Stanford University ACM Team Notebook (2014-15)

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Dinic.cc 1/35

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
// Running time:
//
       O(|V|^2 |E|)
// INPUT:
       - graph, constructed using AddEdge()
       - source and sink
//
// OUTPUT:
       - maximum flow value
//
       - To obtain actual flow values, look at edges with capacity > 0
//
         (zero capacity edges are residual edges).
#include <iostream>
#include <vector>
using namespace std;
typedef long long LL;
struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
  LL rcap() { return cap - flow; }
};
struct Dinic {
  int N;
  vector<vector<Edge> > G;
  vector<vector<Edge *> > Lf;
  vector<int> layer;
  vector<int> Q;
  Dinic(int N) : N(N), G(N), Q(N) {}
  void AddEdge(int from, int to, int cap) {
    if (from == to) return;
    G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
    G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
  LL BlockingFlow(int s, int t) {
    layer.clear(); layer.resize(N, -1);
    layer[s] = 0;
    Lf.clear(); Lf.resize(N);
    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {</pre>
      int x = Q[head++];
      for (int i = 0; i < G[x].size(); i++) {</pre>
        Edge &e = G[x][i]; if (e.rcap() <= 0) continue;</pre>
        if (layer[e.to] == -1) {
          layer[e.to] = layer[e.from] + 1;
          Q[tail++] = e.to;
        if (layer[e.to] > layer[e.from]) {
          Lf[e.from].push back(&e);
      }
    if (layer[t] == -1) return 0;
    LL totflow = 0;
    vector<Edge *> P;
    while (!Lf[s].empty()) {
      int curr = P.empty() ? s : P.back()->to;
      if (curr == t) { // Augment
        LL amt = P.front()->rcap();
        for (int i = 0; i < P.size(); ++i) {</pre>
          amt = min(amt, P[i]->rcap());
```

```
totflow += amt;
        for (int i = P.size() - 1; i >= 0; --i) {
          P[i]->flow += amt;
          G[P[i]->to][P[i]->index].flow -= amt;
          if (P[i]->rcap() <= 0) {</pre>
            Lf[P[i]->from].pop_back();
            P.resize(i);
          }
        }
      } else if (Lf[curr].empty()) { // Retreat
        P.pop_back();
        for (int i = 0; i < N; ++i)
          for (int j = 0; j < Lf[i].size(); ++j)</pre>
            if (Lf[i][j]->to == curr)
              Lf[i].erase(Lf[i].begin() + j);
      } else { // Advance
        P.push_back(Lf[curr].back());
      }
    }
    return totflow;
  LL GetMaxFlow(int s, int t) {
    LL totflow = 0;
    while (LL flow = BlockingFlow(s, t))
      totflow += flow;
    return totflow;
  }
};
```

MinCostMaxFlow.cc 2/35

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.
//
// Running time, O(|V|^2) cost per augmentation
       max flow:
                           O(|V|^3) augmentations
       min cost max flow: O(|V|^4 * MAX_EDGE_COST) augmentations
//
       graph, constructed using AddEdge()
       - source
//
       - sink
//
// OUTPUT:
       - (maximum flow value, minimum cost value)
       - To obtain the actual flow, look at positive values only.
#include <cmath>
#include <vector>
#include <iostream>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;
const L INF = numeric_limits<L>::max() / 4;
struct MinCostMaxFlow {
  int N;
  VVL cap, flow, cost;
  VI found;
  VL dist, pi, width;
  VPII dad;
```

```
MinCostMaxFlow(int N) :
    N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
    found(N), dist(N), pi(N), width(N), dad(N) {}
  void AddEdge(int from, int to, L cap, L cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
  void Relax(int s, int k, L cap, L cost, int dir) {
    L val = dist[s] + pi[s] - pi[k] + cost;
    if (cap && val < dist[k]) {</pre>
      dist[k] = val;
      dad[k] = make_pair(s, dir);
      width[k] = min(cap, width[s]);
    }
  L Dijkstra(int s, int t) {
    fill(found.begin(), found.end(), false);
    fill(dist.begin(), dist.end(), INF);
    fill(width.begin(), width.end(), 0);
    dist[s] = 0;
    width[s] = INF;
    while (s != -1) {
      int best = -1;
      found[s] = true;
      for (int k = 0; k < N; k++) {
        if (found[k]) continue;
        Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
Relax(s, k, flow[k][s], -cost[k][s], -1);
        if (best == -1 || dist[k] < dist[best]) best = k;</pre>
      }
        = best;
    for (int k = 0; k < N; k++)
      pi[k] = min(pi[k] + dist[k], INF);
    return width[t];
  pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
      totflow += amt;
      for (int x = t; x != s; x = dad[x].first) {
        if (dad[x].second == 1) {
          flow[dad[x].first][x] += amt;
          totcost += amt * cost[dad[x].first][x];
        } else {
          flow[x][dad[x].first] -= amt;
          totcost -= amt * cost[x][dad[x].first];
      }
    return make_pair(totflow, totcost);
};
```

PushRelabel.cc 3/35

```
// INPUT:
//
       graph, constructed using AddEdge()
       - source
//
       - sink
//
// OUTPUT:
//
       - maximum flow value
       - To obtain the actual flow values, look at all edges with
         capacity > 0 (zero capacity edges are residual edges).
#include <cmath>
#include <vector>
#include <iostream>
#include <queue>
using namespace std;
typedef long long LL;
struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
struct PushRelabel {
  int N;
  vector<vector<Edge> > G;
  vector<LL> excess;
  vector<int> dist, active, count;
  queue<int> Q;
  PushRelabel(int N) : N(N), G(N), excess(N), dist(N), active(N), count(2*N) {}
  void AddEdge(int from, int to, int cap) {
    G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
    if (from == to) G[from].back().index++;
    G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
  void Enqueue(int v) {
    if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
  void Push(Edge &e) {
    int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
if (dist[e.from] <= dist[e.to] || amt == 0) return;</pre>
    e.flow += amt;
    G[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt;
    Enqueue(e.to);
  void Gap(int k) {
    for (int v = 0; v < N; v++) {
      if (dist[v] < k) continue;</pre>
      count[dist[v]]--;
      dist[v] = max(dist[v], N+1);
      count[dist[v]]++;
      Enqueue(v);
  void Relabel(int v) {
    count[dist[v]]--;
    dist[v] = 2*N;
    for (int i = 0; i < G[v].size(); i++)</pre>
      if (G[v][i].cap - G[v][i].flow > 0)
        dist[v] = min(dist[v], dist[G[v][i].to] + 1);
    count[dist[v]]++;
    Enqueue(v);
  void Discharge(int v) {
    for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i]);</pre>
    if (excess[v] > 0) {
```

```
if (count[dist[v]] == 1)
        Gap(dist[v]);
        Relabel(v);
    }
  }
  LL GetMaxFlow(int s, int t) {
    count[0] = N-1;
    count[N] = 1;
    dist[s] = N;
    active[s] = active[t] = true;
    for (int i = 0; i < G[s].size(); i++) {</pre>
      excess[s] += G[s][i].cap;
      Push(G[s][i]);
    while (!Q.empty()) {
      int v = Q.front();
      Q.pop();
      active[v] = false;
      Discharge(v);
    LL totflow = 0;
    for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;</pre>
    return totflow;
};
```

