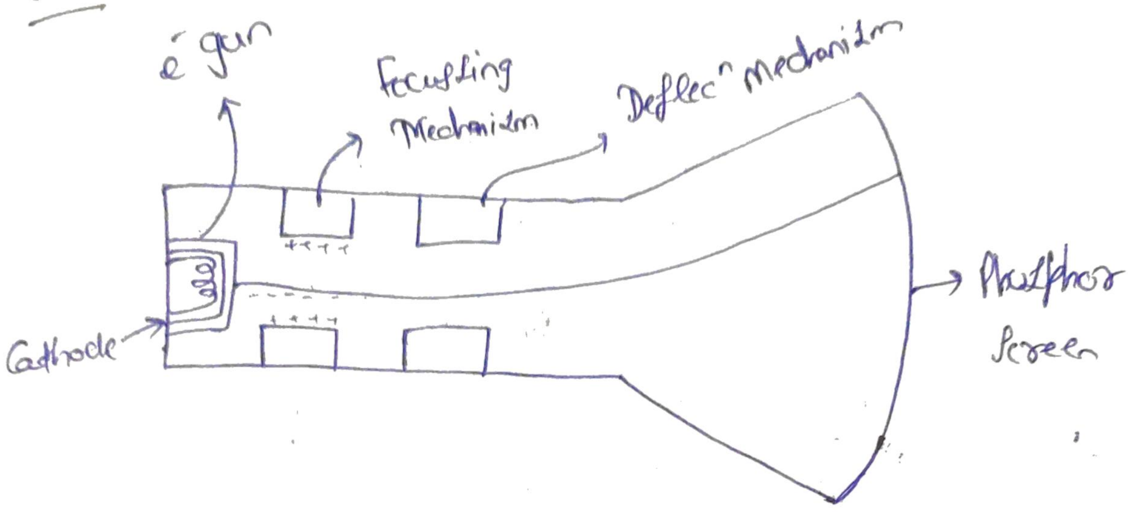
Computer Graphics

CRT



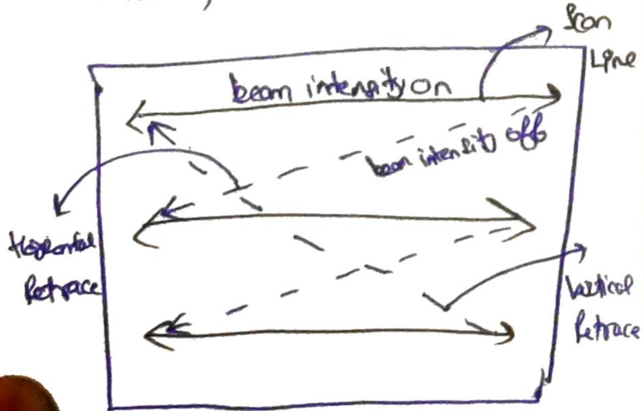
- Properties of CRT

- Electron gun emmits a beam of e⁻, cathode rays.
- The e⁻ beam passes through focussing & deflec^n system, that directed it towards specified positions on the phosphor coated screen.
- When the beam hits the screen, the phosphor emmits a small spot of light at each pos^n contacted by the e⁻ beam.
- It redraws the picture by directing the e⁻ beam back over the same screen points quickly.

{- eg. Old TVs (like Onida, LG) }

Raster Scan

- In this system, e^- beam is swapped across the screen one row at a time from top to bottom.
- As the e^- beam moves across each row, the beam intensity is turned on & off to create a pattern of illuminated spots.
- Picture defn is stored in memory area called "refresh buffer or frame buffer."
- Stored "intensity values" are retrieved from refresh buffer & painted/displayed on the screen, one row at a time.



Random Scan / Vector Scan

- In this technique, the e^- beam is directed only to the part of the screen, where the picture is to be drawn (rather than scanning from left to right & top to bottom, as in raster scan)
- It is also called vector display.
- Pic. defn is stored as a set of line drawing commands in the area of memory, refer to as "refresh display file"
- To display a specified pic., the system cycles through the set of commands in the display file, drawing each component line, in turn.
- After all the line drawing commands are processed, the system cycles back to the I line command

in the end (so, in order to view a line / image / point, continuous effect by avoiding / preventing damaging process)

* Digital Differential Analyzer (DDA)

is a simple line generation algorithm

1) Get the input of 2 end points (x_0, y_0) & (x_1, y_1)
2) Calculate the difference b/w 2 end points.

$$dx = x_1 - x_0$$
$$dy = y_1 - y_0$$

3) Based on the calculated diff in step 2, identify no of steps.

```
if (abs(dx) > abs(dy))
    steps = abs(dx)
else
    steps = abs(dy)
```

4) Calculate increments

$$x_{inc.} = \frac{dx}{(float)\ steps}$$

$$y_{inc} = \frac{dy}{(float)\ steps}$$

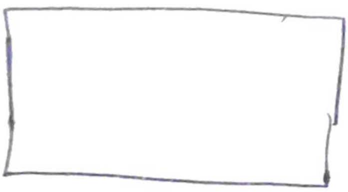
5)
```
for (int v = 0; v < steps; v++) {
    x = x + xinc;
    y = y + yinc;
```

Put pixel (round (x), round (y))
}

1. <u>Pixel</u>

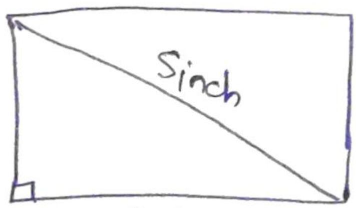
→ It is the smallest unit which is possible in the display.

2. <u>Resolution</u> $\{\underbrace{W}_{\text{width}} \times \underbrace{H}_{\text{height}}\}$



$\Rightarrow W \times H = (1920 \times 1080)$ resolution

3. <u>PPI</u> (Pixels Per Inch)



→ We can find how many pixels are present in the ~~the~~ 5 inches. And after that we can divide it by 5 to get PPI.

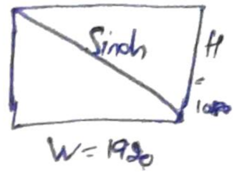
We can calculate the diagonally present pixels, simply by Pythagoras Theorem,

$\Rightarrow \sqrt{H^2 + W^2}$

eg. W: 1920, H = 1080

So, x (diagonally present pixels) = $\sqrt{1920^2 + 1080^2}$
≈ 2202

→ So, PPI = $\frac{2202}{5} \approx \boxed{440 \text{ PPI}}$ for a



④ Aspect Ratio

⇒ $\frac{W}{H}$ → Ratio of the no. of pixels present horizontally to the no. of pixels present vertically.

If, W = H, then A.R = 1:1

other eg. of common Aspect Ratios are :: 4:3, 5:3.

⑤ Frame Buffer

→ It is a memory area which is used to store all the information or data in respect to the visual we are needed to display.

Q: Through DDA, plot a line from points (1,1), (3,3)

{ Initially, we are Standing at (1,1)

So, $x_i = 1$

$y_i = 1$ }

① Slope (m) = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{3-1} = 2/2 = 1$

② Find Δx & Δy.

→ $\Delta x = \frac{\Delta y}{m}$ {From ①} ⇒ $\frac{3-1}{1} = 2$

→ $\Delta y = \Delta x \times m = 2 \times 1 = 2$

③ ∴ $|\Delta x| \geq |\Delta y|$,

assign, $\Delta x = 1$

⇒ $x_{i+1} = x_i + 1 = 1 + 1 = 2$

⇒ $y_{i+1} = y_i + m = 1 + 1 = 2$

⇒ (2, 2)

