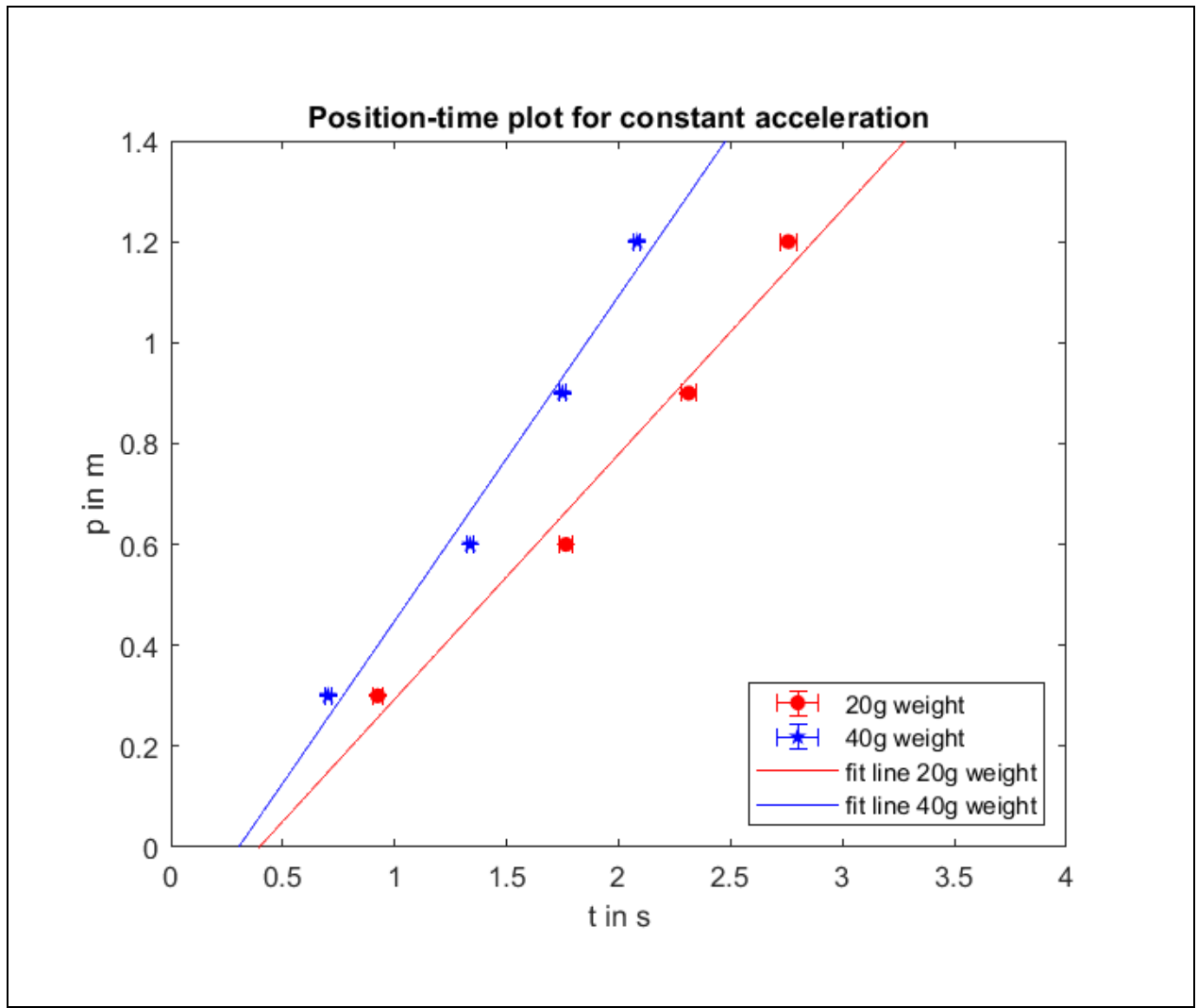


Figure 5: The position-time plots for constant acceleration.

