Experiment Inclined Plane

Tasks

1) Check if the accelerations on the inclined plane depends on the mass.
2) Determine the gravitational acceleration.

1 Theory

The relation between the force that acts on a body and the resulting motion of the body as a consequence of this force is described by Newton's second law

$$\vec{F} = m\,\vec{a}, \tag{1}$$

where $\vec{F}$ is the external force acting on the body, $\vec{a}$ the resulting acceleration of the body and m the mass of the body. According to this equation the acceleration is always in the direction of the external force and the vectors $\vec{F}$ and $m\vec{a}$ have the same length, which leads to the scalar version of Newton's second law:

$$F = m\,a, \tag{2}$$

with $F = |\vec{F}|$ and $a = |\vec{a}|$.

In the case of free fall the external force is the gravitational force $\vec{F}_G$, which on the scale of a laboratory can be assumed as a constant, downward pointing force. The magnitude of the gravitational force acting on the body is proportional to the mass of the body, where the constant of proportionality is $g = 9.81\,\mathrm{m/s^2}$, i.e. $F_G = |\vec{F}_G| = m\,g$. Newton's second law can then be written as $\vec{F}_G = m\,\vec{a}$, or in components:

$$\begin{pmatrix} 0 \\ -mg \\ 0 \end{pmatrix} = m \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}. \tag{3}$$

From this follows, that in free fall there is only a non-vanishing acceleration in y-direction with $a_y = -g$. Or, reading Newton's second law in the other direction: if a body with mass m is accelerated downwards in free fall with an acceleration $\vec{g}$, it experiences a gravitational force $\vec{F}_G = m\,\vec{g}$.

For a constant external force, i.e. if the force vector does not change in time and in space, the resulting movement of the body is according to Newton's second law a linear movement with constant acceleration, where the acceleration points in the same direction as the force. The corresponding vector version of the position time relation for a movement with constant acceleration is:

$$\vec{x}(t) = \frac{1}{2}\vec{a}\,t^2 + \vec{v}_0\,t + \vec{x}_0, \tag{4}$$

where $\vec{x}_0 = \vec{x}(t=0)$ is the initial position of the body, $\vec{v}_0$ its initial velocity and $\vec{a} = \vec{F}/m$ its acceleration due to the external force $\vec{F}$. In the free fall case with no initial velocity the movement is vertical and is described by the y-compontent of the position time equation (4):

$$y(t) = -\frac{1}{2}g\,t^2 + y_0. \tag{5}$$

Before stopwatches were available it was very hard to measure the gravitational acceleration g accurately. For that reason Galileo Galilei thought of a slow motion version of the free fall experiment. This experiment is the inclined plane, because a body that is moving down an inclined plane is accelerated with an acceleration

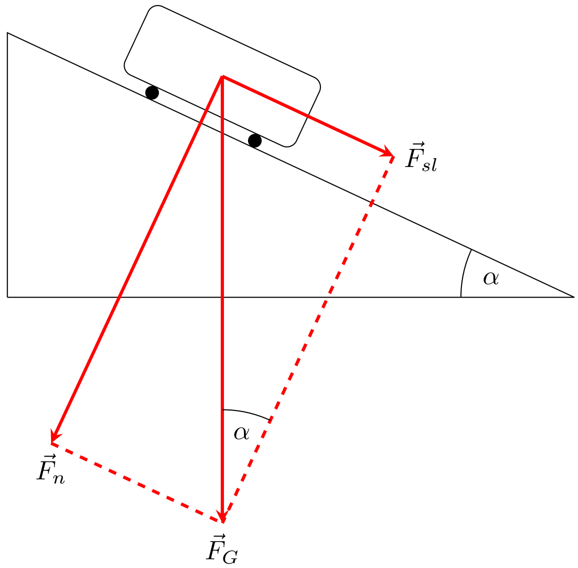

Figure 1: The inclined plane.

smaller than g. If the body is accelerated along the inclined plane, then, according to Newton's second law, there has to be a force in the same direction, i.e. parallel to the inclined plane. This force is called the slope force $\vec{F}_{sl}$ (see figure 1). But since the only external force in this situation is the gravitational force, the slope force must be related to the gravitational force.

