indy256's blog

Dynamic Programming Optimizations

By **indy256**, 2 years ago,

Several recent problems on Codeforces concerned dynamic programming optimization techniques.

The following table summarizes methods known to me.

Name	Original Recurrence	Sufficient Condition of Applicability	Original Complexity	Optimized Co
Convex Hull Optimization1	$dp[i] = min_{j<i}\{dp[j] + b[j] \star a[i]\}$	$b[j] \geq b[j+1]$ optionally $a[i] \leq a[i+1]$	$O(n^2)$	O(
Convex Hull Optimization2	$dp[i][j] = min_{k<j}\{dp[i-1][k] + b[k] * a[j]\}$	$b[k] \geq b[k+1]$ optionally $a[j] \leq a[j+1]$	$O(kn^2)$	O(kn
Divide and Conquer Optimization	$dp[i][j] = min_{k<j}\{dp[i-1][k] + C[k][j]\}$	$A[i][j] \leq A[i][j+1]$	$O(kn^2)$	O(
Knuth Optimization	$dp[i][j] = min_{i<k<j}\{dp[i][k] + dp[k][j]\} + C[i][j]$	$A[i,j-1] \leq A[i,j] \leq A[i+1,j]$	$O(n^3)$	$O(n^2)$

Notes:

- $A[i][j]$ — the smallest k that gives optimal answer, for example in $dp[i][j] = dp[i-1][k] + C[k][j]$
- $C[i][j]$ — some given cost function
- We can generalize a bit in the following way: $dp[i] = min_{j<i}\{F[j] + b[j] * a[i]\}$, where $F[j]$ is computed from $dp[j]$ in constant time.
- It looks like **Convex Hull Optimization2** is a special case of **Divide and Conquer Optimization**.
- It is claimed (in the references) that **Knuth Optimization** is applicable if $C[i][j]$ satisfies the following 2 conditions:
- **quadrangle inequality**: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c],\ a \leq b \leq c \leq d$
- **monotonicity**: $C[b][c] \leq C[a][d],\ a \leq b \leq c \leq d$
- It is claimed (in the references) that the recurrence $dp[j] = min_{i<j}\{dp[i] + C[i][j]\}$ can be solved in $O(nlogn)$ (and even $O(n)$) if $C[i][j]$ satisfies **quadrangle inequality**. **WJMZBMR** described how to solve some case of this problem.

Open questions:

1. Are there any other optimization techniques?
2. What is the sufficient condition of applying **Divide and Conquer Optimization** in terms of function $C[i][j]$? Answered

References:

- *"Efficient dynamic programming using quadrangle inequalities" by F. Frances Yao.* find
- *"Speed-Up in Dynamic Programming" by F. Frances Yao.* find
- *"The Least Weight Subsequence Problem" by D. S. Hirschberg, L. L. Larmore.* find
- *"Dynamic programming with convexity, concavity and sparsity" by Zvi Galil, Kunsoo Park.* find
- *"A Linear-Time Algorithm for Concave One-Dimensional Dynamic Programming" by Zvi Galil, Kunsoo Park.* find

Please, share your knowledge and links on the topic.

dynamic programming, knuth optimization, convex hull optimization

+388 indy256 2 years ago 28

Comments (28)

Write comment?

WJMZBMR

2 years ago, # | ← Rev. 4 +27

Here is another way to optimize some 1D1D dynamic programming problem that I know.

Suppose that the old choice will only be worse compare to the new choice(it is quite common in such kind of problems).

Then suppose at current time we are deal with dp_i, and we have some choice $a_0 < a_1 < a_2, \ldots, a_{k-1} < a_k$. then we know at current time a_i should be better than a_{i+1}. Otherwise it will never be better than a_{i+1},so it is useless.

we can use a deque to store all the a_i.

And Also Let us denote $D(a, b)$ as the smallest i such that choice b will be better than a.

If $D(a_i, a_{i+1}) > D(a_{i+1}, a_{i+2})$,we can find a_{i+1} is also useless because when it overpass a_i,it is already overpass by a_{i+2}.

So we also let $D(a_i, a_{i+1}) < D(a_{i+1}, a_{i+2})$. then we can find the overpass will only happen at the front of the deque.

So we can maintain this deque quickly, and if we can solve $D(a, b)$ in $O(1)$,it can run in $O(n)$.

→ Reply

kingofnumbers

2 years ago, # ^ | +3

could you please give some example problems?

→ Reply

cgy4ever

2 years ago, # | +5

For question 2: The sufficient condition is: $C[a][d] + C[b][c] \geq C[a][c] + C[b][d]$ where $a < b < c < d$.

→ Reply

wanbo

2 years ago, # ^ | 0

```
Is it quadrangle inequalities?
∀i≤ j,w[i, j]+w[i+1, j+1]≤w[i+1, j]+w[i, j+1], and are these two inequalities equivalent except the >= & <=?
```

→ Reply

Sammarize

2 years ago, # | +18

There is one more optimization of dimanic progamming: 101E - Конфеты и Камни (editoral)

→ Reply

kingofnumbers

2 years ago, # | +13

you have put problem "B. Cats Transport" in "Convex Hull Optimization1", actually it belongs to "Convex Hull Optimization2"

→ Reply

indy256

2 years ago, # ^ | +5

fixed

→ Reply

Zlobober

2 years ago, # | ← Rev. 2 +55

For this moment it's the most useful topic of this year. Exactly in the middle: June 30th, 2013.

