Lab partners (name, student #):	**degree course, semester:**
Mangwi Claudia Mongu (27333)	**Bioengineering** **5th Semester**
Sara shafaeian(30832)	**Bioengineering** **First Semester**

Performed the experiment on: **03.11.2021** **Name of the tutor:** ____________ ***Accepted:***

~~Yes~~ No

Submission of the report: ____________ **Checked on:** ____________

Submission of first revision: ____________ **Checked on:** ____________

Submission of second revision: ____________ **Checked on:** ____________

Title:

Inclined plane

Remarks of the tutor:

I) Theory

The purpose of this experiment is to record the arrival time of a cart at different time interval with varying weight. Also to record the position-time curve at varying angle of inclination. In addition position-time plot should be recorded for the two types of movement. The recorded curves are then used to determine if acceleration depends on the mass and incline as well as determine the gravitational acceleration. The curves are described by the following formulas:

F=ma

Fsl = m g sin α and Fn = m g cos α

a = g sin α   α = 90◦   a=g

x(t) = 1 / 2 a t2 + v0 t + x0

y(t) = − 1 2 g t2 + y0

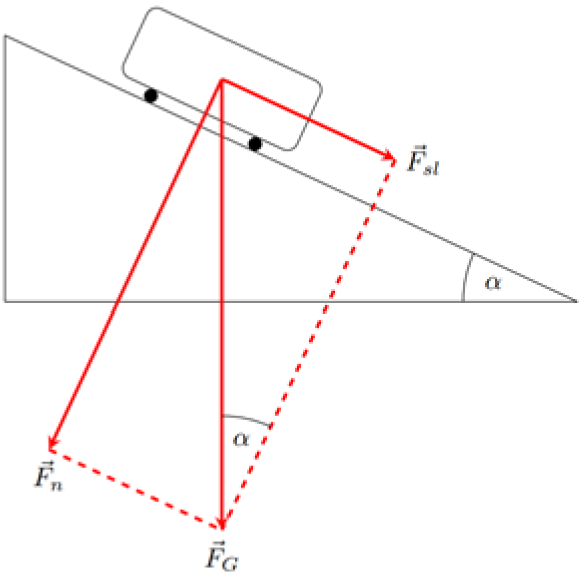


Figure 1: The inclined plane.

II) Setup

The experimental setup consists of a track with a cart. The cart is started with a starter system or accelerated with a pulling weight or by inclining the track. Light barriers are positioned at four positions and with a flag attached to the cart one can measure the arrival time of the cart. Wooden plates are placed under the foot to obtain an incline plane and the angle of inclination can be obtain.

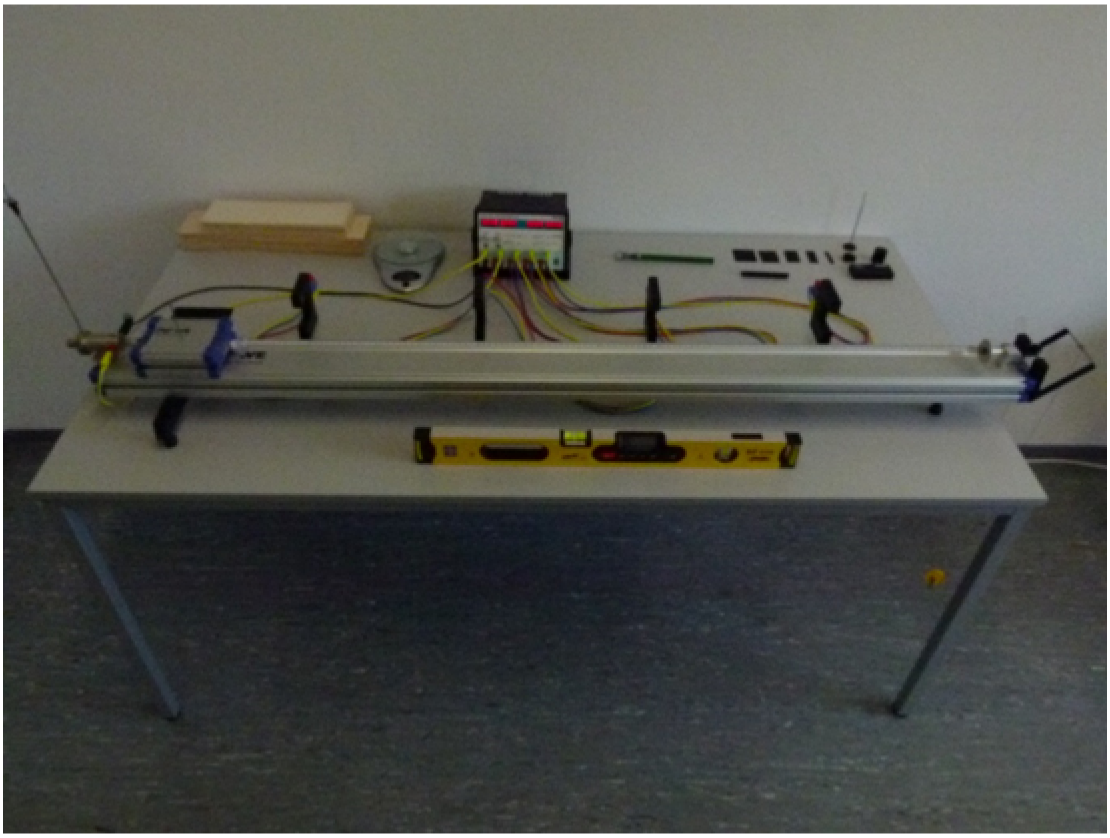


Figure 1: The experimental setup

I) <u>Evaluation</u>

1) Measurement of the acceleration for different weights on the cart

<u>Step 1: The data</u>

The light barriers were fixed at the positions x_1 = 30 cm, x_2 = 60 cm, x_3 = 90 cm and x_4 = 120 cm with an uncertainty of $u(x_i)$ = ±0.1 cm. The following tables contain the N=3 measured arrival times and their mean and uncertainty calculated with the formula

using the student-t-factor = 4.303.