Forces are vector valued quantities, which means that if two forces are acting on the same body, the resulting force, that accelerates the body, is the vector sum of the two single force vectors. And conversely it is possible to write a given force vector as the vector sum of two other force vectors. In the case of the inclined plane one can write the gravitational force as the vector sum of the slope force and the normal force. The normal force $\vec{F}_n$ is the force perpendicular to the inclined plane, that holds the body on the inclined plane (see figure 1):

$$\vec{F}_G = \vec{F}_{sl} + \vec{F}_n. \tag{6}$$

For the length of the slope vector and the length of the normal vector follows then:

$$F_{sl} = m\,g\,\sin\alpha \quad \text{and} \quad F_n = m\,g\,\cos\alpha. \tag{7}$$

The slope force is the accelerating force, because it points in the direction of the acceleration, so that the scalar version of Newton's second law becomes $F_{sl} = ma$, or with (7): $mg\sin\alpha = ma$. This leads then to the following formula for the acceleration down the inclined plane:

$$\boxed{a = g\sin\alpha. \tag{8}}$$

Note, that in the limit $\alpha = 90°$ one has again the free fall case with $a = g$.

2 Experimental Setup

The experimental setup is the same as in the experiment Kinematics. In order to obtain an inclined plane one can put some wooden boards beneath one of the foots of the track.

3 Measurement Procedure

1) Measurement of the acceleration for different weights on the cart

- ☐ Put some wooden plates under the foot of the track where the starter system is mounted, so that you obtain an inclined plane with small inclination angle.
- ☐ Determine the inclination angle using the tape measure.
- ☐ Start the cart with the starter system and let it roll down the inclined track. Measure the arrival times at 30 cm, 60 cm, 90 cm and 120 cm. Do this measurement altogether three times.
- ☐ Repeat the whole measurement then three more times with different additional weights below 400 g on the cart.

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the arrival times and the corresponding measurement uncertainty of the mean value.
- ☐ Create a coordinate system with the arrival times on the horizontal axis and the distance of the waypoints to the start on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the four groups of data points different symbols and colors.
- ☐ Use MATLAB for calculating a fit parabola without linear term through each of the four groups of data points. Plot the four parabolas into the coordinate system with the data points. Determine for each of the four cases the acceleration including the measurement uncertainty.
- ☐ Create a second coordinate system with the total mass of the cart, i.e. mass of the cart plus the mass of the additional weights, on the horizontal axis and the acceleration on the vertical axis. Plot the four data points (with their measurement uncertainties as error bars) in the coordinate system and use MATLAB to calculate the regression line.

Answer the following question:

- Does the acceleration of the cart depend on its weight?
- Does the acceleration agree with the theoretical value?

2) Measurement of the acceleration for different inclination angles

- ☐ Remove the additional weights from the cart.
- ☐ Remove some wooden plates under the foot of the track where the starter system is mounted, so that you obtain an inclined plane with a very small inclination angle.
- ☐ Determine the inclination angle using the tape measure.
- ☐ Start the cart with the starter system and let it roll down the inclined track. Measure the blackout times at 30 cm, 60 cm, 90 cm and 120 cm. Do this measurement altogether three times.
- ☐ Repeat the whole measurement then three more times with stepwise higher inclination angles.

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the blackout times and the corresponding measurement uncertainty of the mean value.

- ☐ Calculate with these mean values the velocity at the waypoints and the corresponding measurment uncertainty of the velocities.

- ☐ Create a coordinate system with the positions of the light barriers on the horizontal axis and the velocities on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the four groups of data points different symbols and colors.

- ☐ Use MATLAB for calculating a fit curve through each of the four groups of data points. Plot the four curves into the coordinate system with the data points.

- ☐ Calculate now for each velocity the corresponding square of the velocity and create a second coordinate system with the positions of the light barriers on the horizontal axis and the square of the velocities on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the four groups of data points different symbols and colors.

- ☐ Use MATLAB for calculating a regression lines through each of the four groups of data points. Plot the four regression lines into the coordinate system with the data points. Determine for each of the four cases the acceleration including the measurement uncertainty.

- ☐ Create a third coordinate system with the sine of the inclination angle on the horizontal axis and the acceleration on the vertical axis. Plot the four data points (with their measurement uncertainties as error bars) in the coordinate system and use MATLAB to calculate a regression line.

Answer the following question:

- What is your measured value for the gravitational acceleration g?