The fit-parabolas are given by:

$$20 \text{ g: } x_{20}(t) = (a_{20} \pm sa_{20})t^2 + (c_{20} \pm sc_{20}) = (0.4853 \pm 0.0255)t^2 + (-0.1913 \pm 0.1007), \quad (21)$$

$$40 \text{ g: } x_{40}(t) = (a_{40} \pm sa_{40})t^2 + (c_{40} \pm sc_{40}) = (0.6437 \pm 0.0339)t^2 + (-0.6533 \pm). \quad (22)$$

Step 3: The results and answer to the question

The measurement value for the acceleration follows by multiplying the fit parameter of the quadratic part by two. The error of the acceleration also has to be doubled:

$$\text{acceleration for 20 g weight: } a_0 \pm u(a_0) = (0.9706 \pm 0.0510) \quad , \quad (23)$$

$$\text{acceleration for 40 g weight: } a_0 \pm u(a_0) = (1.2874 \pm 0.0678) \quad . \quad (24)$$

2b) Velocity-time plot and velocity-position plot for constant acceleration

Step 1: The data

The mean values and uncertainties for the blackout times are calculated as in the case of 1b) just as the velocities and uncertainties of the velocities.

20 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1	0.335	0.202	0.156	0.133
2	0.334	0.201	0.157	0.132
3	0.333	0.201	0.156	0.133
mean $\underline{t_{bl}}$ in s	0.334	0.201	0.156	0.133
$u(\underline{t_{bl}})$ in s	0.003	0.001	0.001	0.001
velocity v in m/s	29.940	49.751	64.100	75.188
$u(v)$ in m/s	0.402	0.556	0.761	0.941
v^2 in $\frac{m^2}{s^2}$	896.411	2475.19	4109.14	5653.23
$u(v^2)$ in $\frac{m^2}{s^2}$	0.162	0.309	0.580	0.885

40 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1	0.248	0.150	0.117	0.100
2	0.245	0.150	0.117	0.100
3	0.244	0.150	0.116	0.100
mean $\underline{t_{bl}}$ in s	0.246	0.150	0.117	0.100
$u(\underline{t_{bl}})$ in s	0.005	0.000	0.001	0.000
velocity v in m/s	40.650	66.667	85.470	100
$u(v)$ in m/s	0.921	0.667	1.124	1
v^2 in $\frac{m^2}{s^2}$	1652.46	4444.44	7305.14	10000
$u(v^2)$ in $\frac{m^2}{s^2}$	0.848	0.444	1.264	1

Table 4: The blackout times for the case of constant acceleration together with the velocities and their uncertainties.

Step 2: The plots

The velocity-time plots are obtained by plotting the velocities of table 4 against the mean arrival times at the corresponding positions given in table 3. In the performed experiment the initial velocity was $v_0 = 0 \frac{m}{s}$. In this case the velocity-time law reduces to

$$v(t) = a_0 t, \quad (25)$$

i.e. a linear regression is performed with a regression line without intercept. The slope of the regression line corresponds then to the acceleration.