MinCostMatching.cc 4/35

```
// Min cost bipartite matching via shortest augmenting paths
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
//
    cost[i][j] = cost for pairing left node i with right node j
    Lmate[i] = index of right node that left node i pairs with
    Rmate[j] = index of Left node that right node j pairs with
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[][] matrix.
#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>
using namespace std;
typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
 int n = int(cost.size());
 // construct dual feasible solution
 VD u(n);
 VD v(n);
 for (int i = 0; i < n; i++) {</pre>
   u[i] = cost[i][0];
   for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);</pre>
  for (int j = 0; j < n; j++) {
   v[j] = cost[0][j] - u[0];
   for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);</pre>
```

```
// construct primal solution satisfying complementary slackness
Lmate = VI(n, -1);
Rmate = VI(n, -1);
int mated = 0;
for (int i = 0; i < n; i++) {</pre>
 for (int j = 0; j < n; j++) {
   if (Rmate[j] != -1) continue;</pre>
    if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {</pre>
      Lmate[i] = j;
      Rmate[j] = i;
      mated++;
      break;
    }
 }
VD dist(n);
VI dad(n);
VI seen(n);
// repeat until primal solution is feasible
while (mated < n) {</pre>
  // find an unmatched left node
  int s = 0;
  while (Lmate[s] != -1) s++;
  // initialize Dijkstra
  fill(dad.begin(), dad.end(), -1);
  fill(seen.begin(), seen.end(), 0);
  for (int k = 0; k < n; k++)
    dist[k] = cost[s][k] - u[s] - v[k];
  int j = 0;
  while (true) {
    // find closest
    i = -1;
    for (int k = 0; k < n; k++) {
      if (seen[k]) continue;
      if (j == -1 || dist[k] < dist[j]) j = k;</pre>
    seen[j] = 1;
    // termination condition
    if (Rmate[j] == -1) break;
    // relax neighbors
    const int i = Rmate[j];
    for (int k = 0; k < n; k++) {
      if (seen[k]) continue;
      const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
      if (dist[k] > new_dist) {
        dist[k] = new_dist;
        dad[k] = j;
    }
  // update dual variables
  for (int k = 0; k < n; k++) {
    if (k == j || !seen[k]) continue;
    const int i = Rmate[k];
    v[k] += dist[k] - dist[j];
    u[i] -= dist[k] - dist[j];
  u[s] += dist[j];
  // augment along path
  while (dad[j] >= 0) {
    const int d = dad[j];
    Rmate[j] = Rmate[d];
    Lmate[Rmate[j]] = j;
    j = d;
  Rmate[j] = s;
```

```
Lmate[s] = j;

mated++;
}

double value = 0;
for (int i = 0; i < n; i++)
   value += cost[i][Lmate[i]];

return value;
}</pre>
```

MaxBipartiteMatching.cc 5/35

```
// This code performs maximum bipartite matching.
// Running time: O(|E| |V|) -- often much faster in practice
     INPUT: w[i][j] = edge between row node i and column node j
     OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
             mc[j] = assignment for column node j, -1 if unassigned
             function returns number of matches made
#include <vector>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
  for (int j = 0; j < w[i].size(); j++) {</pre>
    if (w[i][j] && !seen[j]) {
      seen[j] = true;
      if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {</pre>
        mr[i] = j;
        mc[j] = i;
        return true;
      }
    }
  return false;
int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
  mr = VI(w.size(), -1);
  mc = VI(w[0].size(), -1);
  int ct = 0;
  for (int i = 0; i < w.size(); i++) {</pre>
    VI seen(w[0].size());
    if (FindMatch(i, w, mr, mc, seen)) ct++;
  }
  return ct;
```

MinCut.cc 6/35

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
// O(|V|^3)
//
// INPUT:
// - graph, constructed using AddEdge()
//
// OUTPUT:
// - (min cut value, nodes in half of min cut)
#include <cmath>
#include <vector>
```

```
#include <iostream>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
const int INF = 1000000000;
pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;
  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {</pre>
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];</pre>
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];</pre>
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {</pre>
          best_cut = cut;
          best_weight = w[last];
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
    }
  return make_pair(best_weight, best_cut);
```

GraphCutInference.cc 7/35

```
// Special-purpose {0,1} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
//
          minimize
                           sum_i psi_i(x[i])
// x[1]...x[n] in \{0,1\}
                            + sum_{i < j} phi_{ij}(x[i], x[j])
        psi i : {0, 1} --> R
     phi_{ij} : {0, 1} x {0, 1} --> R
// such that
     phi_{ij}(0,0) + phi_{ij}(1,1) \leftarrow phi_{ij}(0,1) + phi_{ij}(1,0) (*)
// This can also be used to solve maximization problems where the
// direction of the inequality in (*) is reversed.
// INPUT: phi -- a matrix such that phi[i][j][u][v] = phi_{ij}(u, v)
         psi -- a matrix such that psi[i][u] = psi_i(u)
//
          x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution
// To use this code, create a GraphCutInference object, and call the
// DoInference() method. To perform maximization instead of minimization,
// ensure that #define MAXIMIZATION is enabled.
#include <vector>
#include <iostream>
```

```
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
typedef vector<VVVI> VVVVI;
const int INF = 1000000000;
// comment out following line for minimization
#define MAXIMIZATION
struct GraphCutInference {
  int N;
  VVI cap, flow;
  VI reached;
  int Augment(int s, int t, int a) {
    reached[s] = 1;
    if (s == t) return a;
    for (int k = 0; k < N; k++) {
      if (reached[k]) continue;
      if (int aa = min(a, cap[s][k] - flow[s][k])) {
        if (int b = Augment(k, t, aa)) {
          flow[s][k] += b;
          flow[k][s] -= b;
          return b;
      }
    return 0;
  int GetMaxFlow(int s, int t) {
    N = cap.size();
    flow = VVI(N, VI(N));
    reached = VI(N);
    int totflow = 0;
    while (int amt = Augment(s, t, INF)) {
      totflow += amt;
      fill(reached.begin(), reached.end(), 0);
    }
    return totflow;
  }
  int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size():
    cap = VVI(M+2, VI(M+2));
    VI b(M);
    int c = 0;
    for (int i = 0; i < M; i++) {
      b[i] += psi[i][1] - psi[i][0];
      c += psi[i][0];
      for (int j = 0; j < i; j++)</pre>
        b[i] += phi[i][j][1][1] - phi[i][j][0][1];
      for (int j = i+1; j < M; j++) {
        cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j][0][0] - phi[i][j][1][1];
        b[i] += phi[i][j][1][0] - phi[i][j][0][0];
        c += phi[i][j][0][0];
      }
    }
#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
      for (int j = i+1; j < M; j++)
        cap[i][j] *= -1;
      b[i] *= -1;
    }
    c *= -1;
#endif
    for (int i = 0; i < M; i++) {
      if (b[i] >= 0) {
        cap[M][i] = b[i];
      } else {
```

```
cap[i][M+1] = -b[i];
        c += b[i];
      }
    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment(M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif
    return score;
  }
};
int main() {
  // solver for "Cat vs. Dog" from NWERC 2008
  int numcases;
  cin >> numcases;
  for (int caseno = 0; caseno < numcases; caseno++) {</pre>
    int c, d, v;
    cin >> c >> d >> v;
    VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
    VVI psi(c+d, VI(2));
    for (int i = 0; i < v; i++) {</pre>
      char p, q;
      int u, v;
      cin >> p >> u >> q >> v;
      u--; v--;
if (p == 'C') {
        phi[u][c+v][0][0]++;
        phi[c+v][u][0][0]++;
      } else {
        phi[v][c+u][1][1]++;
        phi[c+u][v][1][1]++;
      }
    }
    GraphCutInference graph;
    cout << graph.DoInference(phi, psi, x) << endl;</pre>
  return 0;
}
```

ConvexHull.cc 8/35

```
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm. Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
// INPUT: a vector of input points, unordered.
// OUTPUT: a vector of points in the convex hull, counterclockwise, starting
// with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <assert>
#include <algorithm>
#include <cmath>
using namespace std;
```

```
#define REMOVE_REDUNDANT
typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }</pre>
  bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }
#ifdef REMOVE REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
#endif
void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {</pre>
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);
#ifdef REMOVE REDUNDANT
  if (pts.size() <= 2) return;</pre>
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {</pre>
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push_back(pts[i]);
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  pts = dn;
#endif
```

Geometry.cc 9/35