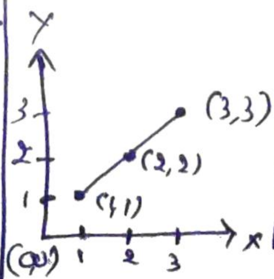
Now,

⇒ $x_{i+1} = x_i + 1 = 2 + 1 = 3$

⇒ $y_{i+1} = y_i + m = 2 + 1 = 3$

⇒ (3,3)

x_i	y_i	x_{i+1}	y_{i+1}
1	1	2	2
2	2	3	3



Case I

• If $|\Delta x| \geq |\Delta y|$

Then, assign, $\Delta x = 1$

→ $x_{i+1} = x_i + \Delta x = x_i + 1$

→ $y_{i+1} = y_i + \Delta y = y_i + m\Delta x$
$= y_i + m$

Case II

• If $|\Delta x| < |\Delta y|$

Then, assign, $\Delta y = 1$

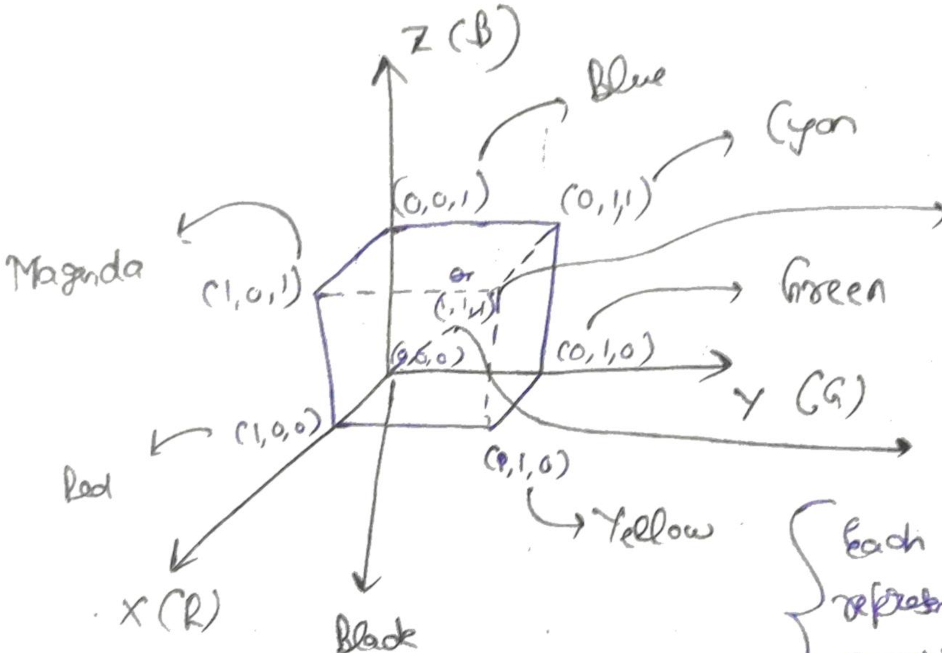
→ $x_{i+1} = x_i + \Delta x = x_i + \frac{\Delta y}{m} = x_i + \frac{1}{m}$

→ $y_{i+1} = y_i + \Delta y = y_i + 1$

RGB Colour Model

- It is an additive color model in which Red, Green & Blue light are added together in various ways to reproduce a broad band of colors.

→ It is represented by R, G, B axis shown below:



{ Each colour is represented by a 3D coordinate (R, G, B) }

→ Purpose of RGB model is for the sensing & representatⁿ or display of images in electronic systems like TV & computers.

→ Applicatⁿ of this is, display of colors on a CRT, LCD, plasma display or omitting light emitting diode (OLED).

→ Each pixel on the screen is built by dividing 3 small & close separated RGB light sources, which is indistinguishable at common viewing distance, tricking the eye to see a given solid color.

CMYK Color Model

(as it subtracts brightness from white)

→ It is a subtractive color model (used in color printing & is also used in describing the printing process itself)

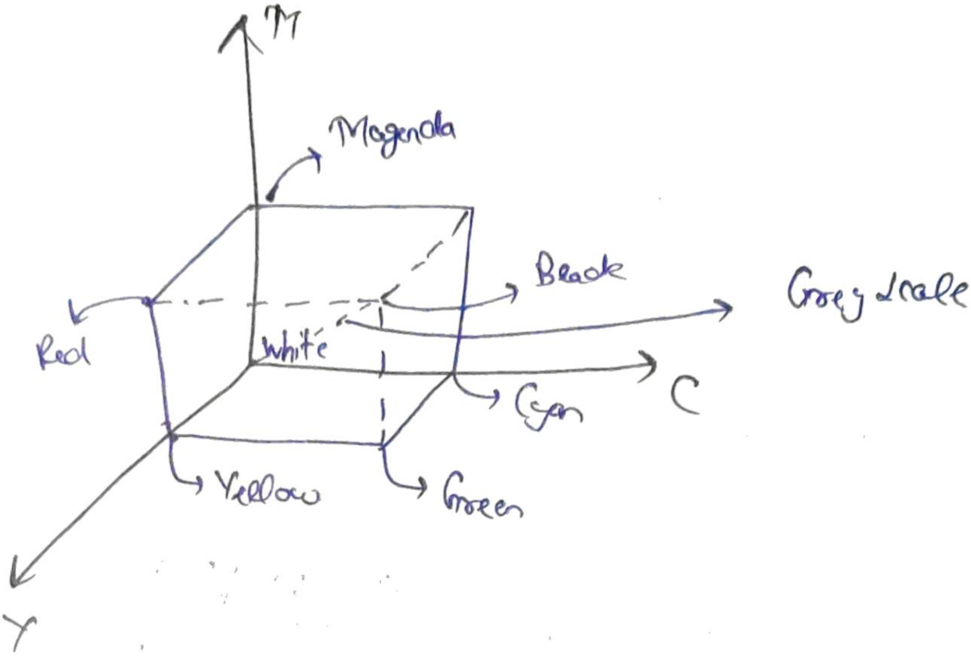
→ CMYK refers to the 4 inks used in some color printing : Cyan, Magenta, Yellow & Key (Black).

→ It works by partially or entirely masking colors on a lighter, usually white Background.

→ In additive color model like RGB col. model, white is the additive combinatn of all primary colors, while black is the absence of light.,

But, in CMYK model, it is the opposite., white is the natural color of the paper, while black is the result of full comb^n of colored inks.

→ To save cost on inks & to produce deeper black tones, unsaturated & dark colors are produced by black ink instead of the combinatn of CMY inks.



To convert RGB representatn to CMY representatn, we can perform matrix transformatn through the formula:

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

To convert CMY to RGB:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

Bresenham Line Drawing Algorithm

- Coordinates will be integers irrespect of floating no. (like we get in DDA)

eg $(1,1), (5,3)$.

① Slope (m) $\rightarrow \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{5-1} = \boxed{0.5}$

② Decision Parameter (P)

Cases	Initially, $\boxed{P = 2\Delta y - \Delta x, \ (m<1)}$	Initially, $\boxed{P = 2\Delta x - \Delta y, \ (m \geq 1)}$
	If, $(m<1)$	If, $(m \geq 1)$
Case I:	if $(P<0)$ $x_{i+1} = x_i + 1$ $y_{i+1} = y_i$ $P_{k+1} = P_k + 2\Delta y$	if $(P<0)$ $x_{i+1} = x_i$ $y_{i+1} = y_i + 1$ $P_{k+1} = P_k + 2\Delta x$
Case II:	if $(P \geq 0)$ $x_{i+1} = x_i + 1$ $y_{i+1} = y_i + 1$ $P_{k+1} = P_k + 2\Delta y - 2\Delta x$	if $(P \geq 0)$ $x_{i+1} = x_i + 1$ $y_{i+1} = y_i + 1$ $P_{k+1} = P_k + 2\Delta x - 2\Delta y$

Now, ∵ $m < 1$

So, calculate P first: $P = 2\Delta y - \Delta x = 2(3-1) - (5-1) = 0$

1) Now, $P \geq 0$

{ $(x_i, y_i) = (1,1)$ initially }

∴, $x_{i+1} = x_i + 1 = 1+1 = 2$

$y_{i+1} = y_i + 1 = 1+1 = 2$

$\Rightarrow (2,2)$ //

Calculate P,

$P_{k+1} = P_k + 2\Delta y - 2\Delta x = 0 + 2(2) - 2(4) = -4$

2) Now, $P < 0$, and $(x_i, y_i) = (2,2)$

$x_{i+1} = x_i + 1 = 2+1 = 3$

$y_{i+1} = y_i = 2$

$\Rightarrow (3,2)$ //

$P_{k+1} = P_k + 2\Delta y = -4 + 2(2) = 0$

3) Now, $P \geq 0$ & $(x_i, y_i) :: (3,2)$

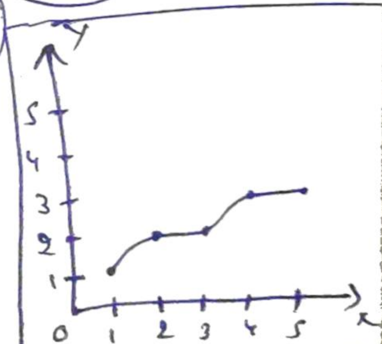
$x_{i+1} = 3+1 = 4$ }
$y_{i+1} = 2+1 = 3$ } $(4,3)$ //

