Project report on

"QUEUING MODEL ANALYSIS ON SUPERMARKET USING MONTE CARLO SIMULATION"

Submitted in partial fulfillment of the requirement for the degree of

B.Sc. (Hons) Mathematics

by

Bhawana Bhatt

20422033

Divyansh Mehra

20422056

Iskandha Goel

Under the guidance of

Dr. Chandan Singh Ujarari

Assistant Professor, Dept. of Mathematics

GRAPHIC ERA HILL UNIVERSITY

DEHRADUN

2020-23

Graphic Era HILL UNIVERSITY
Established by an Act of the State Legislature of Uttarakhand (Adhiniyam Sankhya 12 of 2011)
University under section 2(f) of UGC Act, 1956

CERTIFICATE

The undersigned certify that they have read and recommended to the Department of Mathematics, Graphic Era Hill University, Dehradun, a project entitled, "***Queuing model analysis on Supermarket using Monte Carlo Simulation***" submitted by **Bhawana Bhatt, Divyansh Mehra and Iskandha Goel**, under the guidance of **Dr. Chandan Singh Ujarari,** Assistant Professor, Dept. of Mathematics in partial fulfillment of the requirements of the degree of B.Sc. (Hons) Mathematics prescribed by **Graphic Era Hill University, Dehradun.**

The assistance and help received during the course of this investigation and source of literature have been duly acknowledged.

Date: 16/06/2023
Place: Dehradun

Dr. Chandan Singh Ujarari
(Supervisor)

Dr. Neeraj Dhiman
(Head of Department of Mathematics)

DECLARATION

We, Bhawana Bhatt, Divyansh Mehra and Iskandha Goel, hereby declare that the work presented in our project file titled "***Queuing model analysis on Supermarket using Monte Carlo Simulation***" is entirely our original work. This project was based on our independent research, analysis and synthesis of information gathered from various sources.

All the ideas, concepts, methodologies, and findings presented in this project file are the result of our own intellectual efforts. Any references, citations and sources utilized in the project have been duly acknowledged and properly cited in accordance with the guidance and standards set forth by our educational institution.

By signing this declaration, we affirm the originality and authenticity of our work and accept the consequence of any breach of academic integrity associated with this project.

Bhawana Bhatt

Divyansh Mehra

Iskandha Goel

Date: 16/06/2023

ACKNOWLEDGEMENT

We would like to express our heartfelt gratitude and appreciation to all those who have contributed to the successful completion of our project work titled "***Queuing model analysis on Supermarket using Monte Carlo Simulation***." This project would not have been possible without the support, guidance, and encouragement of numerous individuals, to whom we extend our sincere thanks.

First and foremost, we are deeply indebted to our project supervisor, Dr. Chandan Singh Ujarari, Assistant Professor in the Department of Mathematics at Graphic Era Hill University, Dehradun. His unwavering support, immense knowledge, and valuable insights have been instrumental in shaping this project.

We would also like to extend our gratitude to the Head of the Department of Mathematics, Dr. Neeraj Dhiman, for his support and encouragement. We are thankful for the opportunities provided by the department and the resources made available to us, which have contributed significantly to the successful completion of this project.

We would like to acknowledge the contributions of our fellow classmates and friends who have provided valuable insights, constructive feedback, and encouragement throughout the course of this project. Their discussions and exchange of ideas have been instrumental in refining our understanding of the subject matter and improving the overall quality of our work.

We are grateful to the library staff for their assistance in locating relevant research articles and resources. Their efforts have been indispensable in broadening our knowledge base and providing the necessary references for our project.

Last but not least, we would like to express our deepest appreciation to our family and friends for their unwavering support, understanding, and encouragement throughout this project. Their belief in us and constant motivation has been the driving force behind our accomplishments.

Bhawana Bhatt
Divyansh Mehra
Iskandha Goel

TABLE OF CONTENT

Preface

1. **Introduction**
   1.1. Introduction: An overview
   1.2. Characteristics of queuing models
   1.3. Transient and Steady States
   1.4. Kendall's Notation for representing queuing models
2. **Simulation**
   2.1. Definitions of Simulation
   2.2. Types of Simulation
   2.3. Phases of Simulation model
   2.4. Generation of Random Numbers (or digits)
   2.5. Monte Carlo Method of Simulation
3. **Literature Review**
4. **Research Methodology**
   4.1. Model Description
   4.2. Data collection
   4.3. Description of queuing system
5. **Discussions and Conclusion**
6. **References**

PREFACE

In today's fast-paced world, supermarkets play a vital role in providing essential goods and services to countless customers. With an increasing demand for efficient operations and optimal customer experience, it becomes imperative for supermarkets to streamline their processes, especially when it comes to managing customer queues. The ability to accurately analyze and forecast queue lengths, waiting times, and customer satisfaction can significantly enhance supermarket management strategies.

This project work, titled "***Queuing model analysis on Supermarket using Monte Carlo Simulation***," delves into the intricate world of queuing theory and its application to a real-life supermarket setting. Queuing theory is a branch of operations research that focuses on the mathematical study of waiting lines, allowing us to gain valuable insights into system performance and make informed decisions to improve operational efficiency.

The primary objective of this project is to develop a queuing model specifically tailored to a supermarket environment. By utilizing the Monte Carlo simulation technique, which employs random sampling to analyze statistical phenomena, we aim to comprehensively assess and optimize the queuing system's performance. This simulation-based approach enables us to consider a multitude of factors, such as customer arrival patterns, service times, and queue discipline, all of which contribute to the overall customer experience.

Throughout this project, we will explore the underlying principles of queuing theory and the significance of Monte Carlo simulation in modeling and analyzing complex systems. The simulation results will be analyzed, and meaningful insights will be derived to guide decision-making processes aimed at improving customer satisfaction and resource allocation.

It is important to note that this project work does not focus solely on theoretical concepts but also emphasizes the practical implementation of queuing models through computer simulations. By leveraging computational tools and statistical software, we can replicate real-life supermarket scenarios, collect relevant data, and analyze the results to draw meaningful conclusions.

In conclusion, this project represents a comprehensive investigation into the application of queuing theory and Monte Carlo simulation to supermarket management. The findings and recommendations derived from this research endeavor have the potential to revolutionize the way supermarkets design their queuing systems, leading to enhanced operational efficiency, improved customer satisfaction, and ultimately, a better shopping experience.

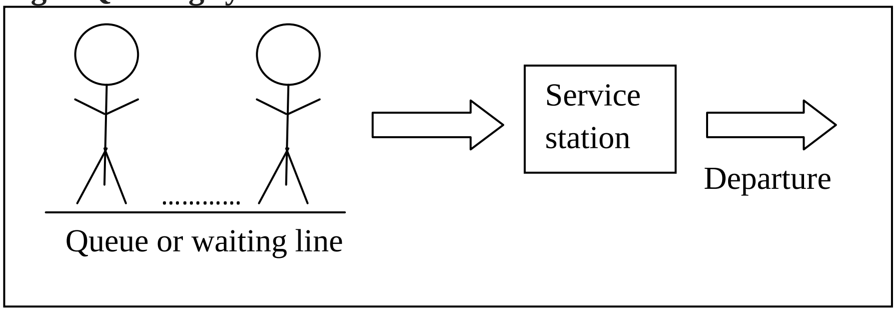
1. INTRODUCTION

1.1 INTRODUCTION: AN OVERVIEW

Queue is a common word that means a waiting line or the act of joining a line. Depending on the server stats, the incoming customer either was at the queue of gets served. If the server at the counter free at the time of arrival, the customer can get served without a waiting time In this process over a period of time, the system may experience customer waiting and/or server In any service system involving queuing situation, the objective is to design the system in such a manner that the average waiting time of the customers is minimized and the percentage utilization of the server is maintained above at a desired level. A queuing system can be described by:

- The input (or arrival pattern)
- The service mechanisms (or service pattern)
- The queue discipline
- Customer behavior

Fig. 1 Queuing system



1.2 CHARACTERISTICS OF QUEUING MODELS

A system consists of one or more servers, an arrival pattern of customers, service pattern, are disciple, the order in which the service is provided and customer behavior. The word 'queue is sometimes used to describe the whole system, but it is, in fact, part of the system that holds the excess customers who cannot be immediately served. Hence, the total number of customers in the system at given time will be equal to the number of customers in the queue plus the number of customers being served. Of course, these numbers will vary with time due to customer arrival and departure, so they are a random process.

1.2.1 The input (or arrival pattern):

It represents the pattern in which customers arrive at the system. Arrivals may also be represented by the inter-arrival time, which is the time period between two successive arrivals. Arrivals may be separated by equal intervals of time, or unequal but definitely known intervals of time, or by unequal intervals of time whose probabilities are known; these are called random arrivals. The rate at which customers arrive at the service station, that is, the number of customers arriving per unit of time is called arrival rate. The assumption regarding the distribution of arrival rate has a great impact on the mathematical model. If the number of customers is very large, the probability of an arrival in the next interval of time does not depend upon the customers already in the system. Hence, the arrival is completely random and it follows the Poisson process with mean equals the average number of arrivals per unit time, represented by λ

1.2.2 The service mechanism (or service pattern):

The service pattern is similar to the arrival pat teen, but there are some important differences. Service time may be a constant or a random variable Distributions of service time which we are following are negative exponential distribution, which is characterized by a single parameter, the mean rate μ or its mean service time $\frac{1}{\mu}$. The servicing system in which the customers may be saved in batches of fixed size or of variable size by the same server is termed as hulk service system. The system in which service depends on the number of waiting customers is termed as state-dependent system.

1.2.3 Capacity of the system:

Some of the queuing processes admit physical limitations to the amount of waiting room, so that when the waiting line exceeds a fixed length, no further customers are allowed to enter until space becomes available by a service completion. This type of situation is termed as finite source queues

1.2.4 Service arrangements:

For providing service to the incoming customers, one or more service points are established the number depends on the number of customers, rate of arrivals, time taken for providing service to a single customer, and so on. Depending on these variables, a service channel is single or multiple. When there are several service channels available to provide service, much depends upon their arrangements. They may be arranged in parallel or in series or a more complex combination of both, depending on the design of the system's service mechanism.

- **Series channels**: A customer must pass successively through all the ordered channels before service is completed, for example, in public offices where parts of the service are done at different service

- **Counters Parallel channels**: A number of channels providing identical service facilities so that several customers may be serviced simultaneously,

A queuing system is called a one-server model when the system has one server only, and a multiple-server model when a system has a number of parallel channels with one server.