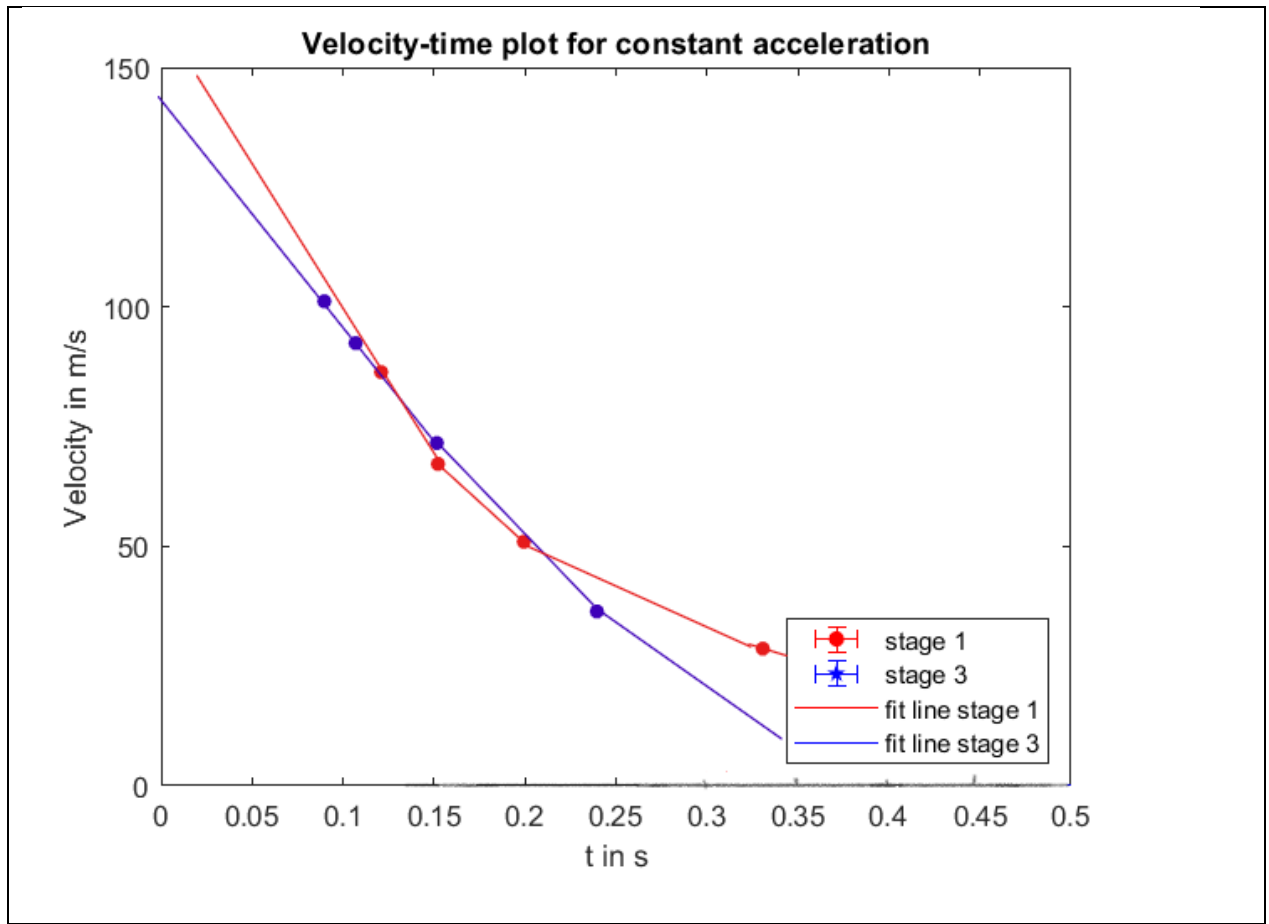


Figure 6: The velocity-time plots for constant acceleration.

The regression lines of the velocity-time plots are given by:

$$20 \text{ g weight: } v_{20}(t) = (a_{20} \pm sa_{20})t + (b_{20} \pm sb_{20}) = (\pm) t, \quad (26)$$

$$40 \text{ g weight: } v_{40}(t) = (a_{40} \pm sa_{40})t + (b_{40} \pm sb_{40}) = (\pm) t. \quad (27)$$

If the velocities are plotted against the positions of the light barriers, one obtains the velocity-position plot. The points are lying on a square root function, because the velocity-position relation in the case of $v_0 = 0 \frac{m}{s}$ is given by

$$v(x) = \sqrt{2a_0(x - x_0)}. \quad (28)$$

Squaring the velocity-position relation gives:

$$v^2(x) = 2a_0(x - x_0), \quad (29)$$

i.e. the function $v^2(x)$ is a linear function in x . Plotting then the squared velocities against the positions of the light barriers gives again data points that lie on a line and the slope of this line is according to equation (29) two times the acceleration. So calculating the regression line gives an alternative way to determine the acceleration.