mass of the cart=(457.4 ±0.1)g , α=(1.6 ±0.1)

a=gsins(α) with uncertainty $u(a)=\sqrt{(gcos(\alpha))2\ (u(\alpha)2}$

Additional mass 1=0g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.947	1.753	2.294	2.731
2	0.949	1.759	2.303	2.741
3	0.941	1.748	2.291	2.729
mean in s	0.9457	1.7533	2.2960	2.7337
$u()$ in s	0.0103	0.0137	0.0155	0.0160

Additional mass 2=50g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.968	1.778	2.321	2.760
2	0.926	1.731	2.273	2.711

3	0.944	1.750	2.292	2.729
mean in s	0.9460	1.7530	2.2953	2.7333
$u()$ in s	0.0523	0.0587	0.0600	0.0616

100 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.933	1.741	2.285	2.723
2	0.947	1.763	2.308	2.748
3	0.970	1.756	2.293	2.727
mean in s	0.9500	1.7533	2.2953	2.7327
$u()$ in s	0.0464	0.0279	0.0290	0.0334

150 g weight				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.946	1.757	2.301	2.740
2	0.942	1.751	2.295	2.734
3	0.918	1.725	2.268	2.706
mean in s	0.9353	1.7443	2.2880	2.7267
$u()$ in s	0.0376	0.0423	0.0437	0.0451

Table 1: The arrival times for different weights on the cart with their mean values and uncertainties.

Step 2: The plots

Now the cart is set on an inclined plane with an inclination angle α= 1.6 degrees. The movement of the cart is then a movement with constant acceleration. The position-time plots are obtained just as in 2a) by plotting the positions against the corresponding mean values of the arrival times. The fit parameter of the quadratic part of the fit parabola corresponds to one half of the acceleration and the intercept of the parabola corresponds to the front edge x_0 of the cart at its start position.

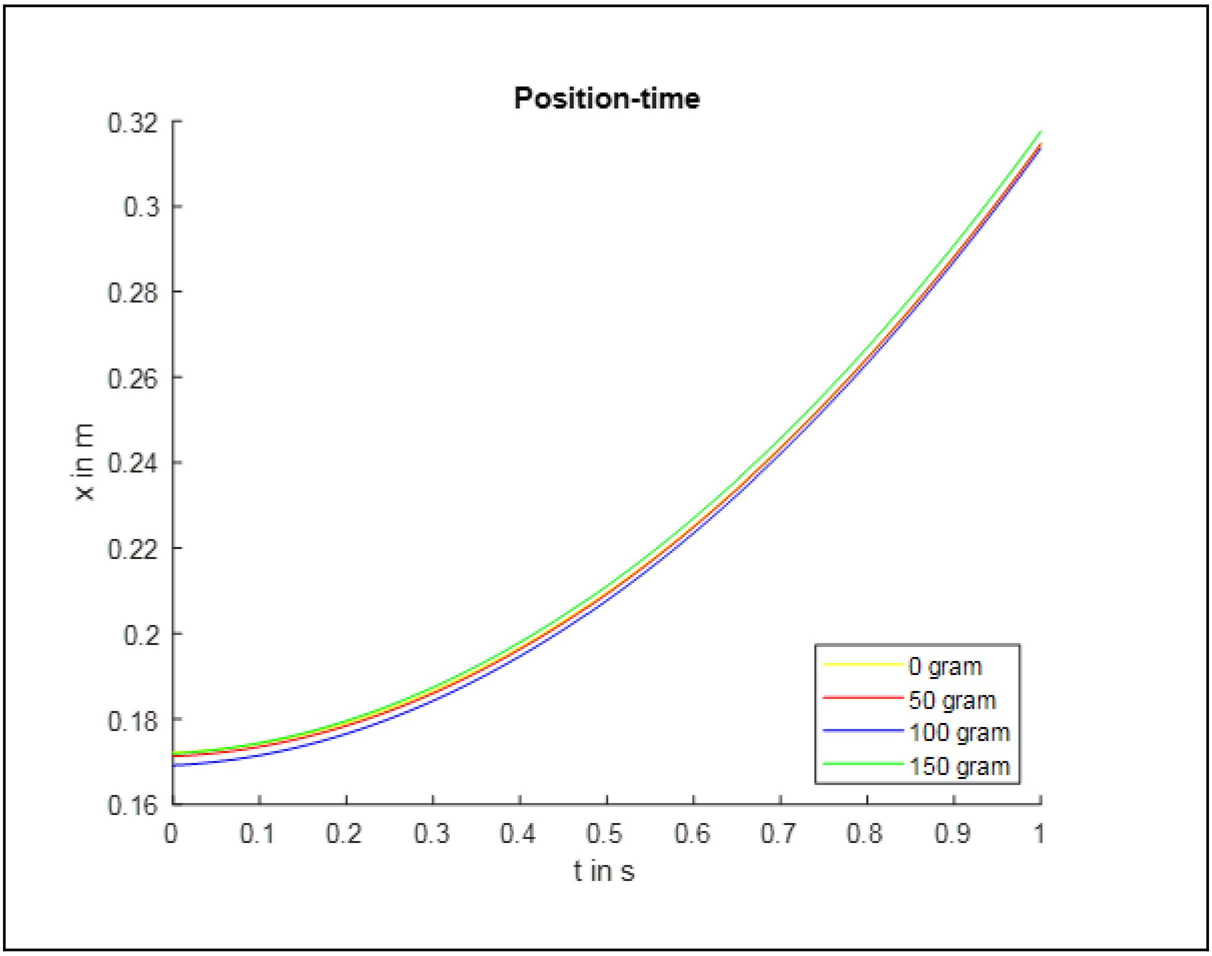
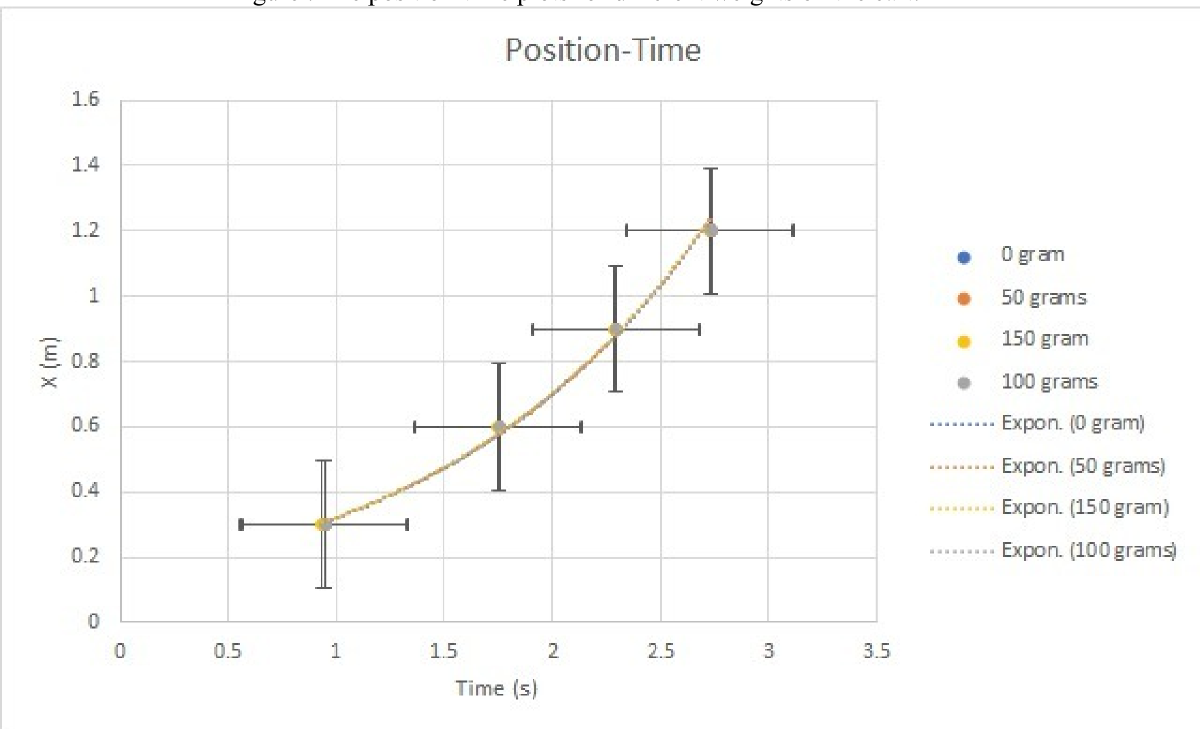