Fig. 2 Queuing system with single queue and single service station

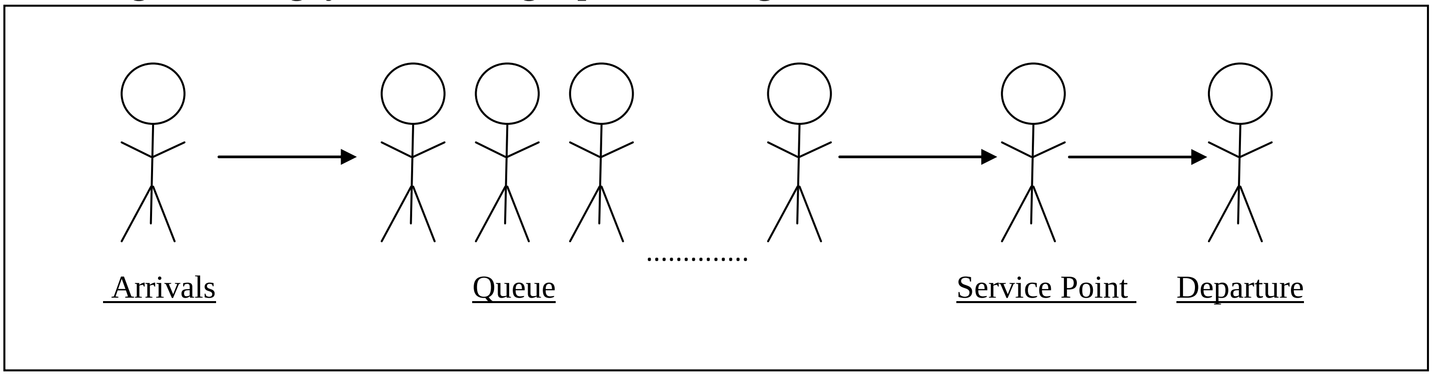
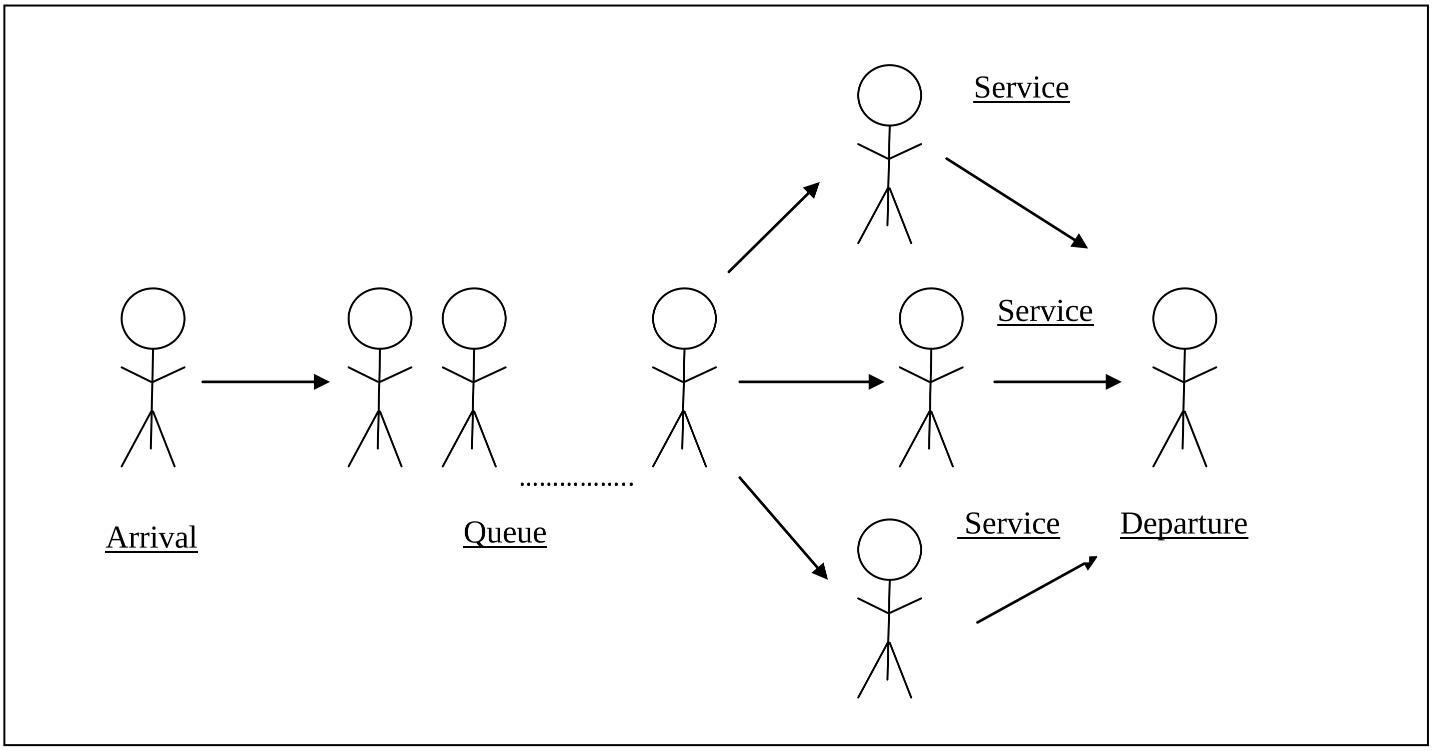


Fig.3 Queuing system with single queue and several service stations, parallel servers



1.2.5 Service time:

The time required for servicing a customer is called service time. Service time may be constant or it may vary with the customer. However, for sake of simplicity, it is assumed that the time required for servicing for all customers is constant. Moreover, since the arrival pattern is assumed to be random, the service time is also taken as random. Hence, the service time follows exponential distribution with mean equal to reciprocal of the mean rate of service. In cases where the assumption of service time following distribution fails, Erlang distribution is applied.

1.2.6 Queue discipline:

It is a rule determining the formation of the queue, the manner of customer's behavior while waiting, and the manner in which they are chosen for service. The following are queue discipline:

(a) **First In First Out (FIFO):** According to it, the customers who come first will be served first. Example: Customers served in ration shop, at the cinema ticket counter.

(b) **Last In First Out (LIFO):** According to it, the service of customers is done in the reverse order in which they arrive, that is, the person entering last is served first. Example: In a big go down, the items which are stored on top (last arrival) are taken out first.

(c) **Service In Random Order (SIRO):** The selection of customer for servicing is random at any particular time.

(d) Service In Priority (SIP): Certain Customers are given priority over others in selection for service. It is of two types: non-emptive priority and, emptive priority.

Non-emptive priority: The customer already getting service is allowed to continue till it is completed even if a priority customer comes midway during the service.

Emptive priority: The service to non-priority customer is stopped as soon as a priority customer arrives.
Example: Treatment for a patient who is critical.

1.2.7 Customer behavior:

Customers generally behave in four ways:

(I) **Balking:** A customer may leave the queue because it is too long and he has no time to wait or there is no sufficient waiting space.

(II) **Reneging:** Some customers wait for some time in the queue but leave due to impatience without getting the service.

(III) **Collusion:** Some of the customers join together and only one of them, instead of all, stay in the queue However, when their turn comes for service, the customers who were in collusion demand service.

(IV) **Jockeying:** Some customers keep shifting from one queue to another to improve their position and to get immediate service.

1.3 TRANSIENT AND STEADY STATES

A system is said to be in transient state when its operating characteristics are dependent on time. A system is said to be in steady state when the behavior of the system is independent of time. Let $P_n(t)$ denote the probability that there are n units in the system at time t; then, in steady state, $\lim_{t\to\infty} P'_n(t) = 0$.

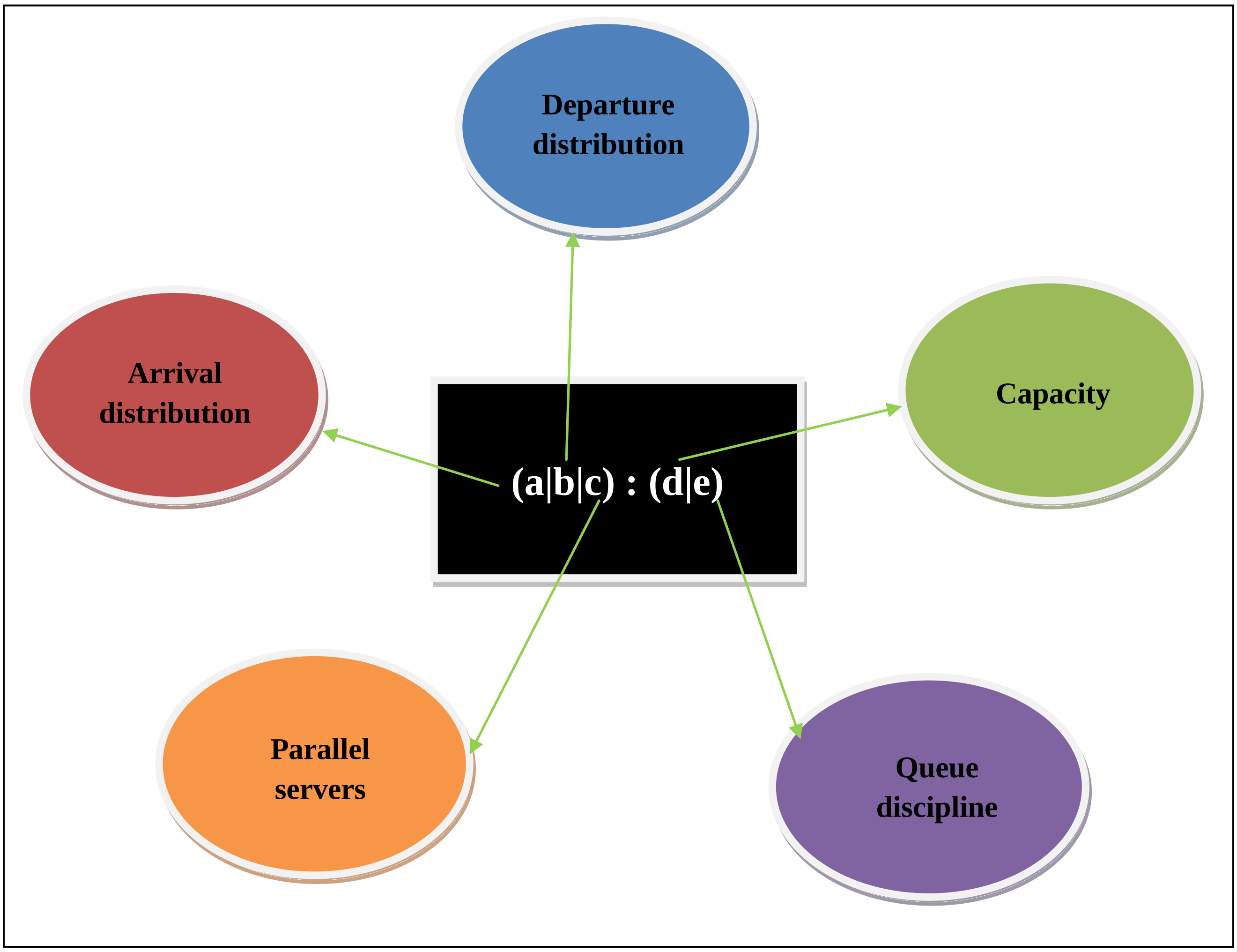
1.4 KENDALL'S NOTATION FOR REPRESENTING QUEUING MODELS

A convenient notation for summarizing the characteristics of the queuing models is given by:

(a|b|c) : (d|e)

Where a = arrival distribution
b = departure (service time) distribution
c = number of parallel servers
d = capacity of the system
e = queue discipline

Fig, 4 Kendall's Notation for representing Queuing Model



2. SIMULATION

2.1 DEFINATIONS OF SIMULATION

Following are a few definitions of simulation:

1. Simulation is a representation of reality through the use of a model or other device which will react in the same manner as reality under a given set of conditions. 2
.
2. Simulation is the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation.

3. According to Donald G. Malcolm, a simulated model may be defined as one which depicts the working of a large-scale system of men, machines, materials and information operating over a period of time in a simulated environment of the actual real world conditions.

4. According to Naylor et al. (1966), simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and

logical relationships necessary to describe the behavior and structure of a complex real world system over extended periods of time.

5. Churchman has defined simulation as follows: "X simulates Y" is true if and only if:

a) X and Y are formal systems;
b) Y is taken to be the real system;
c) X is taken to be a approximation to the real system; and
d) The rules of validity in X are non-error-free; otherwise X will become the real system.

2.2 TYPES OF SIMULATION

Simulation is mainly of two types.

1) Analogue (environmental) simulations: The simple examples cited in 14.1.1 are of simulating the reality in physical form, which we refer as analogue (or environmental) simulation.

2) Computer, system or digital simulation: For complex and intricate problems of managerial decision-making, analogue simulations may not be applicable; moreover, actual experimentation with the system may also be uneconomical. In such situations, the complex system is formulated into a mathematical model for which a computer program is developed and then the problem is solved using a high-speed electronic computer. Such type of simulation is called computer simulation, system simulation or digital simulation. In other words, it is the representation of a system in a form acceptable to a digital computer as opposed to an analogue computer. Electronic circuit simulation uses mathematical models to replicate the behavior of an electronic device or circuit. Simulation software allows for modeling of circuit operation and is an invaluable analysis tool. Simulating a circuit's behavior before actually building it can greatly improve design efficiency by making fault designs and providing insight into the behavior of the electronic circuit designs.

Simulation models can be classified into following four categories:

- **Deterministic models**: In these models input and output variables are not permitted to be random and models are described by exact functional relationship.
- **Stochastic models**: In these models at least one of the variables or functional relationship is given by probability functions.
- **Static models**: These models do not take variable time into consideration.
- **Dynamic models**: These models deal with time varying interaction.

