Energy Methods, FEM

Assignment – 3, $\boxed{4993166}$

Sol: Given

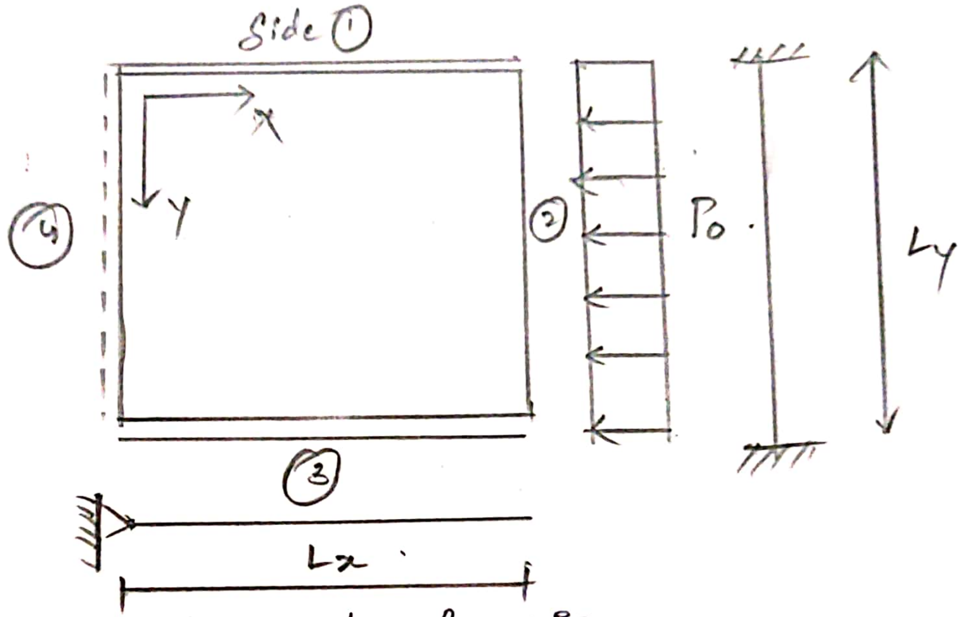
$E = 2\times10^{8}\ kN/m^2$

$\nu = 0.3$

$t = 8mm$

$L_x = 1.8m$

$L_y = 2.3m$



To find the correct ansatz function

Given functions

$$w_1 = a\left(\frac{x}{L_x} - \frac{x^2}{L_x^2}\right)\left(\frac{y}{L_y} - \frac{y^2}{L_y^2}\right) \qquad w_2 = a\cdot\frac{x}{L_x}\left(\frac{y^2}{L_y^2} - \frac{2y^3}{L_y^3} + \frac{y^4}{L_y^4}\right)$$

$$w'_{1x} = a\left[\frac{1}{L_x} - \frac{2x}{L_x^2}\right]\left(\frac{y}{L_y} - \frac{y^2}{L_y^2}\right) \qquad w_{2x} = \frac{a}{L_x}\left(\frac{y^2}{L_y^2} - \frac{2y^3}{L_y^3} + \frac{y^4}{L_y^4}\right)$$

$$w'_{1y} = a\left[\frac{x}{L_x} - \frac{x^2}{L_x^2}\right]\left(\frac{1}{L_y} - \frac{2y}{L_y^2}\right) \qquad w_{2y} = \frac{ax}{L_x}\left(\frac{2y}{L_y^2} - \frac{6y^2}{L_y^3} + \frac{4y^3}{L_y^4}\right)$$

Boundary Conditions:

For side 1 $(x, y=0) \Rightarrow w_N = 0$

$w'_x = 0$

$w'_y = 0$.

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$$W_1 = a(0-0)\left(\frac{x}{L_x}-\frac{x^2}{L_x}\right) = 0.$$

This function does not satisfy side ①

$$w'_{1x} = a\left[\frac{1}{L_x}-\frac{2x}{L_x}\right](0-0) = 0.$$

$$w'_{1y} = a\left[\frac{x}{L_x}-\frac{x^2}{L_x}\right]\left(\frac{1}{L_y}-0\right) \neq 0.$$

$$w_2 = a\cdot\frac{x}{L_x}\left[0-0+0\right] = 0.$$

$$w'_{2x} = a\cdot\frac{x}{L_x}[0] = 0, \quad w'_{2y} = a\frac{x}{L_x}[0] = 0.$$

For side ③:

$$(x, y=l_y) \Rightarrow w_N = 0,\ w'_x = 0,\ w'_y = 0.$$

$$w_1 = a\left[\frac{x}{L_x}-\frac{x^2}{L_x}\right]\left(\frac{L_y}{L_y}-\frac{L_y^2}{L_y^2}\right) = 0.$$

$$w'_{1x} = a\left[\frac{1}{L_x}-\frac{2x}{L_x}\right]\left[\frac{L_y}{L_y}-L_y^2/L_y^2\right] = 0.$$

This function does not satisfy side ①.

