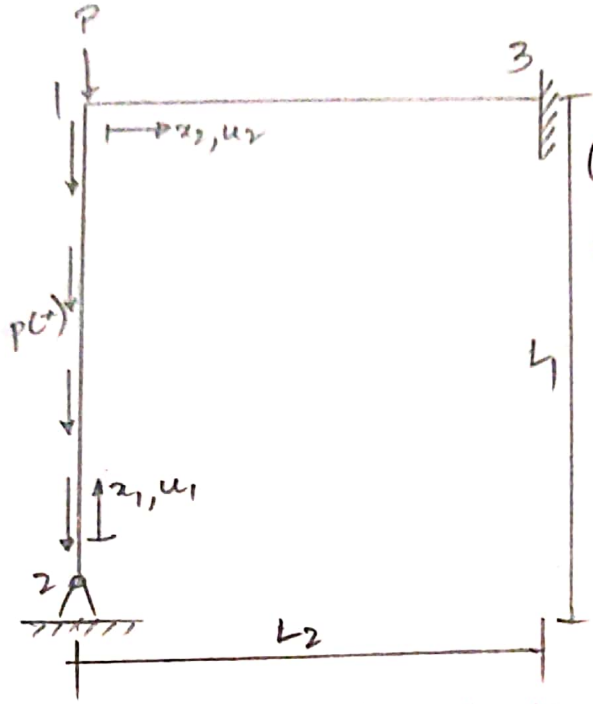
BIWO-03 Energy Methods, FEM

Assignment - 2

Matriculation number - 4993166

1. Given



(21) $I(x_1) = I_0\left(1+\frac{x_1}{L}\right)$

(13) $I(x_1) = I_0$

$L_1 = L_2 = 3m$

$E = 2.1\times10^8\ kN/m^2$

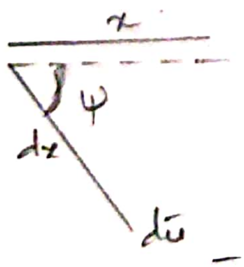
$I_0 = 3\times10^{-4}\ m^4$

$p(x) = P_0 = \frac{P}{2L}$

potential energy of neighbouring state for single beam

$$\Pi_{IN} = \frac{1}{2}\int_0^L EI(x)\bar{v}''(x)^2 dx + \ldots\ldots +$$

$$\approx 1/2\left(\delta^2\Pi_0\right)_N \approx \delta\left(\Pi_{IN}+\Pi_{EN}\right)_N$$



$$d\bar{u} = dx - dx\cos\psi$$

$$\approx dx\,(1-\cos\psi)$$

$$\approx dx\cdot\frac{\psi^2}{2} = dx\cdot\frac{\bar{v}'^2}{2}$$

$$\bar{u}(x) = \int_0^x \frac{v'(x)^2}{2}dx + \bar{u}(0)$$

$$u(L) = -\int_0^L \frac{v'(x)^2}{x}dx + \bar{u}(0)$$

$$\Pi_{EN} = -W_{Iex} = -\int_0^L -p(x)\cdot u(x)dx - \left(-P\bar{u}(L)\right)$$

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$$\therefore \Pi_{ex} = -\int_0^L p(x) \int_0^x \left(\frac{v'(x)^2}{2} dx \right) dx - p \int_0^L \frac{v'(x)^2}{2} dx.$$

Overall potential is

$$\Pi_{IN,EN} = \Pi_{IN} + \Pi_{EN}$$

$$= \frac{1}{2} E \int_0^{L_1} I(x_1)\, v_1''(x_1)^2 dx + \frac{1}{2} E \int_0^{L_2} I(x_2)\, v_2''(x_2)^2 dx_2$$

$$- \int_0^{L_1} p(x_1) \left[\int_0^x \frac{v_1'(x_1)^2}{2} dx_1 \right] dx_1 - p \int_0^{L_1} \frac{v_1'(x_1)^2}{2} dx_1$$

Using Ritz method to solve the problem.

