Experiment Kinematics

Tasks

1) Record a position-time plot for uniform velocity and uniform acceleration.
2) Record a velocity-time plot for uniform velocity and uniform acceleration.

1 Theory

In kinematics the movement of bodies in space is described by their position, their velocity and their acceleration. If the movement takes place along a line, the position can be determined by one coordinate, for example the x-coordinate. A movement along the x-axis means, that the body is at different points of time t in different positions x, so that the movement itself is described by a function $x(t)$. This function $x(t)$ is the position-time law of the movement.

For the determination of the velocity of the body one needs its position at two points of time t_1 and t_2. The difference $\Delta x = x(t_2) - x(t_1)$ is the covered distance and the difference $\Delta t = t_2 - t_1$ is the needed time. The mean velocity of the body in this interval is then given by the difference quotient

$$\overline{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}. \tag{1}$$

This formula gives only the mean velocity in the time interval Δt, because the velocity does not have to be constant during this time interval.

The velocity of the body at a given point of time t is the instantaneous velocity $v(t)$. It can be obtained by calculating in (1) the limit $\Delta t \to 0$. The limit of the difference quotient is the derivative of the position-time law:

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{d}{dt} x(t). \tag{2}$$

I.e. the velocity-time law $v(t)$ is the derivative of the position-time law: $v(t) = \frac{d}{dt}x(t) = \dot{x}(t)$.

The acceleration gives the change in the velocity. It can be calculated in the same way as the velocity by considering the instantaneous velocity at two points of time t_1 and t_2. The difference $\Delta v = v(t_2) - v(t_1)$ is the change of the velocity in the time interval $\Delta t = t_2 - t_1$ and the mean acceleration is the difference quotient:

$$\overline{a} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \tag{3}$$

The instantaneous acceleration at a given point of time is then obtained by calculating the limit:

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d}{dt} v(t). \tag{4}$$

The acceleration-time law $a(t)$ is the derivative of the velocity-time law, and the second derivative of the position-time law: $a(t) = \frac{d}{dt}v(t) = \dot{v}(t) = \ddot{x}(t)$.

For a known position-time law one can calculate the velocity-time law and the acceleration-time law by differentiating. The inverse direction is then obtained by integration and the corresponding integration constants can be calculated from the initial condition. This should be demonstrated for the two most important types of movement, the uniform movement and the movement with constant acceleration:

- **<u>The movement with constant velocity</u>**
  If the velocity of a movement is constant, the velocity-time law is given by:
  $$v(t) = v(t=0) = v_0. \tag{5}$$
  The acceleration-time law is then obtained by differentiating and the position-time law is obtained by integration:
  $$x(t) = \int v_0 \, dt = v_0 t + C \quad \Leftarrow \quad v(t) = v_0 \quad \Rightarrow \quad a(t) = \frac{d}{dt} v_0 = 0 \tag{6}$$
  The integration constant C is given by the initial position:
  $$x(t=0) = x_0 = C. \tag{7}$$

- **<u>The movement with constant acceleration</u>**
  If the acceleration of the movement is constant, the acceleration-time law is given by:
  $$a(t) = a(t=0) = a_0. \tag{8}$$
  The velocity-time law and the position-time law are obtained by integration:
  $$a(t) = a_0 \quad \Rightarrow \quad v(t) = \int a_0 \, dt = a_0 t + C_1 \quad \Rightarrow \quad x(t) = \int (a_0 t + C_1) \, dt = \frac{a_0}{2} t^2 + C_1 t + C_2 \tag{9}$$
  The integration constants C_1 and C_2 are given by the initial velocity and the initial position:
  $$v(t=0) = v_0 = C_1 \qquad \text{und} \qquad x(t=0) = x_0 = C_2. \tag{10}$$

Summarized one has:

	constant velocity	constant acceleration
position-time law	$x(t) = v_0 t + x_0$	$x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0$
velocity-time law	$v(t) = v_0$	$v(t) = a_0 t + v_0$
acceleration-time law	$a(t) = 0$	$a(t) = a_0$

Besides the description of velocity and acceleration in dependence of time it is also possible to describe these quantities in dependence of the position. For the case of a movement with constant velocity, the velocity is not only the same for all time values, but also in all positions, i.e. the velocity-position law is $v(x) = v_0$ and since there is no acceleration anywhere, the acceleration-position law is $a(x) = 0$.