```
// C++ routines for computational geometry.
#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>
using namespace std;
double INF = 1e100;
double EPS = 1e-12;
struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT \& p) : x(p.x), y(p.y)
                                      {}
  PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)
                               const { return PT(x*c, y*c ); }
```

```
PT operator / (double c)
                               const { return PT(x/c,
                                                        y/c ); }
};
double dot(PT p, PT q)
                            { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)
                           { return dot(p-q,p-q); }
double cross(PT p, PT q)
                          { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p)
                       { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;</pre>
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;</pre>
  if (r > 1) return b;
  return a + (b-a)*r;
// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                           double a, double b, double c, double d)
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;</pre>
bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}
// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||</pre>
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;</pre>
    if (dot(c-a, c-b) > 0 \&\& dot(d-a, d-b) > 0 \&\& dot(c-b, d-b) > 0)
      return false;
    return true;
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
\ensuremath{//}\xspace compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
```

```
assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){</pre>
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
      p[j].y \le q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
      c = !c;
  }
  return c;
}
// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)</pre>
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
    return false;
}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;</pre>
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;</pre>
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret:
}
// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {</pre>
```

```
int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  return area / 2.0;
}
double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){</pre>
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {</pre>
    for (int k = i+1; k < p.size(); k++) {</pre>
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == 1 || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  return true;
int main() {
  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;</pre>
  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;</pre>
  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;</pre>
  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;</pre>
  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "</pre>
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "</pre>
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;</pre>
  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;</pre>
  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "</pre>
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;</pre>
  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "</pre>
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;</pre>
  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;
  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;</pre>
  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;</pre>
```

```
vector<PT> v;
v.push_back(PT(0,0));
v.push_back(PT(5,0));
v.push_back(PT(5,5));
v.push_back(PT(0,5));
// expected: 1 1 1 0 0
cerr << PointInPolygon(v, PT(2,2)) << " "</pre>
       << PointInPolygon(v, PT(2,0)) << " "</pre>
       << PointInPolygon(v, PT(0,2)) << " "</pre>
       << PointInPolygon(v, PT(5,2)) << " "</pre>
       << PointInPolygon(v, PT(2,5)) << endl;
// expected: 0 1 1 1 1
cerr << PointOnPolygon(v, PT(2,2)) << " "</pre>
       << PointOnPolygon(v, PT(2,0)) << " "</pre>
       << PointOnPolygon(v, PT(0,2)) << " "</pre>
       << PointOnPolygon(v, PT(5,2)) << " "</pre>
       << PointOnPolygon(v, PT(2,5)) << endl;
// expected: (1,6)
                 (5,4)(4,5)
//
                 blank line
//
                 (4,5)(5,4)
//
                 blank line
//
                 (4,5) (5,4)
vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " ";</pre>
                                                                       "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
// area should be 5.0
// centroid should be (1.1666666, 1.166666)
PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
vector<PT> p(pa, pa+4);
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;</pre>
cerr << "Centroid: " << c << endl;</pre>
return 0;
```

JavaGeometry.java 10/35

```
// In this example, we read an input file containing three lines, each
// containing an even number of doubles, separated by commas. The first two
// Lines represent the coordinates of two polygons, given in counterclockwise // (or clockwise) order, which we will call "A" and "B". The last line
// contains a list of points, p[1], p[2], ...
//
// Our goal is to determine:
      (1) whether B - A is a single closed shape (as opposed to multiple shapes)
      (2) the area of B - A
     (3) whether each p[i] is in the interior of B - A
// INPUT:
     0 0 10 0 0 10
     0 0 10 10 10 0
     8 6
     5 1
// OUTPUT:
     The area is singular.
      The area is 25.0
     Point belongs to the area.
```

```
Point does not belong to the area.
import java.util.*;
import java.awt.geom.*;
import java.io.*;
public class JavaGeometry {
    // make an array of doubles from a string
    static double[] readPoints(String s) {
        String[] arr = s.trim().split("\\s++");
        double[] ret = new double[arr.length];
        for (int i = 0; i < arr.length; i++) ret[i] = Double.parseDouble(arr[i]);</pre>
        return ret:
   }
   // make an Area object from the coordinates of a polygon
    static Area makeArea(double[] pts) {
        Path2D.Double p = new Path2D.Double();
        p.moveTo(pts[0], pts[1]);
        for (int i = 2; i < pts.length; i += 2) p.lineTo(pts[i], pts[i+1]);</pre>
        p.closePath();
        return new Area(p);
   }
    // compute area of polygon
    static double computePolygonArea(ArrayList<Point2D.Double> points) {
        Point2D.Double[] pts = points.toArray(new Point2D.Double[points.size()]);
        double area = 0;
        for (int i = 0; i < pts.length; i++){</pre>
            int j = (i+1) % pts.length;
            area += pts[i].x * pts[j].y - pts[j].x * pts[i].y;
        return Math.abs(area)/2;
   }
    // compute the area of an Area object containing several disjoint polygons
    static double computeArea(Area area) {
        double totArea = 0;
        PathIterator iter = area.getPathIterator(null);
        ArrayList<Point2D.Double> points = new ArrayList<Point2D.Double>();
        while (!iter.isDone()) {
            double[] buffer = new double[6];
            switch (iter.currentSegment(buffer)) {
            case PathIterator.SEG MOVETO:
            case PathIterator.SEG_LINETO:
                points.add(new Point2D.Double(buffer[0], buffer[1]));
            case PathIterator.SEG_CLOSE:
                totArea += computePolygonArea(points);
                points.clear();
                break:
            iter.next();
        return totArea;
   }
   // notice that the main() throws an Exception -- necessary to
    // avoid wrapping the Scanner object for file reading in a
    // try { ... } catch block.
   public static void main(String args[]) throws Exception {
        Scanner scanner = new Scanner(new File("input.txt"));
           Scanner scanner = new Scanner (System.in);
        double[] pointsA = readPoints(scanner.nextLine());
        double[] pointsB = readPoints(scanner.nextLine());
        Area areaA = makeArea(pointsA);
        Area areaB = makeArea(pointsB);
        areaB.subtract(areaA);
        // also,
             areaB.exclusiveOr (areaA);
        //
             areaB.add (areaA);
        //
             areaB.intersect (areaA);
```

```
// (1) determine whether B - A is a single closed shape (as
             opposed to multiple shapes)
        boolean isSingle = areaB.isSingular();
        // also,
             areaB.isEmpty();
        if (isSingle)
            System.out.println("The area is singular.");
            System.out.println("The area is not singular.");
        // (2) compute the area of B - A
        System.out.println("The area is " + computeArea(areaB) + ".");
        // (3) determine whether each p[i] is in the interior of B - A
        while (scanner.hasNextDouble()) {
            double x = scanner.nextDouble();
            assert(scanner.hasNextDouble());
            double y = scanner.nextDouble();
            if (areaB.contains(x,y)) {
                System.out.println ("Point belongs to the area.");
            } else {
                System.out.println ("Point does not belong to the area.");
        }
        // Finally, some useful things we didn't use in this example:
             Ellipse2D.Double ellipse = new Ellipse2D.Double (double x, double y,
       //
                                                               double w, double h);
        //
               creates an ellipse inscribed in box with bottom-left corner (x,y)
               and upper-right corner (x+y,w+h)
        //
             Rectangle2D.Double rect = new Rectangle2D.Double (double x, double y,
                                                                double w, double h);
        //
               creates a box with bottom-left corner (x,y) and upper-right
       //
               corner (x+y,w+h)
        // Each of these can be embedded in an Area object (e.g., new Area (rect)).
    }
}
```

Geom3D.java 11/35