$$w'_{1y} = a\left[\frac{x}{L_x}-\frac{x^2}{L_x}\right]\left[\frac{1}{L_y}-\frac{2L_y}{L_y^2}\right] \neq 0.$$

$$w_2 = a\frac{x}{L_x}\left[\frac{L_y^2}{L_y^2}-\frac{2L_y^3}{L_y^3}+\frac{L_y^4}{L_y^4}\right] = \frac{ax}{L_x}\left[1-2+1\right] = 0.$$

$$w'_{2x} = \frac{ax}{L_x}\left[\frac{L_y^2}{L_y^2}-\frac{2L_y^3}{L_y^3}+\frac{L_y^4}{L_y^4}\right] = 0.$$

$$w'_{2y} = \frac{ax}{L_x}\left[\frac{2L_y}{L_y^2}-\frac{6L_y^2}{L_y^3}+\frac{4L_y^3}{L_y^4}\right] = \frac{ax}{L_x}\left[\frac{2}{L_y}-\frac{6}{L_y}+\frac{4}{L_y}\right] = 0.$$

For side (4) $(x=0, y) \Rightarrow w_N = 0$

$w'_x \neq 0,\ w'_y = 0$.

$$w_1 = a(0-0)\left(\frac{y}{L_y} - \frac{y^2}{L_y^2}\right) = 0.$$

$$w'_{1x} = a\left[\frac{1}{L_x} - 0\right]\left[\frac{y}{L_y} - \frac{y^2}{L_y^2}\right] \neq 0.$$

$$w'_{1y} = a\left[0 - 0\right]\left[\frac{1}{L_y} - \frac{2y}{L_y^2}\right] = 0.$$

$$w_2 = \frac{a}{L_x}\cdot 0\cdot\left[\frac{y^2}{L_y^2} - \frac{2y^3}{L_y^3} + \frac{y^4}{L_y^4}\right] = 0.$$

$$w'_{2x} = \frac{a}{L_x}\left[\frac{y^2}{L_y^2} - \frac{2y^3}{L_y^3} + \frac{y^4}{L_y^4}\right] \neq 0.$$

$$w'_{2y} = \frac{a}{L_x}\cdot 0\left[\frac{2y}{L_y^2} - \frac{6y^2}{L_y^3} + \frac{4y^3}{L_y^4}\right] = 0.$$

The second Ansatz function is the only one which satisfies all the sides boundary conditions.

To find overall potential energy we have Π_i, Π_{ex}

$$\boxed{\Pi_{Ii,N} = \int_0^{L_y}\int_0^{L_x} \frac{B}{2}\left[(\bar{w}_{xx}+\bar{w}_{yy})^2 - 2(1-\nu)(w_{xx}\cdot w_{yy} - \bar{w}_{xy}^2)\right]dx\,dy}$$

For Ansatz function.

$$w_{xx} = 0$$

$$w_{yy} = \frac{ax}{L_x}\left[\frac{2}{L_y^2} - \frac{12y}{L_y^3} + \frac{12y^2}{L_y^4}\right]$$

$$w_{xy} = \frac{a\cdot 1}{L_x}\left[\frac{2y}{L_y^2} - \frac{6y^2}{L_y^3} + \frac{4y^3}{L_y^4}\right].$$

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The above equation is converted to.

$$= \int_0^{L_y}\int_0^{L_x} \frac{B}{2}\left[\bar{w}_{xx}^2 + 2\cdot w_{xx}\cdot w_{yy} + w_{yy}^2 - 2(1-\nu)\left(w_{xx}\cdot w_{yy} - w_{xy}^2\right)\right] dx\,dy$$

$$w_{xx} = 0$$

$$= \int_0^{L_y}\int_0^{L_x} \frac{B}{2}\left[\underbrace{\bar{w}_{yy}^2}_{I_1} + 2(1-\nu)\underbrace{\bar{w}_{xy}^2}_{I_2} \right] dx\,dy .$$

$$I_1 = \int_0^{L_y}\int_0^{L_x} \bar{w}_{yy}^2 = \int_0^{L_y}\int_0^{L_x}\left[a\frac{x}{L_x}\left(\frac{2}{L_y^2} - \frac{12y}{L_y^3} + \frac{12y^2}{L_y^4}\right)\right]^2 dx\,dy$$

$$= a^2 \int_0^{L_y}\int_0^{L_x} \frac{x^2}{L_x^2}\left[\frac{4}{L_y^4} - \frac{48y}{L_y^5} + \frac{192y^2}{L_y^6} - \frac{288y^3}{L_y^7} + \frac{144y^4}{L_y^8}\right] dx\,dy$$

$$= \frac{a^2 L_x^{\cancel{3}}}{3\cancel{L_x^2}}\left[\frac{4L_y}{L_y^4} - \frac{24L_y^2}{L_y^5} + \frac{64L_y^3}{L_y^6} - \frac{72L_y^4}{L_y^7} + \frac{144L_y^5}{5L_y^8}\right]$$