$P_{k+1} = 0 + 2(2) - 2(4) = -4$

4) $P < 0$, $(x_i, y_i) = (4,3)$

$x_{i+1} = 5$ }
$y_{i+1} = 3$ } $(5,3)$ //

$P_{k+1} = -4 + 2(2) = 0$ //

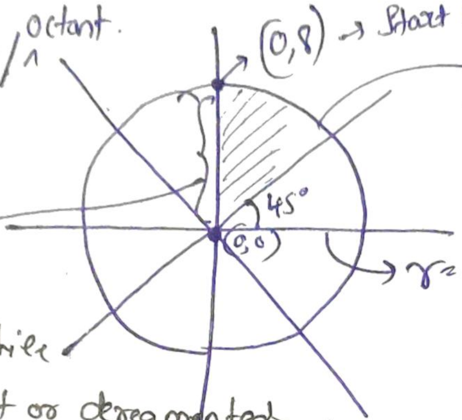


P	x_i	y_i	x_{i+1}	y_{i+1}
0	1	1	2	2
-4	2	2	3	2
0	3	2	4	3
-4	4	3	$\boxed{5}$	$\boxed{3}$

Bresenham Circle Drawing Algorithm

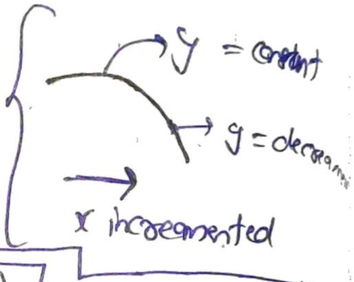
Given: Center : (0,0) , radius = 8 (r)

- We always divide the circle into 8 equal parts with the angle of 45° each and compute the algorithm for only one sector/octant.



(Always) compute for that sector/octant

~~In this case~~, when we draw the circle, 'x' is always incremented, while 'y' can either be constant or decremented.



① Start Pt. → $x_0 = 0$, $y_0 = 8$ $(= r)$ $\boxed{(0,8)}$

② Decision Parameter (P) = $\boxed{3-2R}$ (Initially) = $3 - 2(8)$ = $\boxed{-13}$

Cases

Case I (P < 0)	Case II P(≥ 0)
$x_{i+1} = x_i + 1$ // Incremented	$x_{i+1} = x_i + 1$ // Incremented
$y_{i+1} = y_i$ // Constant	$y_{i+1} = y_i - 1$ // Decremented
$P_{i+1} = P_i + 4x_{i+1} + 6$	$P_{i+1} = P_i + 4(x_{i+1} - y_{i+1}) + 10$

Here, $\underline{P<0}$, So, $x_{i+1} = x_i + 1 = 0+1 = 1$

1) $y_{i+1} = y_i = 8$ $\Big\}$ $\boxed{(1,8)}$

$P_{i+1} = P_i + 4x_{i+1} + 6 = -13 + 4(1) + 6$
$= -3$

2) Now $\underline{P<0 \ , \ (1,8)}$

So, $x_{i+1} = 1+1 = 2$
$y_{i+1} = 8$ $\Big\}$ $\boxed{(2,8)}$
$P_{i+1} = -3 + 4(2) + 6 = 11$

3) Now, $\underline{P \geq 0 \ , \ (2,8)}$

So, $x_{i+1} = 2+1 = 3$
$y_{i+1} = 8-1 = 7$ $\Big\}$ $\boxed{(3,7)}$
$P_{i+1} = 11 + 4(3-7) + 10 = 11 - 16 + 10 = 5$

4) Now, $\underline{P \geq 0 \ , (3,7)}$

So, $x_{i+1} = 3+1 = 4$
$y_{i+1} = 7-1 = 6$ $\Big\}$ $\boxed{(4,6)}$
$P_{i+1} = 5 + 4(4-6) + 10 = 5 + (-8) + 10 = 7$

5) Now $\underline{P \geq 0, (4,6)}$

So, $x_{i+1} = 4+1 = 5$
$y_{i+1} = 6-1 = 5$ $\boxed{(5,5)}$
$P_{i+1} = 7 + 4(5-5) + 10 = 17$

We don't need to perform further steps, $\because$ we get $\boxed{x_{i+1} \geq y_{i+1}}$

Now we get the required points for the required sector.
Its time to get all points. (all four quadrants) ~~of quadrant~~

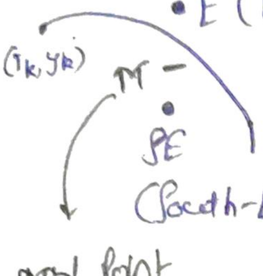
Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
(x, y)	(-x, y)	(-x, -y)	(x, -y)
(0, 8)	(0, 8)	(0, -8)	(0, -8)
(1, 8)	(-1, 8)	(-1, -8)	(1, -8)
(2, 8)	(-2, 8)	(-2, -8)	(2, -8)
(3, 7)	(-3, 7)	(-3, -7)	(3, -7)
(4, 6)	(-4, 6)	(-4, -6)	(4, -6)
flip (5, 5)	(-5, 5)	(-5, -5)	(5, -5)
(6, 4)	(-6, 4)	(-6, -4)	(6, -4)
(7, 3) flip	(-7, 3)	(-7, -3)	(7, -3)
(8, 2) (mirror image)	(-8, 2)	(-8, -2)	(8, -2)
(8, 1)	(-8, 1)	(-8, -1)	(8, -1)
(8, 0)	(-8, 0)	(-8, 0)	(8, 0)

Mid Point Circle Algorithm

{8-way symmetry, ∴ we plot only one octant points}

→ $x^2+y^2-r^2=0$ {Center: (0,0)}

Result: $\{(x')^2+(y')^2-r^2\}$
- → 0 {Point lies on the circle}
- → < 0 {" " inside the circle}
- → > 0 {" " outside " "}



• E (East Point) $\{(\underbrace{x_{k+1}}_{\text{Increment}}, \underbrace{y_k}_{\text{Constant}})\}$

SE (South-East Point) $\{(\underbrace{x_{k+1}}_{\text{Increment}}, \underbrace{y_{k-1}}_{\text{Decremented}})\}$

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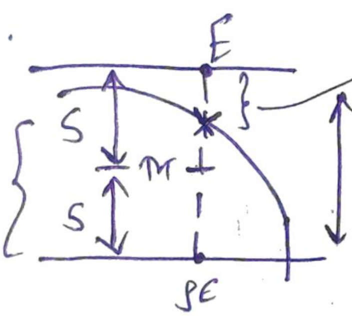
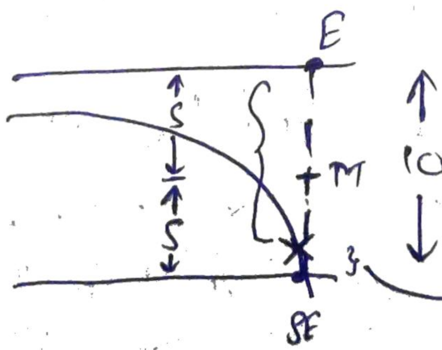
eg. Suppose mid pt. is lying $\boxed{\text{inside the circle}}$ & dist. b/w E & SE is 10.



→ Then we $\boxed{\text{choose E pt.}}$ — y remains constant

∵ it lies closer to the circle

eg. Suppose mid pt. lies $\boxed{\text{outside the circle}}$



→ Then we $\boxed{\text{choose SE pt.}}$ — y is decremented

∵ it lies closer to the circle.

$E(x_k+1, y_k)$, $SE(x_k+1, y_k-1)$

<u>Midpoint</u>: $M(x_m, y_m) = \left(\frac{x_k+1+x_k+1}{2}, \frac{y_k+y_k-1}{2}\right)$

$\Rightarrow M(x_m, y_m) = M\left(x_k+1, \frac{2y_k-1}{2}\right)$ //

* <u>Decision Parameter</u>: $\boxed{P_k = x_m^2 + y_m^2 - r^2}$ {Eq^n of circle}

$\Rightarrow P_k = (x_k+1)^2 + \left(\frac{2y_k-1}{2}\right)^2 - r^2$ — (1)

$\Rightarrow P_{k+1} = (x_{k+1}+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$ — (2)

Subtract (1) from (2)

$\Rightarrow P_{k+1} - P_k = (x_{k+1}+1)^2 - (x_k+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2$

We know $\{x_{k+1} = x_k + 1\}$

$= (x_k+2)^2 - (x_k+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2$

$= \cancel{x_k^2} + 4 + 4x_k - \cancel{x_k^2} - 1 - 2x_k + y_{k+1}^2 + \cancel{\frac{1}{4}} - y_{k+1} - y_k^2 - \cancel{\frac{1}{4}} + y_k$

$$P_{k+1} = P_k + 2x_k + 3 + y_{k+1}^2 - y_{k+1} - y_k^2 + y_k$$ //

If, $P_k < 0$ {mid pt. lies inside the circle}, {then we choose East point., & y remains constant}