Figure : The position-time plots for different weights on the cart.



NOTE: Because the error command does not work in MATLAB, I redrawn the rule in Excel to specify the errorbar.

The fit-parabolas are given by:

weight 1: $x_1(t) = (a_1 \pm sa_1)t^2 + (b_1 \pm sb_1) = (0.1346 \pm 0.00235)t^2 + (0.1721 \pm 0.0072)$

weight 2: $x_1(t) = (a_2 \pm sa_2)t^2 + (b_2 \pm sb_2) = (0.1344 \pm 0.0001)t^2 + (0.1713 \pm 0.0003)$

weight 3: $x_3(t) = (a_3 \pm sa_3)t^2 + (b_3 \pm sb_3) = (0.1343 \pm 0.0035)t^2 + (0.1691 \pm 0.01065)$

weight 4: $x_4(t) = (a_4 \pm sa_4)t^2 + (b_4 \pm sb_4) = (0.1340 \pm 0.0016)t^2 + (0.1719 \pm 0.0049)$

Acceleration for different weights on the cart:

(a1 ± u(a1)) = (0.2692 ±0.0047)m/s2

(a2 ± u(a2)) = (0.2688 ±0.0002)m/s2

(a3 ± u(a3)) = (0.2686 ±0.007)m/s2

(a4 ± u(a4)) = (0.268 ± 0.0032)m/s2

The measured values for the acceleration are obtained as in 2a) from the quadratic part of the fit-parabola. The accelerations are then plotted against the weight of the cart. If the acceleration does not depend on the weight of the cart, the data points should lie on a line with slope zero. This is checked by performing a linear regression with the data points.