2.3 PHASES OF SIMULATION MODEL

A simulation model consists of two basic phases:

Phase 1: Data Generation: Data generation involves the sample observation of variables and can be carried out with the help of any of the following methods:
a) Using random numbers.
b) Resorting to mechanical devices, and

c) Using electronic computers.

Phase II: Book Keeping: The book-keeping phase of a simulation model deals with updating the system when new events occur, monitoring and recording the system states as and when they change, and keeping track of qualities of our interest (such as, idle time and waiting time) to compute the measures of effectiveness.

Steps in Simulation Process:

Step 1: Identify the measure of effectiveness.

Step 2: Decide the variables which influence the measure of effectiveness, choose those variables which affect the measure of effectiveness significantly.

Step 3: Determine the probability distribution for each variable in step 2 and construct the cumulative probability distribution.

Step 4: Choose an appropriate set of random numbers.

Step 5: Consider each random number as a decimal value of the game cumulative probability distribution.

Step 6: Use the simulated values so generated into the formula derived from the measure of effectiveness.

Step 7: Repeat (5) and (6) until the sample is large enough to arrive at a satisfactory and reliable decision.

2.4 GENERATION OF RANDOM NUMBERS (OR DIGITS)

In the mid-square method of generating pseudo-random numbers, a four-digit number is taken. By squaring it a high digit figure is obtained from which the middle four digits are picked. This yields the second random number and the process is repeated and a sequence of pseudo- random numbers is obtained. For instance, if

$$X_1 = 2421$$
$$X_1^2 = 05\underline{8612}41$$
$$X_2 = 8612$$
$$X_2^2 = 74\underline{1665}44$$
$$X_3 = 1665$$
$$X_3^2 = 02\underline{7722}25$$
$$X_4 = 7722 \qquad \text{and so on.}$$

2.4.1 Generation of Random Numbers (or Digits) from Probability Distributions

There are a couple of methods to generate a random number based on a probability density function. These methods involve transforming a uniform random number in some way. Because of this, the methods work equally well in generating both pseudo-random and true random numbers. The first method, called the inversion method, involves integrating up to an area greater than or equal to the random number (which should be generated between 0 and 1 for proper distributions). The second method, called the acceptance-rejection method, involves choosing the x and y values and testing whether the function of x is greater than the y value. If it is so, the x value is accepted. Otherwise, the x value is rejected and the algorithm tries again.

The Chi-square test of goodness of fit is employed to check if the sequence of numbers is generated from a (0, 1) uniform distribution. The randomness or independence test is used to check that the successive numbers are not correlated. One of the most effective methods for this purpose is the poker test. Most computer systems have a sub-routine available for generating random numbers.

In inventory models, the variables include customer's demand and delivery times, which may also be probabilistic. In this type of situations, for generating random numbers the manual method consisting of the following steps can be used.

1) Collect the data related to the current problem.
2) Construct a frequency distribution with this data.
3) Construct the relative frequency distribution.
4) Assign a coding system that relates the identified events to generate random numbers.
5) Select a suitable method for obtaining the required random numbers.
6) Match the random numbers to the assigned events and tabulate the results.
7) Repeat step (6) until the desired number of simulation runs has been generated.

2.5 MONTE CARLO METHOD OF SIMULATION

The Monte-Carlo method is a simulation technique in which statistical distribution functions are created by using a series of random numbers. This approach has the ability to develop many months or years of data in a matter of a few minutes on a digital computer. The method is generally used to solve problems which cannot be adequately represented by mathematical model or where the solution of the model is not possible by analytical method.

Monte-Carlo simulation yields a solution which should be very close to the optimal, but not necessarily the exact solution. However, it should be not noted that this technique yields a solution that converges to the optimal or correct solution as the number of simulated trials lead to infinity.

2.5.1 Steps in Monte-Carlo simulation:

Step 1: Clearly define the problem

- **(1)** Identify the objectives of the problem.
- **(2)** Identify the main factors which have the greatest effect on the objectives of the problem.

Step 2: Construct an appropriate model

- **(1)** Specify the variables and parameters of the model.
- **(2)** Formulate the appropriate decision rules, that is, state the conditions under which the experiments is to be performed.
- **(3)** Identify the type of distribution that will be used-models use either theoretical distributions or empirical distributions to state the patterns and the occurrence associated with the variables.

(4) Specify the manner in which time will change.
(5) Define the relationship between the variables and parameters.

Step 3: Prepare the model for experimentation
(1) Define the starting conditions for the simulation.
(2) Specify the number of runs of simulation to be made.

Step 4: Using step 1 to 3, experiment with the model
(1) Define a coding system that will correlate the factors defined in step 1 with the random numbers to be generated for the simulation.
(2) Select a random number generator and create the random numbers to be used in the simulation.
(3) Associate the generated random numbers with the factors identified in step 1 and coded in step 4(1).

Step 5: Summarize and examine the results obtained in step 4.

Step 6: Evaluate the results of the simulation.
(1) Select the best course of action.

3. LITERATURE REVIEW

Applebaum, W. (1951) aimed to ascertain who buys where, what, when and how. In addition, such studies try to learn about customer response to sales promotion de-vices. The results of these studies were useful in the solution of an array of marketing problems. These studies were gaining importance in marketing research. It was safe to predict that interest in them will increase in the next decade. in this paper the authors summarized their experiences with such studies in grocery stores. However, the principles and techniques discussed here were also applicable to other types of retail stores. The "why" of customer behavior was a separate and exceedingly difficult subject; it was not treated here. A knowledge of customer behavior should have been preceded any con-sideration of the reasons for the behavior.

Van Dijk, N. M. (1997) aimed to address and illustrate that queuing theory has a wider potential than perceived, while at the same time a variety of practical problems, both in daily-life and industry, were open for fundamental research. To this end, this paper: (i) Highlighted *basic queuing insights* for daily-life purposes. (ii) Provided *exact and bounding results* for queuing network applications. (iii) Presented several practical illustrations (case studies) taken from areas as: Daily-life situations (postal offices and supermarkets); Transportation (railways and air-traffic); Administrative logistics (reengineering); Telecommunications (call-centres).

Bennett, R. (1998) took a data of shoppers leaving four supermarket outlets (two in one of London’s most prosperous boroughs and two in London’s poorest) were questioned about their attitudes towards having to wait in queues, including express checkout lines. The last of interview questions incorporated items designed to assess whether customers possessed Type A or Type B personality traits. It emerged that although Type A inclination did help explain certain shoppers’ perspectives on waiting-in-line, customers’ locations in either a rich or a poor neighbourhood and their subjective evaluations of whether their families were better-off or worse-off than other families in the area were more influential as determinants of attitudes concerning queues.

Wallace, D. W., et al. (2004) investigated customer retailer loyalty in the context of multiple channel retailing strategies. Results showed that multiple channel retail strategies enhance the portfolio of service outputs provided to the customer, thus enhancing customer satisfaction and customer retailer loyalty. These results suggested that multiple channel retailing can be a useful strategy for building customer retailer loyalty.

Luczak, M. J., & McDiarmid, C. (2006) used M/M/1 model taking n queues, each with a single server. Customers arrive in a Poisson process at rate λn, where $0<\lambda<1$. Upon arrival each customer selects $d\geq 2$ servers uniformly at random and joins the queue at a least-loaded server among those chosen. Service times were independent exponentially distributed random variables with mean 1. The authors showed that the system was rapidly mixing, and then investigated the maximum length of a queue in the equilibrium distribution. The authors proved that with probability tending to 1 as $n\to\infty$ the maximum queue length takes at most two values, which were $\ln(\ln n)/\ln d+O(1)$.

Nafees, A. (2007) contained the analysis of Queuing systems for the empirical data of supermarket checkout service unit as an example. This paper aimed to estimate the waiting time and length of queue(s). The authors used queuing simulation to obtain a sample performance result and were more interested in obtaining estimated solutions for multiple queuing models. This paper described a queuing simulation for a multiple server process as well as for single queue models. This study required an empirical data which may include the variables like, arrival time in the queue of checkout operating unit (server), departure time, service time, etc. A questionnaire was developed to collect the data for such variables and the reaction of the ICA Supermarket from the customers separately. This model was developed for a sales checkout operation in ICA supermarket, Borlänge. The model designed for this example was multiple queues multiple-server model. The model contained five servers which were checkout sales counters; attached to each server was a queue.

Zhiwei, N., (2009) established some simulation models of queuing systems, based on Anylogic6.0 simulation software, with different queuing disciplines including first come first service, last come first service and random service. Compared with the theoretical values, the accuracy of the experiment data was verified. Finally, with comparative analyzes of experiment data, the authors showed that under a special condition, the difference of the performance of the queuing systems with different queuing disciplines was limited.

Li, B., & Wang, D. (2010) developed a new the mathematical model about service system based on queuing theory. Then it analyzed and optimized the model, which realized rational allocation about human resources and made the system at the best operation state. In a word, the paper developed a $M/M/C/\infty/\infty$ model for the checkout employee allocation in supermarket, which provided the theoretical basis for scientific management of supermarkets.

Jie, Y. (2010) studied about waiting cost of every unit time and customer, on the condition of adding cost of a serviceman, on the scope of free time and time customer expending on system.

Madan, M., & Kumari, S. (2012) investigated the detailed information about the growth of retailing industry in India. It examined the growing awareness and brand consciousness among people across different socio-economic classes in India, and how the urban and semi-urban retail markets were witnessing significant growth. It explored the role of the

Government of India in the industry's growth and the need for further reforms. In India, the vast middle class and its almost untapped retail industry were the key attractive forces for global retail giants wanting to enter newer markets, which in turn will help the retail industry in India to grow faster. This paper included growth of retail sector in India, strategies, strength, and opportunities of retail stores, retail format in India, latest trends, and opportunities and challenges. This paper concluded with the impact of the entry of global players into the retail industry in India. It also highlighted the challenges faced by the industry in near future.

Pathirana, A. D., & Peiris, T. U. I. (2012) investigated the characteristics of waiting lines and proceeds to investigate the factors affecting customer satisfaction concerning waiting lines in supermarkets located in the Colombo district. Since customers were particularly concerned about the time that they spend in the queue, this was very crucial. Customers become frustrated and turned away because of the time spent in the queue. This study further investigated the factors that can be used to overcome the issue frustrated customers through the results of queuing theory characteristics. In this regard the results revealed that the customer frustrations due to overestimated waiting can be abolished through promotional activities, improving customer perceptions regarding waiting time, showing positive responses to reduce waiting lines, improving the environment around the waiting lines, and reducing perceived waiting duration.

Rani, G., & Srinivasan, A. (2012) used the M/M/2/K queueing Model with controllable arrival rates, different service rates and feedback was considered. The steady state solution and system characteristics were derived for this model. The analytical results were numerically illustrated and the effect of the nodal parameters on the system characteristics were studied, and relevant conclusions were presented.

Chai, C. F. (2013) provided reference for decision at the issue of optimizing the quantity of service cashier desks, improving service efficiency, and decreasing operating costs. It analyzed Supermarket cashier system queuing issues through establishing M/M/C queuing model on basis of operation research.

Joshua, O. O. (2013) was a revelation to novice researchers since it explores Markovian queueing model in real life situation. The fundamental of Markovian Queueing model as birth and death process was hereby reviewed in this article, with fundamental results applications in M / M / 1, M / M / S, M / M / 1/ K, and M / M / s / K. Here the authors re-examined; Average Number of Customers and average number of times in the system, waiting in the queue, in service respectfully. The summaries of these results were also tabulated.

Zhao, T., & He, C. (2013) explained that supermarket operators to facilitate the purchase installed substantial number of tourists pay POS machine, to reduce the customer waiting time. Introduction of excess POS is used to increase the cost of the supermarket operators. Used queue theory by comparing the calculated cash and credit card payment waiting time. With the results of guided supermarket operator reasonable arrangement of POS machine number.