Hence, $y_{k+1} = y_k$

$$\Rightarrow \boxed{P_{k+1} = P_k + 2x_k + 3} + \cancel{y_{k+1}^2} - \cancel{y_k} - \cancel{y_k^2} + \cancel{y_k}$$

If, $P_k > 0$ {mid pt. lies outside the circle, then choose South east pt., value of y is decremented}

Hence, $y_{k+1} = y_k - 1$

$$\Rightarrow P_{k+1} = P_k + 2x_k + 3 + (y_k - 1)^2 - \cancel{y_k}^{+1} - y_k^2 + \cancel{y_k}$$

$$= P_k + 2x_k + 3 + \cancel{y_k^2} + 1 - 2y_k + 1 - \cancel{y_k^2}$$

$$\Rightarrow \boxed{P_{k+1} = P_k + 2x_k - 2y_k + 5}$$

Initially, we take (x_0, y_0) as $(0, r)$ {like before} {I octant}

So, for initial value of P (P_0), substitute $(x_k, y_k) = (0, r)$.

$$P_k = x_m^2 + y_m^2 - r^2 = (x_k + 1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

$$\Rightarrow P_{k_0} = (0+1)^2 + \left(r - \tfrac{1}{2}\right)^2 - r^2 \Rightarrow \boxed{P_{k_0} = 5/4 - r}$$

Midpoint Ellipse Algorithm {Elongated Circle} {Centred at (0,0)}

$$\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right]$$

a = major axis; b = minor axis

major axis → Length → 2a

minor axis → Length → 2b

{Let, $r_x = a$, radius along x-axis,
$r_y = b$, " " y " }

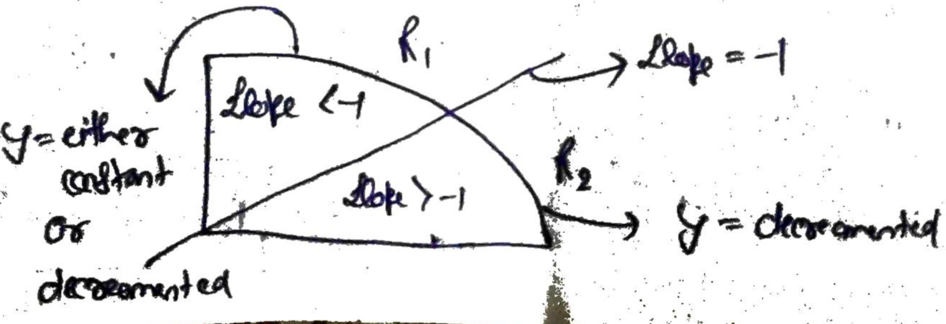
So, Major axis = $2r_x$ {Semimajor = r_x}
Minor axis = $2r_y$ {" minor = r_y}

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1 \Rightarrow \underline{x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0}$$

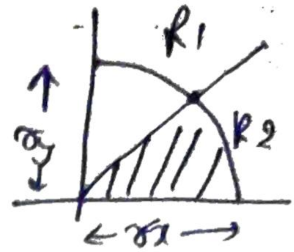
Ily, here also, ~~#~~ mid point lies on ~~circle~~ ellipse, if = 0
" " inside the ~~circle~~ ", if < 0
" " outside " " " > 0

- But like circle, ellipse doesn't have 8-way symmetry.
So, ~~instead of~~ plotting only of an octant points wouldn't work,
we have to plot a complete quadrant points.

Quadrant:



* Quadrant 1 Region 1 (R1)



- Start Point : $(0, r_y)$ ✓

- Slope of Curve < -1

- Take unit steps in the x direc^n till boundary b/w the R1 & R2 is reached.

  ↳ $x \to x+1$

* Quadrant 1 Region 2 (R2)

- Slope of curve > -1 → $y \to y-1$

- Take unit steps in -ve y direc^n till the end of quadrant.

{On the boundary b/w R1 & R2, slope = -1}

→ **Slope of Curve**

$$\frac{d}{dx}\left[x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2\right] = 0$$

$$\cancel{2}x\, r_y^2 + \cancel{2}y \frac{dy}{dx} r_x^2 = 0$$

$$\boxed{dy/dx = \frac{-r_y^2 x}{r_x^2 y}} = -1 \Rightarrow \boxed{r_y^2 x = r_x^2 y}$$

* $\boxed{R1} \rightarrow (x_k+1, y_k), (x_k+1, y_k-1) \rightarrow M(x_k+1, y_k - \frac{1}{2})$

Eq^n: $r_y^2 x_m^2 + r_x^2 y_m^2 - r_x^2 r_y^2 = 0 = P1_k$

* Decision Parameter

$$P1_k = r_y^2 (x_k+1)^2 + r_x^2 (y_k - \tfrac{1}{2})^2 - r_x^2 r_y^2 \quad -(1)$$

$$P1_{k+1} = r_y^2 ((x_k+1)+1)^2 + r_x^2 (y_{k+1} - \tfrac{1}{2})^2 - r_x^2 r_y^2 \quad -(2)$$

$\{x_{k+1} = x_k + 1\}$

- Subtract (1) from (2)

$$P1_{k+1} - P1_k = r_y^2 [\cancel{x_k^2} + 4 + 4x_k - \cancel{x_k^2} - 1 - 2x_k] +$$
$$r_x^2 [y_{k+1}^2 + \tfrac{1}{4} - y_{k+1} - y_k^2 - \tfrac{1}{4} + y_k]$$

$$\Rightarrow P1_{k+1} - P1_k = r_y^2 [2x_k + 3] + r_x^2 [y_{k+1}^2 - y_k^2 - y_{k+1} + y_k]$$

$$y_{k+1} = \begin{cases} y_k & ; \text{if } P1_k < 0 \text{ (inside the ellipse)} \\ y_k - 1 & ; \text{if } P1_k \geq 0 \text{ (outside the ellipse)} \end{cases}$$

Case I $\{P1_k < 0\}$

$$\boxed{P1_{k+1} = P1_k + r_y^2 \cdot 2x_{k+1} + r_y^2}$$

$\{x_{k+1} = x_k + 1\}$

Case 2 $\{P1_k \geq 0\}$

$$\boxed{P1_{k+1} = P1_k + 2x_{k+1} r_y^2 + r_y^2 - 2y_{k+1} r_x^2}$$

* Initial Decision Parameter ($P1_0$) $\left\{ \text{start pt. of R1} \right\}$ $(0, r_y)$

let $(0, r_y)$ in, $P1_k = r_y^2 (x_k + 1)^2 + r_x^2 \left(y_k - \frac{1}{2}\right)^2 - r_x^2 r_y^2$

$\Rightarrow P1_0 = r_y^2 (1)^2 + r_x^2 \left(r_y - \frac{1}{2}\right)^2 - r_x^2 r_y^2$

$$\Rightarrow \boxed{P1_0 = r_y^2 + r_x^2/4 - r_y r_x^2}$$

---

* $\boxed{R2}$ → $(x_k, y_k - 1)$, $(x_k + 1, y_k - 1)$ → $M\left(x_k + \frac{1}{2}, y_k - 1\right)$

Eqn: $r_y^2 x_m^2 + r_x^2 y_m^2 - r_x^2 r_y^2 = 0 = P2_k$

* Decision Parameter

$r_y^2 \left(x_k + \frac{1}{2}\right)^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2 = P2_k$

$P2_k = r_y^2 \left[x_k^2 + \frac{1}{4} + x_k\right] + r_x^2 \left[y_k^2 + 1 - 2y_k\right] - r_x^2 r_y^2$ — (1)

$P2_{k+1} = r_y^2 \left[x_{k+1} + \frac{1}{2}\right]^2 + r_x^2 \left[y_{k+1} - 1\right)^2 - r_x^2 r_y^2$ — (2)

- Subtract (1) from (2)

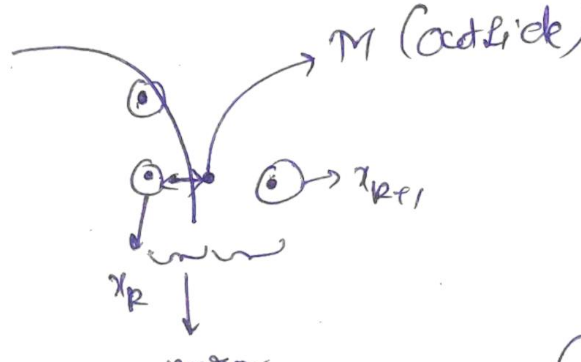
$\Rightarrow P2_{k+1} - P2_k = r_y^2 \left[x_{k+1}^2 + \cancel{\frac{1}{4}} + x_{k+1} - x_k^2 - \cancel{\frac{1}{4}} - x_k\right] +$