In the case of a movement with constant acceleration, the acceleration has not only always the same value a_0, but also everywhere, i.e. the acceleration-position law is in this case $a(x) = a_0$. For the derivation of the velocity-position law in the case of constant acceleration one solves the velocity-time law $v(t) = a_0 t + v_0$ for t and inserts this into the position-time law. Solving then for the velocity gives then the velocity-position law $v(x) = \sqrt{v_0^2 + 2a(x - x_0)}$.

Summarized one has:

	constant velocity	constant acceleration
velocity-position law	$v(x) = v_0$	$v(x) = \sqrt{v_0^2 + 2a_0(x - x_0)}$
acceleration-position law	$a(x) = 0$	$a(x) = a_0$

2 Experimental Setup

Given is a track and a cart (see figure 1). The cart can perform different types of movement. The first possibility is to push the cart with the startersystem that is fixed at one end of the track. The startersystem has different stages that allows to apply pushes with different strength to the cart. The second possibility is to pull the cart with a pulling weight, where the weight is hanging on a loose pulley (see figure 2). Thirdly one can put some wooden boards that can be placed under one end of the track so that one obtains a inclined plane.

For the measurement there are four light barriers that are connected with a timer. The timer has differnt modes, that allow to measure for example the arrival times or the blackout times of the light barriers. For this purpose has the cart at its side a flag that interrupts the light barriers.

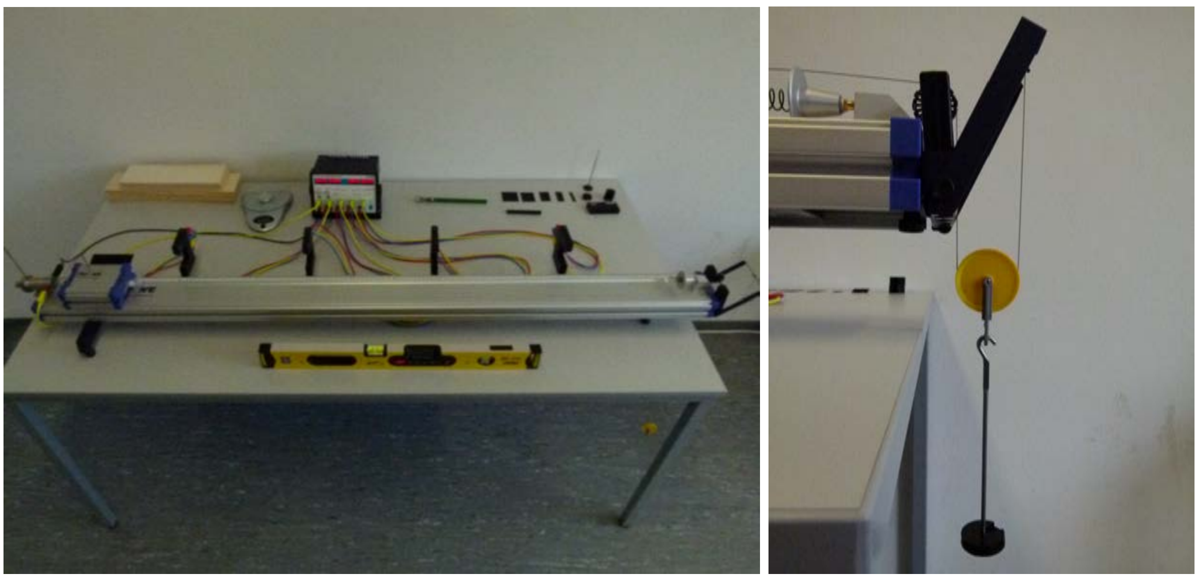

Figure 1: Left the experimental setup and right the loose pulley and the pulling weight.

3 Measurement Procedure

0) Adjustment of the track

- ☐ Adjust the track so that the cart reaches the end of the track when started in stage 1. Turn the feet for that purpose.
- ☐ Place the light barriers at 30 cm, 60 cm, 90 cm und 120 cm. Make sure that the opening of the light barrier is correctly adjusted at its corresponding waypoint.
- ☐ Find out what the different modes of the timer measure.