1. $\bar{v}_N(x) = \sum a_j \varphi_j(x)$
2. $\delta \Pi_{IN} = 0$
3. $\frac{\partial \Pi}{\partial a_j} \Rightarrow \underline{K} \cdot \underline{a} = 0$

   where $a_j \neq 0$ if $\det \underline{K} = 0 \Rightarrow P_{crit}$

Boundary conditions are :

Beam 1

$v_1(0) = 0, \quad v_1(L_1) = 0$

$v_1'(0) \neq 0, \quad v_1'(L_1) = +\varphi(1) \neq 0$

Beam 2

$v_2(0) = 0, \quad v_2(L_2) = 0$

$\cancel{v_2'(0) \neq 0}, \quad v_2'(0) = +\varphi(1) \neq 0$

$v_2'(L_2) = 0.$

Ansatz functions

$$v_1(x_1) = \left(-2x + \frac{2x_1^2}{L_1} \right) \varphi(1)$$

$$v_1'(x_1) = \left[-2 + \frac{4x_1}{L_1} \right] \varphi(1)$$

$$v_1''(x_1) = \left(\frac{4}{L_1} \right) \varphi(1)$$

$$v_2(x_2) = \left(2x_2 - \frac{4x_2^2}{L_2} + \frac{2x_2^3}{L_2^2} \right) \varphi(1)$$

$$v_2'(x_2) = \left[2 - \frac{8x_2}{L_2} + \frac{6x_2^2}{L_2^2} \right] \varphi(1)$$

$$v_2''(x_2) = \left[\frac{-8}{L_2} + \frac{12x_2}{L_2^2} \right] \varphi(1).$$

$$\therefore I_1 = \frac{1}{2}E\int_0^{L_1} I(x_1)\, v_1''(x_1)^2\, dx$$

$$= \frac{1}{2}EI_0\int_0^{L_1}\left(1+\frac{x_1}{L_1}\right)\left(\frac{16}{L_1^2}\right)\psi(1)^2\, dx_1$$

$$= \frac{1}{2}EI_0\,\psi(1)^2\int_0^{L_1}\left[\frac{16}{L_1^2}+\frac{x\cdot 16}{L_1^3}\right]dx_1$$

$$= \frac{1}{2}EI_0\,\psi(1)^2\left[\frac{16x}{L_1^2}+\frac{8x^2}{L_1^3}\right]_0^{L_1}$$

$$= \frac{1}{2}EI_0\,\psi(1)^2\left[\frac{16}{L_1}+\frac{8}{L_1}\right] = \frac{1}{2}EI_0\,\psi(1)^2\left(\frac{24}{L_1}\right).$$

$$= \frac{1}{2}\times 2.1\times 10^8\times 3\times 10^{-4}\times\frac{24}{3}\psi(1)^2$$

$$= 25.2\times 10^4\,\psi(1)^2.$$

$$I_2 = \frac{1}{2}E\int_0^{L_2} I(x_2)\cdot v_1''(x_2)^2\, dx_2$$

$$= \frac{1}{2}EI_0\int_0^{L_2}\left(\frac{-8}{L_2}+\frac{12x_2}{L_2^2}\right)^2\psi(1)^2\, dx_2$$

$$= \frac{1}{2}EI_0\,\psi(1)^2\int_0^{L}\left[\frac{64}{L_2^2}+\frac{144x_2^2}{L^4}-\frac{192x_2}{L^3}\right]dx_2$$

$$= \frac{1}{2}EI_0\,\psi(1)^2\left[\frac{64x_2}{L_2^2}+\frac{48x_2^3}{L_2^4}-\frac{96x_2^2}{L_2^3}\right]_0^{L_2}$$

$$= \frac{1}{2}EI_0\,\psi(1)^2\left[\frac{16}{L}\right] = \frac{1}{2}\times 2.1\times 10^8\times 3\times 10^{-4}\times\frac{16}{3}\psi(1)^2$$

$$= 16.8\times 10^4\,\psi(1)^2$$

$$I_3 = \int_0^L P(x_1)\left(\int_0^x \frac{v_1'(x_1)^2}{2}\,dx_1\right)dx_1 = \int_0^L \frac{P}{2L}\left(\int_0^x \left(\frac{-2+4x}{L}\right)^2\Big/2\;\psi(1)^2\,dx_1\right)dx_1$$

$$= \frac{P}{4L}\int_0^L\int_0^x\left(-2+\frac{4x}{L}\right)^2\psi(1)^2\,dx_1\,dx_1$$

$$= \frac{P}{4L}\psi(1)^2\int_0^L\int_0^x\left[4+\frac{16x_1^2}{L_1^2}-\frac{16x_1}{L_1}\right]dx_1\,dx_1$$