$r_x^2 \left[y_{k+1}^2 + \cancel{1} - 2y_{k+1} - y_k^2 - \cancel{1} + 2y_k\right]$

$\Rightarrow \left\{ y_{k+1} = y_k - 1 \right\}$

$$\Rightarrow P2_{k+1} - P2_k = r_y^2[\ ] + r_x^2\left[\left(\cancel{y_k^2}+1-\cancel{2y_k}\right) - 2(y_k-1) - \cancel{y_k^2} + \cancel{2y_k}\right]$$

$$\Rightarrow P2_{k+1} - P2_k = r_y^2\left[x_{k+1}^2 + x_{k+1} - x_k^2 - x_k\right] + r_x^2\left[-2y_{k+1}+1\right]$$



(outside)

Case I, if $\{P2_k > 0\}$

$[x_{k+1} = x_k]$

$$\Rightarrow \boxed{P2_{k+1} = P2_k - 2y_{k+1} r_x^2 + r_x^2}$$

(inside/on)

Case 2, if $\{P2_k \leq 0\}$

$[x_{k+1} = x_k + 1]$

$$\Rightarrow \boxed{P2_{k+1} = P2_k + r_y^2(2x_{k+1}) - 2y_{k+1} r_x^2 + r_x^2}$$

* Initial Decision Parameter {or start pt. of R2} (x, y)

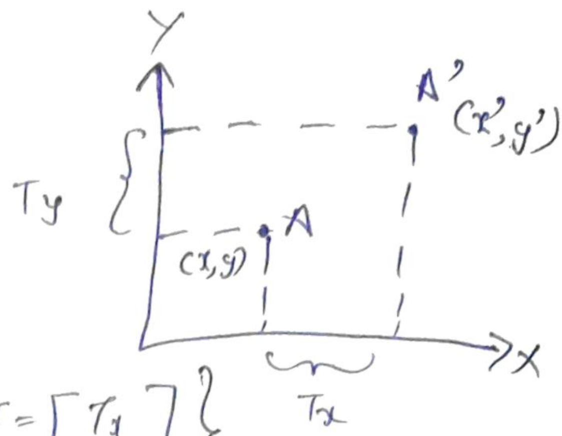
- is obtained by putting last pt. of R1 in the eq^n:

$$P2_k = r_y^2 \left(x_k + \tfrac{1}{2}\right)^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2$$

$$\Rightarrow \boxed{P2_0 = r_y^2 \left(x + \tfrac{1}{2}\right)^2 + r_x^2 (y-1)^2 - r_x^2 r_y^2}$$

— x —

2D Translation (Shift)



$x' = x + T_x$

$y' = y + T_y$

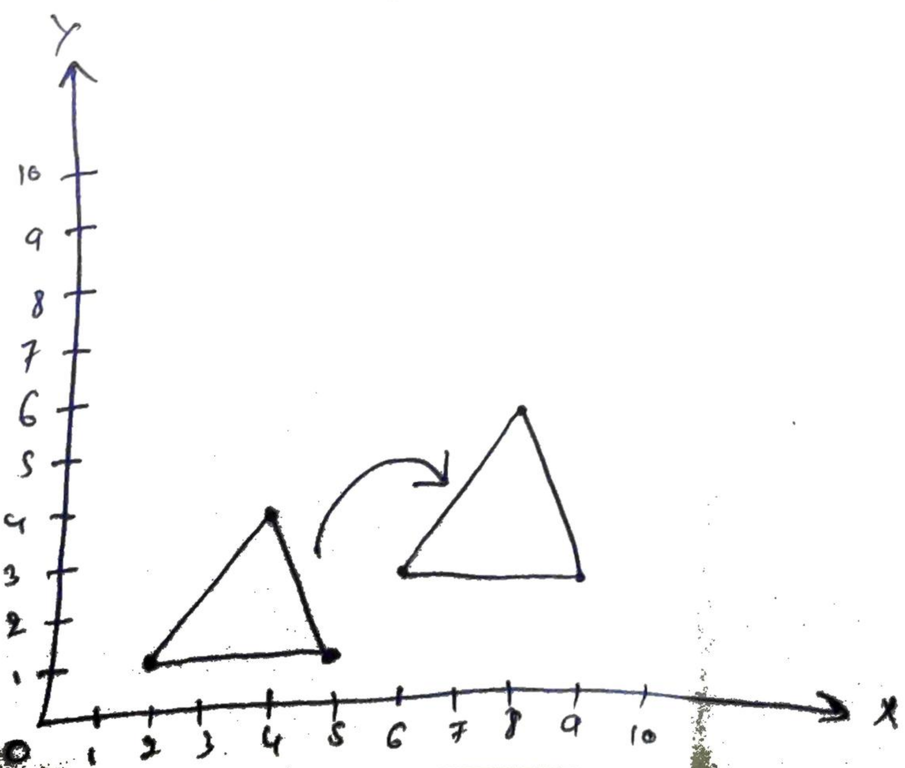
$\boxed{A' = A + T} \rightarrow$ Additⁿ

$$\left\{ A' = \begin{bmatrix} x' \\ y' \end{bmatrix} ; \; A = \begin{bmatrix} x \\ y \end{bmatrix} ; \; T = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \right\}$$

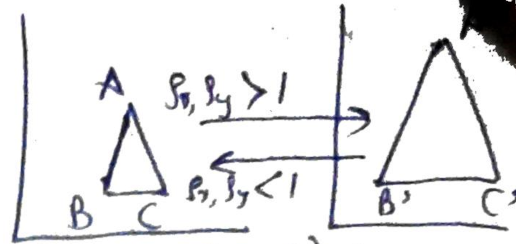
<u>Q</u>: Shift 4 units in x-axis & 2 units in y-axis

A(2,1), B(5,1), C(4,4) → Δ

$$\left.\begin{aligned} A' &= A + T = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} \\ B' &= B + T = \begin{bmatrix} 5 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix} \\ C' &= C + T = \begin{bmatrix} 4 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix} \end{aligned}\right\}$$



2D Scaling (Zoom In / Out)



If (C=1), then no change.

S_x, S_y → Scaling factors

$$\left.\begin{aligned} A' &= A.S \\ B' &= B.S \\ C' &= C.S \end{aligned}\right\} \rightarrow \text{Multiplic}^n$$

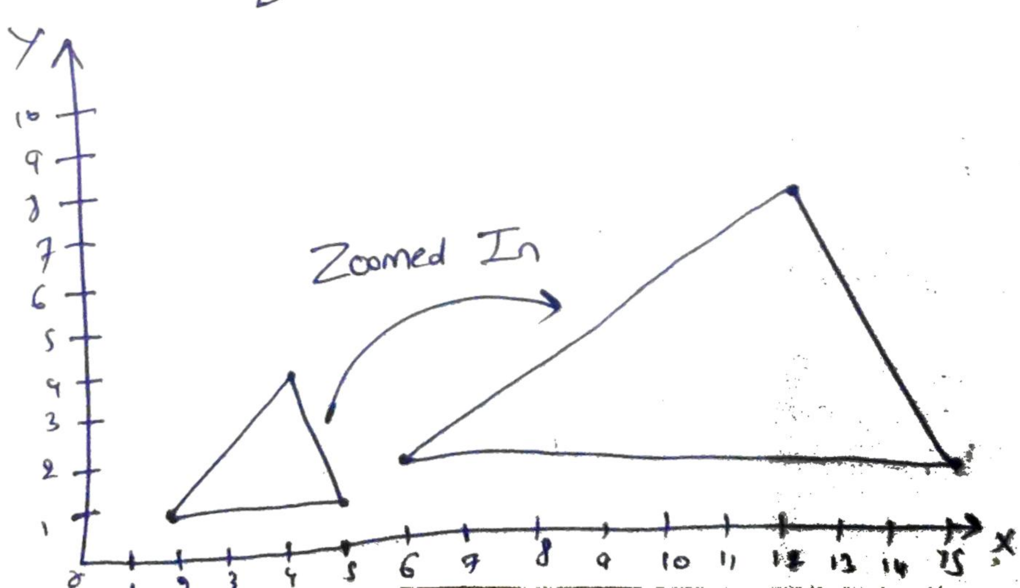
$$\left[A' = [x', y']_{1\times2} \; ; \; A = [x, y]_{1\times2} \; ; \; S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}_{2\times2} \right]$$

$$\{ x' = x.S_x \; ; \; y' = x.S_y \}$$

Q: Scale 3 units in x direcⁿ & 2 units in y-direcⁿ?