1a) Position-time plot for constant velocity

- ☐ Choose the timer mode for the arrival times.
- ☐ Start the cart in stage 1. Write down the arrival times of the cart at the four waypoints. Do this measurement altogether three times.

- ☐ Do the whole measurement again for a cart that is started in stage 3.

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the arrival time and the corresponding measurement uncertainty (see: Formulas for data analysis, equation (1)).
- ☐ Create a coordinate system with the arrival times on the horizontal axis and the distance from the start on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the same coordinate system. Use for the group of data points for stage 1 and the group of data points for stage 3 different symbols and colors.
- ☐ Use MATLAB for calculating a regression line through the group of data points for stage 1 and a second regression line through the group of data points for stage 3. Plot the two regression lines into the coordinate system with the data points and give the formulas of the two regression lines, including the uncertainties for the slope and the intercept.

Answer the following questions:

- Which velocity (including the measurement uncertainty) has the cart when it is started in stage 1 and stage 3?
- Where is the front edge of the cart when it is started in the two stages (including the measurement uncertainty)?

1b) Velocity-time plot and velocity-position plot for a constant velocity

- ☐ Set the timer to the mode for the blackout times.
- ☐ Start the cart in stage 1 and write down the blackout times at the four waypoints. Do this measurement altogether three times.
- ☐ Repeat the whole measurement of three blackout times at four waypoints for a cart that is stated in stage 3.

Evaluation:

- ☐ Calculate for each group of three measurements the mean value of the blackout time and the corresponding measurement uncertainty of the mean value (see: Formulas for data analysis, equation (1)).
- ☐ Calculate from each mean value of the blackout time the corresponding velocity of the cart and the measurement uncertainty of the velocity (see: Formulas for data analysis, equation (6)).
- ☐ Create a coordinate system with the arrival times measured in 1a) on the horizontal axis and the velocities on the vertical axis. Plot all calculated data points (with their measurement uncertainties as error bars) in this coordinate system. Use for the group of data points for stage 1 and the group of data points for stage 3 different symbols and colors.
- ☐ Use MATLAB for calculating a regression line through the group of data points for stage 1 and a second regression line through the group of data points for stage 3. Plot the two regression lines into the coordinate system with the data points and give the formulas of the two regression lines, including the uncertainties for the slope and the intercept.

- ☐ Create a second coordinate system with the positions of the light barriers on the horizontal axis and the velocities calculated above on the vertical axis. Plot all data points (with their measurement uncertainties as error bars) in this coordinate system. Use for the group of data points for stage 1 and the group of data points for stage 3 different symbols and colors.
- ☐ Use MATLAB for calculating a regression line through the group of data points for stage 1 and a second regression line through the group of data points for stage 3. Plot the two regression lines into the coordinate system with the data points and give the formulas of the two regression lines, including the uncertainties for the slope and the intercept.

Answer the following questions:

- Is the velocity of the cart in the two cases constant? If not, why not? How big is the acceleration (including the measurement uncertainty) of the cart?
- What is the initial velocity of the cart in the two cases (including the measurement uncertainty)?

2a) Position-time plot for constant acceleration

- ☐ Set the timer to the mode for the arrival times.
- ☐ Unfasten the screw beneath the track, turn the starter system around and fasten the screw again.
- ☐ Connect the thread with the loose pulley to the cart. Hang a weight of 20 g (10 g for the weight holder and one additional 10 g weight) on the loose pulley (see figure 1 right).
- ☐ Start the cart with the starter system and measure the arrival times at the four waypoints. Do this measurement altogether three times.
- ☐ Repeat the whole measurement then for a weight of 40 g (10 g for the weight holder and three additional 10 g weights).