$$= \frac{a^2 L_x}{3}\left(\frac{4}{5L_y^3}\right) = \frac{4a^2L_x}{15L_y^3} .$$

$$I_2 = \int_0^{L_y}\int_0^{L_x} \bar{w}_{xy}\,dy\,dx = \int_0^{L_y}\int_0^{L_x}\left[a\cdot\frac{1}{L_x}\left(\frac{2y}{L_y^2} - \frac{6y^2}{L_y^3} + \frac{4y^3}{L_y^4}\right)\right]^2 dx\,dy$$

$$= a^2 \int_0^{L_y}\int_0^{L_x} \frac{1}{L_x^2}\left[\frac{4y^2}{L_y^4} - \frac{24y^3}{L_y^5} + \frac{52y^4}{L_y^6} - \frac{48y^5}{L_y^7} + \frac{16y^6}{L_y^8}\right] dx\,dy$$

$$= a^2\cdot\frac{L_x}{L_x^2}\left[\frac{4L_y^3}{3L_y^4} - \frac{6L_y^4}{L_y^5} + \frac{52L_y^5}{5L_y^6} - \frac{8L_y^6}{L_y^7} + \frac{16L_y^7}{7L_y^8}\right]$$

$$= a^2\cdot\frac{1}{L_x}\cdot\frac{2}{105L_y} = \frac{2a^2}{105L_xL_y} .$$

$\therefore$ Overall p.E is $\Pi = \Pi_i + \Pi_e$.

$$\Pi_i = \frac{B}{2}\left[\frac{4a^2 L_x}{15 L_y^3} + 2(1-\nu)\frac{2a^2}{105 L_x L_y}\right]$$

$$\Pi_e = -\int_0^{L_y} +P(y)\cdot u(x=L_x, y)\, dy$$

$$= -\frac{1}{2}P\int_0^{L_y}\int_0^{L_x} \bar{w}_x^2\, dx\, dy$$

$$= \frac{-P}{2}\int_0^{L_y}\int_0^{L_x}\left(a\cdot\frac{1}{L_x}\left[\frac{y^2}{L_y^2} - \frac{2y^3}{L_y^3} + \frac{y^4}{L_y^4}\right]\right)^2 dy\, dx.$$

$$= \frac{-Pa^2}{2}\cdot\frac{L_x}{L_x^2}\left[\frac{L_y^5}{5L_y^4} - \frac{2L_y^6}{3L_y^5} + \frac{6L_y^7}{7L_y^6} - \frac{L_y^8}{2L_y^7} + \frac{L_y^9}{9L_y^8}\right]$$

$$= \frac{-Pa^2 L_y}{1260 L_x}.$$

$$\Pi_{Total} = \Pi_i + \Pi_e$$

$$= \frac{B}{2}\left[\frac{4a^2 L_x}{15 L_y^3} + 2(1-\nu)\frac{2a^2}{105 L_x L_y}\right] - \frac{Pa^2 L_y}{1260 L_x}$$

$$= a^2\left[\frac{B}{2}\left(\frac{4}{15}\frac{L_x}{L_y^3} + 2(1-\nu)\frac{2}{105 L_x L_y}\right) - \frac{P L_y}{1260 L_x}\right]$$

Equilibrium Condition

$$\frac{\partial \Pi}{\partial a} = 0 \Rightarrow 2a\left[\frac{B}{2}\left(\frac{4L_x}{15 L_y^3} + \frac{4(1-\nu)}{105 L_x L_y}\right) - \frac{P L_y}{1260 L_x}\right] = 0.$$

$$\boxed{\therefore\ P_{crit} = B\left[\frac{4L_x}{15 L_y^3} + 2(1-\nu)\frac{2}{105 L_x L_y}\right]\cdot\frac{630 L_x}{L_y}}$$

$$= 630B \times \frac{4}{15}\frac{L_x^2}{L_y^4} + 2(1-\nu)\frac{4}{105 L_y^2}$$

$$B = \frac{Et^3}{12(1-\nu^2)} = \frac{2\times10^8 \times (0.005)^3}{12(1-0.3)^2} = 9.377 \text{ kN.m}$$

$$P_{crit} = 630(9.377)\left[\frac{4(1.8)^2}{15(2.3)^4} + 2(1-0.3)\frac{4}{105(2.3)^2}\right].$$

$$= 5907.51\left[0.03087 + 0.04095\right]$$

$$\boxed{P_{crit} = 241.951 \text{ kN/m}}$$

If ribs are added to plate its buckling stiffness increases and results in higher potential energy.

$$\Pi_{Ii,N} = \Pi_{i,N}(\text{plate}) + \sum \Pi_{Ii,N}(\text{rib})$$

$$\Pi_{Ii,N} = a_1^2 C_0, \quad \Pi_{Ie,N} = -P_0 a_1^2 C_1$$

$$\Pi_{I,N} = \Pi_{Ii,N} + \Pi_{Ie,N} = a_1^2 C_0 - P_0 a_1^2 C_1$$

$$= a_1^2 (C_0 - P_0 C_1)$$

$$\delta\Pi_{I,N} = 2a_1(C_0 - P_0 C_1) = 0$$

$$\boxed{P_0 = \frac{C_0}{C_1}}$$

Thus $\Pi_{I,N}$ increasing results in higher stiffness and positical.

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