A(2,1), B(5,1), C(4,4) → Δ ; $S_x = 3$, $S_y = 2$

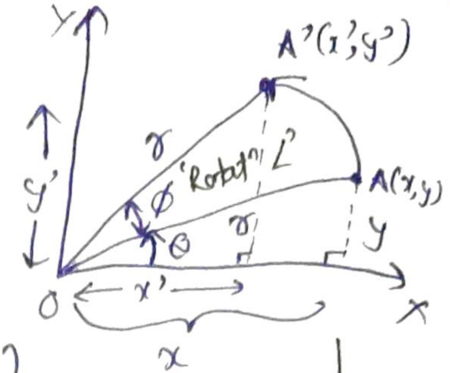
$$\left.\begin{aligned} A' &= [2 \; 1] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = [6, 2] \\ B' &= [5 \; 1] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = [15, 2] \\ C' &= [4, 4] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = [12, 8] \end{aligned}\right\}$$



2D Rotatⁿ

$\cos\theta = x/r \Rightarrow \boxed{x = r\cos\theta}$

IIly, $\boxed{y = r\sin\theta}$



$\cos(\theta+\phi) = \dfrac{x'}{r} \Rightarrow x' = r\cos(\theta+\phi)$

$\sin(\theta+\phi) = y'/r \Rightarrow y' = r\sin(\theta+\phi)$

$\boxed{\text{Anticlockwise}}$ (then we take the angles)

* $x' = r[\cos\theta\cos\phi - \sin\theta\sin\phi]$

$\Rightarrow \boxed{x' = x\cos\phi - y\sin\phi}$

* $y' = r[\sin\theta\cos\phi + \cos\theta\sin\phi]$

$\Rightarrow \boxed{y' = y\cos\phi + x\sin\phi}$

Note: If we do rotⁿ in $\boxed{\text{clockwise}}$ direcⁿ, then the angles are considered / taken as -ve. ∴,

$x' = x\cos(-\phi) - y\sin(-\phi)$

$\Rightarrow \boxed{x' = x\cos\phi + y\sin\phi}$

$y' = y\cos(-\phi) + x\sin(-\phi)$

$\Rightarrow \boxed{y' = y\cos\phi - x\sin\phi}$

• For Anticlockwise direcⁿ rotⁿ:

$A' = R.A$

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

• For Clockwise Rotⁿ

$$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$$

Q: $A(10, 5)$, $\phi = 45°$ (Anticlockwise)

$A' = R.A$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{bmatrix} \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 10/\sqrt{2} - 5/\sqrt{2} \\ 10/\sqrt{2} + 5/\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5/\sqrt{2} \\ 15/\sqrt{2} \end{bmatrix} = A'$$

for Clockwise, $\begin{bmatrix} 15/\sqrt{2} \\ -5/\sqrt{2} \end{bmatrix} = A'$

Homogenous Coordinate

Translation Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

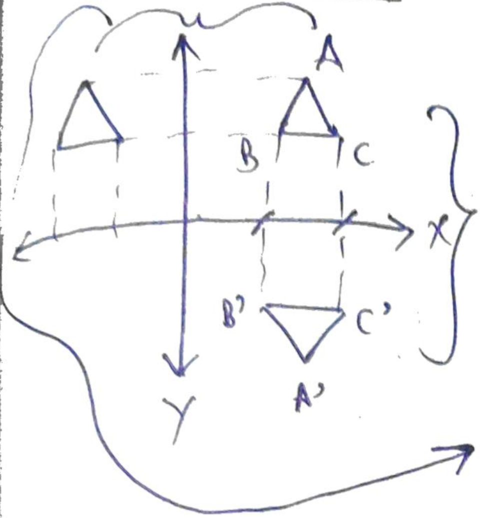
Scaling Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D Reflection



Reflecⁿ on X-axis :

$x' = x$
$y' = -y$ $\rightarrow$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$A' = \underset{\text{(Reflecⁿ)}}{R} \cdot A$

Reflecⁿ on Y-axis :

$x' = -x$
$y' = y$ $\rightarrow$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$A' = R \cdot A$

for homogenous coordinate system, it will be :

- for X-axis Reflection:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- for Y-axis Reflecⁿ:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q: A(3,4), B(2,3), C(4,3)

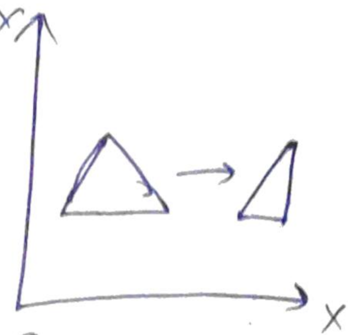
1) Reflection on X-axis

$A' = [3, -4]$; $B' = [2 \ -3]$; $C' = [4 \ -3]$

2) Reflecⁿ on Y-axis

$A' = [-3 \ 4]$; $B = [-2 \ 3]$; $C' = [-4 \ 3]$

2D Shearing {Trying to change the shape of the object}



- Shearing in X-axis

$$\begin{cases} X' = X + \boxed{(Sh_x) \times (Y)} \\ Y' = Y \rightarrow \text{remains unchanged} \end{cases}$$

- Shearing in Y-axis

$$\begin{cases} X' = X \rightarrow \text{unchanged} \\ Y' = Y + \boxed{(Sh_x) \times (X)} \end{cases}$$

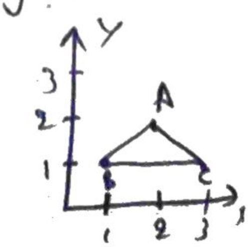
$\left\{ Sh_x , Sh_y \right.$ ↓ Shearing Parameters

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \rightarrow \text{in X-axis}$$

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} \rightarrow \text{in Y-axis}$$

Q: A(2,2), B(1,1), C(3,1), $Sh_x = 1$, $Sh_y = 1$

Perform Shearing in both X & Y axes.



1) in X-axis.

$$A' = \begin{bmatrix} 2 + 1 \times 2 = 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad ; \quad C' = \begin{bmatrix} 3 + 1 \times 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 + 1 \times 1 = 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

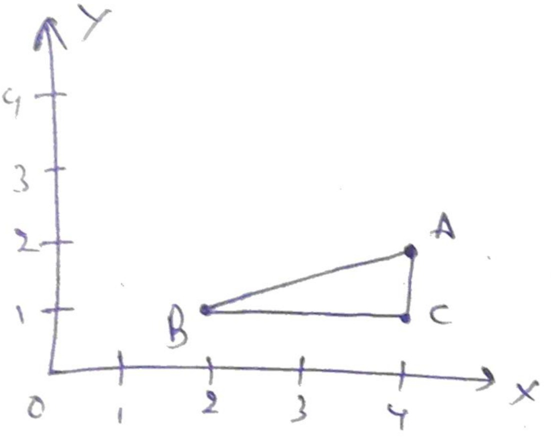
// in Y-axis

$$A' = \begin{bmatrix} 2 \\ 2+1\times 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 \\ 1+1\times 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

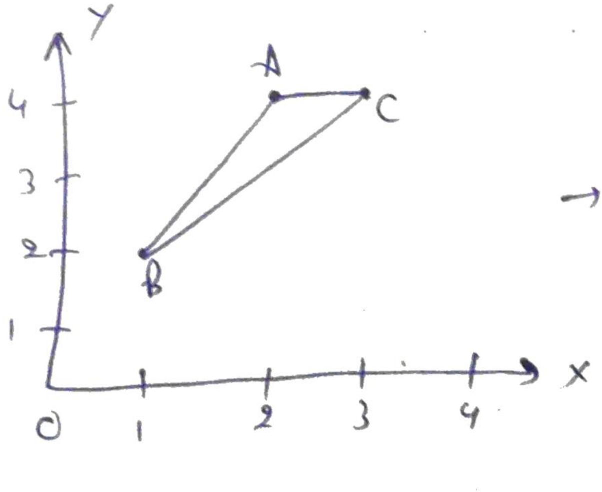
$$C' = \begin{bmatrix} 3 \\ 1+1\times 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

→ Lets display the shearing along x-axis.