```
public class Geom3D {
  // distance from point (x, y, z) to plane aX + bY + cZ + d = 0
  public static double ptPlaneDist(double x, double y, double z,
      double a, double b, double c, double d) {
    return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
 }
 // distance between parallel planes aX + bY + cZ + d1 = 0 and
 // aX + bY + cZ + d2 = 0
  public static double planePlaneDist(double a, double b, double c,
      double d1, double d2) {
    return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
 }
 // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2)
  // (or ray, or segment; in the case of the ray, the endpoint is the
 // first point)
  public static final int LINE = 0;
  public static final int SEGMENT = 1;
  public static final int RAY = 2;
  public static double ptLineDistSq(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz,
      int type) {
    double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);
```

```
double x, y, z;
    if (pd2 == 0) {
      x = x1;
      y = y1;
      z = z1;
    } else {
      double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1)) / pd2;
      x = x1 + u * (x2 - x1);

y = y1 + u * (y2 - y1);
      z = z1 + u * (z2 - z1);
      if (type != LINE && u < 0) {</pre>
        x = x1;
        y = y1;
        z = z1;
      if (type == SEGMENT && u > 1.0) {
        x = x2;
        y = y2;
        z = z2;
      }
    }
    return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
  public static double ptLineDist(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz,
    return Math.sqrt(ptLineDistSq(x1, y1, z1, x2, y2, z2, px, py, pz, type));
  }
}
```

Delaunay.cc 12/35

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
// Running time: O(n^4)
// INPUT:
             x[] = x-coordinates
             y[] = y-coordinates
//
// OUTPUT:
             triples = a vector containing m triples of indices
                        corresponding to triangle vertices
#include<vector>
using namespace std;
typedef double T;
struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};
vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
        int n = x.size();
        vector<T> z(n);
        vector<triple> ret;
        for (int i = 0; i < n; i++)</pre>
            z[i] = x[i] * x[i] + y[i] * y[i];
        for (int i = 0; i < n-2; i++) {</pre>
            for (int j = i+1; j < n; j++) {
                 for (int k = i+1; k < n; k++) {</pre>
                     if (j == k) continue;
                     double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j]-z[i]);
                     double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k]-z[i]);
                     double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j]-y[i]);
                     bool flag = zn < 0;</pre>
                     for (int m = 0; flag && m < n; m++)</pre>
                         flag = flag && ((x[m]-x[i])*xn +
```

```
(y[m]-y[i])*yn +
                                          (z[m]-z[i])*zn <= 0);
                     if (flag) ret.push_back(triple(i, j, k));
            }
        }
        return ret;
}
int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(\&xs[0], \&xs[4]), y(\&ys[0], \&ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);
    //expected: 0 1 3
                0 3 2
    int i;
    for(i = 0; i < tri.size(); i++)</pre>
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

Euclid.cc 13/35

```
// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;
typedef vector<int> VI;
typedef pair<int,int> PII;
// return a % b (positive value)
int mod(int a, int b) {
  return ((a%b)+b)%b;
// computes gcd(a,b)
int gcd(int a, int b) {
  int tmp;
  while(b){a%=b; tmp=a; a=b; b=tmp;}
  return a;
// computes Lcm(a,b)
int lcm(int a, int b) {
  return a/gcd(a,b)*b;
// returns d = gcd(a,b); finds x,y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
  int xx = y = 0;
  int yy = x = 1;
  while (b) {
   int q = a/b;
    int t = b; b = a%b; a = t;
    t = xx; xx = x-q*xx; x = t;
    t = yy; yy = y-q*yy; y = t;
  }
  return a;
// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
  int x, y;
  VI solutions;
```

```
int d = extended_euclid(a, n, x, y);
  if (!(b%d)) {
    x = mod(x*(b/d), n);
    for (int i = 0; i < d; i++)
      solutions.push_back(mod(x + i*(n/d), n));
  }
  return solutions;
// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
  int x, y;
  int d = extended_euclid(a, n, x, y);
  if (d > 1) return -1;
  return mod(x,n);
// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y).
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
  int s, t;
  int d = extended_euclid(x, y, s, t);
  if (a%d != b%d) return make_pair(0, -1);
  return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
// Chinese remainder theorem: find z such that
//z \% x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i(x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
  PII ret = make_pair(a[0], x[0]);
  for (int i = 1; i < x.size(); i++) {</pre>
    ret = chinese_remainder_theorem(ret.second, ret.first, x[i], a[i]);
    if (ret.second == -1) break;
  return ret;
}
// computes x and y such that ax + by = c; on failure, x = y = -1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
  int d = gcd(a,b);
  if (c%d) {
    x = y = -1;
  } else {
    x = c/d * mod_inverse(a/d, b/d);
    y = (c-a*x)/b;
}
int main() {
  // expected: 2
  cout << gcd(14, 30) << endl;</pre>
  // expected: 2 -2 1
  int x, y;
  int d = extended_euclid(14, 30, x, y);
cout << d << " " << x << " " << y << endl;</pre>
  // expected: 95 45
  VI sols = modular_linear_equation_solver(14, 30, 100);
  for (int i = 0; i < (int) sols.size(); i++) cout << sols[i] << " ";</pre>
  cout << endl;</pre>
  // expected: 8
  cout << mod_inverse(8, 9) << endl;</pre>
  // expected: 23 56
               11 12
  int xs[] = {3, 5, 7, 4, 6};
  int as[] = {2, 3, 2, 3, 5};
  PII ret = chinese_remainder_theorem(VI (xs, xs+3), VI(as, as+3)); cout << ret.first << " " << ret.second << endl;
  ret = chinese_remainder_theorem (VI(xs+3, xs+5), VI(as+3, as+5));
```

```
cout << ret.first << " " << ret.second << endl;

// expected: 5 -15
linear_diophantine(7, 2, 5, x, y);
cout << x << " " << y << endl;
}</pre>
```

GaussJordan.cc 14/35

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
     (1) solving systems of linear equations (AX=B)
     (2) inverting matrices (AX=I)
     (3) computing determinants of square matrices
// Running time: O(n^3)
// INPUT:
             a[][] = an nxn matrix
//
             b[][] = an nxm matrix
//
// OUTPUT:
                    = an nxm matrix (stored in b[][])
             A^{-1} = an nxn matrix (stored in a[][])
             returns determinant of a[][]
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;
  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])</pre>
      for (int k = 0; k < n; k++) if (!ipiv[k])</pre>
        if (pj == -1 \mid | fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }</pre>
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;
    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;</pre>
    for (int p = 0; p < m; p++) b[pk][p] *= c;</pre>
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;</pre>
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
  }
  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);</pre>
```

```
return det;
int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = \{ \{1,2,3,4\}, \{1,0,1,0\}, \{5,3,2,4\}, \{6,1,4,6\} \};
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  double det = GaussJordan(a, b);
  // expected: 60
  cout << "Determinant: " << det << endl;</pre>
  // expected: -0.233333 0.166667 0.133333 0.0666667
                0.166667 0.166667 0.333333 -0.333333
  //
  //
                //
                0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;</pre>
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
  cout << a[i][j] << ' ';</pre>
    cout << endl;</pre>
  // expected: 1.63333 1.3
                -0.166667 0.5
  //
                2.36667 1.7
  //
                -1.85 -1.35
  cout << "Solution: " << endl;</pre>
  for (int i = 0; i < n; i++) {</pre>
    for (int j = 0; j < m; j++)
  cout << b[i][j] << ' ';</pre>
    cout << endl;</pre>
  }
}
```

ReducedRowEchelonForm.cc 15/35

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
// Running time: O(n^3)
// INPUT:
             a[][] = an nxm matrix
// OUTPUT:
             rref[][] = an nxm matrix (stored in a[][])
             returns rank of a[][]
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const double EPSILON = 1e-10;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
int rref(VVT &a) {
  int n = a.size();
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
    int j = r;
```

