CSE 211: Discrete Mathematics **(Due: 12/11/19)**

Homework #2

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Sets (2+2+2+2+2=10 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 - 6x + 8 = 0}

(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)
a)$x = 2 \ and \ x = 4 \rightarrow \{2, 4\}$
b)$y = [2, 3]$
c){4, 2, 5, 4}
d){4, 2}
e){4, 2}
so a=d=e

Problem 2: Cartesian Product of Sets (15 points)

Explain why (A × B) × (C × D) and A × (B × C) × D are not the same.
(Solution)
Assume that $a \epsilon A$, $b \epsilon B$, $c \epsilon C$, $d \epsilon D$
(A × B) × (C × D) = $\{(a, b)\}$ X $\{(c, d)\}$
$=\{\{(a, c), (a, d), (b, c), (b, d)\}\}$
A × (B × C) × D = $\{a\}$ X $\{(b, c)\}$ X $\{d\}$
$=\{(a, b, c), (b, c, d)\}$

Problem 3: Cartesian Product of Sets in Algorithms (25 points)

Let A, B and C be sets which have different cardinalities. Let (p, q, r) be each triple of A × B × C where p ∈ A, q ∈ B and r ∈ C. Design an algorithm which finds all the triples that are satisfying the criteria: p ≤ q and q ≥ r. Write the pseudo code of the algorithm in your solution.

For example: Let the set A, B and C be as A = { 3, 5, 7 }, B = { 3, 6 } and C = { 4, 6, 9 }. Then the output should be : { (3, 6, 4), (3, 6, 6), (5, 6, 4), (5, 6, 6) }.

(Note: Assume that you have sets of A, B, C as an input argument.)

(Solution)

Algorithm 1: Pseudo Code of Your Algorithm

Input: The sets of A, B, C
while $i < n$ **do**
  if $a_i < b_i \ \Lambda \ c_i < b_i$ **then**
    $p_i \leftarrow a_i$
    $q_i \leftarrow b_i$
    $r_i \leftarrow c_i$
    $i \leftarrow i + 1$
    $(\ p_i \ , \ q_i \ , \ r_i \ ) \ _i$
  else
    $i \leftarrow i + 1$
  end
end

Problem 4: Relations (3+3+3+3+3+3+3=21 points)

Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where (x, y) ∈ R if and only if
(a) x ≠ y.
(Solution)
transitive
symmetric

(b) xy ≥ 1.
(Solution)
reflexitive
transitive
symmetric

(c) x = y + 1 or x = y - 1.
(Solution)
reflexive
symmetric
transitive

(d) x is a multiple of y.
(Solution)
reflexice
antisymmetric
transitive

(e) x and y are both negative or both nonnegative.
(Solution)
reflexitive
transitive
symmetric

(f) x ≥ y^2.
(Solution)
antisymmetric
transitive

(g) x = y^2.
(Solution)
transitive
antisymmetric

Problem 5: Functions (15 points)

If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer.
(Solution)
we assume that by conradiction
Let's say g(x)=g(y) which means its one to one
fog(x)=fog(y)
that' means fog(x) and fog(y) reaches the same value and it proof its not one to one
so we can say g() is also one to one

Problem 6: Inverse of Functions (7+7=14 points)

Let f be the function from ℝ to ℝ defined by f(x) = x^2. Find
(a) f^{-1} ({ x | 0 < x < 1 })
(Solution)
$f(x) = x^2$ $y = x^2$
so $x = y^2 \sqrt{x} = y$
$f^{-1} = \sqrt{x}$
$0 < \sqrt{x} < 1$ $x = (0, 1)$

(b)f^{-1} ({ x | x > 4 })
(Solution)
$\sqrt{x} > 4$
$x > 16$
$x = (16, \infty)$