→ y-coordinates remains unchanged

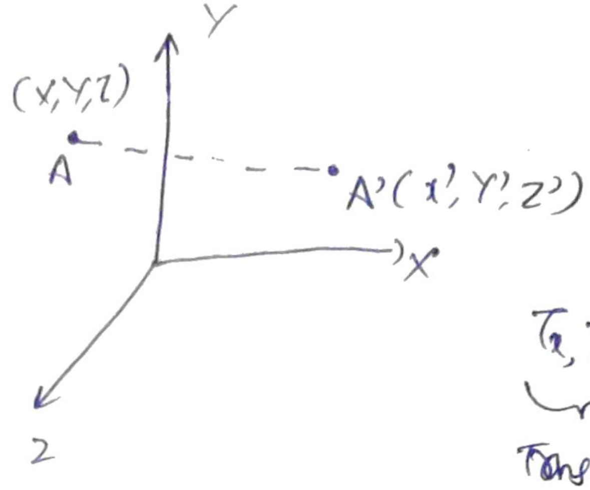
→ along y-axis



→ x-coordinates remains unchanged

* 3D Translat^{n}

$$\left.\begin{aligned} X' &= X + T_x \\ Y' &= Y + T_y \\ Z' &= Z + T_z \end{aligned}\right\} \text{Add}^n$$



T_x, T_y, T_z ⇝ Translat^{n} vectors

Homogenous coordinate system:-

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Translat}^n \text{ Matrix}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Q: A(0,3,6)
B(4,5,9)
C(3,4,8)

$T_x = 3$ units
$T_y = 2$ units
$T_z = 4$ units

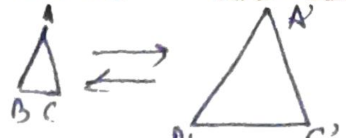
$$A' = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 0+3 \\ 3+2 \\ 6+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 10 \end{bmatrix}$$

$$B' = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 4+3 \\ 5+2 \\ 9+4 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 13 \end{bmatrix}$$

$$C' = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 3+3 \\ 4+2 \\ 8+4 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 12 \end{bmatrix}$$

* 3D Scaling

$$A' = \begin{cases} x' = x \cdot S_x \\ y' = y \cdot S_y \\ z' = z \cdot S_z \end{cases} \quad \text{Multiplic}^n$$



$\underbrace{S_x, S_y, S_z}_{\text{Scaling Factors}}$

$$\text{If} \begin{cases} > 1 \rightarrow \text{Zoom In} \\ < 1 \rightarrow \text{Zoom Out} \\ = 1 \rightarrow \text{no change} \end{cases}$$

Homogenous Coordinate System:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→ Scaling Matrix

Q: A(3,1,2) $S_x = 2$
B(1,2,2) $S_y = 2$
C(0,1,1) $S_z = 3$

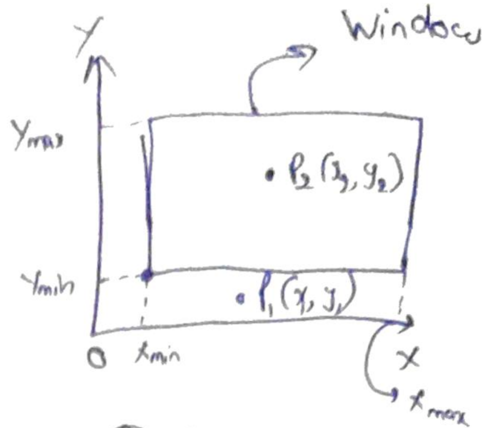
$$A' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 3\times2 \\ 1\times2 \\ 2\times3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 6 \end{bmatrix}$$

$$B' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1\times2 \\ 2\times2 \\ 2\times3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$C' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 0\times2 \\ 1\times2 \\ 1\times3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

Point Clipping

Lets check wheather a pt. lies inside / outside the window



⇒ $\boxed{\text{Inside}}$

1) $X \geq X_{min}$
2) $X \leq X_{max}$
3) $Y \geq Y_{min}$
4) $Y \leq Y_{max}$

{all 4 should satisfy for a pt to lie inside the window}

• BL (x_{min}, y_{min}) → Bottom Left of window

• UR (x_{max}, y_{max}) → Upper Right of window

Q. Lets say BL(2,3), UR(9,11). Check for a pt. (5,1)

1) $5 \geq 2$ ✓
2) $5 \leq 9$ ✓
3) $1 \geq 3$ → X
4)

{outside lie}

• Check for (6,9)

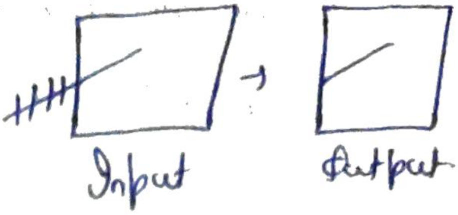
1) $6 \geq 2$ ✓
2) $6 \leq 9$ ✓
3) $9 \geq 3$ ✓
4) $9 \leq 11$ ✓

{Inside the window}

Cohen Sutherland Algorithm

{(LCA) = line clipping Algo.}

TBRL {Top, Bottom, Right, Left}



- Divide a window in 9 parts:

1001	1000	1010
0001	(0,0,0,0)	0010
0101	0100	0110

→ (TBRL) ↓ Region Code

(I) Line Inside (Completely) → Accept

Let say AB is a line, if the region codes of both A & B are '0000', then it lies completely inside.

(II) Line Outside (Completely) → Reject

Let CD be a line, then if logical AND of region codes of C & D is 'not 0000', then it lies completely outside eg., C = 0001, D = 0001 ⇒

$$\begin{array}{r} 0001 \\ 0001 \\ \hline 0001 \neq 0000 \end{array}$$

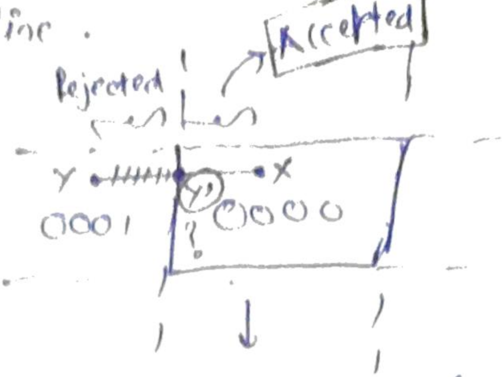
(III) Partially Inside (~~one end~~ pt. ~~lies inside & other lies outside~~)

- When the logical AND of region codes of both end pts. is '0000'. eg: C = 0001, D = 0110 ⇒

$$\begin{array}{r} 0001 \\ 0110 \\ \hline 0000 \end{array}$$ → Partially inside

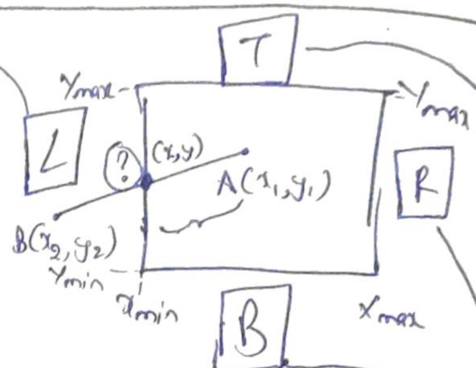
For (1111) case, the line inside the window is accepted irrespective of the entire line.

eg X: 0000
Y: 0001



→ Then we have to find that intersection point
"Y'"

Left Boundary Case : $\boxed{X = X_{min}}$



$$\left.\begin{aligned} m &= y_2 - y_1 / x_2 - x_1 \\ m &= y - y_1 / x - x_1 \end{aligned}\right\} \text{Same slopes}$$

$$m = \frac{y - y_1}{x_{min} - x_1} \Rightarrow \boxed{y = y_1 + m(x_{min} - x_1)}$$

Right Boundary Case : $\boxed{X = X_{max}}$

$$\boxed{y = y_1 + m(x_{max} - x_1)}$$

Top : $\boxed{Y = Y_{max}}$

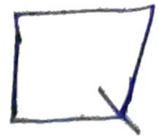
$$m = \frac{y - y_1}{x - x_1} = \frac{y_{max} - y_1}{x - x_1} \Rightarrow \boxed{x = x_1 + (y_{max} - y_1)/m}$$

Bottom : $\boxed{Y = y_{min}}$

$$\Rightarrow \boxed{x = x_1 + \frac{(y_{min} - y_1)}{m}}$$

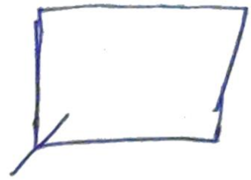
Now, for $\boxed{\text{Corner Cases:}}$

1) <u>Bottom Right Corner:</u>



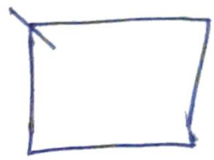
$$\boxed{\begin{aligned} X &= X_{max} \\ Y &= Y_{min} \end{aligned}}$$

2) <u>Bottom Left Corner:</u>



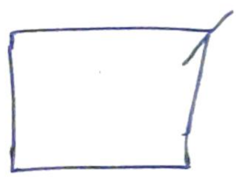
$$\boxed{\begin{aligned} X &= X_{min} \\ Y &= Y_{min} \end{aligned}}$$

3) <u>Top Left Corner:</u>



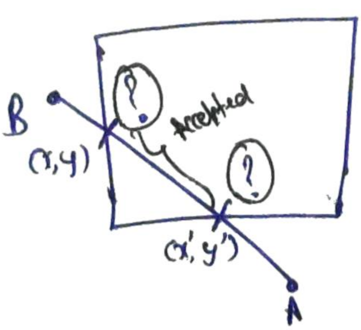
$$\boxed{\begin{aligned} X &= X_{min} \\ Y &= Y_{max} \end{aligned}}$$