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the arrival time and the corresponding measurement uncertainty of the mean value (see: Formulas for data analysis, equation (1)).
- ☐ Create a coordinate system with the arrival times on the horizontal axis and the distance of the waypoints to the start on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the group of data points with a weight of 20 g and the group of data points with a weight of 40 g different symbols and colors.
- ☐ Use MATLAB for calculating a fit parabola without linear term through the group of data points with a weight of 20 g and a second fit parabola without linear term through the group of data points with a weight of 40 g. Plot the two parabolas into the coordinate system with the data points and give the formulas of the two fit parabolas, including the uncertainties of the parameters.

Answer the following questions:

- Which values for the acceleration do you obtain in the two cases (including the measurement uncertainties)?

2b) Velocity-time plot and velocity-position plot for constant acceleration

- ☐ Set the timer to the mode for the blackout times.

- ☐ Hang a weight of 20 g (10 g for the weight holder and one additional 10 g weight) on the pulley.
- ☐ Start the cart with the starter system and measure the blackout times at the four waypoints. Do this measurement altogether three times.
- ☐ Repeat the whole measurement for a weight of 40 g (10 g for the weight holder and three 10 g weights).

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the blackout time and the corresponding measurement uncertainty of the mean value (see: Formulas for data analysis, equation(1)).
- ☐ Calculate with these mean values the velocity at the waypoints and the corresponding measurement uncertainty of the velocities (see: Formulas for data analysis, equation (6)).
- ☐ Create a coordinate system with the arrival times measured in 2a) on the horizontal axis and the velocities on the vertical axis. Plot all calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the group of data points with a weight of 20 g and the group of data points with a weight of 40 g different symbols and colors.
- ☐ Use MATLAB for calculating a regression line through the data points for the 20 g weight and a second regression line through the data points for the 40 g weight. Use in both cases a regression line without constant term. Plot the two regression lines into the coordinate system with the data points and give the formulas of the two regression lines, including the uncertainties for the slope.
- ☐ Create a second coordinate system with the positions of the light barriers on the horizontal axis and the velocities on the vertical axis. Plot the data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the group of data points with a weight of 20 g and the group of data points with a weight of 40 g different symbols and colors.
- ☐ Create a third coordinate system with the positions of the light barriers on the horizontal axis and the squared velocities on the vertical axis. Plot the data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the group of data points with a weight of 20 g and the group of data points with a weight of 40 g different symbols and colors.
- ☐ Use MATLAB for calculating a regression line through the data points for the 20 g weight and a second regression line through the data points for the 40 g weight. Plot the two regression lines into the coordinate system with the data points and give the formulas of the two regression lines, including the uncertainties for the slope and the intercept.

Answer the following question:

- Which value for the acceleration (including the measurement uncertainty) do you get in the two cases? Compare the values with the one measured in 2a) and explain any differences.

4 Preparatory Exercises

a) Derive the acceleration-position law for constant acceleration: $v(x) = \sqrt{v_0^2 + 2a(x - x_0)}$.

b) Derive the formula for the measurement uncertainty of the velocity $u(v)$ (equation (6) in Formulas for data analysis).

c) Derive the formula for the measurment uncertainty of the squared velocity $u(v^2)$.

Setup No.	Names:	Date:
		Sign. supervisor:

1a) Position-time plot for constant velocity

stage 1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

stage 3				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

Sketch of the position-time plot:

slope =
intercept =

Formula of the position-time law:

1b) Veloctity-time plot and velocity-position plot for constant velocity

length of the flag=(__________________ ± __________________) _____

stage 1				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

stage 3				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

Sketch of the velocity-time plot:

slope =
intercept=

Sketch of the velocity-position plot:

slope =
intercept=

Formula of the velocity-time law:

Formula of the velocity-position law:

2a) Position-time plot for constant acceleration

20 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

40 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

Sketch of the position-time plot:

Formula of the position-time law:

2b) Velocity-time plot and velocity-position plot for constant acceleration

20 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

40 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

Sketch of the velocity-time plot:

slope =
intercept =

Sketch of the v^2-position plot:

slope =
intercept =

Formula of the velocity-time law:

Formula of the v^2-position law:

Results Experiment Kinematics

1a) Position-time plot for constant velocity

- Regression line for stage 1 and stage 3:

$$x_1(t) = (\mathtt{a}_1 \pm \mathtt{sa}_1)t + (\mathtt{b}_1 \pm \mathtt{sb}_1) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$
$$x_3(t) = (\mathtt{a}_3 \pm \mathtt{sa}_3)t + (\mathtt{b}_3 \pm \mathtt{sb}_3) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$