→ Reply

MarioYC

2 years ago, # | +8

this one seemed a nice dp with optimization to me:https://www.hackerrank.com/contests/monthly/challenges/alien-languages

→ Reply

gen → ACM ICPC 2015-2016, Baltic Selection Contest (Online Mirror)

Shafaet → [HackerRank] CodeStorm on 29th October / Awesome Prizes

minimario → Why I have Ten Post on the Front Page

PrinceOfPersia → Codeforces Round #326 (Editorial)

King_Alexander → Doubt regarding virtual participation

DanAlex → Matrix

maximaxi → The best contest start times

AFGhazy → Binary Search without Infinite Loop

GlebsHP → New coordinator presents Codeforces Round #327

ndatta → Need integration related problem link

MerelyMHP → Why most programmer prefer C++ than C?

asifruetcse10 → Where can I run a virtual practice contest?

muratt → Are there any judges which contains COCI problems ?

PrinceOfPersia → Hello 2015 Editorial

misis_VShtefanko → Codeforces compiler wrong answer.

Rustin → Help needed.

Detailed →

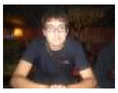
hogloid

2 years ago, # | ← Rev. 4 +29

The problem mentioned in the article (Breaking Strings) is "Optimal Binary Search Tree Problem" , traditional one.

It can be solved by simple DP in O(N^3), by using Knuth's optimization , in O(N^2) . But it still can be solved in O(NlogN) — http://poj.org/problem?id=1738 (same problem but bigger testcases) (I don't know how to solve it. I hear the algorithm uses meld-able heap)

→ Reply



Giorgos_Christoglou

2 years ago, # | +20

Convex Hull Optimization 1 Problems:

- APIO 2010 task Commando
- TRAKA
- ACQUIRE
- SkyScrapers (+Data Structures)

Convex Hull Optimization 2 Problems:

- BAABO

Convex Hull Optimization 3 Problems (No conditions for a[] array and b[] array) :

- GOODG
- BOI 2012 Day 2 Balls
- Cow School
- Solution-Video

→ Reply

zscefn

2 years ago, # | 0

For some reason I cannot open the links with firefox because they go over the Top Rated table.

→ Reply

indy256

2 years ago, # ^ | +4

Try to zoom out, pressing Ctrl + -

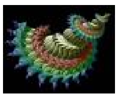
→ Reply

Monyura

2 years ago, # | ← Rev. 2 +8

One more problem where Knuth Optimization is used:
Andrew Stankevich Contest 10, Problem C.
BTW, does anybody know how to insert a direct link to a problem from gyms?

→ Reply

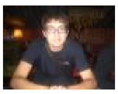


mbrc

15 months ago, # | 0

I need some problems to solve on Divide and Conquer Optimization. Where can I find them? An online judge / testdata available would be helpful.

→ Reply

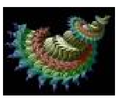


Giorgos_Christoglou

15 months ago, # ^ | +1

Check this one : Guardians of the Lunatics

→ Reply



15 months ago, # ^ | 0

Learnt Divide and Conquer Optimization just from there. :P That is why I'm asking for more problems to practice. :D

→ Reply

mbrc → Reply


sifrit98

4 weeks ago, # ^ | 0

Is this the best complexity for this problem? Can't we do any better? Can't we somehow turn the logL needed into a constant?
→ Reply

micklepru

3 weeks ago, # ^ | 0

We can, using that `opt[i-1][j] <= opt[i][j] <= opt[i][j+1]`.

Key thing is to see that opt function is monotone for both arguments. With that observation, we don't need to use binary search.

Check out my submission.
→ Reply

92anurag

15 months ago, # | +3

can anyone provide me good editorial for dp with bitmask .
→ Reply


icalFikr

15 months ago, # | 0

Has matrix-exponent optimizations been included here?
→ Reply


Farsid

12 months ago, # | +2

Can matrix chain multiplication problem b also optimized by knuth optimization? If not, dn why?
→ Reply


indy256

12 months ago, # ^ | +3

Quote from the first of the references above:

> *The monotonicity property for the division points does not hold for the matrix multiplication chain problem...*
>
> *Consider the matrices M1,M2,M3,M4 with dimensions 2x3, 3x2, 2x10, and 10x1, respectively. As can be easily verified, the proper order to compute M1M2M3 is to parenthesize it as (M1M2)M3, while the optimal computation of M1M2M3M4 corresponds to M1(M2(M3M4)).*

The second reference gives $O(n^2)$ dynamic programming solution, based on some properties of the matrix chain multiplication problem.

There is also an $O(n * \log n)$ algorithm by Hu and Shing.
→ Reply


Thomas_Ahle

6 months ago, # ^ | 0

Link to the Hu and Shing algorithm?
→ Reply

mayankp

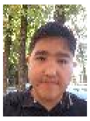
7 months ago, # | +1

What are some recent USACO questions that use this technique or variations of it?
→ Reply

4 months ago, # | ← Rev. 6 0

Can this problem be solved using convex hull optimization?

Can this problem be solved using convex hull optimization?



EXM_KG

You are given a sequence A of N positive integers. Let's define "value of a splitting" the sequence to K blocks as a sum of maximums in each of K blocks. For given K find the minimal possible value of splittings.

$N <= 10^5$

$K <= 100$

```
Input:     Output:
5 2        6
1 2 3 4 5
```

→ Reply

na2a

4 months ago, # ^ | 0

I don't think so, but I guess it can be solved by Divide And Conquer optimization.

→ Reply

vdmedragon

3 months ago, # | 0

Could you elaborate a little me more in the "Convex Hull Optimization2" and other sections for the clearer notations.

For example, You have "k" — a constant in O(kn^2). So the first dimension is of the length K and the second dimension is of the length N?

I think it would be clearer if you can write dp[n], dp[k][n] ... instead of dp[i], dp[i][j] .

Best regards,

→ Reply