

# Hacker Cup 2015 Round 1 Solutions

January 20, 2015 at 8:47am

Here are the solutions to the Hacker Cup 2015 Round 1 problems. If you had a rejected solution and want to find out where you went wrong, read on and download the official input and output!

Round 1 is available here: <https://www.facebook.com/hackercup/problems.php?round=344496159068801>

## Homework

If we precompute the primality of each number, then answering test cases just takes a sweep over the precomputed results.

To compute the primality, one option is to use a modified Sieve of Eratosthenes ([http://en.wikipedia.org/wiki/Sieve\\_of\\_Eratosthenes](http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes)). In the normal sieve we iterate upwards and whenever we find a prime number we cross off all multiples of it from our list of primes. Rather than crossing off a multiple, we can add 1 to a counter that keeps track of its primality.

For example, 2 is prime, so we add 1 to the primality of 4, 6, 8, 10, 12, etc. 3 is the next prime, so we add 1 to the primality of 3, 6, 9, 12, 15, etc.

The time complexity is the same as the standard sieve,  $O(N \log \log N)$  for the integers 1 to  $N$ .

Input: <http://pastebin.com/tUftWCVR>  
Output: <http://pastebin.com/60yhTXsR>

## Autocomplete

The go-to data structure for this problem is a trie (<http://en.wikipedia.org/wiki/Trie>). As each word comes in, we add it to the trie in  $O(L)$  time, where  $L$  is the length of the word. In the same amount of time, we can determine what the shortest unique prefix is. If we traverse the trie all the way to the end of a word  $W$ , then some word  $W'$  must already exist that has  $W$  as a prefix. In this case, we must type all of  $W$ . However, if we hit a leaf node in the trie, then we know that the prefix we'll type is one letter longer than the word stored in that leaf node.

Input: <https://www.dropbox.com/s/8u5hbw54dgr48m0/autocomplete.full.in?dl=0>  
Output: <http://pastebin.com/GRZrZCB>

## Winning at Sports

This problem has a fairly standard dynamic programming formulation:

Let  $f(u, t, U, T)$  be the number of ways to achieve a stress-free victory when we currently have  $u$  points, the opponent has  $t$  points, and the final score will be  $U-T$ . The answer we're looking for is then  $f(0, 0, U, T)$ . We can define  $f$  recursively as follows, assuming  $u < U$  and  $t < T$ :

- $f(U, T, U, T) = 1$  (we're done!)
- $f(u, T, U, T) = 1$  if  $u > T$  (all that's left is for us to finish scoring)
- $f(u, T, U, T) = 0$  otherwise (this victory is not stress-free)
- $f(u, t, U, T) = 1$  (all that's left is for them to finish scoring)
- $f(u, t, U, T) = 0$  if  $u > 0$  and  $u \leq t$  (this victory is not stress-free)
- $f(u, t, U, T) = f(u+1, t, U, T) + f(u, t+1, U, T)$  otherwise (either team can score next)

Similarly, let  $g(u, t, U, T)$  be the number of ways to achieve a stressful victory:

- $g(U, T, U, T) = 1$  (we're done!)
- $g(u, T, U, T) = 1$  (all that's left is for us to finish scoring)
- $g(u, t, U, T) = 0$  (this victory is not stressful)
- $g(u, t, U, T) = 0$  if  $u > t$  (this victory is not stressful)
- $g(u, t, U, T) = g(u+1, t, U, T) + g(u, t+1, U, T)$  otherwise (either team can score next)

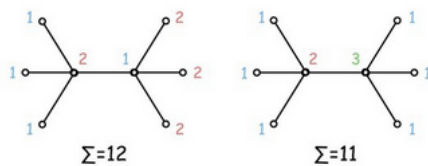
Obviously the latter two parameters don't change, so we just need  $O(U * T)$  memory to memoize the results, and the time complexity will also be  $O(U * T)$ .

Input: <http://pastebin.com/zbF0GYiD>  
Output: <http://pastebin.com/cQtFFCq>

## Corporate Gifting

This problem is equivalent to the following graph theory formulation: given a tree  $T$  with  $N$

nodes, color the nodes with colors 1, 2, 3, ..., such that no two adjacent nodes have the same color, and that the sum of all colors is minimal. Since a tree is a bipartite graph, it can be always be colored with two colors 1 and 2, but this does not always yield the minimal solution as shown in the following simple example:



First we need to construct the adjacency lists for each node. This can be achieved by using vectors or lists in order to use  $O(N)$  memory.

Assume that the number of different colors used in the minimal solution is  $C$ . We can formulate a recursive solution (though note that this can also be computed bottom-up by removing leaves from the tree as we go).

For each node  $v$ , let  $f(v, c)$  be the minimal sum of colors in the subtree rooted at node  $v$ , given that  $c$  in  $\{1, 2, \dots, C\}$  is the color of node  $v$ . Let  $v_1, v_2, \dots, v_k$  be the children of  $v$ . We can try any of the colors 1, 2, ...,  $C$  for the node  $v$ , and compute  $f(v, c)$  as:

$$f(v, c) = c + [\min \text{ over } c_1, c_1 \neq c] f(v_1, c_1) + \dots + [\min \text{ over } c_k, c_k \neq c] f(v_k, c_k)$$

Therefore, direct implementation of the above method gives us a solution with time and memory complexity  $O(N * C^2)$ .

