CSE 211: Discrete Mathematics **(Due: 27/10/19)**

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Conditional Statements (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)

Converse: If I will stay at home then it snows tonight
Contrapositive: If I will not stay at home tonight then it does not snow tonight
Inverse: If it does not snow tonight then I will not stay at home

(b) I go to the beach whenever it is a sunny summer day.
(Solution)

Converse: It is a sunny summer day whenever I go to the beach
Contrapositive: It is not a sunny summer day whenever I do not go to the beach
Inverse: I do not go to the beach whenever it is not a sunny summer day

(c) When I stay up late, it is necessary that I sleep until noon.
(Solution)

Converse: When It is necessary that I sleep until noon, I stay up late
Contrapositive: When It is not necessary that I sleep until noon, I do not stay up late
Inverse: When I do not stay up late, It is not necessary that I sleep until noon

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.
(a) (p $\oplus$ $\neg$ q)
(Solution)

p	q	$\neg q$	$p \oplus \neg q$
1	1	0	1
1	0	1	0
0	1	0	0
0	0	1	1

(b) (p $\iff$ q) $\oplus$ ($\neg$ p $\iff$ $\neg$ r)
(Solution)

$\neg p$	p	q	r	$\neg r$	$p \Leftrightarrow q$	$\neg p \Leftrightarrow \neg r$	$(p \Leftrightarrow q) \oplus (\neg p \Leftrightarrow \neg r)$
0	1	1	1	0	1	1	0
0	1	1	0	1	1	0	1
0	1	0	1	0	0	1	1
0	1	0	0	1	0	0	0
1	0	1	1	0	0	0	0
1	0	1	0	1	0	1	1
1	0	0	1	0	1	0	1
1	0	0	0	1	1	1	0

(c) (p ⊕ q) ⇒ (p ⊕ ¬ q)
(Solution)

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \Rightarrow (p \oplus \neg q)$
1	1	0	0	1	1
1	0	1	1	0	0
0	1	0	1	0	0
0	0	1	0	1	1

Problem 3: Logic in Algorithms (10+10+10=30 points)

If x = 1 before the statement is reached, what is the value of x after each of these statements is encountered in a computer program? Why? Show your work step by step.

(a) for i $\Leftarrow$ 1 **to** 10 **do**

if x + 2 = 3 **then** x := x + 1

end

(Solution)
for i=1 x+2=3 is an true statement so we increase x and new x value is 2
for i=2 4=3 is false satement
for i=3 5=3 is false satement
for i=4 6=3 is false satement
for i=5 7=3 is false satement
for i= 6 8=3 is false satement
for i= 7 9=3 is false satement
for i=8 10=3 is false satement
for i=9 11=3 is false satement
for i=10 statement is false
so the last x value is still 2

(b) for i $\Leftarrow$ 1 **to** 5 **do**

if (x + 1 = 2) XOR (x + 2 = 3) **then** x := x + 1

end

(Solution)
for i=1 2=2 XOR 3=3 is false so x=1
for i=2 3=2 XOR 4=3 is false so x=1
for i=3 4=2 XOR 5=3 is false so x=1
for i=4 5=2 XOR 6=3 is false so x=1
for i=5 6=2 XOR 7=3 is false so x=1
finally the x value is still 1

(c) for i $\Leftarrow$ 1 **to** 4 **do**

if (2x + 3 = 5) AND (3x + 4 = 7) **then** x := x + 1

end

(Solution)
for i=1 5=5 AND 7=7 is a true statement so we increase x 1 new x is x=2
for i=2 7=5 AND 10=7 is a false statement so x still 2
for i=3 9=5 AND 13=7 is a false statement
for i=4 11=5 AND 16=7 is a false statement
finally x value is still 2

Problem 4: Proof by contradiction (20 points)

Show that at least three of any 25 days chosen must fall in the same month of the year using a proof by contradiction. Explain your work step by step.

(Solution)

since inverse of atleast theree days is smaller three days we can express a number which is smaller than three
Let's chose 2 days for each month(to be fewer than at least 3days)
There are 12 months in a year
12x2=24
but the given number of days 25 so we have to chose one month more
but we have only 12 month so one of the month has three days
finally we have at least one month which has three days

Problem 5: Proof by contraposition (20 points)

Show that if $n^3 + 5$ is odd, then n is even using a proof by contraposition. Explain your work step by step.
Note: Assume that n is an integer.

(Solution)

the contraposition of statement is "If n is odd then $n^3 + 5$ is even" so
we assume n is odd
since n=2k+1
$n^3 + 5$ =$(2k + 1)^3$ +5
$n^3 + 5$ =8k^3 +12k^2 + 6k +1 +5
$n^3 + 5$ =8k^3 +12k^2 + 6k +6
$n^3 + 5$ =2$(4k^3 + 6k^2 + 3k + 3)$
let's say $(4k^3 + 6k^2 + 3k + 3)$ is p
$n^3 + 5$ =2p
2p is even $n^3 + 5$ is also even
since "If n is odd then $n^3 + 5$ is even" is true which is the contraposition of given statement
then the original statement is also true.