Varma, I. G., & Agarwal, M. R. (2014) aimed to identify the factors that affected the online buying behavior of women particularly homemakers in Western suburbs of Mumbai. The question that raised was whether this segment was also buying online or remained untapped by the e-retailers? An empirical study through online survey was conducted on 56 homemakers in western suburbs of Mumbai across the age group of 25-45 yrs. This research aimed to analyze the buying attitudes of homemakers, influence of social media on their buying behavior and other factors that affected their purchase decision online. It also attempted to explore the brick and mortar buying behavior vs. online buying behavior of the target audience. This study had its limitations as the sample size was too small to provide definitive information for marketing strategies to be developed, however it provided cues on which marketers can go into further depth to commission large scale research, given the opportunity that obviously exists.

Prasad, S. V., & Badshah, V. H. (2014) proved that single queue – multi server model was better than multi queue – multi server model and the authors discussed the relation between these two models and derive the mathematical equations, and the queuing number, the optimal number of checkout stands, were investigated by means of the queuing model, and also derive the mathematical equations of servicing cost and waiting time (cost) of the customers. So that the waiting time of customer was reduced, the customer satisfaction was increased. It was proved that this model of the queuing system was feasible by the example. The results were effective and practical.

Igwe, A., Onwuere, J. U. J., & Egbo, O. P. (2014) examined efficient queue management in the study of Wenjie, X., Senbiao, L., & Lihong, H. (2015). The “M/M/I” model was applied owing, inter alia, to its relative simplicity as well as its relevance to the firms under-studied: the City Company, Nobis Ltd, Dobbiac and Eke Ltd; particularly the service unit of each of the firms. The analysis revealed that on the aggregate, queue management in Makurdi town, the capital of Benue State was grossly inadequate, inefficient, and ineffective. The paper had consequently made suggestions to help mitigate the prevailing queuing problems in Makurdi town, which was relevant to many cities in developing countries having similar challenges.

Rana, S. S., et al. (2015) aimed at investigating how much differential impact these factors have on the purchase intention of customers to shop at hyper markets. Primary data were collected through survey with structured questionnaire from 150 customers in Kedah and Perlis states in Malaysia. Correlations and multiple regression analyzes were employed to estimate relationships between independent and dependent variables. The results showed that brand image had the highest impact on purchase intention of customers followed by the quality of products sold at the stores and social influence. So, marketers should give importance on these factors to influence customers to shop at hypermarkets.

Priyangika, J. S. K. C., & Cooray, T. M. J. A. (2015) tried to estimate the waiting time and length of queue(s). Queuing simulation was used to obtain a sample performance result and estimated solutions for multiple queuing models were also interested. This study required an empirical data which may include the variables like, arrival time in the queue of checkout operating unit (server), departure time, service time, etc. A questionnaire was developed to collect the data for such variables and the reaction of the Supermarket from the customers separately. This model was developed for a sales checkout operation in the supermarket. The model designed for this research was multiple queues multiple-server model. The model contained five servers which were checkout sales counters; attached to each server was a queue. In any service system, a queue formed whenever current demand exceeded the existing capacity to serve. This occurred when the checkout operation unit was too busy to serve the arriving costumers, immediately.

Wenjie, X., et al. (2015) presented a simulation model of supermarket queuing theory based on M/M/1 model. The authors compared queuing systems under different number of goods, one was single queuing system, the other was double-queuing system. Then, the optimal cut-off number of goods was developed according to the analysis of economic cost and the optimal cashier number in both queuing systems. The simulation results showed that the double queuing system further shortened the staying time of customers, significantly reduced the operating cost, and improved the operational efficiency of the supermarket.

Macharia, I. (2016) aimed to show that queuing theory satisfies the model when tested with the real-case scenario. The data used in this research was obtained from NSNB archives as well as through observation. The data was analyzed basing on the queuing theory. Little's theorem becomes very essential in deriving certain quantities using a M/M/4 queuing model. The quantities analyzed includes arrival rate, service rate, average utilization of the system, average number of customers in the system, average number of customers in the queue, the average time a customer spends in the system, the average time a customer spends waiting in the queue before being served and the probability of having n customers in the system at some time t. All these quantities were calculated and compared to the observed values to produce a suitable solution to the growing queue length in the supermarket. This research was concluded by discussing the importance of the queuing model analysis to a busy supermarket premises.

Rema, V., (2017) attempted to understand consumer demographics, key factors driving buying behavior, impact of loyalty programs on customer loyalty intentions at a well-established brick and mortar supermarket. A structured questionnaire was administered to 100 shoppers at a selected Supermarket in Bengaluru city. Statistical tests of chi-square and correlation had been used. Convenience and ease of access, variety of product categories, quality and availability of products were emerged as key factors driving customer purchase behavior. There was a significant impact of store loyalty programs on customers of different age groups. Empirical data of arrival and service times during the checkout process was recorded through observation technique to analyze the performance measures of the multi-channel multi-queue system at the checkout counters. Monte Carlo Simulation technique was used to analyze the operating characteristics of queues, efficiency, and server utilization. These factors played a predominant role in understanding customer satisfaction during the checkout process and helped the Management to evaluate service efficiency at the checkout points.

Jhala, N., & Bhathawala, P. (2017) examined efficient queue management in XYZ supermarket as case study. One of the expected gains from studying queuing systems was to review the efficiency of the models in terms of utilization and waiting length, hence increasing the number of queues so customers did not have to wait longer when servers were too busy. In other words, the authors tried to estimate the waiting time and length of queue(s), was the aim of this study. Through this, the author tried to estimate that at given time how many servers could be needed so that the total cost was optimized.

Rajesh, G., & Rajan, N. (2018) enabled the sales personnel to undertake a self-assessment of the quality of services offered by them in the organized retail markets in Kerala. Data were collected from 90 sales personnel from private stores Supply.co and Consumer fed. Their responses were assessed through the SERVQUAL scale. These findings indicated that except in respect of promotional services such as parking facilities in all other aspects the quality of services were found to be same according to the study. The authors examined that the satisfaction of the customer could be obtained only by providing high quality services. For this the staff had to be made quality-conscious in all dealings through systematic training by business management experts.

Koeswara, S., et al. (2018) used multiple lines queuing system (M/M/S) to analyze. The calculation was done manually and by using the software "Quantitative for Windows" with waiting lines module. The result of the research revealed that the performances of the existing queuing system was not optimal due to the usage of the payment facility and the cashier skill were still low.

Wang, J., & Zhou, Y. P. (2018) studied how queue configuration affects human servers' service time by comparing dedicated queues with shared queues using field data from a natural experiment in a supermarket. The authors hypothesized that queue configuration may affect servers' service rate through several mechanisms such as pooling. To investigate these impacts, the authors took advantage of the supermarket's checkout layout and use a data set containing both checkout transaction details and queue information collected from video recordings in the supermarket. The queue configuration's direct effect and its indirect queue length effected function independently of each other. In aggregation, the social loafing effect dominated, and servers slowed down (a 6.86% increase in service time) in shared queues.

Li, L., & Xu, J. (2018) showed the related theory of queuing theory was applied to optimize the supermarket cashier service system. Some relevant data was obtained by collecting and then M/M/C queuing model was applied to cashier service system of a supermarket in Urumqi. According to the actual data, the author optimized the number of the cash register in different periods. From the angles of supermarkets and customers, the author got the most optimal number of the register, which reduced the costs of the supermarket and improve customer satisfaction.

Raj, S. Y., et al. (2018) focused on developing an alternative methodology to the existing one by using time study and queuing theory. With help of results, a new methodology was proposed to increase the productivity, swiftness of shopping. The proposal was to departmentalise the billing process and implant smart billing concepts like localised self-initiated billing counters or customized trolleys to bill products. This new study resulted in time saving in the queuing and billing process. The cart was designed using CREO 2.0 software and made smart with the help of wireless sensing devices. With the burgeoning role of wireless sensing devices such as bar-code scanners, this application can be extended to the consumer market such as a supermarket where manual-billing process becomes cumbersome and at the same time involves a hefty waste of time and labour.

Devi, A. (2018) considered M/M/c queuing model and applied to solve a problem in a supermarket of generated crowds by buyers on the cash boxes. The performance of the supermarket was analyzed using three cash boxes with arriving rate of 50 customers/ hour and service rate 18 customers/ hour. Also, it was considered whether it was necessary to include additional cash boxes to reduce the waiting queue and time.

Olamide, E. I. (2019) took an example of where queue can be formed was the banking sector. The data used for this project was a primary source collected through direct observation from First Bank of Nigplc Samuel Falodun branch, Akure, for a period of ten working days. The M│M│C model was used. The analytical formulas of queue theory were used in the estimation of the parameters. The underlying assumption of queuing theory ensures the arrival and service rates were Poisson and exponentially distributed, respectively. The results obtained from the estimation of parameters showed that service rate was less than the arrival rate, and the traffic intensity i.e., the probability that the servers were idle was 0.06 since $\rho < 1$ it implies that the service rate μ utilizes 94% of the system and remains idle for 6% of the time. If there was a cost incurred for every time spent, then by adding one more server, the cost and customer waiting times will be reduced.

Arvidsson, A., & Hassani, L. (2020) aimed to explore the possibility of designing a Smart Cart application prototype with a user-centred approach based on Human-Computer Interaction (HCI) to be extended upon previous proposals. User data, which was used to analyze and find the points in design, had been gathered by a questionnaire with 275 participants and interviews with 3 people. This was used together with information from a literature review to design the Smart Cart app prototype, which was a visualisation of the study results. The prototype was supported by an analysis which showed why it was important to involve users in the design process and what should be considered when doing so. The result of this study also supported that when users accepted and were familiar with certain functionalities in applications, they were more likely to adopt the application. Lastly, the study concluded that most of the participants had a positive attitude towards applications in smart shopping and had similar desires of functions and appearance.

Luo, R., & Shi, Y. (2020) analyzed the behavior of the customers, got the balance of the customer arrival rate in these two queueing systems. Further, this paper analyzed the optimal pricing problem of supermarket, and got the average optimal price the supermarket shall set up. Finally, by comparing two kinds of queuing system, this paper gave the supermarket the determination when the cashier weighing merged, and when to separate the two conditions.

Junior, J. V. P., et al. (2020) presented a case study related to the management of queues in a supermarket and the problems found in the organization of this process. To reduce the time that customers remain in the checkout lines, an analysis of the supermarket was carried out through the discrete simulation technique, linked to the Arena software. This technique was classified as quantitative because it made it possible to measure entities and predict the action of the environment in a way that mimics the reality of the queues at the site. Through the simulated scenario, it was possible to identify the flaws in the process and the cause of the queuing. Through the results of the simulation, it was observed that the average length of stay in the queue would be reduced by 88.23% if it contemplates the inclusion of 2 to 3 employees to perform the service.

Meiyappan, S., et al. (2020) examined customers waiting to get service from server were portrayed by queue and called waiting line. Unsatisfied customer due to lengthy queue can be potential loss to any service organization. In single queue single server, the above-mentioned potential loss was higher. So, the authors dealt with single queue multi server. This motivated them for the study among single queue multi server and multi queue multi server which one was best according to the expected total cost. In this paper, the authors revealed the total cost with assumption of service cost and waiting cost was better in single queue multiple servers than multiple queue multiple servers in supermarket checkouts counters.

Pallikkara,V., et al.(2021) showed that impulse buying at the store checkout area was minimal and sporadic for most of the product categories at the checkout. Impulse buying at the checkout was instigated by factors such as store environment, credit card availability, momentary mood, in-store promotion, offers and discounts and large merchandise. The study had important implications for retail stores by emphasising on the choice of merchandise offered for sale at the checkout area. Further, the investigation revealed that Indian shoppers are health-conscious and cautious about their purchase at the checkout rather than being impulsive.