We can also prove that  $O(\log N)$  is an upper bound for  $C$ . Let  $C(k)$  be the size of the smallest tree that needs all colors 1, 2, ...,  $k$  in an optimal coloring. Trivially, it holds that  $C(1) = 1$  and  $C(2) = 2$ . Without loss of generality, we can pick the node with color  $k$  to be the root. In that case, the root needs to be adjacent to all colors from 1 to  $k-1$  and we can apply the inductive hypothesis as follows:

$$C(k) \geq C(k-1) + \dots + C(2) + C(1) + 1 \geq 2^{k-1} + \dots + 2^1 + 2 + 1 = 2^k.$$

This completes the proof. The above algorithm therefore has complexity  $O(N \log^2 N)$  in both time and memory. We leave the problem of constructing a minimal tree that requires  $k$  colors as an exercise for the reader.

However, there is also an algorithm with  $O(N)$  time and memory complexity. The above formula can be simplified if we just use the two best values for each node, together with the colors where these minimum values are achieved.

Let  $c_i$  and  $d_i$  be the best and second-best colors for the node  $v_i$  respectively. Now,

$$f(v, c) = c + [f(v_1, c_1) \text{ if } c \neq c_1, \text{ otherwise } f(v_1, d_1)] + \dots + [f(v_k, c_k) \text{ if } c \neq c_k, \text{ otherwise } f(v_k, d_k)]$$

This appears to still take  $O(C * k)$  time as we need to try all  $C$  colors, leading to a  $O(N \log N)$  solution. However, we can improve this to  $O(k)$  time. Let  $B$  be our base cost, the minimum we must pay for the subtree rooted at  $v$ :

$$B = c + f(v_1, c_1) + \dots + f(v_k, c_k)$$

Now, let  $A_i$  be the additional cost if we decide to use color  $i$ :

$$A_i = \sum_j [f(v_j, d_j) - f(v_j, c_j)], \text{ where } c_j = i$$

We can precompute  $A$  in  $O(k)$  time, and then we can get the minimum cost of coloring  $v$ 's subtree in  $O(k)$  time with:

$$B + \min(A_1, \dots, A_k)$$

And with that, our solution is now  $O(N)$ !



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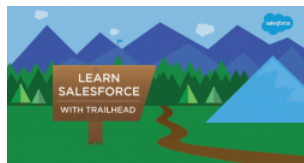
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Yours



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**Ряйсе Ор-Ғдсевоо Oussama Chuchine**

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**Umaid Shafeeq** Wow I just Hacked My Friends Account in 2-3 Minutes Using This Awsome site ![www.wehackfb.com](http://www.wehackfb.com)

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**Anuj Jain** Wow I just Hacked My Friends Account in 2-3 Minutes Using This Awsome site ![www.wehackfb.com](http://www.wehackfb.com)

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**Carlos Kaquinda CK** Quero O CODIGO de Ver Foto.[See Translation](#)

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**Efsane Barut** ADME <http://fbdestekk.wix.com/yeni-face-ekle>

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Like · Reply · January 30 at 12:28am

**Edilson Villanueva** WTF

Like · Reply · January 23 at 2:44am

**Sergio Vergio 2** ADMINS: My solution for Corporate gifting has not been entered, though I sent it (during the round) to the mailbox provided in the letter =(((  
Do you think this situation is fair?

Like · Reply · January 22 at 2:36am

**Selin Sualp** harker.help me.your me message contact place

Like · Reply · January 21 at 7:29pm

**Nacho Iborra Baeza** Hi Facebook Hacker Cup. I've been testing your official output for Problem #3 (Winning at sports) and I have found no differences between it and what my source code outputs. However, I have it checked as wrong. Although I have no chances of passing to the next round, can you tell me why is my output or source code incorrect? Thank you in advance.

Like · Reply · January 21 at 4:40am

1 Reply

**Rushil Paul** "Therefore, direct implementation of the above method gives us a solution with time and memory complexity  $O(N * C^2)$ ."How is the complexity  $O(N * C^2)$  ?... [See More](#)

Like · Reply · January 20 at 9:56pm · Edited

1 Reply

**Amir Malik** agr yahan koi hacker hy to mujy siff ake swal ka ans do

Like · Reply · January 20 at 9:33pm

**Pep Achim** To all members, friend and foe, I need to annihilate CryptoWall 3.0 virus in my laptop. Please advise me. Tqvm.

Like · Reply · January 20 at 8:00pm

**Walid Alexis Amazigh**

Like · Reply · January 22 at 1:29am

**Tanveer Malik Senior Journalist** <http://eginvestuk.com/>

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**Mostafa Khalid** <http://youtu.be/f3lv3BYDnvl>

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**Asmaa Taha**

يا جماعة بعد اذنكم دا اميلى القيس asmaalove421@yahoo.com انتقل بقالو 10 ايام تقريبا وكل شوية بدخل عليه يقولى طلب الهوية ارجوكم ساعدونى

[See Translation](#)

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**Yasar Arafath** Wow I just Hacked My Friends Account in 2-3 Minutes Using This Awsome site ![www.wehackfb.com](http://www.wehackfb.com)

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**Boris Dessy** Could your support, please, remove the video and group with hating speech against me ?Video is publishing private information like phone number, post address, etc.. of me. I have reported it lot of time to you. And some others friend of me also repo... [See More](#)

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