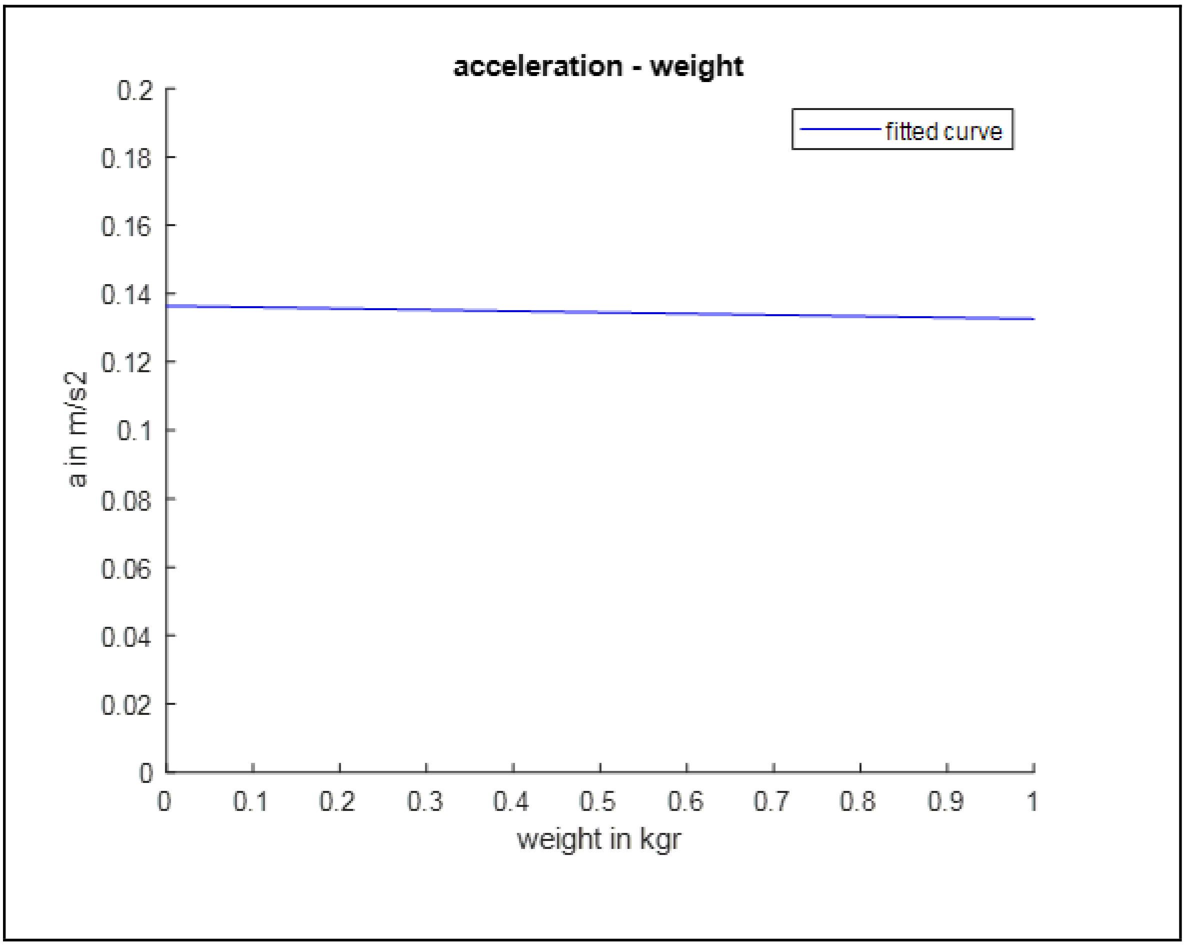
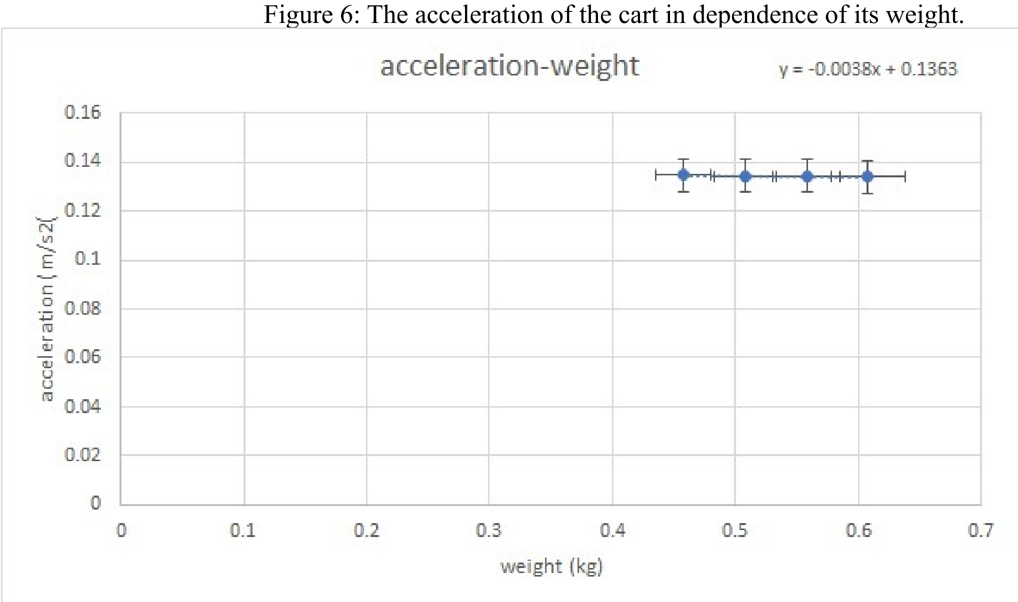


Figure 6: The acceleration of the cart in dependence of its weight.



NOTE: Because the error command does not work in MATLAB, I redrawn the rule in Excel to specify the errorbar.

The regression line is given by:

$a(m) = (a \pm sa)m + (b \pm sb) = (-0.0076 \pm 0.004554)m + (0.2726 \pm 0.0025)$m/s2

Step 3: The results

The evaluation shows, that the acceleration does not depend on the mass of the cart.

The mean value of the four measured accelerations is 0.134325 m/s^2

2) Measurement of the acceleration for different inclination angles

Step 1: The data

The mean values and uncertainties for the arrival times are calculated with different angles of inclination.