4 Preparatory Exercises

a) Prove geometrically that the angle between the gravitational force vector $\vec{F}_G$ and the normal force vector $\vec{F}_n$ is equal to the inclination angle α (see figure 1).

b) Write down the three components of the slope vector $\vec{F}_{sl}$ and of the normal vector $\vec{F}_n$ and then prove equation (6).

Setup No.	Names:	Date:
		Sign. of Supervisor:

1) Measurement of the acceleration for different weights on the cart

mass of the cart=(________ ± ________) _____, α=(________ ± ________)

additional mass 1 = 0 g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

additional mass 2 = ____________				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

additional mass 3 = ____________				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

additional mass 4 = ____________				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1				
2				
3				

2) Measurement of the acceleration for different inclination angles

inclination angle 1 =__________±__________				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

inclination angle 2 =__________±__________				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

inclination angle 3 =__________±__________				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

inclination angle 4 =__________±__________				
	30 cm	60 cm	90 cm	120 cm
measurement	blackout times in s	blackout times in s	blackout times in s	blackout times in s
1				
2				
3				

Sketch of the velocity-position plot:

Sketch of the v^2-position plot:

slope =
intercept =

sketch of the a-$\sin\alpha$ plot:

slope =
intercept =

Formula of the velocity-position law:

Formula of the v^2-position law:

Formula of the a-$\sin\alpha$ law:

Results Experiment Inclined Plane

1) Measurement of the acceleration for different weights on the cart

- Fit parabolas for different weights on the cart:

$$
\begin{aligned}
x_1(t) &= (\mathtt{a}_1 \pm \mathtt{sa}_1)t^2 + (\mathtt{c}_1 \pm \mathtt{sc}_1) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)t^2 + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\\
x_2(t) &= (\mathtt{a}_2 \pm \mathtt{sa}_2)t^2 + (\mathtt{c}_2 \pm \mathtt{sc}_2) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)t^2 + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\\
x_3(t) &= (\mathtt{a}_3 \pm \mathtt{sa}_3)t^2 + (\mathtt{c}_3 \pm \mathtt{sc}_3) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)t^2 + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\\
x_4(t) &= (\mathtt{a}_4 \pm \mathtt{sa}_4)t^2 + (\mathtt{c}_4 \pm \mathtt{sc}_4) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)t^2 + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)
\end{aligned}
$$

- Acceleration for different weights on the cart:

$$
\begin{aligned}
(a_1 \pm u(a_1)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
(a_2 \pm u(a_2)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
(a_3 \pm u(a_3)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
(a_4 \pm u(a_4)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
\\
\text{mean value : } (a \pm u(a)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_
\end{aligned}
$$

- Calculated acceleration:

$$a = \_\_\_\_\_\_ \;\; \_\_\_$$

- Regression line for the a-m plot:

$$a(m) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)m + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)$$

2) Measurement of the acceleration for different inclination angles

- Regression lines for different inclination angles:

$$
\begin{aligned}
v_1^2(x) &= (\mathtt{a}_1 \pm \mathtt{sa}_1)x + (\mathtt{c}_1 \pm \mathtt{sc}_1) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)x + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\\
v_2^2(x) &= (\mathtt{a}_2 \pm \mathtt{sa}_2)x + (\mathtt{c}_2 \pm \mathtt{sc}_2) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)x + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\\
v_3^2(x) &= (\mathtt{a}_3 \pm \mathtt{sa}_3)x + (\mathtt{c}_3 \pm \mathtt{sc}_3) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)x + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\\
v_4^2(x) &= (\mathtt{a}_4 \pm \mathtt{sa}_4)x + (\mathtt{c}_4 \pm \mathtt{sc}_4) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)x + (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)
\end{aligned}
$$

- Acceleration for different inclination angles:

$$
\begin{aligned}
(a_1 \pm u(a_1)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
(a_2 \pm u(a_2)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
(a_3 \pm u(a_3)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_\\
(a_4 \pm u(a_4)) &= (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_
\end{aligned}
$$

- Regression line for the a - $\sin\alpha$ plot:

$$a(\sin\alpha) = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\sin\alpha$$

- Gravitational acceleration:

$$g = (\_\_\_\_\_\_ \pm \_\_\_\_\_\_)\_\_\_$$