$$= \frac{P}{4L}\psi(1)^2\int_0^L\left[4x_1+\frac{16x_1^3}{3L_1^2}-\frac{8x_1^2}{L_1}\right]dx_1$$

$$= \frac{P}{4L}\psi(1)^2\left[\frac{4x_1^2}{2} + \frac{16x^4}{4\times 3L_1^2} - \frac{8x_1^3}{3L_1}\right]_0^{L_1}$$

$$= \frac{P}{4L}\psi(1)^2\left[2L_1^2 + \frac{4L_1^2}{3} - \frac{8L_1^2}{3}\right]$$

$$= \frac{P}{4L}\psi(1)^2\left[\frac{2L_1^2}{3}\right]$$

$$= \frac{PL}{6E}\psi(1)^2.$$

$$I_4 = P\int_0^L \frac{\vartheta_1'(x_1)^2}{2}dx = \frac{1}{2}P\psi(1)^2\int_0^L\left[-2+\frac{4x_1}{L}\right]^2 dx_1$$

$$= \frac{1}{2}P\psi(1)^2\left[4L_1 + \frac{16L_1^3}{3L_1^2} - \frac{8L_1^2}{L}\right]$$

$$= \frac{1}{2}P\psi(1)^2\left[\frac{4L}{3}\right].$$

$$\Pi_{IN} = \frac{12}{L}EI_0\psi(1)^2 + \frac{8}{L}EI_0\psi(1)^2 - \frac{L^2}{3}P_0\psi(1)^2 - \frac{2L}{3}P\cdot\psi(1)^2$$

(Take $L_1 = L_2 = 3m = L$).

$$\frac{\partial\Pi}{\partial\psi(1)} = 0$$

$$\frac{\partial}{\partial\psi}\left[\frac{20}{L}EI_0\psi(1)^2 - \frac{L^2}{3}P_0\psi(1)^2 - \frac{2L}{3}P\cdot\psi(1)^2\right] = 0.$$

$$\left[\frac{20}{L}EI_0\psi(1) - \frac{L^2}{3}P_0\psi(1) - \frac{2L}{3}\cdot P\cdot\psi(1)\right]2 = 0.$$

$$\left[\frac{20}{L}EI_0 - \frac{L^2}{3}P_0 - \frac{2L}{3}P\right]2\psi(1) = 0.$$

$$K\cdot\psi(1) = 0 \Rightarrow K = 0.$$

$$\frac{20}{L}EI_0 - \frac{L^2}{3}P_{0crit} - \frac{2L}{3}P_{crit} = 0.$$

but

$$P_{0crit} = f_{crit}\cdot P_0$$

$$P_{crit} = f_{crit}\cdot P$$

$$\frac{20}{L}EI_0 - \frac{L^2}{3}f_{crit}P_0 - \frac{2L}{3}f_{crit}P = 0.$$

$$f_{crit} = \frac{+(20/L)EI_0}{+\left(\frac{L^2}{3}\right)P_0 + \left(\frac{2L}{3}\right)P}$$

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$$P_{crit} = \frac{\left(\frac{20}{L}\right) EI_0}{\frac{K^2}{3} \times \frac{P}{24} + \frac{2K}{3} P}$$

$$P_{crit} = \frac{420000}{(5/2)P}$$

$$\boxed{P_{crit} = 168000\,kN}$$

Comparison:

From Seminar

$$K = \frac{18}{5} EI_0 \times \frac{2}{L} + \frac{17}{40} PL - \frac{17}{140} P_0 L^2 = 0$$

$$P_0 = P/2L$$

$$K = \frac{36}{5} \frac{EI_0}{L} + \frac{17}{40} PL - \frac{17}{280} PL = 0.$$

$$P_{crit} = 23.718 \frac{EI_0}{L^2}$$

$$P_{crit} = \frac{23.718 \times 2.1 \times 10^8 \times 3 \times 10^{-4}}{3 \times 3}$$

$$\boxed{P_{crit} = 166026\,kN}$$

The P_{crit} value in Seminar is less than value we obtained in assignment.

$$P_{crit\,(seminar)} < P_{crit\,(assignment)}.$$

It depends upon the degree of ansatz function which varies for one problem to other. The greater the degree of function the closer is the approximate solution to the real solution.