Rahaman, A., et al. (2021) adopted the Single queue and multi-channel models for the study of the existing structure of the Grocery Bazaar (GB) supermarket with three servers/machines. The data set was analyzed using R-studio software and the following estimates were obtained for Single Queuing Model ($M/M/1$): Arrival Rate, Service Rate, Traffic Intensity, Average Time in the System, Average Time each Customer wait in the queue, Average Number of Customers waiting, Number of Customers in the System, Expected Average Total Time, and Expected Number to be Served. Also, under the Multi-Channel Queuing System/Model ($M/M/C$), this paper examined the Service Rate, Traffic Intensity, Probability of having zero Customer in the System, Average expected Queue Length, Average Number of Customers in the System, Expected Total Time, Waiting Time, Expected Number of Customers in the Queue, Probability of Queuing on the Arrival and Probability of not Queuing on the Arrival. The data collected was tested to show if it followed a Poisson and exponential distributions of arrival and service rate using chi square goodness of fit. The derived results revealed that the acquired data were statistically reliable to a great extent and suggestions were made at the end of the study on how to improve the process of queue /queuing problem in the Grocery Bazaar (GB) supermarket.

Pinto, P., & Hawaldar, I. T. (2022) investigated the customer idle time and its implication on emotional discomfort resulting from crowding stress. Accordingly, 385 respondents (shoppers) visiting the leading organized retailers located in major localities in Bengaluru were approached. The responses were analyzed using a Chi-squared test and Pearson correlation. The outcome revealed that irrespective of age and gender, customers visiting the offline retail outlets experienced emotional discomfort due to the idleness during the checkout. The young customers aged 18-30 dislike waiting in the queue at the checkout compared to older customers. In contrast, gender did not affect the inclination to wait. The study concluded that customers desire to avert an unproductive use of time, thus lowering their emotional discomfort.

Safuri, A. A. M., & KAMARDAN, M. G. (2022) focused on a queuing simulation of a supermarket using Arena Simulation Software. The goal of this research was to learn more about Arena Simulation Software and to examine the supermarket's queuing problem. Throughout this study, three factors were examined: Average Waiting Time of Customer (Wq), Average Number of Waiting Customers (Lq), and Instantaneous Utilization. The simulation model result showed that the (Wq) was 13.84 minutes, (Lq) was 3 customers and the Instantaneous Utilization was 89%. These conditions were well utilized. They also tried to simulate the outcome if the supermarket adds another server. From the analysis the authors suggested that the supermarket did not need to add another cashier and use just one cashier to serve customers.

Murad, S. S., et al. (2022) aimed to provide a comprehensive background on social distancing as well as effective technologies that can be used to facilitate the social distancing practice. Scenarios of enabling wireless and emerging technologies were presented, which were especially effective in monitoring and keeping distance amongst people. In addition, detailed taxonomy was proposed summarizing the essential elements such as implementation type, scenarios, and technology being used. This research reviews and analyzes existing social distancing studies that focus on employing various kinds of technologies to fight the Coronavirus disease (COVID-19) pandemic. The main goal of this study was to identify and discuss the issues, challenges, weaknesses, and limitations found in the existing models and/or systems to provide a clear understanding of the area.

De Melo, G. A., et al. (2022) aimed to optimize the cashier service macroprocess in a supermarket located in the Alto Paranaiba. Decision-making tools were applied, such as brainstorming, process mapping, Ishikawa diagram and PDCA cycle. The simulation type was based on the M/M/1 model, in the context of Queuing Theory. Thus, factors such as customer satisfaction and increased productivity were sought through systematic control and continuous improvement of the cashiers' service macroprocess. Finally, the authors carried out the alignment between the PDCA cycle and the Routine Management brought an effective contribution, represented by the standardization of the macroprocess, by the construction of Standard Operating Procedures (SOPs).

Liu, Q., & Chen, H. (2022) found that the arrival time of supermarket customers obeyed Poisson distribution, and the service time obeyed exponential distribution through queuing theory. By calculating the average arrival rate of customers, the authors got the service intensity of the system in each period and master the use of the supermarket cashier system in each period. Then the authors used process analysis to shorten the cashier time, reduce customer waiting, optimize the number of cashiers according to the specific situation, and determine the optimal number of open cashiers in each period.

Mushi, A. R., & Kagoya, S. M. (2022) explained that OR aims at deriving optimal or better solutions to maximize various attributes such as sales or profits and/or to minimize attributes such as costs, losses, or risks. Many research on OR have been reported to supermarkets operations in the world. These include supermarket customer behaviors, productivity modelling, store simulations, promotion planning, and others. It was important to find out specific consumer behaviors and cultural factors that may influence better performances on supermarkets and the extent of their considerations when planning business models. This may help to explain some of the factors of supermarket business failures in the region. The paper intended to review OR supermarkets applications around the world, collected data on specific features of the East African market, and proposed measures suitable to the regional markets.

Hua, S., et al. (2022) collected the person times of the supermarket in different time periods, tested the service rate with K-S test, and calculated various index values, so as to put forward suggestions for optimizing the number of people and configuring a reasonable number of settlement service people in the supermarket based on M/M/C model. To optimize the management of the queue length, the average service time, and the time interval to the cashier in the process of supermarket queuing settlement *was observed.*

Sasi, M. A., et al. (2023) investigated efficient queue management in FPS using M/M/C queueing model. The major goal of this research was to minimize customer waiting time in the FPS with the minimal number of servers. It demonstrated the application of the simulation modelling as a method for optimizing the number of servers in a specific FPS in Kerala, India. The model was developed to provide the outcomes of system parameters like arrival rate, waiting time and server utilization.

4. RESEARCH METHODOLOGY:

Owing to variety of products and variety in modes of shopping, consumers today have become very demanding. The efficient management of queues in a supermarket is crucial for providing satisfactory customer experience. By accurately analyzing queue behaviour, supermarkets can optimize their operations and minimize customer waiting time.

This research paper focuses on employing Monte Carlo Simulation to evaluate the queuing dynamics and determine the optimal number of servers required to achieve maximum efficiency within the studied supermarket. A well established super mart of Dehradun is chosen located in Subhashnagar. The choice of this location stems from the understanding that students constitutes a significant customer base for the supermarket, making it an ideal setting to examine queuing dynamics and optimise service delivery. The sampling design used was Simple Random Sampling.

One of the key drivers of customer satisfaction may be associated with waiting time at the checkout counters. Delays at the billing or checkout points are of common occurrence in supermarkets which could prevent the customer from coming back to the store. With an objective to model the checkout process, data on customer arrival rate, service time, and other relevant parameters is collected to construct an accurate representation of supermarket's queuing system.

Detailed analysis was carried out using MS EXCEL and ORIGIN and key findings are presented as graphs. By simulating different scenarios and analyzing the results, the research aims to identify potential bottlenecks, evaluate system performance, and propose strategies for enhancing customer satisfaction and operational efficiency in supermarket.

4.1 MODEL DESCRIPTION: MULTI-SERVICE MODEL (M|M|s): (∞|FIFO)

When there are n units in the system, following two cases will arrive:

1. $n \le s$, all the customers may be served simultaneously. There will be no queue. (s-n) number of servers remains idle. In this case $\mu_n = n\mu$, n= 0, 1, 2, ..., s.
2. $n \ge s$, all the servers are busy and the maximum number of customers waiting in queue will be (n-s). Then $\mu_n = s\mu$, n= 0, 1, 2, ...

The steady state difference equations are:

for n = 0,

$$P_0(t + \Delta t) = P_0(t)\,(1 - \lambda\Delta t) + P_1(t)\,\mu\Delta t, \tag{4.1.1}$$

for n = 1, 2, ..., s-1

$$P_n(t + \Delta t) = P_n(t)\,(1 - (\lambda + \mu t)\,\Delta t) + P_{n-1}(t)\,\lambda\Delta t + P_{n+1}(t)\,(n + 1)\,\mu\Delta t, \tag{4.1.2}$$

and

for n = s, s+1, ...

$$P_n(t + \Delta t) = P_n(t)\,[(1 - (\lambda + s\mu)\,\Delta t)] + P_{n-1}(t)\,\lambda\Delta t + P_{n+1}(t)\,\mu\Delta t, \tag{4.1.3}$$

Dividing these equations by Δt and taking $\Delta t \rightarrow 0$, the difference equations are

for n = 0

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t), \tag{4.1.4}$$

for n = 1, 2, ..., s-1

$$P_n'(t) = -(\lambda + n\mu) P_n(t) + \lambda P_{n-1}(t) + (n + 1) \mu P_{n+1}(t) \quad , \qquad (4.1.5)$$

for n = s, s+1, s+2, ...

$$P_n'(t) = -(\lambda + s\mu) P_n(t) + \lambda P_{n-1}(t) + s\mu P_{n+1}(t) \quad , \qquad (4.1.6)$$

In steady state condition, when $t \rightarrow \infty$, $P_n(t) \rightarrow P_n$, $P_n'(t) \rightarrow 0$, for all n. So, the above equations become

$$0 = -\lambda P_0 + \mu P_1, \quad \text{for } n = 0 \qquad (4.1.7)$$

$$0 = -(\lambda + n\mu) P_n + \lambda P_{n-1} + (n + 1) \mu P_{n+1}, \quad \text{for } n = 1, 2, ..., s\text{-}1 \qquad (4.1.8)$$

$$0 = -(\lambda + s\mu) P_n + \lambda P_{n-1} + s\mu P_{n+1}, \quad \text{for } n \geq s \qquad (4.1.9)$$

From (4.1.7) it follows that

$$P_1 = \frac{\lambda}{\mu} P_0 \qquad (4.1.10)$$

Substituting n = 1 in (4.1.8), we get

$$P_2 = \frac{\lambda}{2\mu} P_1 = \left(\frac{1}{2!}\right)\left(\frac{\lambda}{\mu}\right)^2 P_0 \qquad (4.1.11)$$

Substituting n = 2 in (4.1.8), we get

$$P_3 = \left(\frac{1}{3!}\right)\left(\frac{\lambda}{\mu}\right)^3 P_0 \qquad (4.1.12)$$

In general, we get

$$P_n = \frac{\lambda}{n\mu} P_{n-1} = \left(\frac{1}{n!}\right)\left(\frac{\lambda}{\mu}\right)^n P_0, \quad \text{for } 1 \leq n \leq s \qquad (4.1.13)$$

From (4.1.9) we get

$$s\mu P_{s+1} = (\lambda + s\mu) P_s - \lambda P_{s-1}$$

$$P_{s+1} = \frac{(\lambda + s\mu)}{s\mu} P_s - \frac{\lambda}{s\mu} P_{s-1}$$

$$= \left(\frac{\lambda}{s\mu}\right) P_s + P_s - \left(\frac{\lambda}{s\mu}\right) P_{s-1}$$

$$= \left(\frac{\lambda}{s\mu}\right) P_s + \frac{\lambda}{s\mu} P_{s-1} - \left(\frac{\lambda}{s\mu}\right) P_{s-1} \qquad \text{by (4.1.13)}$$

$$= \left(\frac{\lambda}{s\mu}\right) P_s$$

$$P_{s+1} = \frac{1}{n} . \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^{s+1} P_0 \qquad (4.1.14)$$

Similarly, we get

$$P_{s+2} = \left(\frac{\lambda}{s\mu}\right) P_{s+1} = \frac{1}{s^2} . \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s+2} P_0 \qquad (4.1.15)$$

In general,

$$P_n = P_{s+(n-s)} = \frac{1}{s^{n-s}} . \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^n P_0, \quad \text{for } n \geq s \qquad (4.1.16)$$

Moreover,

$$\sum_{n=0}^{\infty} P_n = 1 \quad \Rightarrow \quad \sum_{n=0}^{s-1} P_n + \sum_{n=s}^{\infty} P_n = 1 \qquad (4.1.17)$$

That is,

$$\sum_{n=0}^{s-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0\right] + \sum_{n=s}^{\infty}\left[\frac{1}{s^{n-s}}.\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^n P_0\right] = 1$$
$$P_0\left[\sum_{n=0}^{s-1}\frac{s^n}{n!}\left(\frac{\lambda}{s\mu}\right)^n + \frac{1}{s!}\sum_{n=s}^{\infty}\frac{s^n}{s^{n-s}}.\left(\frac{\lambda}{s\mu}\right)^n\right] = 1$$
$$P_0\left[\sum_{n=0}^{s-1}\frac{s^n}{n!}\rho^n + \frac{s^s}{s!}\sum_{n=s}^{\infty}\rho^n\right] = 1, \quad \text{where } \rho = \frac{\lambda}{s\mu}$$
$$P_0 = \left[\sum_{n=0}^{s-1}\frac{(s\rho)^n}{n!} + \frac{s^s}{s!}\sum_{n=s}^{\infty}\rho^n\right]^{-1} \tag{4.1.18}$$