Inclination angle 1 = 1.6±0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.939	1.745	2.287	2.724
2	0.946	1.750	2.291	2.721

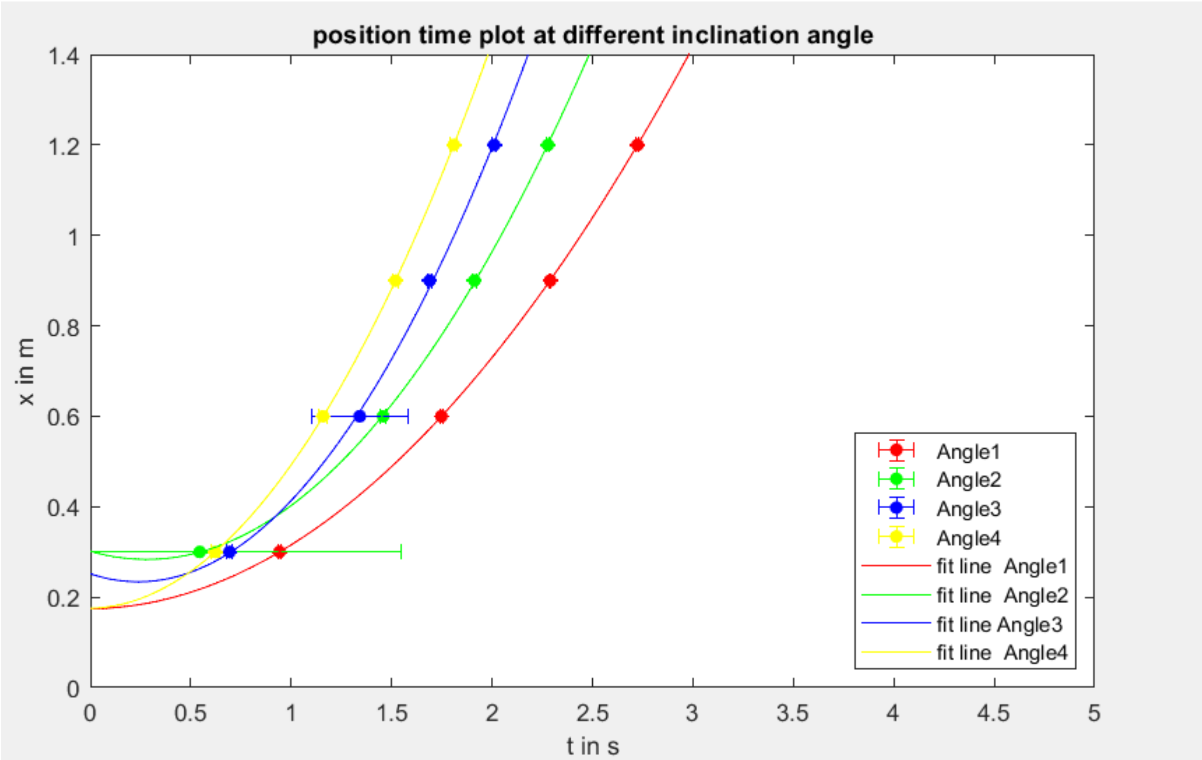
3	0.940	1.747	2.289	2.726
mean in s	0.941	1.747	2.289	2.724
u() in s	0.009	0.006	0.050	0.006

Inclination angle 2 =2.3 ±0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.0782	1.460	1.913	2.279
2	0.784	1.461	1.915	2.281
3	0.771	1.452	1.908	2.274
mean in s	0.544	1.458	1.912	2.278
u() in s	1.003	0.012	0.009	0.009

Inclination angle 3 = 3.0±0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.689	1.286	1.686	2.007
2	0.690	1.286	1.686	2.008

3	0.699	1.452	1.695	2.016
mean in s	0.693	1.341	1.689	2.010
$u()$ in s	0.014	0.238	0.013	0.012
Inclination angle 4 = 3.8±0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.631	1.167	1.527	1.817
2	0.617	1.156	1.517	1.807
3	0.615	1.153	1.514	1.803
mean in s	0.621	1.159	1.519	1.809
$u()$ in s	0.022	0.018	0.017	0.018

The fit-parabolas are given by:



Angle 1: $x_1(t) = (a_1 \pm sa_1)t^2 + (b_1 \pm sb_1)$ =(0.1363 ± 0.0242) + (0.1743 ± 0.0731)

Angle 2: $x_1(t) = (a_2 \pm sa_2)t^2 + (b_2 \pm sb_2)$ =(0.2299 ± 0.1952) + (0.3014 ± 0.326)

Angle 3: $x_3(t) = (a_3 \pm sa_3)t^2 + (b_3 \pm sb_3)$ =(0.309± 0.6334) ± (0.2514 ± 1.0326)

Angle 4: $x_4(t) = (a_4 \pm sa_4)t^2 + (b_4 \pm sb_4)$ =(0.307 ± 0.0212) ± (0.1753 ± 0.0282)

Acceleration for different angles on the cart:

(a1 ± u(a1)) = (0.2726 ±0.0484)m/s^2

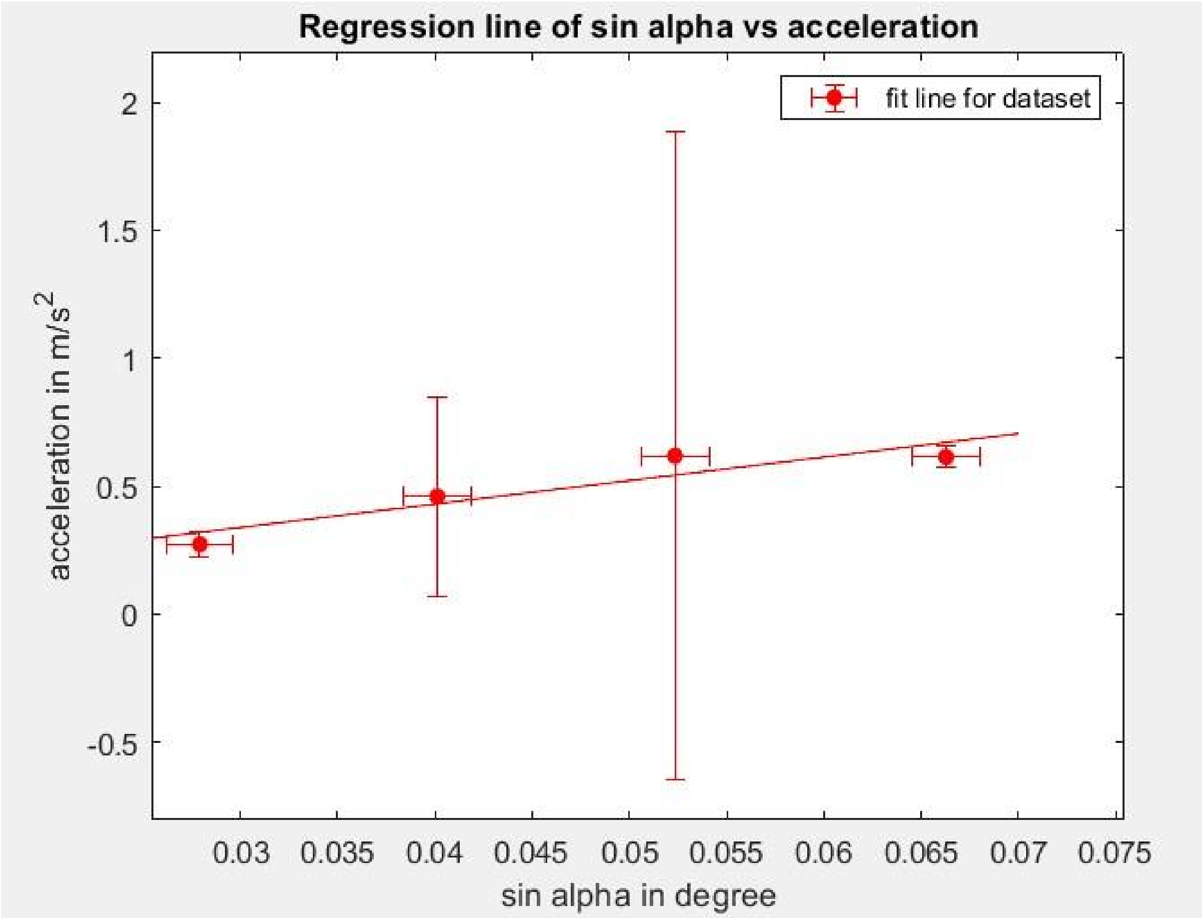
(a2 ± u(a2)) = (0.4598 ± 0.3904)m/s^2

(a3 ± u(a3)) = (0.6180 ± 1.2668)m/s^2

(a4 ± u(a4)) = (0.6140 ± 0.0424)m/s^2

Discussion

The measured values for the acceleration are obtained as in the plot from the quadratic part of the fit-parabola. The accelerations are then plotted against the sin α of the cart. A linear regression line is use to check the hypothesis. If the acceleration does not depend on the angle the inclined plane, the data points lie on a line with slope zero.



Regression line of acceleration against sin α

* Regression line for a -sin α - plot:

a(sin α)= (9.165 ±11.6265)sin α

* Gravitational acceleration:

g = (9.165±0.01)m/s^2

RESULT

The regression line of the acceleration-sin α is a curve with a positive slope which implies that the acceleration of a cart is dependent on the angle of inclination of surface. Experimental value g = (9.165 ±0.01)m/s^2 .Possible sources of error during recording and calculation of data as well as approximation of values.

Experiment Inclined Plane

Tasks

1) Check if the accelerations on the inclined plane depends on the mass.
2) Determine the gravitational acceleration.

1 Theory

The relation of a force that acts on a body and the resulting motion of the body as a consequence of this force is described by Newton's second law

$$\vec{F} = m\,\vec{a}, \tag{1}$$

where $\vec{F}$ is the external force acting on the body, $\vec{a}$ the resulting acceleration of the body and m the mass of the body. This equation means, that the acceleration is always in the direction of the external force and that the vectors $\vec{F}$ and $m\vec{a}$ have the same length, which gives the scalar version of Newton's second law:

$$F = m\,a, \tag{2}$$

with $F = |\vec{F}|$ and $a = |\vec{a}|$.

In the case of a free fall the external force is the gravitational force $\vec{F}_G$, which on the scale of the laboratory can be assumed as a constant, downward pointing force. The magnitude of the gravitational force the body experiences is proportional to the mass of the body, where the constant of proportionality is $g = 9.81\,\mathrm{m/s^2}$, i.e. $F_G = |\vec{F}_G| = m\,g$. Newton's second law can then be written as $\vec{F}_G = m\,\vec{a}$, or in components:

$$\begin{pmatrix} 0 \\ -mg \\ 0 \end{pmatrix} = m \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}. \tag{3}$$

From this follows, that in free fall there is only a non-vanishing acceleration in y-direction with $a_y = -g$. Or, reading Newton's second law in the other direction: if a body with mass m is accelerated downwards in free fall with an acceleration $\vec{g}$, it experiences a gravitational force $\vec{F}_G = m\,\vec{g}$.

For a constant external force, i.e. if the force vector does not change in time and in space, the resulting movement of the body is according to Newton's second law a linear movement with constant acceleration, where the acceleration points in the same direction as the force. The corresponding vector version of the position time relation for a movement with constant acceleration is:

$$\vec{x}(t) = \frac{1}{2}\vec{a}\,t^2 + \vec{v}_0\,t + \vec{x}_0, \tag{4}$$

where $\vec{x}_0 = \vec{x}(t=0)$ is the initial position of the body, $\vec{v}_0$ its initial velocity and $\vec{a} = \vec{F}/m$ its acceleration due to the external force $\vec{F}$. In the free fall case with no initial velocity the movement is vertical and is described by the y-compontent of the position time equation (4):

$$y(t) = -\frac{1}{2}g\,t^2 + y_0. \tag{5}$$

Before stopwatches were available it was very hard to measure the gravitational acceleration g accurately. For that reason Galileo Galilei thought of a slow motion version of the free fall experiment. This experiment