4) <u>Top Right Corner:</u>



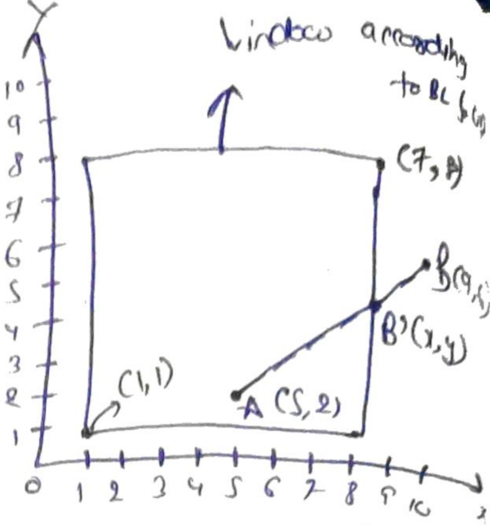
$$\boxed{\begin{aligned} X &= X_{max} \\ Y &= Y_{max} \end{aligned}}$$

If both points lie outside, but a part lie inside (which is to be accepted) then we will find 2 intersection points instead of 1, i.e.,



Q: BL(1,1), UR(7,8)

Points, A(5,2), B(9,6) {Line}



$X_{min} = 1$, $Y_{min} = 1$

$X_{max} = 7$, $Y_{max} = 8$

(T B R L)

For pt. A, region code is 0000

" " B, " " " 0010

& = 0000 → Partially Inside

Correction:

right boundary must be upto Coordinate x=7 not x=8.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{9-5} = 1$$

- Right Boundary Case: $x = x_{max} = 7$

$$\text{So, } m = \frac{y - y_1}{x - x_1} = \frac{y - y_1}{7 - x_1}$$

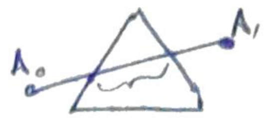
$$\Rightarrow 1 = \frac{y-2}{7-5} \Rightarrow y - 2 = 2 \Rightarrow y = 4$$

So, the req pt. B' is $\boxed{B'(7,4)}$

Cyrus Beck Algorithm {(CBA)}

- We were restricted to a rectangular clipping window in Cohen Sutherland's LCA, but in this algo we can clip $\boxed{\text{any polygon}}$

- Lets take a triangle.



Parametric eqⁿ
't' = Ac
at $t=0$,
$A_t = A_0$]
at $t=1$
$A_t = A_1$]

- Lets 'N_i' be a normal vector to any edge of the polygon.

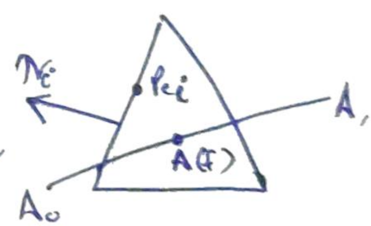
- Lets 'P_{ei}' be any point ~~to~~ on a particular edge of the polygon.

=) Let $A(t)$ be any point on the line

and, $\boxed{A(t) = A_0 + t(A_1 - A_0)}$

$\Rightarrow \{(x_t, y_t)\}$

* Then, we can conclude that,



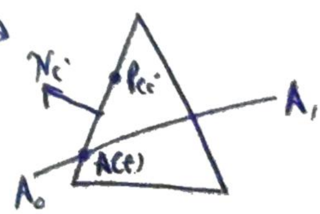
$\Rightarrow N_i(A(t) - P_{ei})$

→ 0 [intersection pt.]

→ >0 [A(t) outside the CW (clipping window)]

→ <0 [A(t) inside the CW]

→ $N_i\left[[A_0 + t(A_1 - A_0)] - P_{ei}\right] = 0$



$\Rightarrow N_i A_0 + N_i t A_1 - N_i t A_0 - N_i P_{ei} = 0$

$\Rightarrow N_i t A_1 - N_i t A_0 = N_i P_{ei} - N_i A_0$

$$\Rightarrow \boxed{t = \frac{N_i(P_{ei} - A_0)}{N_i(A_1 - A_0)}}$$

Lets, d (denominator) = $\boxed{N_i \cdot (A_1 - A_0)} = d$

Case I : if $d < 0$ → entering pt.
$d > 0$ → leaving pt.

In this algorithm, we find t_1, t_2, t_3 for each edge e_1, e_2, e_3 respectively ∵ we are considering a general polygon {triangle {3 sides}}

Suppose,

- $e_1 \rightarrow t_1 \Rightarrow$ entering pt
- $e_2 \rightarrow t_2 \Rightarrow$ leaving pt
- $e_3 \rightarrow t_3 \Rightarrow$ leaving pt

} compare for the smallest one

- But, if we get 2 entering (or multiple) points, then we compare / take for the greatest one.

⇒ Lets, say we get two respective points "t_1" as leaving pt. & "t_2" as entering pt.

then, just put 't_1' in eqn $A(t) = A_0 + t(A_1 - A_0)$ to get $\boxed{(x_{t_1}, y_{t_1})}$ → one end pt.

& " 't_2' " " " " " " " $\boxed{(x_{t_2}, y_{t_2})}$ → another end pt.

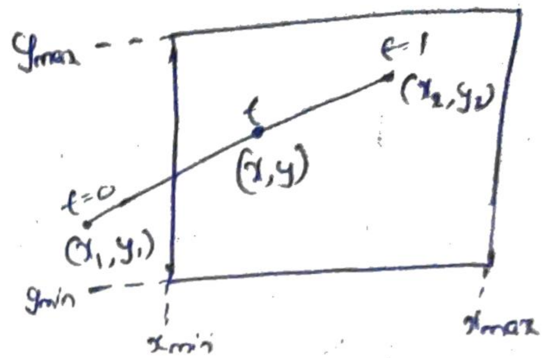
Liang Barsky Algorithm (LCA)

- Applicable on a $\boxed{\text{rectangular}}$ clipping window.

- Line Parametric eq^n:

$$\left.\begin{aligned} x &= (1-t)\,x_1 + t x_2 \\ y &= (1-t)\,y_1 + t y_2 \end{aligned}\right\}$$

$$\boxed{\begin{aligned} x &= x_1 + t(x_2 - x_1) \\ y &= y_1 + t(y_2 - y_1) \end{aligned}}$$



As per Point clipping algo.

$$\left.\begin{aligned} x_{min} &\le x \le x_{max} \\ y_{min} &\le y \le y_{max} \end{aligned}\right\}$$

$$\Rightarrow \quad x_1 + t\Delta x \geq x_{min} \qquad ; \qquad y_1 + t\Delta y \geq y_{min}$$

$$x_1 + t\Delta x \le x_{max} \qquad\qquad y_1 + t\Delta y \le y_{max}$$

$$t\Delta x \geq x_{min} - x_1 \qquad\qquad t\Delta y \geq y_{min} - y_1$$

$$\boxed{\begin{aligned} t\Delta x &\le x_{max} - x_1 \\ -t\Delta x &\le x_1 - x_{min} \end{aligned}} \qquad \boxed{\begin{aligned} t\Delta y &\le y_{max} - y_1 \\ -t\Delta y &\le y_1 - y_{min} \end{aligned}}$$

- General eq^n form: $\boxed{t p_K \le q_K}$ $\{K = 1, 2, 3, 4\}$

Here, $p_1 = -\Delta x$ $\quad p_3 = -\Delta y$

$p_2 = \Delta x$ $\quad p_4 = \Delta y$

IIIly, $q_1 = x_1 - x_{min}$ $\quad q_3 = y_1 - y_{min}$

$q_2 = x_{max} - x_1$ $\quad q_4 = y_{max} - y_1$

→ If, $P_k = 0$ = Line Parallel {for $k = 1, 2, 3, 4$}

→ If, $q_k < 0$ = Line Outside

→ If, $P_k \neq 0$

- if, $P_k < 0$, then find value t_1

  then, new value of $t_1 = \max\left(0, \frac{q_k}{p_k}\right)$

- if, $P_k > 0$, then find value t_2

  then, new value of $t_2 = \min\left(1, \frac{q_k}{p_k}\right)$

→ Now, check $(t_1 > t_2)$, then line is completely outside & it is rejected.

→ ~~Now~~ if $(t_1 < t_2)$, then we will clip it,

$$\Rightarrow \boxed{\begin{aligned} x &= x_1 + t_1 \Delta x \\ y &= y_1 + t_1 \Delta y \end{aligned}}$$