```
for (int i = r + 1; i < n; i++)
       if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
     if (fabs(a[j][c]) < EPSILON) continue;</pre>
     swap(a[j], a[r]);
     T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
for (int i = 0; i < n; i++) if (i != r) {</pre>
       T t = a[i][c];
       for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];</pre>
     }
    r++;
  }
  return r;
int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16, 2, 3, 13},
{ 5, 11, 10, 8},
    { 9, 7, 6, 12},
{ 4, 14, 15, 1},
{13, 21, 21, 13}};
  VVT a(n);
  for (int i = 0; i < n; i++)</pre>
    a[i] = VT(A[i], A[i] + m);
  int rank = rref(a);
  // expected: 3
  cout << "Rank: " << rank << endl;</pre>
  // expected: 1 0 0 1
                  0 1 0 3
  //
                  0 0 1 -3
  //
                  0 0 0 3.10862e-15
  //
                  0 0 0 2.22045e-15
  cout << "rref: " << endl;</pre>
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
  cout << a[i][j] << ' ';</pre>
     cout << endl;</pre>
  }
}
```

FFT new.cpp 16/35

```
#include <cassert>
#include <cstdio>
#include <cmath>
struct cpx
  cpx(){}
  cpx(double aa):a(aa),b(0){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b;
  double modsq(void) const
    return a * a + b * b;
  cpx bar(void) const
    return cpx(a, -b);
};
cpx operator +(cpx a, cpx b)
  return cpx(a.a + b.a, a.b + b.b);
```

```
cpx operator *(cpx a, cpx b)
  return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
cpx operator /(cpx a, cpx b)
  cpx r = a * b.bar();
  return cpx(r.a / b.modsq(), r.b / b.modsq());
cpx EXP(double theta)
  return cpx(cos(theta),sin(theta));
const double two_pi = 4 * acos(0);
// in:
           input array
// out:
           output array
// step:
           {SET TO 1} (used internally)
           length of the input/output {MUST BE A POWER OF 2}
// size:
// dir:
           either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{\size - 1} in[j] * \exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
  if(size < 1) return;</pre>
  if(size == 1)
    out[0] = in[0];
    return;
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, size / 2, dir);
  for(int i = 0 ; i < size / 2 ; i++)</pre>
    cpx even = out[i];
    cpx odd = out[i + size / 2];
    out[i] = even + EXP(dir * two pi * i / size) * odd;
    out[i + size / 2] = even + EXP(dir * two pi * (i + size / 2) / size) * odd;
  }
}
// Usage:
// f[0...N-1] and g[0...N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum \ of \ f[k]g[n-k] \ (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product).
// To compute h[] in O(N log N) time, do the following:
    1. Compute F and G (pass dir = 1 as the argument).
    2. Get H by element-wise multiplying F and G.
     3. Get h by taking the inverse FFT (use dir = -1 as the argument)
        and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.
int main(void)
  printf("If rows come in identical pairs, then everything works.\n");
  cpx a[8] = \{0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0\};
  cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
  cpx A[8];
  cpx B[8];
  FFT(a, A, 1, 8, 1);
  FFT(b, B, 1, 8, 1);
  for(int i = 0; i < 8; i++)
  {
    printf("%7.21f%7.21f", A[i].a, A[i].b);
  printf("\n");
  for(int i = 0; i < 8; i++)
    cpx Ai(0,0);
    for(int j = 0; j < 8; j++)
```

```
Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
  printf("%7.21f%7.21f", Ai.a, Ai.b);
printf("\n");
cpx AB[8];
for(int i = 0; i < 8; i++)
AB[i] = A[i] * B[i];</pre>
cpx aconvb[8];
FFT(AB, aconvb, 1, 8, -1);
for(int i = 0; i < 8; i++)
  aconvb[i] = aconvb[i] / 8;
for(int i = 0; i < 8; i++)
  printf("%7.21f%7.21f", aconvb[i].a, aconvb[i].b);
printf("\n");
for(int i = 0; i < 8; i++)
  cpx aconvbi(0,0);
  for(int j = 0; j < 8; j++)</pre>
    aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
  printf("%7.21f%7.21f", aconvbi.a, aconvbi.b);
printf("\n");
return 0;
```

Simplex.cc 17/35

```
// Two-phase simplex algorithm for solving linear programs of the form
       maximize
       subject to
                   Ax <= b
                    x >= 0
// INPUT: A -- an m x n matrix
         b -- an m-dimensional vector
//
         c -- an n-dimensional vector
          x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution (infinity if unbounded
           above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).
#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>
using namespace std;
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9;
struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;
  LPSolver(const VVD &A, const VD &b, const VD &c):
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
```

```
for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }
  void Pivot(int r, int s) {
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] / D[r][s];
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {</pre>
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 \mid | D[x][j] < D[x][s] \mid | D[x][j] == D[x][s] && N[j] < N[s]) s = j;
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {</pre>
        if (D[i][s] < EPS) continue;</pre>
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
      if (r == -1) return false;
      Pivot(r, s);
    }
  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {</pre>
        int s = -1:
        for (int j = 0; j <= n; j++)</pre>
          if (s == -1 \mid | D[i][j] < D[i][s] \mid | D[i][j] == D[i][s] && N[j] < N[s]) s = j;
        Pivot(i, s);
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) \times [B[i]] = D[i][n + 1];
    return D[m][n + 1];
};
int main() {
  const int m = 4;
  const int n = 3;
  DOUBLE A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  DOUBLE _b[m] = \{ 10, -4, 5, -5 \};
  DOUBLE _{c[n]} = \{ 1, -1, 0 \};
  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(A[i], A[i] + n);
  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);
```

```
cerr << "VALUE: " << value << endl; // VALUE: 1.29032
cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
cerr << endl;
return 0;
}</pre>
```

FastDijkstra.cc 18/35

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
// Running time: O(|E| Log |V|)
#include <queue>
#include <stdio.h>
using namespace std;
const int INF = 20000000000;
typedef pair<int,int> PII;
int main(){
  int N, s, t;
  scanf ("%d%d%d", &N, &s, &t);
  vector<vector<PII> > edges(N);
  for (int i = 0; i < N; i++){
    int M:
    scanf ("%d", &M);
    for (int j = 0; j < M; j++){
      int vertex, dist;
      scanf ("%d%d", &vertex, &dist);
      edges[i].push_back (make_pair (dist, vertex)); // note order of arguments here
    }
  // use priority queue in which top element has the "smallest" priority
  priority_queue<PII, vector<PII>, greater<PII> > Q;
  vector<int> dist(N, INF), dad(N, -1);
  Q.push (make_pair (0, s));
  dist[s] = 0;
  while (!Q.empty()){
    PII p = Q.top();
    if (p.second == t) break;
    Q.pop();
    int here = p.second;
    for (vector<PII>::iterator it=edges[here].begin(); it!=edges[here].end(); it++){
      if (dist[here] + it->first < dist[it->second]){
        dist[it->second] = dist[here] + it->first;
        dad[it->second] = here;
        Q.push (make_pair (dist[it->second], it->second));
      }
    }
  printf ("%d\n", dist[t]);
  if (dist[t] < INF)</pre>
    for(int i=t;i!=-1;i=dad[i])
      printf ("%d%c", i, (i==s?'\n':' '));
  return 0;
```

SCC.cc 19/35

```
#include<memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
http://web.stanford.edu/~liszt90/acm/notebook.html#file13
```

```
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
  int i;
  v[x]=true;
  for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
  stk[++stk[0]]=x;
void fill_backward(int x)
{
  int i;
  v[x]=false;
  group_num[x]=group_cnt;
  for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
void add_edge(int v1, int v2) //add edge v1->v2
  e [++E].e=v2; e [E].nxt=sp [v1]; sp [v1]=E;
  er[ E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
void SCC()
{
  int i;
  stk[0]=0;
  memset(v, false, sizeof(v));
  for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);</pre>
  group_cnt=0;
  for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[i]);}
```