Hence the steady state distribution of arrival is

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, n = 0, 1, \ldots.., s-1 \\ \frac{1}{s!}.\frac{1}{s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, n = s, s+1, \ldots. \end{cases} \tag{4.1.19}$$

4.1.1 Measures of the model:

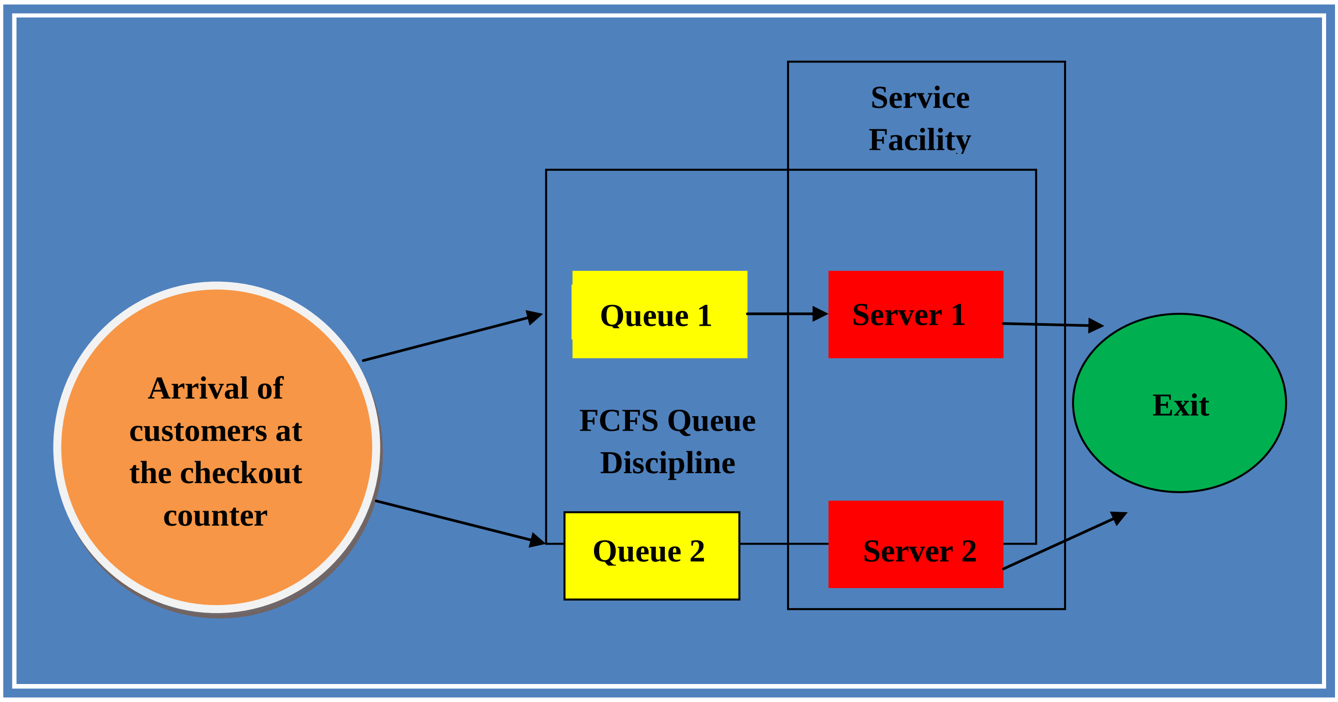
1. Average length of queue, $L_q = P_s.\frac{\rho}{1-\rho^2}$, where $P_s = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0$
2. Average number of units in the system, $L_s = L_q + \frac{\lambda}{\mu}$
3. Expected waiting time in the system, $W_S = \frac{L_s}{\lambda}$
4. Expected waiting time in the queue, $W_q = \frac{L_q}{\lambda}$
5. Expected length of non empty queue (L | L > 0) = $\frac{s\mu}{s\mu-\lambda} = \frac{1}{1-\rho}$
6. Expected waiting time of customer who has to wait (w | w > 0) = $\frac{1}{s\mu-\lambda}$
7. Probability of waiting time of a customer who has to wait [w > 0] = $\frac{P_s}{1-\rho}$
8. Probability that there will be someone waiting = $\frac{P_s\rho}{1-\rho}$
9. Average number of idle servers = s – average number of customers served
10. Efficiency = $\frac{Average\ number\ of\ customers\ served}{Total\ number\ of\ customers\ served}$

4.2 DATA COLLECTION:

Data is collected from a supermarket located in Subhashnagar, Dehradun, during a specified time period from 04:00 p.m. to 6:00 p.m. The store had two server and two parallel queues. A multi-channel, multi-queuing system was analysed through Monte Carlo simulation technique. The purpose was to understand the different performance measures or indicators to enable the supermarket develop efficient queue management techniques.

A queuing system can be described as customers arriving for service, waiting for the service if it is not immediate, and if having waited for service, leaving the system after being served. In the present study, the queuing system is described by arrival of customers at two checkout/billing counters in the supermarket, waiting in the queues, being served at one of the two counters and then leaving the Supermarket, as shown in the conceptual framework below.

Fig.5. Conceptual framework of the Queuing process at the Checkout Counter of the Supermarket.



4.2 **DESCRIPTION OF QUEUING SYSTEM**:

There are four basic characteristics of queuing process that describe a queuing system:

1) **Arrival pattern:** A customer's arrival at the store and at the checkout/ billing counter is a random/stochastic phenomenon. The data for the arrival times of customers are taken from the daily database of the mart and the time between successive arrivals (inter-arrival times) is computed. The arrival rate 'λ' which indicates the number of customers arriving at the checkout counter per unit time (per minute) is computed.

2) **Service pattern:** The service process is governed by the service rate 'μ', which indicates the number of customers served per unit time (per minute).

3) **Number of service channels:** The supermarket has two servers at two checkout counters serving the arriving customers. Hence this is a case of multi-server queuing system, fed by two queues, one for each server.

4) **Queue Discipline:** Queue discipline refers to the manner in which customers in queue are selected for service. The queue discipline here is first come, first served (FCFS).

4.3 ANALYSIS FOR FIRST AND SECOND QUEUE OF SYSTEM MODEL

Observation data was collected from the store. The arrival time, service time, number of customers in queue and departure time were observed and recorded for the two queues generated at the two servers in the checkout unit. Data was recorded for 21 customers at each of the two queues. Monte Carlo simulation is used to model the queuing system in the supermarket. The simulation replicates the customer arrival process and their subsequent interactions with the available servers. MS Excel's RANDBETWEEN function is employed to generate random numbers that simulate trials up to 50.

4.3.1 Analysis for queue 1:

Based on the observation data collected with respect to arrival times, inter-arrival times were computed. For the first channel (server), computations of the inter-arrival time probability distribution, service time distribution, waiting time distribution and detailed Monte Carlo Simulation using random number generation is shown from tables 1 to 4. Random number generator was used to simulate the trials based on the probability distribution for inter-arrival times, service times and waiting times. Key operating characteristics i.e. arrival rate, service rate, average waiting time for a customer in the queue and in the system (i.e. time spent in the queue and at service) and service facility utilization (percentage of the time the servers are busy) are computed for first queue. An average measure of operating characteristics has been computed based on the results.

Table 1:- Customer Inter-arrival time probability distribution for queue one (at the first server).

Inter Arrival time (in minutes)	Frequency	Cumulative frequency	Probability	Cumulative probability	Random number Interval
0	3	3	0.14	0.14	00-13
1	2	5	0.09	0.23	14-22
2	3	8	0.14	0.37	23-36
3	1	9	0.05	0.42	37-41
4	4	13	0.20	0.62	42-61
6	3	16	0.14	0.76	62-75
7	1	17	0.05	0.81	76-80
8	1	18	0.05	0.86	81-85
10	1	19	0.05	0.91	86-90
11	2	21	0.09	1.00	91-99
Total	**21**		**1.00**		

Table 2:- Service time distribution for server one

Service time (in minutes)	Frequency	Cumulative frequency	Probability	Cumulative Frequency	Random Number Interval
2	4	4	0.20	0.20	00-19
3	5	9	0.23	0.43	20-42
4	3	12	0.14	0.57	43-56
5	3	15	0.14	0.71	57-70
6	2	17	0.10	0.81	71-80
7	2	19	0.10	0.91	81-90
8	1	20	0.04	0.95	91-94
9	1	21	0.04	0.99	95-98
Total	**21**		**0.99 ≈ 1.00**		

Table 3:- Customer waiting time distribution for server one.

Waiting time (in minute)	Frequency	Cumulative frequency	Probability	Cumulative probability	Random number Interval
0	8	8	0.38	0.38	00-37
1	5	13	0.24	0.62	38-61
2	2	15	0.09	0.71	62-70
3	2	17	0.09	0.80	71-79
4	1	18	0.05	0.85	80-84
5	1	19	0.05	0.90	85-89
7	1	20	0.05	0.95	90-94
11	1	21	0.05	1.00	95-99
Total	**21**		**1.00**		

Table 4:- Detailed simulation of the queuing process for server one.

Trial	Random Number		Inter Arrival time	Service time distribution	Arrival time	Service time		Idle time	Customer waiting time
	Arrival time	Service time				start	end		
1	-	-	0	2	04:00	04:00	04:02	0	0
2	-	-	2	4	04:02	04:02	04:06	0	0
3	-	-	6	5	04:08	04:08	04:13	2	0
4	-	-	7	3	04:15	04:15	04:18	2	0
5	-	-	0	6	04:15	04:18	04:24	0	3
6	-	-	8	2	04:23	04:24	04:26	0	1
7	-	-	4	4	04:27	04:27	04:31	1	0
8	-	-	2	3	04:29	04:31	04:34	0	2
9	-	-	6	5	04:35	04:35	04:40	1	0
10	-	-	1	3	04:36	04:40	04:43	0	4
11	-	-	4	4	04:40	04:43	04:47	0	3
12	-	-	10	7	04:50	04:50	04:57	3	0
13	-	-	6	2	04:56	04:57	04:59	0	1
14	-	-	1	3	04:57	04:59	05:02	0	2
15	-	-	4	6	05:01	05:02	05:08	0	1
16	-	-	0	5	05:01	05:08	05:13	0	7
17	-	-	11	3	05:12	05:13	05:16	0	1
18	-	-	3	2	05:15	05:16	05:18	0	1
19	-	-	11	9	05:26	05:26	05:35	8	0
20	-	-	4	8	05:30	05:35	05:43	0	5
21	-	-	2	7	05:32	05:43	05:50	0	11
22	93	35	11	3	05:43	05:50	05:53	0	7
23	27	24	2	3	05:45	05:53	05:55	0	8
24	95	29	11	3	05:56	05:56	05:59	1	0
25	23	11	2	2	05:58	05:59	06:01	0	1
26	51	77	4	6	06:02	06:02	06:08	1	0
27	67	43	6	4	06:08	06:08	06:12	0	0

28	32	86	2	7	06:10	06:12	06:19	0	2
29	64	49	6	4	06:16	06:19	06:23	0	3
30	49	80	4	6	06:20	06:23	06:29	0	3
31	83	30	8	3	06:28	06:29	06:32	0	1
32	22	84	1	7	06:29	06:32	06:39	0	3
33	11	88	2	7	06:31	06:39	06:46	0	8
34	08	50	2	4	06:33	06:46	06:50	0	13
35	43	12	4	2	06:37	06:50	06:52	0	13
36	59	7	4	2	06:41	06:52	06:54	0	11
37	64	20	6	3	06:47	06:54	06:57	0	7
38	35	13	2	2	06:49	06:57	06:59	0	8
39	66	26	6	3	06:55	06:59	07:02	0	4
40	11	22	0	3	06:55	07:02	07:05	0	7
41	42	67	4	5	06:59	07:05	07:10	0	6
42	52	34	4	3	07:03	07:10	07:13	0	7
43	88	20	10	3	07:13	07:13	07:16	0	0
44	41	16	3	2	07:16	07:16	07:18	0	0
45	14	58	1	5	07:17	07:18	07:23	0	1
46	94	08	11	2	07:28	07:28	07:30	5	0
47	61	45	4	4	07:32	07:32	07:36	2	0
48	16	77	1	6	07:33	07:36	07:43	0	3
49	65	47	6	4	07:39	07:43	07:47	0	4
50	49	54	4	4	07:43	07:47	07:51	0	4