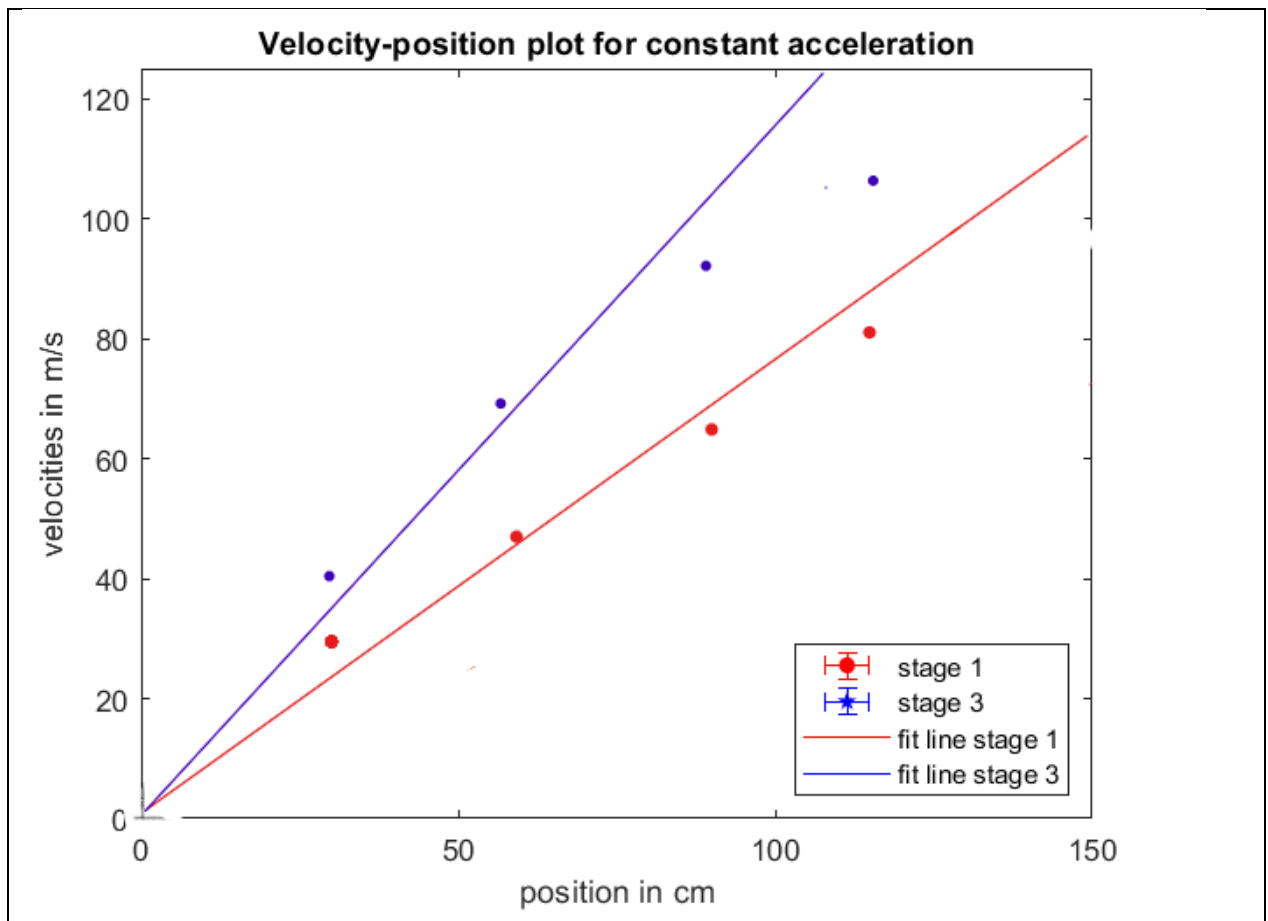


Figure 7: The velocity-position plot for constant acceleration.

