School Consolidation model:

School consolidation is the act of merging schools in order to provide better facilities and enhance mono-grade teaching. The proposed mathematical model minimizes the number to students that are transferred in order to close the schools. It is ensured that the students will not need to travel a maximum additional distance of D1. There will not be more than one school within D2 distance of every school.

In a special case where a small school having enrolment less than N_{min} exist, it will only close if it has a school within D3 distance. All schools having enrolment beyond N_{max} will be retained provided they are more than D2 distance from each other, where D2 is optional for current model. The values of N_{min} & N_{max} are considered same in the current model.

Further, an upper bound to the grade wise capacity for each school can be specified.

The model gives the output of the school status (open/close), total students in each grade of the open schools, the number of students transferred from old school to new school.

The model has various constraints mathematically given in the annexure explained as follows:

1. Access constraint: It ensures that the students do not travel a maximum additional distance D1.
2. Logical constraints: It ensures that transfer only takes place to one open school from any number of the closed schools.
3. Small school closing constraint: It forces those small schools to close that have low enrolment and have its nearest school within D3 distance.
4. Large school opening constraint: It forces large schools to open if they are more than D2 from each other (Kept as an option right now).
5. No school distance constraint: There will be at most one school within D2.
6. Flow balance constraint: It ensures that all students are allocated to schools and to the same grade.
7. Capacity constraint: It ensures that the total students in any grade after consolidation does not exceed the capacity.

Decision Variables: $x_{i,g,j}$ (1,if transfer takes place from to I to j of grade g)and y_j (1 if school is open after consolidation) both binary indicating the transfer and open/close decision of the schools and $n_{j,g}$ (where n is integer multiplier of 30 to ensure lower deviations from 30:1 PTR).

Annexure: Mathematical model

S. No	Type of constraint	Constraint	Model 1	Model 2
1	Access constraint	$x_{i,g,j} * d_{i,j} <= D1 \quad \forall$ i, j, g	Yes	Yes
2	Logical constraints	$x_{i,g,j} <= y_j \quad \forall$ i, j, g $\sum_j x_{i,g,j} <= 1 - y_i \quad \forall$ i, g	Yes	Yes
3	Small school closing constraints	$(\sum_g N_{j,g} - N_{\min}) u_j >= 0$ $(d\min_j - D3) v_j >= 0 \quad \forall$ j $y_j = u_j + v_j$	No (Not part of current model)	No (Not part of current model)
4	Large school opening constraint	$(\sum_g N_{j,g} - N_{\max})(1 - y_j) <= 0$ $\forall$ j	Yes	Yes
5	No school distance constraint	$(d_{i,j} - D2) >= (y_i + y_j - 2) * M$ $\forall$ (i, j)ϵ S, S is school pair having small-small & small-large school size combination	Yes	No
6	Flow balance constraint	$O_{j,g} = (\sum_i N_{i,g} x_{i,g,j}) + N_{j.g} y_j$ $\forall$ j, g $\sum_j O_{j,g} = \sum_i N_{i,g} \quad \forall$ g	Yes	Yes
7	Capacity constraint	$C_{j,g} y_j >= O_{j,g} \quad \forall$ j, g	Yes	Yes
	Objective	Minimize	$\sum_i \sum_g \sum_j N_{i,g} x_{i,g,j}$	$\sum_j y_j$

Additional models:

Model	Constraint	Objective
Model 3	Model 1 constraints and $Z>=(O_{j,g}-n_{j,g}*STR)$ & $Z>=-(O_{j,g}-n_{j,g}*STR)$	Minimize $\sum_i \sum_g \sum_j N_{i,g} x_{i,g,j}$ + cz
Model 4	Model 1 constraints	Minimize $\sum_i \sum_g \sum_j N_{i,g} x_{i,g,j} d_{i,j}$
Model 5	Model 1 constraints and $Z>=(O_{j,g}-n_{j,g}*STR)$ & $Z>=-(O_{j,g}-n_{j,g}*STR)$	Minimize $\sum_i \sum_g \sum_j N_{i,g} x_{i,g,j} d_{i,j}$ + cz
Model 6	Model 2 constraints and $Z>=(O_{j,g}-n_{j,g}*STR)$ & $Z>=-(O_{j,g}-n_{j,g}*STR)$	$\sum_j y_j$ + cz
Model 7	Model 2 constraints and $Z>=(O_{j,g}-n_{j,g}*STR)$ & $Z>=-(O_{j,g}-n_{j,g}*STR)$	$\sum_i \sum_g \sum_j N_{i,g} x_{i,g,j}$ + cz+M* $\sum_j y_j$
Model 8	Model 2 constraints	$\sum_i \sum_g \sum_j N_{i,g} x_{i,g,j}$ + M* $\sum_j y_j$

Where STR is student teacher ratio whose default value is 30.