- Velocities for stage 1 and stage 3:

$$(v_1 \pm u(v_1)) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$
$$(v_3 \pm u(v_3)) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$

- Front edge of cart at start position in stage 1 and stage 3:

$$(b_1 \pm u(b_1)) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$
$$(b_3 \pm u(b_3)) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$

1b) Veloctity-time plot and velocity-position plot for constant velocity

- Velocity-time plot: regression line for stage 1 and stage 3:

$$v_1(t) = (\mathtt{a}_1 \pm \mathtt{sa}_1)t + (\mathtt{b}_1 \pm \mathtt{sb}_1) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$
$$v_3(t) = (\mathtt{a}_3 \pm \mathtt{sa}_3)t + (\mathtt{b}_3 \pm \mathtt{sb}_3) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$

- Velocity-time plot: acceleration for stage 1 and stage 3:

$$(a_1 \pm u(a_1)) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$
$$(a_3 \pm u(a_3)) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$

- Velocity-time plot: initial velocity for stage 1 and stage 3:

$$(v_{0,1} \pm u(v_{0,1})) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$
$$(v_{0,3} \pm u(v_{0,3})) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$

- Velocity-position plot: regression line for stage 1 and stage 3:

$$v_1(x) = (\mathtt{a}_1 \pm \mathtt{sa}_1)x + (\mathtt{b}_1 \pm \mathtt{sb}_1) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)x + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$
$$v_3(x) = (\mathtt{a}_3 \pm \mathtt{sa}_3)x + (\mathtt{b}_3 \pm \mathtt{sb}_3) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)x + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$

2a) Position-time plot for constant acceleration

- Fit parabolas for a pulling weight of 20 g and 40 g:

$$x_{20}(t) = (\mathtt{a}_{20} \pm \mathtt{sa}_{20})t^2 + (\mathtt{c}_{20} \pm \mathtt{sc}_{20}) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t^2 + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$
$$x_{40}(t) = (\mathtt{a}_{40} \pm \mathtt{sa}_{40})t^2 + (\mathtt{c}_{40} \pm \mathtt{sc}_{40}) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t^2 + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)$$

- Acceleration for a pulling weight of 20 g and 40 g:

$$(a_{20} \pm u(a_{20})) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$
$$(a_{40} \pm u(a_{40})) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_$$

2b) Velocity-time plot and velocity-position plot for constant acceleration

- Velocity-time plot: regression lines for a pulling weight of 20 g and 40 g:

$$
\begin{aligned}
v_{20}(t) &= (\mathtt{a}_{20} \pm \mathtt{sa}_{20})t + (\mathtt{b}_{20} \pm \mathtt{sb}_{20}) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_) \\
v_{40}(t) &= (\mathtt{a}_{40} \pm \mathtt{sa}_{40})t + (\mathtt{b}_{40} \pm \mathtt{sb}_{40}) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)
\end{aligned}
$$

- Velocity-time plot: acceleration for a pulling weight of 20 g and 40 g:

$$
\begin{aligned}
(a_{20} \pm u(a_{20})) &= (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_ \\
(a_{40} \pm u(a_{40})) &= (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_
\end{aligned}
$$

- v^2-position plot: regression line for pulling weight of 20 g and 40 g:

$$
\begin{aligned}
v_{20}^2(x) &= (\mathtt{a}_{20} \pm \mathtt{sa}_{20})x + (\mathtt{b}_{20} \pm \mathtt{sb}_{20}) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_) \\
v_{40}^2(x) &= (\mathtt{a}_{40} \pm \mathtt{sa}_{40})x + (\mathtt{b}_{40} \pm \mathtt{sb}_{40}) = (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)t + (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)
\end{aligned}
$$

- v^2-position plot: acceleration for pulling weight of 20 g and 40 g:

$$
\begin{aligned}
(a_{20} \pm u(a_{20})) &= (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_ \\
(a_{40} \pm u(a_{40})) &= (\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_)\_\_\_\_
\end{aligned}
$$