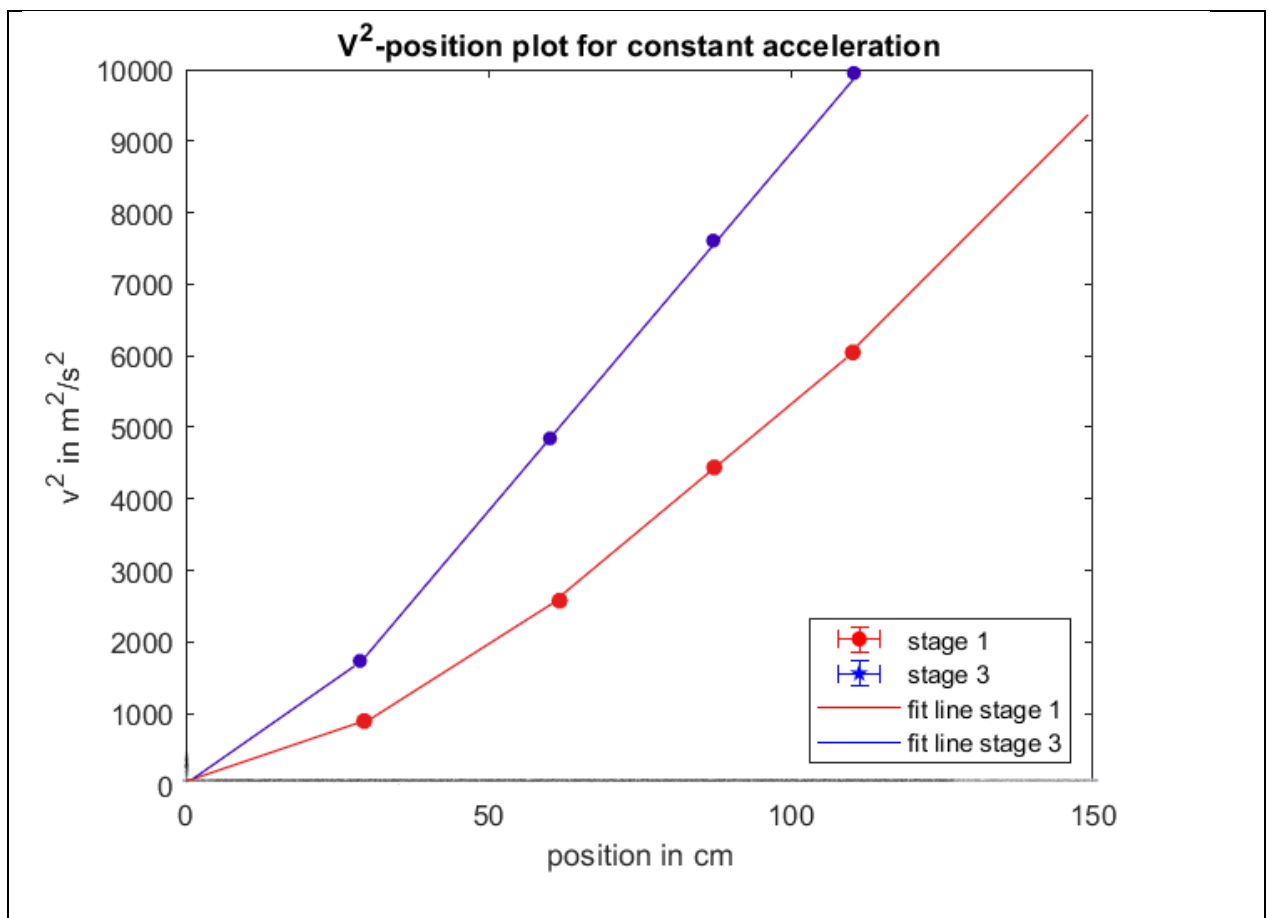


Figure 8: The v^2 -position plot for constant acceleration.

The regression lines of the v^2 -position plots are given by:

$$20 \text{ g: } v_{20}^2(x) = (a_1 \pm sa_1)x + (b_1 \pm sb_1) = (\pm)x + (\pm), \quad (30)$$

$$40 \text{ g: } v_{40}^2(x) = (a_3 \pm sa_3)x + (b_3 \pm sb_3) = (\pm)x + (\pm). \quad (31)$$

Step 3: The results and answer to the question

The measurement value for the acceleration can be determined with the velocity-time plot and the v^2 -position plot. The measurement results for the acceleration follow in the case of the velocity-time plot directly from the slope and the case of the v^2 -position the acceleration is the slope divided by two:

Determination of the acceleration with the velocity-time plot:

$$\text{acceleration for 20 g weight: } a_0 \pm u(a_0) = (\pm) \text{ m/s}^2, \quad (32)$$

$$\text{acceleration for 40 g weight: } a_0 \pm u(a_0) = (\pm) \text{ m/s}^2. \quad (33)$$

Determination of the acceleration with the v^2 -position plot:

$$\text{acceleration for 20 g weight: } a_0 \pm u(a_0) = (\pm) \text{ m/s}^2, \quad (34)$$

$$\text{acceleration for 40 g weight: } a_0 \pm u(a_0) = (\pm) \text{ m/s}^2. \quad (35)$$