EulerianPath.cc 20/35

```
struct Edge;
typedef list<Edge>::iterator iter;
struct Edge
{
        int next vertex;
        iter reverse_edge;
        Edge(int next_vertex)
                :next_vertex(next_vertex)
                { }
};
const int max_vertices = ;
int num vertices;
list<Edge> adj[max_vertices];
                                        // adjacency list
vector<int> path;
void find_path(int v)
{
        while(adj[v].size() > 0)
                int vn = adj[v].front().next_vertex;
                adj[vn].erase(adj[v].front().reverse_edge);
                adj[v].pop_front();
                find_path(vn);
        path.push_back(v);
}
void add_edge(int a, int b)
{
        adj[a].push_front(Edge(b));
        iter ita = adj[a].begin();
        adj[b].push_front(Edge(a));
        iter itb = adj[b].begin();
        ita->reverse_edge = itb;
```

}

```
itb->reverse_edge = ita;
```

SuffixArray.cc 21/35

```
// Suffix array construction in O(L Log^2 L) time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
// INPUT:
            string s
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
            of substring s[i...L-1] in the list of sorted suffixes.
            That is, if we take the inverse of the permutation suffix[],
            we get the actual suffix array.
#include <vector>
#include <iostream>
#include <string>
using namespace std;
struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<pair<int,int>,int> > M;
  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);</pre>
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {</pre>
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 \&\& M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
  }
  vector<int> GetSuffixArray() { return P.back(); }
  // returns the length of the longest common prefix of s[i\dotsL-1] and s[j\dotsL-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
};
int main() {
  // bobocel is the 0'th suffix
  // obocel is the 5'th suffix
      bocel is the 1'st suffix
        ocel is the 6'th suffix
  //
         cel is the 2'nd suffix
          el is the 3'rd suffix
  //
           l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();
  // Expected output: 0 5 1 6 2 3 4
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";</pre>
  cout << endl;</pre>
  cout << suffix.LongestCommonPrefix(0, 2) << endl;</pre>
```

BIT.cc 22/35

```
#include <iostream>
using namespace std;
#define LOGSZ 17
int tree[(1<<LOGSZ)+1];</pre>
int N = (1 << LOGSZ);
// add v to value at x
void set(int x, int v) {
  while(x <= N) {</pre>
    tree[x] += v;
    x += (x & -x);
  }
}
// get cumulative sum up to and including x
int get(int x) {
  int res = 0;
  while(x) {
    res += tree[x];
    x -= (x \& -x);
  }
  return res;
}
// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
  int idx = 0, mask = N;
  while(mask && idx < N) {</pre>
    int t = idx + mask;
    if(x >= tree[t]) {
      idx = t;
      x -= tree[t];
    mask >>= 1;
  return idx;
```

UnionFind.cc 23/35

```
//union-find set: the vector/array contains the parent of each node int find(vector <int>& C, int x){return (C[x]==x) ? x : C[x]=find(C, C[x]);} //C++ int find(int x){return (C[x]==x)?x:C[x]=find(C[x]);} //C
```

KDTree.cc 24/35

```
#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>
using namespace std;
// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();
// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};
bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}
// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
    return a.x < b.x;</pre>
}
// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;</pre>
}
// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
}
// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;
    bbox(): x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}
    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {</pre>
            }
    }
    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
                                return pdist2(point(x0, y0), p);
            if (p.y < y0)
                                return pdist2(point(x0, y1), p);
            else if (p.y > y1)
            else
                                return pdist2(point(x0, p.y), p);
        else if (p.x > x1) {
                                return pdist2(point(x1, y0), p);
            if (p.y < y0)
                                return pdist2(point(x1, y1), p);
            else if (p.y > y1)
            else
                                return pdist2(point(x1, p.y), p);
        else {
            if (p.y < y0)
                                return pdist2(point(p.x, y0), p);
            else if (p.y > y1)
                                return pdist2(point(p.x, y1), p);
            else
                                return 0;
        }
    }
};
// stores a single node of the kd-tree, either internal or leaf
```

```
struct kdnode
{
                    // true if this is a leaf node (has one point)
    bool leaf;
                    // the single point of this is a leaf
    point pt;
                    // bounding box for set of points in children
    bbox bound:
    kdnode *first, *second; // two children of this kd-node
    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }
    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp)
    {
        // compute bounding box for points at this node
        bound.compute(vp);
        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
        else {
            // split on x if the bbox is wider than high (not best heuristic...)
            if (bound.x1-bound.x0 >= bound.y1-bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
                sort(vp.begin(), vp.end(), on_y);
            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size()/2;
            vector<point> vl(vp.begin(), vp.begin()+half);
            vector<point> vr(vp.begin()+half, vp.end());
            first = new kdnode(); first->construct(v1);
            second = new kdnode(); second->construct(vr);
    }
};
// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kdnode *root;
    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    ~kdtree() { delete root; }
    // recursive search method returns squared distance to nearest point
    ntype search(kdnode *node, const point &p)
        if (node->leaf) {
            // commented special case tells a point not to find itself
              if (p == node->pt) return sentry;
              else
                return pdist2(p, node->pt);
        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);
        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {</pre>
            ntype best = search(node->first, p);
            if (bsecond < best)</pre>
```

```
best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)</pre>
                best = min(best, search(node->first, p));
            return best;
        }
    }
    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
};
// some basic test code here
int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    kdtree tree(vp);
    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand()%100000, rand()%100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y << ")"
             << " is " << tree.nearest(q) << endl;</pre>
    }
    return 0;
}
```

splay.cpp 25/35

```
#include <cstdio>
#include <algorithm>
using namespace std;
const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
  Node *ch[2], *pre;
  int val, size;
  bool isTurned;
} nodePool[N_MAX], *null, *root;
Node *allocNode(int val)
  static int freePos = 0;
  Node *x = &nodePool[freePos ++];
  x->val = val, x->isTurned = false;
  x \rightarrow ch[0] = x \rightarrow ch[1] = x \rightarrow pre = null;
  x \rightarrow size = 1;
  return x;
inline void update(Node *x)
  x \rightarrow size = x \rightarrow ch[0] \rightarrow size + x \rightarrow ch[1] \rightarrow size + 1;
inline void makeTurned(Node *x)
  if(x == null)
```

```
return;
  swap(x->ch[0], x->ch[1]);
  x->isTurned ^= 1;
inline void pushDown(Node *x)
{
  if(x->isTurned)
  {
    makeTurned(x->ch[0]);
    makeTurned(x->ch[1]);
     x->isTurned ^= 1;
  }
}
inline void rotate(Node *x, int c)
  Node *y = x \rightarrow pre;
  x \rightarrow pre = y \rightarrow pre;
  if(y->pre != null)
    y \rightarrow pre \rightarrow ch[y == y \rightarrow pre \rightarrow ch[1]] = x;
  y \rightarrow ch[!c] = x \rightarrow ch[c];
  if(x->ch[c] != null)
    x \rightarrow ch[c] \rightarrow pre = y;
  x \rightarrow ch[c] = y, y \rightarrow pre = x;
  update(y);
  if(y == root)
     root = x;
}
void splay(Node *x, Node *p)
  while(x->pre != p)
     if(x->pre->pre == p)
       rotate(x, x == x->pre->ch[0]);
     else
     {
       Node *y = x->pre, *z = y->pre;
       if(y == z -> ch[0])
         if(x == y \rightarrow ch[0])
            rotate(y, 1), rotate(x, 1);
            rotate(x, 0), rotate(x, 1);
       }
       else
       {
         if(x == y \rightarrow ch[1])
            rotate(y, 0), rotate(x, 0);
            rotate(x, 1), rotate(x, 0);
    }
  update(x);
void select(int k, Node *fa)
  Node *now = root;
  while(1)
     pushDown(now);
     int tmp = now->ch[0]->size + 1;
     if(tmp == k)
       break;
     else if(tmp < k)</pre>
       now = now -> ch[1], k -= tmp;
     else
       now = now - > ch[0];
  splay(now, fa);
Node *makeTree(Node *p, int 1, int r)
```

```
if(1 > r)
    return null;
  int mid = (1 + r) / 2;
  Node *x = allocNode(mid);
  x \rightarrow pre = p;
  x \rightarrow ch[0] = makeTree(x, 1, mid - 1);
  x \rightarrow ch[1] = makeTree(x, mid + 1, r);
  update(x);
  return x;
int main()
  int n, m;
  null = allocNode(0);
  null->size = 0;
  root = allocNode(0);
  root->ch[1] = allocNode(oo);
  root->ch[1]->pre = root;
  update(root);
  scanf("%d%d", &n, &m);
  root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
  splay(root->ch[1]->ch[0], null);
  while(m --)
  {
    int a, b;
    scanf("%d%d", &a, &b);
    a ++, b ++;
    select(a - 1, null);
    select(b + 1, root);
    makeTurned(root->ch[1]->ch[0]);
  for(int i = 1; i <= n; i ++)</pre>
    select(i + 1, null);
    printf("%d ", root->val);
}
```