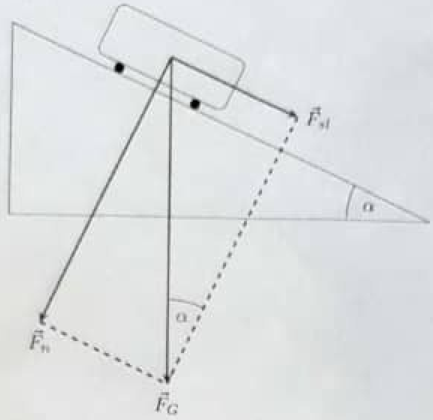


Figure 1: The inclined plane.

is the inclined plane, because a body that is moving down an inclined plane with small inclination angle α is accelerated with an acceleration much smaller than g. If there is an accelerated movement along the direction of the inclined plane, then, according to Newton's second law, there has to be a force in the same direction, i.e. parallel to the inclined plane. This force is called the slope force $\vec{F}_{sl}$ (see figure 1). But since the only physical force in this situation is gravity and the body is surely going down the inclined plane due to gravity, the slope force must be related to the gravitational force.

Forces are vector valued quantities, which means that if two forces are acting on the same body, the resulting force, that accelerates the body, is the vector sum of the two single force vectors. And conversely it is possible to write a given force vector as the vector sum of two other force vectors. In the case of the inclined plane one can write the gravitational force as the vector sum of the slope force and the normal force. The normal force $\vec{F}_n$ is the force perpendicular to the inclined plane, that holds the body on the inclined plane (see figure 1):

$$\vec{F}_G = \vec{F}_{sl} + \vec{F}_n. \tag{6}$$

For the length of the slope vector and the length of the normal vector follows then:

$$F_{sl} = m\,g\,\sin\alpha \quad \text{and} \quad F_n = m\,g\,\cos\alpha. \tag{7}$$

The slope force is the accelerating force, because it points in the direction of the acceleration, so that the scalar version of Newton's second law becomes $F_{sl} = ma$, or with (7): $mg\sin\alpha = ma$. This leads then to the following formula for the acceleration down the inclined plane:

$$\boxed{a = g\sin\alpha. \tag{8}}$$

Note, that in the limit $\alpha = 90^\circ$ one has again the free fall case with $a = g$.

2 Experimental Setup

The experimental setup is the same as in the experiment Kinematics. In order to obtain an inclined plane one can put some wooden boards beneath one of the foots of the track.

3 Measurement Procedure

1) Measurement of the acceleration for different weights on the cart

- ☐ Put some wooden plates under the foot of the track where the starter system is mounted, so that you obtain an inclined plane with small inclination angle.
- ☐ Determine the inclination angle using the tape measure.
- ☐ Start the cart with the starter system and let it roll down the inclined track. Measure the arrival times at 30 cm, 60 cm, 90 cm and 120 cm. Do this measurement altogether three times.
- ☐ Repeat the whole measurement then three more times with different additional weights below 400 g on the cart.

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the arrival times and the corresponding measurement uncertainty of the mean value.
- ☐ Create a coordinate system with the arrival times on the horizontal axis and the distance of the waypoints to the start on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the four groups of data points different symbols and colors.
- ☐ Use MATLAB for calculating a fit parabola without linear term through each of the four groups of data points. Plot the four parabolas into the coordinate system with the data points. Determine for each of the four cases the acceleration including the measurement uncertainty.
- ☐ Create a second coordinate system with the total mass of the cart, i.e. mass of the cart plus the mass of the additional weights, on the horizontal axis and the acceleration on the vertical axis. Plot the four data points (with their measurement uncertainties as error bars) in the coordinate system and use MATLAB to calculate the regression line.

Answer the following question:

- Does the acceleration of the cart depend on its weight?
- Does the acceleration agree with the theoretical value?

2) Measurement of the acceleration for different inclination angles

- ☐ Remove the additional weights from the cart.
- ☐ Put some more wooden plates under the foot of the track where the starter system is mounted, so that you obtain an inclined plane with a different inclination angle.
- ☐ Determine the inclination angle using the tape measure.
- ☐ Start the cart with the starter system and let it roll down the inclined track. Measure the arrival times at 30 cm, 60 cm, 90 cm and 120 cm. Do this measurement altogether three times.

- ☐ Repeat the whole measurement then three more times with stepwise higher inclination angles.

Evaluation:

- ☐ Calculate for each group of three measurement values the mean value of the arrival times and the corresponding measurement uncertainty of the mean value.
- ☐ Create a coordinate system with the arrival times on the horizontal axis and the distance of the waypoints to the start on the vertical axis. Plot the calculated data points (with their measurement uncertainties as error bars) in the coordinate system. Use for the four groups of data points different symbols and colors.
- ☐ Use MATLAB for calculating a fit parabola without linear term through each of the four groups of data points. Plot the four parabolas into the coordinate system with the data points. Determine for each of the four cases the acceleration including the measurement uncertainty.
- ☐ Create a second coordinate system with the sine of the inclination angle on the horizontal axis and the acceleration on the vertical axis. Plot the four data points and as a fifth data point your measurement without additional mass in 1) (with their measurement uncertainties as error bars) in the coordinate system and use MATLAB to calculate a regression line without intercept.

Answer the following question:

- What is your measured value for the gravitational acceleration g?

4 Preparatory Exercises

a) Prove geometrically that the angle between the gravitational force vector $\vec{F}_G$ and the normal force vector $\vec{F}_n$ is again the inclination angle α (see figure 1).

b) Write down the three components of the slope vector $\vec{F}_{sl}$ and of the normal vector $\vec{F}_n$ and then prove equation (6).

$\boxed{\alpha = 1.6}$

Setup No.	Names	Date:
		Sign. of Supervisor: P. H.

