THE DRAW

To understand the concept of incentives, let us begin with the example of Draw. The Draw is the process that assigns Stanford students in their dorms and rooms.

THE PROCESS

- Each student submits a ranked list. Each item in the list is a type of room in a specific dorm or house and they are ordered from most preferred (at the top) to least preferred. (No ties allowed) This method is used to express preference in the Draw.

- Each student is assigned a number in {1, 2, . . . , 3500}.

- For i = 1, 2, . . . , 3500:
  Student i is assigned to her favorite choice among the options still available. In effect, the algorithm goes down student i's ranked list (from top to bottom), giving her the first option that is available.

ANALYZING DRAW

To review a system such as the Draw one needs to look at various jargons and a systematic way of thinking. So we will first look at some terms and later revert back to our original question. "Is Draw a good mechanism"?

I. PARETO OPTIMALITY

Glossary Pareto optimal (adj.): the property of an outcome that you can't make anyone better off without making someone else worse off.

A Pareto optimal situation seems ideal. It is fair that nobody is benefitted while making anyone else worse. It is preferred that a good mechanism has this quality. However, the converse is not true. We simply can not say that a mechanism is good if it is Pareto optimal. Let us look at the Draw in the view of this new concept.

PROPOSITION 1.1: THE OUTCOME OF THE Draw IS PARETO OPTIMAL.
Proof:
Call the outcome of the Draw the "old" outcome. Consider some alternative "new" outcome. Assume that no student is worse off in the new assignment than in the old (otherwise the new assignment is no threat to the Draw's Pareto optimality). The plan is to show that the two assignments must be identical. Thus every assignment different from the outcome of the Draw makes someone worse off, and this establishes Pareto optimality. We'll prove the following statement by induction on i (assuming no one is worse off in the new assignment): the first i students are assigned identically in the two assignments. The base case (with i = 0) is trivial, since two empty assignments coincide. For the inductive step, fix a value of i ≥ 1.

By the inductive hypothesis, the first i − 1 students are assigned identically in the two assignments. Thus, the remaining options for student i in the new assignment are precisely the options remaining when i is considered in the Draw. The Draw gives student i her favorite option among those remaining; if the new assignment does anything different, then student i is worse off. This completes the inductive step and the proof.

The above proposition must be taken as a sanity check. If Draw was not a reasonable mechanism, there would have been complaints about it already.
Until now, we assumed that the preferences submitted by the students are their true preferences. However, in real life, students may build up strategies and fill in dishonest preferences to achieve their true goals. Let us take a look at the same.

II. STRATEGYPROOF MECHANISMS

Glossary
strategyproof (synonym: truthful) (adj.): the property of a mechanism that honesty is always the best policy, meaning that lying about your preferences cannot make you better off.

PROPOSITION 1.2: THE DRAW IS STRATEGYPROOF.
Proof:
Draw numbers are generated independently.So students can't affect the draw number i that they get. They also can't affect the choices made by the i − 1 students before them — their choices are independent of the ranked list that the ith student submits. Thus, the available options at the time you are considered by the Draw does not depend on your submitted list. Since the Draw awards you your highest-ranked available option, it pays to be honest — any lie in your ranked list could only cause you to instead receive a remaining option that is not your favorite.

By now we have used the term mechanism for Draw. What exactly does it mean?

Glossary
mechanism (n.): a procedure for making a decision or taking an action, as a function of what people want (i.e., of participants' preferences).

We see that the students are assigned various rooms on the basis of a "ranked list". This is a special mechanism known as 'serial dictatorship'.

Glossary
serial dictatorship (n.): a mechanism that orders the participants, and in this order allows each player to dictate their favorite feasible option (given the choices made by previous players).

After this analysis we see that Draw works fine as of now. No serious problem exists. It is Pareto optimal and strategyproof. However rarely a mechanism like this is built at the first stage itself. We usually make a system and correct the flaws as they come by. This is exactly what happened with the Draw. It had been modified most recently in 2009. Let us take a look at the draw mechanism functioning in the mid 90’s.

THE DRAW: THE MID-90’S VERSION

THE PROCESS:

- Each student submits a ranked list of at most 8 options, out of the roughly 60 possibilities.

- Same as before (modulo unimportant details): each student is assigned a number from some range, and these numbers are chosen independently of the submitted lists.

- Same as before. But if none of student i’s 8 options are still available, then the student is unassigned, or assigned by default to the least popular dorm.

Looking at the old mechanism, can you say it was strategyproof? No. To submit the preferences in an honest manner may really be disadvantageous. Imagine you fill out all the popular and most sought after houses in your order of preferences. A situation may arise if you get a very large ranked number where all your preferred houses have been already occupied. You will automatically get sorted out to the least popular dorm. However, if one acts smartly one can fill the popular houses at the top followed by some mild houses as a backup. In case you are unlucky and end up at a large number, you may still get a decent dorm. Therefore this mechanism isn’t strategyproof unlike the new one.

WHY DO WE NEED STRATEGYPROOF MECHANISMS?

We see that the strategyproof property of a mechanism does not seem as important as other properties like Pareto optimality. Why do we need to bother about it altogether? Although strategyproof property is not necessary, it enhances a system and helps the system attain its ultimate goal. Just like in the Draw, in the old scenario one has to think of multiple ways to play with the mechanism in order to get the preferred dorm. However the new mechanism requires one to soul-search oneself and see what one truly wants and leave the rest to probability. It seems a quality we would want most mechanisms to have.

Another issue with the old mechanism was ‘regret’. A participant may feel in the future that one should have submitted a different list in order to be at a better dorm. However in the second mechanism it is just the choice of the participant and rest is pure probability. It leaves out no traces of regret.

THE CONCLUSION

Comparing the two versions of the Draw, we see a vivid illustration that the rules of the game matter. Seemingly small changes to a system can make a big difference in the experience of the participants, and in how well the system functions. We’ll see this same phenomenon over and over again, as we move from one application domain to another.