CSE 211: Discrete Mathematics **(Due: 24/12/19)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework4 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Nonhomogeneous Linear Recurrence Relations (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.

(Solution)
$a_{n-1} = -2^n$ for $a_n = -2^{n+1}$
when we place it in the $a_n = 3a_(n-1) + 2^n$ relation
$a_n = 3(-2^n) + 2^n$ and we get
$a_n = -2^{n+1}$ and it shows the results are same for a_n

(b) Find the solution with $a_0 = 1$.

(Solution)
$a_n = 3a_(n-1) + 2^n$

$r = 3 + a^p$

r=3

$a_n = c_1(3)^n + a^p$

lets assume A is the our const value

$A.2^n = 3.A.2^{n-1} + 2^n$

$2.A = 3.A + 2$

$A = -2$

$a_n = c_1(3)^n - 2.2^n$

$a_0 = 1 = c_1 - 2$

so $c_1 = 3$

$a_n = 3(3)^n - 2.2^n$

Problem 2: Linear Recurrence Relations (35 points)

Find all solutions of the recurrence relation a_n = $7a_{n-1}$ - $16a_{n-2}$ + $12a_{n-3}$ + n4^n with a_0 = -2, a_1 = 0, and a_2 = 5.

(Solution)
$r^3 = 7r^2 - 16r + 12$

$(r-2)^2(r-3) = 0$

$r_1 = r_2 \neq r_3$

$2 = 2 \neq 3$

$a_n = c_1 2^n + c_2 2^n n + c_3 3^n$

$a_n = (AnB)4^n$

$a_{n-1} = (A(n-1)B)4^n$

$a_{n-2} = (A(n-2)B)4^{n-2}$

$a_{n-3} = (A(n-3)B)4^{n-3}$

if we put them in the relation and divide them by 4^{n-3}

$(An+B)4^3 = 7(A(n-1)+B)4^2 - 16(A(n-2)+B)4 + 12(A()n-3) + n4^3$

if we simplify the equation

$-4An - 20A - 64n + 4B = 0$

$4B - 20A + (-4A - 64)n = 0$

$A = 16 and B = -80$

$a_n^p = (16n - 80)4^n$

$a_n = c_1 2^n + c_2 2^n n + c_3 3^n + (16n - 80)4^n$

for a_0

$-2 = c_1 + c_3 - 80$

for a_1

$0 = c_1 2 + c_2 2 + c_3 3 + -256$

for a_2

$5 = c_1 4 + c_2 8 + c_3 9 + -768$

$c_1 = 17 c_2 = \frac{39}{2} c_3 = 61$

$a_n = 17.2^n + 39.2^{n-1}n - 61.3^n + (16n + 80)4^n$

Problem 3: Linear Homogeneous Recurrence Relations (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.

(Solution)
$r^2 = 2r - 2$

$r^2 - 2r + 2 = 0$

we can fid the roots by using delta which is $b^2 - 4ac$

$\Delta = (-2).(-2) - 4.1.2 \rightarrow -4$

$\frac{2 \pm 2\sqrt{-1}}{2}$
$1 \pm i$

$a_n = c_1(1 + i)^n + c_2(1 - i)^n$

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

(Solution)
$a_0 = 1 = c_1 + c_2$

$a_1 = 2 = c_1(1 + i) + c_2(1 - i)$

if we apply first equation to second equation like

$1 - c_1 = c_2$

$c1 + c_1 i + 1 - i - c_1 + c_1 i$

$2c_1 i - i + 1 = 2$

$c_1 = \frac{-i+1}{2}$

$1 = \frac{1-i}{2} + c_2$

$c_2 = \frac{i+1}{2}$

so the relation is

$a_n = \frac{-i+1}{2}(1 + i)^n + \frac{i+1}{2}(1 - i)^n$