4.3.1.1 Computation of Operating Characteristics of Queue One:

- Mean Inter-arrival time = $\frac{\text{Total inter arrival time}}{\text{Total number of arrivals}} = \frac{223}{50}$ = **4.46 minutes per customer**
- Arrival rate, $\lambda_1 = \frac{1}{Mean\ inter\ arrival\ time} = \frac{1}{4.46}$ = **0.22 customers per minute**
- Mean Service time = $\frac{Total\ service\ time}{Number\ of\ customers\ served} = \frac{205}{50}$ = **4.1 minutes per customer**
- Service rate, $\mu_1 = \frac{1}{Mean\ service\ time} = \frac{1}{4.1}$ = **0.24 customers per minute**
- Mean waiting time of customers in queue = $\frac{Sum\ of\ customers\ waiting\ time}{Total\ arrivals} = \frac{202}{50}$
  = **4.04 minutes per customer**
- Mean time a customer spends in the system (checkout counter)
  = Average serving time + Average waiting time in the queue
  = 4.10 + 4.04 = **8.14 minutes**
- Service facility utilization at server one = $\frac{\lambda}{\mu} = \frac{0.22}{0.24}$ = **0.9167**

4.3.2 Analysis for queue 2:

Now for the second channel (server), similar computations are shown from tables 5 to 8. Key operating characteristics i.e. arrival rate, service rate, average waiting time for a customer in the queue and in the system (i.e. time spent in the queue and at service) and service facility utilization (percentage of the time the servers are busy) are computed for second queue. An average measure of operating characteristics has been computed based on the results.

Table 5:- Customer Inter-arrival time probability distribution for queue two (at the second server).

Inter Arrival time (in minutes)	Frequency	Cumulative frequency	Probability	Cumulative probability	Random number Interval
0	1	1	0.05	0.05	00-05
1	3	4	0.14	0.19	05-18
2	2	6	0.09	0.28	19-27
3	1	7	0.05	0.33	28-32
4	2	9	0.09	0.42	33-41
5	4	13	0.19	0.61	42-60
6	1	14	0.05	0.66	61-65
8	3	17	0.14	0.80	66-79
10	2	19	0.09	0.89	80-88
13	1	20	0.05	0.94	89-93
15	1	21	0.05	0.99	94-98
Total	**21**		**0.99 ≈ 1.00**		

Table 6:- Service time distribution for server two.

Service time (in minutes)	Frequency	Cumulative frequency	Probability	Cumulative Frequency	Random Number Interval
1	2	2	0.09	0.09	00-08
2	2	4	0.09	0.18	09-17
3	3	7	0.14	0.32	18-31
4	5	12	0.24	0.56	32-55
5	6	18	0.29	0.85	56-84
6	2	20	0.09	0.94	85-93
7	1	21	0.05	0.99	94-98
Total	**21**		**0.99 ≈ 1.00**		

Table 7:- Customer waiting time distribution for server two.

Waiting time (in minute)	Frequency	Cumulative frequency	Probability	Cumulative probability	Random number Interval
0	10	10	0.48	0.48	00-47
1	2	12	0.09	0.57	48-56
2	2	14	0.09	0.66	57-65
3	1	15	0.05	0.71	66-70
4	3	18	0.14	0.85	71-84
6	1	19	0.05	0.90	85-89

7	2	21	0.09	0.99	90-98
Total	**21**		**0.99≈1.00**		

Table 8:- Detailed simulation of the queuing process for server two.

Trial	Random Number		Inter Arrival time	Service time distribution	Arrival time	Service time		Idle time	Customer waiting time
	Arrival time	Service time				start	end		
1	-	-	5	5	04:05	04:05	04:10	5	0
2	-	-	8	4	04:13	04:13	04:17	3	0
3	-	-	6	3	04:19	04:19	04:22	2	0
4	-	-	8	6	04:27	04:27	04:33	5	0
5	-	-	5	4	04:32	04:33	04:37	0	1
6	-	-	3	5	04:35	04:37	04:42	0	2
7	-	-	4	3	04:39	04:42	04:45	0	3
8	-	-	10	6	04:49	04:49	04:55	4	0
9	-	-	2	1	04:51	04:55	04:56	0	4
10	-	-	13	5	05:04	05:04	05:09	8	0
11	-	-	1	4	05:05	05:09	05:13	0	4
12	-	-	8	3	05:13	05:13	05:16	0	0
13	-	-	5	4	05:18	05:18	05:22	2	0
14	-	-	2	5	05:20	05:22	05:27	0	2
15	-	-	0	1	05:20	05:27	05:28	0	7
16	-	-	15	7	05:35	05:35	05:42	7	0
17	-	-	1	5	05:36	05:42	05:47	0	6
18	-	-	4	2	05:40	05:47	05:49	0	7
19	-	-	10	5	05:50	05:50	05:55	1	0
20	-	-	1	2	05:51	05:55	05:57	0	4
21	-	-	5	4	05:56	05:57	06:01	0	1
22	37	40	4	4	06:00	06:01	06:05	0	1
23	6	85	1	6	06:01	06:05	06:11	0	4
24	19	37	2	4	06:03	06:11	06:15	0	8
25	41	10	4	2	06:07	06:15	06:17	0	8
26	59	19	5	3	06:12	06:17	06:20	0	5
27	70	55	8	4	06:20	06:20	06:24	0	0
28	45	28	5	3	06:25	06:25	06:28	1	0
29	76	7	8	1	06:33	06:33	06:34	5	0
30	88	51	10	4	06:43	06:43	06:47	9	0
31	52	6	5	1	06:48	06:48	06:49	1	0
32	86	71	10	5	06:58	06:58	07:03	9	0
33	24	89	2	6	07:00	07:03	07:09	0	3
34	82	14	10	2	07:10	07:10	07:12	1	0
35	25	52	2	4	07:12	07:12	07:16	0	0
36	41	77	4	5	07:16	07:16	07:21	0	0
37	4	24	0	3	07:16	07:21	07:24	0	5
38	37	65	4	5	07:20	07:24	07:29	0	4
39	54	92	5	6	07:25	07:29	07:35	0	4

40	94	39	15	4	07:40	07:40	07:44	5	0
41	44	46	5	4	07:45	07:45	07:49	1	0
42	35	27	4	3	07:49	07:49	07:52	0	0
43	63	64	6	5	07:55	07:55	08:00	3	0
44	18	75	1	5	07:56	08:00	08:05	0	4
45	11	22	1	3	07:57	08:05	08:08	0	8
46	83	98	10	7	08:07	08:08	08:15	0	1
47	55	18	5	3	08:12	08:15	08:18	0	3
48	1	74	0	5	08:12	08:18	08:23	0	5
49	2	38	0	4	08:12	08:23	08:27	0	4
50	77	18	8	3	08:20	08:27	08:30	0	7

4.3.2.1 Computation of Operating Characteristics of Queue Two:

- Mean Inter-arrival time = $\frac{\text{Total inter arrival time}}{\text{Total number of arrivals}} = \frac{245}{50}$ = **4.9 minutes per customer**
- Arrival rate, $\lambda_2 = \frac{1}{Mean\ inter\ arrival\ time} = \frac{1}{4.9}$ = **0.20 customers per minute**
- Mean Service time = $\frac{Total\ service\ time}{Number\ of\ customers\ served} = \frac{198}{50}$= **3.96 minutes per customer**
- Service rate, $\mu_2 = \frac{1}{Mean\ service\ time} = \frac{1}{3.96}$ = **0.25 customers per minute**
- Mean waiting time of customers in queue = $\frac{Sum\ of\ customers\ waiting\ time}{Total\ arrivals} = \frac{115}{50}$
  = **2.3 minutes per customer**
- Mean time a customer spends in the system (checkout counter)
  = Average serving time + Average waiting time in the queue
  = 3.96 + 2.3 = **6.26 minutes**
- Service facility utilization at server one = $\frac{\lambda}{\mu} = \frac{0.20}{0.25}$ = **0.8**

4.3.3 Computation of the average of the Operating Characteristics of the queuing system represented at the checkout point with both servers:

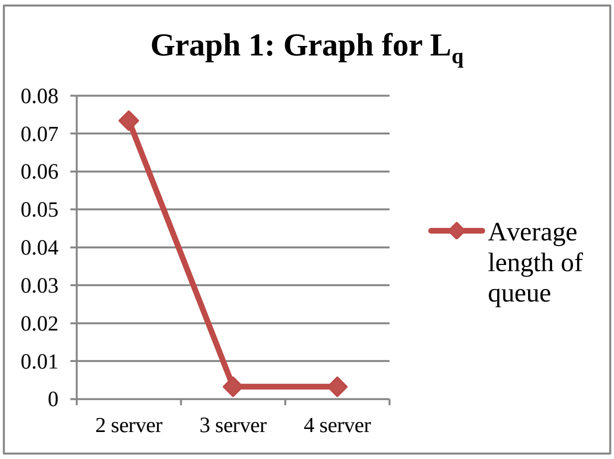
- Mean inter-arrival time = $\frac{4.90 + 4.46}{2}$ = **4.68 minutes per customer**
- Mean Arrival rate, $\lambda = \frac{1}{4.68}$ = **0.21 customers per minute**
- Mean service time = $\frac{3.96 + 4.10}{2}$ = **4.03 minutes per customer**
- Mean Service rate, $\mu = \frac{1}{4.03}$ = **0.25 customers per minute**
- Mean Waiting time in Queue = $\frac{4.04 + 2.30}{2}$ = **3.17 minutes per customer**
- Mean Waiting time in System(Queue and in service) = $\frac{8.14 + 6.26}{2}$
  = **7.2 minutes per customer**

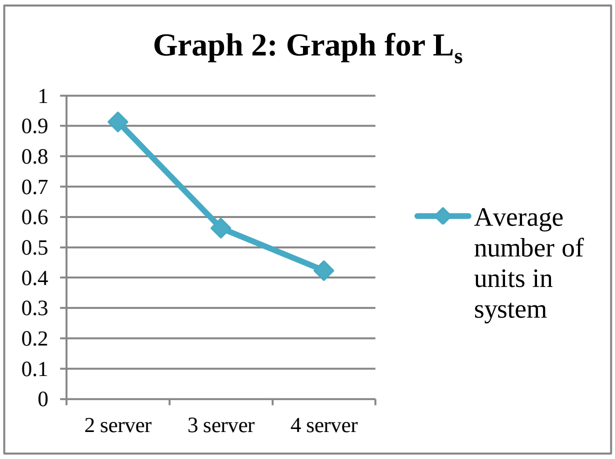
We see that the average time a customer has to wait in the system (queue and in service) is **7.2 minutes** which may eventually lead the customer to either leave the queue and move to next queue because it is too long and he has no time to wait (balking), or leave due to impatience without getting the service (reneging). The above results provide insights into the relationship between the number of servers and queue metric such as queue length and waiting time so by varying the number of servers we can identify the optimal server quantity that minimizes these metrics and maximizes efficiency. Now we will compare the results for three and four servers which will also increase the service rate by 1.5 and 2 times respectively keeping the arrival rate constant i.e. **0.21 customers per minute.**

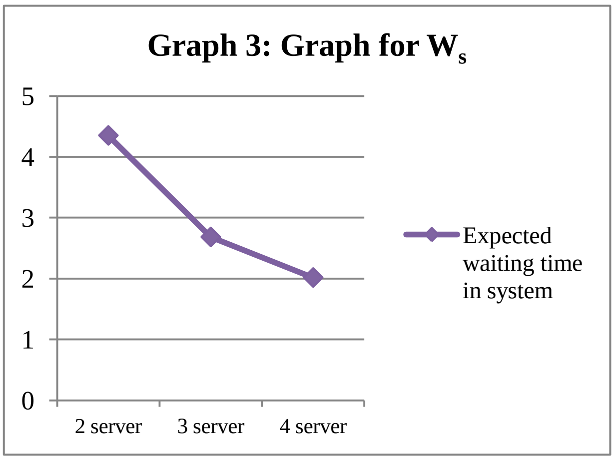
Table 9:- Comparison of measures for queuing model for 2, 3and 4 servers.