SegmentTreeLazy.java 26/35

```
public class SegmentTreeRangeUpdate {
        public long[] leaf;
        public long[] update;
        public int origSize;
        public SegmentTreeRangeUpdate(int[] list)
                origSize = list.length;
                leaf = new long[4*list.length];
                update = new long[4*list.length];
                build(1,0,list.length-1,list);
        public void build(int curr, int begin, int end, int[] list)
                if(begin == end)
                        leaf[curr] = list[begin];
                else
                        int mid = (begin+end)/2;
                        build(2 * curr, begin, mid, list);
                        build(2 * curr + 1, mid+1, end, list);
                        leaf[curr] = leaf[2*curr] + leaf[2*curr+1];
                }
        public void update(int begin, int end, int val) {
                update(1,0,origSize-1,begin,end,val);
        public void update(int curr, int tBegin, int tEnd, int begin, int end, int val){
                if(tBegin >= begin && tEnd <= end)</pre>
                        update[curr] += val;
                        leaf[curr] += (Math.min(end,tEnd)-Math.max(begin,tBegin)+1) * val;
                        int mid = (tBegin+tEnd)/2;
                        if(mid >= begin && tBegin <= end)</pre>
```

```
update(2*curr, tBegin, mid, begin, end, val);
                if(tEnd >= begin && mid+1 <= end)</pre>
                         update(2*curr+1, mid+1, tEnd, begin, end, val);
        }
public long query(int begin, int end)
        return query(1,0,origSize-1,begin,end);
public long query(int curr, int tBegin, int tEnd, int begin, int end)
        if(tBegin >= begin && tEnd <= end)</pre>
                 if(update[curr] != 0)
                         leaf[curr] += (tEnd-tBegin+1) * update[curr];
                         if(2*curr < update.length){</pre>
                                  update[2*curr] += update[curr];
                                  update[2*curr+1] += update[curr];
                         update[curr] = 0;
                return leaf[curr];
        }
        else
                 leaf[curr] += (tEnd-tBegin+1) * update[curr];
                 if(2*curr < update.length){</pre>
                         update[2*curr] += update[curr];
                         update[2*curr+1] += update[curr];
                update[curr] = 0;
                 int mid = (tBegin+tEnd)/2;
                long ret = 0;
                if(mid >= begin && tBegin <= end)</pre>
                         ret += query(2*curr, tBegin, mid, begin, end);
                if(tEnd >= begin && mid+1 <= end)</pre>
                         ret += query(2*curr+1, mid+1, tEnd, begin, end);
                return ret;
        }
}
```

LCA.cc 27/35

}

```
const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;
vector<int> children[max nodes];
                                          // children[i] contains the children of node i
int A[max_nodes][log_max_nodes+1];
                                           // A[i][j] is the 2^j-th ancestor of node i, or -1 if that ancestor does not exist
int L[max_nodes];
                                           // L[i] is the distance between node i and the root
// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if(n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>= 8; p += 8; }
    if (n >= 1<< 4) { n >>= 4; p += 4; }
if (n >= 1<< 2) { n >>= 2; p += 2; }
    if (n >= 1<< 1) {
                                  p += 1; }
    return p;
}
void DFS(int i, int 1)
    L[i] = 1;
    for(int j = 0; j < children[i].size(); j++)</pre>
        DFS(children[i][j], l+1);
}
int LCA(int p, int q)
    // ensure node p is at least as deep as node a
    if(L[p] < L[q])
        swap(p, q);
```

```
// "binary search" for the ancestor of node p situated on the same level as q
    for(int i = log_num_nodes; i >= 0; i--)
        if(L[p] - (1 << i) >= L[q])
             p = A[p][i];
    if(p == q)
        return p;
    // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
        if(A[p][i] != -1 && A[p][i] != A[q][i])
             p = A[p][i];
             q = A[q][i];
        }
    return A[p][0];
}
int main(int argc,char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes=lb(num_nodes);
    for(int i = 0; i < num_nodes; i++)</pre>
        int p;
        // read p, the parent of node i or -1 if node i is the root
        A[i][0] = p;
        if(p != -1)
             children[p].push_back(i);
             root = i;
    }
    // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
    for(int i = 0; i < num_nodes; i++)</pre>
             if(A[i][j-1] != -1)
                 A[i][j] = A[A[i][j-1]][j-1];
                 A[i][j] = -1;
    // precompute L
    DFS(root, 0);
    return 0;
}
```

LongestIncreasingSubsequence.cc 28/35

```
// Given a list of numbers of length n, this routine extracts a
// Longest increasing subsequence.
//
// Running time: O(n log n)
//
// INPUT: a vector of integers
// OUTPUT: a vector containing the longest increasing subsequence
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;
typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;
#define STRICTLY_INCREASNG
VI LongestIncreasingSubsequence(VI v) {
```

```
VPII best;
  VI dad(v.size(), -1);
  for (int i = 0; i < v.size(); i++) {</pre>
#ifdef STRICTLY INCREASING
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
      dad[i] = dad[it->second];
      *it = item;
    }
  }
 VI ret;
  for (int i = best.back().second; i >= 0; i = dad[i])
   ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
```

Dates.cc 29/35

```
// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.
#include <iostream>
#include <string>
using namespace std;
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};
// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}
// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;
  x = jd + 68569;
  n = 4 * x / 146097;
  x = (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x = 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
```