1) Measurement of the acceleration for different weights on the cart

mass of the cart=(457.4 ± 0.1) ____ , α=(1.6 ± 0.1)

additional mass 1 = 0 g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.947	1.753	2.294	2.731
2	0.949	1.759	2.303	2.741
3	0.941	1.748	2.291	2.729

additional mass 2 = 50 g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.968	1.778	2.321	2.760
2	0.926	1.731	2.273	2.711
3	0.944	1.750	2.292	2.729

additional mass 3 = 100 g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.933	1.741	2.285	2.723
3	0.947	1.763	2.308	2.748
2	0.970	1.756	2.293	2.727

additional mass 4 = 150 g				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.946	1.757	2.301	2.790
2	0.942	1.751	2.295	2.734
3	0.918	1.725	2.268	2.706

$a = g \sin \alpha$

2) **Measurement of the acceleration for different inclination angles**

inclination angle 1 = 1.6 ± 0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.939	1.745	2.287	2.724
2	0.946	1.750	2.291	2.727
3	0.940	1.747	2.289	2.726

$\sin\alpha = \frac{4\,cm}{96}$

$\alpha = 2.65$

inclination angle 2 = 2.3 ± 0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.782	1.460	1.913	2.279
2	0.784	1.461	1.915	2.281
3	0.771	1.452	1.908	2.274

$\sin\alpha = \frac{5.1}{96}$

$\alpha = 3.45$

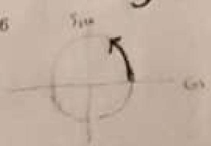
inclination angle 3 = 3.0 ± 0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.689	1.286	1.686	2.007
2	0.690	1.286	1.686	2.008
3	0.699	1.295	1.695	2.016

$\sin\alpha = \frac{6.2}{96}$

$\alpha = 41$

inclination angle 4 = 3.8 ± 0.1				
	30 cm	60 cm	90 cm	120 cm
measurement	arrival times in s	arrival times in s	arrival times in s	arrival times in s
1	0.631	1.167	1.577	1.817
2	0.617	1.156	1.517	1.807
3	0.615	1.153	1.514	1.803

$a = g\sin\alpha$



Results Experiment Inclined Plane

1) Measurement of the acceleration for different weights on the cart

- Fit parabolas for different weights on the cart:

$$x_1(t) = (\mathrm{a}_1 \pm \mathrm{sa}_1)t^2 + (\mathrm{c}_1 \pm \mathrm{sc}_1) = (0.1346 \pm 0.00235)t^2 + (0.1721 \pm 0.0072)$$
$$x_2(t) = (\mathrm{a}_2 \pm \mathrm{sa}_2)t^2 + (\mathrm{c}_2 \pm \mathrm{sc}_2) = (0.1344 \pm 0.0001)t^2 + (0.1713 \pm 0.0003)$$
$$x_3(t) = (\mathrm{a}_3 \pm \mathrm{sa}_3)t^2 + (\mathrm{c}_3 \pm \mathrm{sc}_3) = (0.1343 \pm 0.0035)t^2 + (0.1691 \pm 0.01065)$$
$$x_4(t) = (\mathrm{a}_4 \pm \mathrm{sa}_4)t^2 + (\mathrm{c}_4 \pm \mathrm{sc}_4) = (0.1340 \pm 0.0016)t^2 + (0.1719 \pm 0.0049)$$

- Acceleration for different weights on the cart:

$$(a_1 \pm u(a_1)) = (0.2692 \pm 0.0047)\ m/s^2$$
$$(a_2 \pm u(a_2)) = (0.2688 \pm 0.0002)\ m/s^2$$
$$(a_3 \pm u(a_3)) = (0.2686 \pm 0.007)\ m/s^2$$
$$(a_4 \pm u(a_4)) = (0.2680 \pm 0.0032)\ m/s^2$$

$$\text{mean value}: (a \pm u(a)) = (0.26865 \pm 0.0039 66)\ m/s^2$$

- Calculated acceleration:

$$a = 0.26865\ m/s^2$$

- Regression lines for a-m-plot:

$$a(m) = (-0.0076 \pm 0.004554 + 0.2726 \pm 0.0025$$

2) Measurement of the acceleration for different inclination angles

- Fit parabolas for different inclination angles:

$$x_1(t) = (\mathrm{a}_1 \pm \mathrm{sa}_1)t^2 + (\mathrm{c}_1 \pm \mathrm{sc}_1) = (0.1363 \pm 0.0242)t^2 + (0.1743 \pm 0.0731)$$
$$x_2(t) = (\mathrm{a}_2 \pm \mathrm{sa}_2)t^2 + (\mathrm{c}_2 \pm \mathrm{sc}_2) = (0.2299 \pm 0.1952)t^2 + (0.3014 \pm 0.326)$$
$$x_3(t) = (\mathrm{a}_3 \pm \mathrm{sa}_3)t^2 + (\mathrm{c}_3 \pm \mathrm{sc}_3) = (0.3091 \pm 0.6182)t^2 + (0.2514 \pm 0.5029)$$
$$x_4(t) = (\mathrm{a}_4 \pm \mathrm{sa}_4)t^2 + (\mathrm{c}_4 \pm \mathrm{sc}_4) = (0.3091 \pm 0.6182)t^2 + (0.1753 \pm 0.0563)$$

- Acceleration for different inclination angles:

$$(a_1 \pm u(a_1)) = (0.2726 \pm 0.0484)\ m/s^2$$
$$(a_2 \pm u(a_2)) = (0.4598 \pm 0.3904)\ m/s^2$$
$$(a_3 \pm u(a_3)) = (0.6180 \pm 1.2668)\ m/s^2$$
$$(a_4 \pm u(a_4)) = (0.6140 \pm 0.0424)\ m/s^2$$

- Regression line for a - $\sin\alpha$ - plot:

$$a(\sin\alpha) = (9.165 \pm 11.6265)\sin\alpha$$

- Gravitational acceleration:

$$g = (9.165 + 0.01)\ m/s^2$$