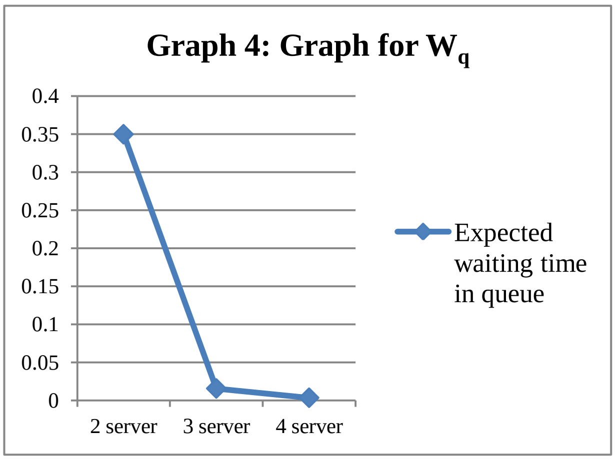
Number of servers	**2**	**3**	**4**
Server Utilization, ρ	0.42	0.187	0.105
P_0	0.408	0.570	0.657
P_s	0.144	0.0167	0.00852
Average length of queue, L_q	0.0734	0.00324	0.003236
Average number of units in system, L_s	0.9134	0.5632	0.4232
Expected waiting time in system, W_s	4.3495	2.6819	2.0152
Expected waiting time in queue, W_q	0.3495	0.01543	0.015409
Expected waiting time of a customer who has to wait, (w|w>0)	3.4482	1.0927	0.5587
Probability of waiting time of a customer who has to wait, [w>0]	0.2482	0.0205	0.000952
Probability that there will be someone waiting	0.1042	0.0038	0.0000996

The graphical representation of the comparison between various parameters for the 2, 3 and 4 servers is shown below:









5. DISCUSSIONS AND CONCLUSION

The findings of the research paper provide valuable insights into the queue behavior within the studied supermarket. The analysis demonstrates the relationship between the number of servers and key performance metrics. The data for two servers was thoroughly processed and compared through both graphical and analytical methods. Through this research paper we concluded that if we increase the number of servers then we get a better overall performance from the system.

After extensive analysis, we observed that the three-server and four-server models exhibited comparable performance in terms of the aforementioned metrics. This implies that adding an extra server did not significantly improve the queuing system's overall performance. Thus, from a cost-benefit perspective, implementing a three-server model may be a more feasible option for the supermarket, as it can achieve similar outcomes without incurring the additional expenses associated with an extra server. However, it is important to acknowledge the limitations of our research. Our study focused on a specific context and may not be directly applicable to other scenarios or industries. Additionally, our findings are based on simulations and assumptions, which may not perfectly replicate real-world conditions. In conclusion, our research highlights the importance of considering queuing theory and carefully evaluating the number of servers in a supermarket setting.

By comparing the three-server and four-server models, we determined that they yielded similar results in terms of performance metrics. This knowledge can assist supermarket managers in making informed decisions regarding their queuing systems, optimizing resource allocation, and improving customer satisfaction. Further research can be conducted to explore additional factors that could impact queuing system performance and to validate our findings in different settings.

6. REFERENCES

1. Pallikkara, V., Pinto, P., Hawaldar, I. T., & Pinto, S. (2021). Impulse buying behaviour at the retail checkout: An investigation of select antecedents. Business: Theory and Practice, 22(1), 69-79.

2. Pinto, P., & Hawaldar, I. T. (2022). Perceived idle wait and associated emotional discomfort: An analysis of retail waiting experience.

3. Safuri, A. A. M., & KAMARDAN, M. G. (2022). Queue Management and Application in Supermarket. Enhanced Knowledge in Sciences and Technology, 2(1), 352-360.

4. Varma, I. G., & Agarwal, M. R. (2014). Online buying behaviour of homemakers in Western Suburbs of Mumbai and social media influence. IOSR Journal of Business and Management, 16(8), 42-65.

5. Arvidsson, A., & Hassani, L. (2020). Design of a Smart Cart App for Automated Shopping in Supermarkets.

6. Murad, S. S., Yussof, S., & Badeel, R. (2022). Wireless Technologies for Social Distancing in the Time of COVID-19: Literature Review, Open Issues, and Limitations. Sensors, 22(6), 2313.

7. V.Rema, Karabi Sikdar, Dhanjit Mehdi. International Journal of Advanced Research, 311-324, (June 2017)

8. Igwe, A., Onwuere, J. U. J., & Egbo, O. P. (2014). Efficient queue management in supermarkets: a case study of Makurdi Town, Nigeria. European Journal of Business and Management, 6(39), 185-192.

9. Rana, S. S., Osman, A., & Othman, Y. H. (2015). Factors affecting purchase intention of customers to shop at hypermarkets. Mediterranean Journal of Social Sciences, 6(3), 429.

10. Madan, M., & Kumari, S. (2012). Determinants of retail customer satisfaction: a study of organised retail outlets in Delhi. Delhi Business Review, 13(1), 117-126.

11. Wallace, D. W., Giese, J. L., & Johnson, J. L. (2004). Customer retailer loyalty in the context of multiple channel strategies. Journal of retailing, 80(4), 249-263.

12. Priyangika, J. S. K. C., & Cooray, T. M. J. A. (2015). Analysis of the sales checkout operation in supermarket using queuing theory.

13. Applebaum, W. (1951). Studying customer behavior in retail stores. Journal of marketing, 16(2), 172-178.

14. Afande, F. O., & John, K. (2015). Factors affecting customer loyalty of supermarkets in Nyeri Town, Kenya. Journal of Marketing and Consumer Research, 11(1), 1-13.

15. Jhala, N., & Bhathawala, P. (2017). Analysis and application of queuing theory in Supermarkets.

16. Nafees, A. (2007). Queuing theory and its application: analysis of the sales checkout operation in ICA supermarket. Department of Economics and Society University of Dalarna, 2, 20.

17. Li, B., & Wang, D. (2010). Configuration issues of cashier staff in supermarket based on queuing theory. In Information Computing and Applications: International Conference, ICICA 2010, Tangshan, China, October 15-18, 2010. Proceedings, Part II 1 (pp. 334-340). Springer Berlin Heidelberg.

18. Luo, R., & Shi, Y. (2020, January). Analysis and optimization of supermarket operation mode based on queuing theory: queuing and pricing of personalized service. In Proceedings of the 2020 4th International Conference on Management Engineering, Software Engineering and Service Sciences (pp. 221-224).

19. Koeswara, S., Kholil, M., & Pratama, Z. (2018, November). Evaluation on Application of Queuing Theory On Payment System in the Supermarket “Saga” Padang Pariaman West Sumatra. In IOP Conference Series: Materials Science and Engineering (Vol. 453, No. 1, p. 012045). IOP Publishing.

20. Wenjie, X., Senbiao, L., & Lihong, H. (2015, July). Simulation model of supermarket queuing system. In 2015 34th Chinese Control Conference (CCC) (pp. 8819-8823). IEEE.

21. RAHAMAN, A., Bello, A. H., Abdulganiyu, T. O., & Eebo, E. A. (2021). Application of Queuing Theory: Analysis of Services of Grocery Bazaar (GB) Supermarket, Akesan, Lasu Igando, Lagos State. Nigeria. International Research Journal of Education and Technology, 1(05), 60-88.

22. Jie, Y. (2010). The optimal supermarket service. International Journal of Business and Management, 5(2), 128-131.

23. Chai, C. F. (2013). Problem analysis and optimizing of setting service desks in supermarket based on M/M/C queuing system. In The 19th International Conference on Industrial Engineering and Engineering Management: Assistive Technology of Industrial Engineering (pp. 833-841). Springer Berlin Heidelberg.

24. Pathirana, A. D., & Peiris, T. U. I.(2012) Waiting Lines of Supermarkets in Colombo District: Queuing Theory Model and Customer Satisfaction Analysis.

25. Wang, J., & Zhou, Y. P. (2018). Impact of queue configuration on service time: Evidence from a supermarket. Management Science, 64(7), 3055-3075.

26. Junior, J. V. P., da Silva, A. M., & de Moraes, D. G. (2020). Discrete simulation applied to queue management in a supermarket. Independent Journal of Management & Production, 11(5), 1667-1684.

27. Macharia, I. (2016). Customer service analysis at naivas supermarket narok branch using queuing theory.

28. Zhao, T., & He, C. (2013). Supermarket application based on queueing theory. In Proceedings of the International Conference on Information Engineering and Applications (IEA) 2012: Volume 3 (pp. 545-551). Springer London.

29. de Melo, G. A., Peixoto, M. G. M., Barbosa, S. B., & de Mendonça, M. C. A. (2022). The cashier service macroprocess analysis in a supermarket organization: a quality management and simulation case study. DYNA, 89(223), 19-26.

30. Liu, Q., & Chen, H. (2022, April). Optimization of J supermarket cashier service system based on M/M/c model. In Proceedings of the 4th International Conference on Management Science and Industrial Engineering (pp. 478-484).

31. Mushi, A. R., & Kagoya, S. M. (2022). Operations research contribution to the performance of supermarkets in East Africa. In Supermarket Retailing in Africa (pp. 104-128). Routledge.

32. Li, L., & Xu, J. (2018). Optimal Design and Control of Cash Register Service System of Supermarkets.

33. Prasad, S. V., & Badshah, V. H.(2-14). A Mathematical Model for Super Market Service using Queuing Theory and its Application.

34. Bennett, R. (1998). Queues, customer characteristics and policies for managing waiting-lines in supermarkets. International Journal of Retail & Distribution Management, 26(2), 78-87.

35. Van Dijk, N. M. (1997). Why queuing never vanishes. European Journal of Operational Research, 99(2), 463-476.

36. Raj, S. Y., Karthee, K., Sundaram, S. K., Nishal, M., & Prasad, K. R. (2018). Increasing reliability and productivity in hypermarkets by queuing theory analysis and a smart shopping cart-based system. International Journal of Business Excellence, 14(4), 545-556.

37. Zhiwei, N., Xiaochun, L., & Dongyuan, L. (2009, June). Simulation of Queuing Systems with Different Queuing Disciplines Based on Anylogic. In 2009 International Conference on Electronic Commerce and Business Intelligence (pp. 164-167). IEEE.

38. Devi, A. Study about Queuing Theory and its Applications.

39. Luczak, M. J., & McDiarmid, C. (2006). On the maximum queue length in the supermarket model.

40. Mariyappan, S., Alexander, C., & Anbalagan, S. (2020). Mathematical modeling in comparative relation of single queue multi server and multi queue multi server queuing models. Malaya Journal of Matematik (MJM), (1, 2020), 331-334.

41. Hua, S., Wu, L., & Zhang, Y. (2022). RESEARCH ON SUPERMARKET SETTLEMENT PERSONNEL ALLOCATION AND OPTIMIZATION BASED ON QUEUING THEORY. Harbin Gongye Daxue Xuebao/Journal of Harbin Institute of Technology, 54(2), 273-275.

42. Rani, G., & Srinivasan, A. (2012). M/M/c/k Interdependent Queueing Model with controllable Arrival Rates and feedback. In Proceedings Of The Heber International Conference On Application Of Mathematics And Statistics (pp. 271-277).

43. Joshua, O. O. (2013). On Markovian Queueing Model as Birth-Death Process. Global Journal of Science Frontier Research, 21-40.

44. Sasi, M. A., Subramanian, T., & Ravichandran, S. K. (2023). QUEUEING SYSTEM IN FAIR PRICE SHOP OF KERALA TO MITIGATE WAITING TIME AND OPTIMIZATION OF SERVERS.