LogLan.java 30/35

```
// Code which demonstrates the use of Java's regular expression libraries.
// This is a solution for
//
     Loglan: a logical language
//
     http://acm.uva.es/p/v1/134.html
// In this problem, we are given a regular language, whose rules can be
// inferred directly from the code. For each sentence in the input, we must
// determine whether the sentence matches the regular expression or not. The
// code consists of (1) building the regular expression (which is fairly
// complex) and (2) using the regex to match sentences.
import java.util.*;
import java.util.regex.*;
public class LogLan {
    public static String BuildRegex (){
        String space = " +";
        String A = "([aeiou])";
        String C = "([a-z&&[^aeiou]])";
String MOD = "(g" + A + ")";
String BA = "(b" + A + ")";
        String DA = "(d" + A + ")";
        String LA = "(1" + A + ")";
        String NAM = "([a-z]*" + C + ")";
        String PREDA = "(" + C + C + A + C + A + "|" + C + A + C + C + A + ")";
        String predstring = "(" + PREDA + "(" + space + PREDA + ")*)";
        String predame = "(" + LA + space + predstring + "|" + NAM + ")";
String preds = "(" + predstring + "(" + space + A + space + predstring + ")*)";
        String predclaim = "(" + predname + space + BA + space + preds + "|" + DA + space +
             preds + ")";
        String verbpred = "(" + MOD + space + predstring + ")";
        String statement = "(" + predname + space + verbpred + space + predname + "|" +
             predname + space + verbpred + ")"
        String sentence = "(" + statement + "|" + predclaim + ")";
        return "^" + sentence + "$";
    }
    public static void main (String args[]){
        String regex = BuildRegex();
        Pattern pattern = Pattern.compile (regex);
        Scanner s = new Scanner(System.in);
        while (true) {
             // In this problem, each sentence consists of multiple lines, where the last
             // line is terminated by a period. The code below reads lines until
            // encountering a line whose final character is a '.'. Note the use of
            //
             //
                   s.length() to get length of string
                   s.charAt() to extract characters from a Java string
             //
                   s.trim() to remove whitespace from the beginning and end of Java string
             //
             //
             // Other useful String manipulation methods include
```

```
s.compareTo(t) < 0 if s < t, lexicographically</pre>
                  s.indexOf("apple") returns index of first occurrence of "apple" in s
                  s.lastIndexOf("apple") returns index of last occurrence of
                  s.replace(c,d) replaces occurrences of character c with d
            //
                  s.startsWith("apple) returns (s.indexOf("apple") == 0)
                  s.toLowerCase() / s.toUpperCase() returns a new lower/uppercased string
            //
            //
                  Integer.parseInt(s) converts s to an integer (32-bit)
                  Long.parseLong(s) converts s to a long (64-bit)
                  Double.parseDouble(s) converts s to a double
            String sentence = "";
            while (true){
                sentence = (sentence + " " + s.nextLine()).trim();
                if (sentence.equals("#")) return;
                if (sentence.charAt(sentence.length()-1) == '.') break;
            }
            // now, we remove the period, and match the regular expression
            String removed_period = sentence.substring(0, sentence.length()-1).trim();
               (pattern.matcher (removed_period).find()){
                System.out.println ("Good");
              else {
                System.out.println ("Bad!");
        }
    }
}
```

Primes.cc 31/35

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
  if(x<=1) return false;</pre>
  if(x<=3) return true;</pre>
  if (!(x%2) || !(x%3)) return false;
  LL s=(LL)(sqrt((double)(x))+EPS);
  for(LL i=5;i<=s;i+=6)</pre>
    if (!(x%i) || !(x%(i+2))) return false;
  return true;
  Primes less than 1000:
        2
               3
                     5
                                  11
                                         13
                                               17
                                                      19
                                                             23
                                                                   29
                                                                          31
                                                                                 37
       41
              43
                     47
                           53
                                  59
                                         61
                                               67
                                                      71
                                                             73
                                                                   79
                                                                          83
                                                                                 89
                                                            137
       97
             101
                   103
                          107
                                 109
                                        113
                                              127
                                                     131
                                                                  139
                                                                         149
                                                                                151
      157
             163
                   167
                          173
                                 179
                                        181
                                              191
                                                     193
                                                            197
                                                                  199
                                                                         211
                                                                                223
      227
             229
                   233
                          239
                                 241
                                        251
                                              257
                                                     263
                                                            269
                                                                  271
                                                                         277
                                                                                281
      283
             293
                   307
                          311
                                 313
                                        317
                                              331
                                                     337
                                                            347
                                                                  349
                                                                         353
                                                                                359
      367
             373
                    379
                          383
                                 389
                                        397
                                              401
                                                     409
                                                            419
                                                                  421
                                                                         431
                                                                                433
             443
                          457
                                                     479
                                                                         499
      439
                   449
                                 461
                                        463
                                              467
                                                            487
                                                                  491
                                                                                503
      509
             521
                    523
                          541
                                 547
                                        557
                                                            571
                                                                  577
                                                                         587
                                              563
                                                     569
                                                                                593
      599
             601
                   607
                          613
                                 617
                                        619
                                              631
                                                     641
                                                            643
                                                                  647
                                                                         653
                                                                                659
      661
             673
                   677
                          683
                                 691
                                        701
                                              709
                                                     719
                                                            727
                                                                  733
                                                                         739
                                                                                743
      751
             757
                    761
                          769
                                 773
                                        787
                                              797
                                                     809
                                                            811
                                                                  821
                                                                         823
                                                                                827
      829
             839
                   853
                          857
                                 859
                                        863
                                              877
                                                     881
                                                            883
                                                                  887
                                                                         907
                                                                                911
      919
             929
                   937
                          941
                                 947
                                        953
                                              967
                                                     971
                                                            977
                                                                  983
                                                                         991
                                                                                997
// Other primes:
      The largest prime smaller than 10 is 7.
      The largest prime smaller than 100 is 97.
      The largest prime smaller than 1000 is 997.
      The largest prime smaller than 10000 is 9973.
      The largest prime smaller than 100000 is 99991.
      The largest prime smaller than 1000000 is 999983.
      The largest prime smaller than 10000000 is 9999991.
//
      The largest prime smaller than 100000000 is 99999989.
```

IO.cpp 32/35

```
#include <iostream>
#include <iomanip>
using namespace std;
int main()
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);</pre>
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);
    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);
    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);
    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

KMP.cpp 33/35

```
Searches for the string w in the string s (of length k). Returns the
O-based index of the first match (k if no match is found). Algorithm
runs in O(k) time.
#include <iostream>
#include <string>
#include <vector>
using namespace std;
typedef vector<int> VI;
void buildTable(string& w, VI& t)
  t = VI(w.length());
  int i = 2, j = 0;
  t[0] = -1; t[1] = 0;
  while(i < w.length())</pre>
    if(w[i-1] == w[j]) { t[i] = j+1; i++; j++; }
    else if(j > 0) j = t[j];
    else { t[i] = 0; i++; }
  }
}
int KMP(string& s, string& w)
```

```
5/16/2016
    int m = 0, i = 0;
    VI t;
    buildTable(w, t);
    while(m+i < s.length())</pre>
      if(w[i] == s[m+i])
      {
        i++;
        if(i == w.length()) return m;
      }
      else
      {
        m += i-t[i];
        if(i > 0) i = t[i];
    return s.length();
 int main()
  {
    string a = (string) "The example above illustrates the general technique for assembling "+
      "the table with a minimum of fuss. The principle is that of the overall search: "+
      "most of the work was already done in getting to the current position, so very "+
      "little needs to be done in leaving it. The only minor complication is that the "+
      "logic which is correct late in the string erroneously gives non-proper "+
      "substrings at the beginning. This necessitates some initialization code.";
    string b = "table";
    int p = KMP(a, b); cout << p << ": " << a.substr(p, b.length()) << " " << b << endl;
```

LatLong.cpp 34/35

```
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
#include <iostream>
#include <cmath>
using namespace std;
struct 11
  double r, lat, lon;
};
struct rect
{
  double x, y, z;
};
11 convert(rect& P)
{
 11 Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));
  return Q;
rect convert(11& Q)
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);
```

```
return P;
}
int main()
{
  rect A;
  11 B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}</pre>
```

EmacsSettings.txt 35/35

```
;; Jack's .emacs file
(global-set-key "\C-z"
(global-set-key "\C-x\C-p"
(global-set-key "\C-x\C-o"
                                        'scroll-down)
                                        '(lambda() (interactive) (other-window -1)) )
                                        'other-window)
(global-set-key "\C-x\C-n"
                                        'other-window)
(global-set-key "\M-." 'end-of-buffer)
(global-set-key "\M-," 'beginning-of-buf-
(global-set-key "\M-g" 'goto-line)
(global-set-key "\C-c\C-w" 'compare-windows)
                                        'beginning-of-buffer)
(tool-bar-mode 0)
(scroll-bar-mode -1)
(global-font-lock-mode 1)
(show-paren-mode 1)
(setq-default c-default-style "linux")
(custom-set-variables
  (compare-ignore-whitespace t)
```

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