1)

a) $\log_2 n^2 + 1 \in O(n)$

$$\lim_{x\to\infty} \frac{(\log_2(x^2)+1)}{x} \Rightarrow \frac{\log_2(x^2)}{x} + \frac{1}{x} \Rightarrow \underbrace{\lim_{x\to\infty}\left(\frac{\log_2(x^2)}{x}\right)}_{0} + \underbrace{\lim_{x\to\infty}\left(\frac{1}{x}\right)}_{0}$$

$0 + 0 = \underline{\underline{0}}$ $\quad \lim_{x\to\infty}\left(\frac{f(n)}{g(n)}\right) = 0$; if $f(n) \in o(g(n))$

$f(n)$ grows slower than $g(n)$ $\rightarrow$ if its little-oh, it can be big-Oh

so it provides $\log_2 n^2 + 1 \in O(n)$ True

b) $\sqrt{n(n+1)} \in \Omega(n)$

$$\lim_{x\to\infty}\left(\frac{\sqrt{x(x+1)}}{x}\right) \Rightarrow \lim_{x\to\infty}\left(\frac{\sqrt{x\,x\,(1+\frac{1}{x})}}{x}\right) \Rightarrow \lim_{x\to\infty}\left(\frac{\sqrt{\frac{x+1}{x}}\,\cancel{x}}{\cancel{x}}\right)$$

$$\lim_{x\to\infty}\sqrt{\frac{x+1}{x}} \Rightarrow \sqrt{\lim_{x\to\infty}\left(\frac{x+1}{x}\right)} \Rightarrow$$ from rule $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

$$\Rightarrow \underbrace{\sqrt{\lim_{x\to\infty}\left(1 + \underbrace{\lim_{x\to\infty}\left(\frac{1}{x}\right)}_{0}\right)}}_{1} \Rightarrow \sqrt{1} = 1$$

$\lim_{x\to\infty}\left(\frac{f(n)}{g(n)}\right) = 1$, $f(n) \in \sim(g(n))$ strictly asymptotic

$f(n) \in \Theta(g(n)) \iff f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$ True

c) $n^{n-1} \in \Theta(n^n)$

there exist c_1, c_2, n_0 such that and $\forall n \geq n_0$ (contradiction)

$c_1 \cdot n^n \leq n^{n-1} \leq c_2 n^n$

$c_1 n^n \leq n^{n-1}$ $\quad n^{n-1} \leq c_2 \cdot n^n$

$c_1 n \leq 1$ $\quad 1 \leq c_2 n$

$c_1 \leq \frac{1}{n}$ $\quad \frac{1}{n} \leq c_2$

Only if $n \to \infty$, $c_1 = 0$ provides this equation

so $n^{n-1} \notin \Theta(n^n)$ False

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d) $O(2^n+n^3) \subset O(4^n)$

Since from equation $O(4^n)$ is proper superset of $O(2^n+n^3)$ we can check $O(2^n+n^3) \in O(4^n)$

$$\lim_{x\to\infty}\left(\frac{2^x+x^3}{4^x}\right) \Rightarrow \lim_{x\to\infty}\left(\left(\frac{1}{2}\right)^x+\frac{x^3}{4^x}\right) \Rightarrow \underbrace{\lim_{x\to\infty}\left(\left(\frac{1}{2}\right)^x\right)}_{0} + \underbrace{\lim_{x\to\infty}\left(\frac{x^3}{4^x}\right)}_{0} = 0$$

$\lim_{x\to\infty}\left(\frac{f(n)}{g(n)}\right) = 0$; if $f(n) \in o\,g(n)$ for little oh $f(n)$ grows slower than $g(n)$

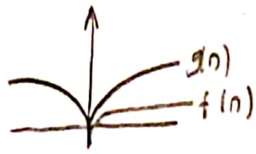
since it provides little-oh, its also provides Big-Oh which means True.

e) $O(2\log_3 \sqrt[3]{n}) \subset O(3\log_2 n^2)$

Since from equation $O(3\log_2 n^2)$ is proper superset of $O(2\log_3 \sqrt[3]{n})$ we can check $O(2\log_3 \sqrt[3]{n}) \in O(3\log_2 n^2)$

$$\lim_{x\to\infty}\left(\frac{2\log_3 \sqrt[3]{x}}{3\log_2 x^2}\right) \Rightarrow \frac{2}{3}\lim_{x\to\infty}\left(\frac{\log_3 x}{6\log_2 x}\right) \Rightarrow \frac{1}{9}\lim_{x\to\infty}\left(\frac{\frac{1}{x\ln 3}}{\frac{1}{x\ln 2}}\right) \Rightarrow \frac{1}{9}\cdot\lim_{x\to\infty}\frac{\ln 2}{\ln 3} = \frac{\ln(2)}{9\ln 3}$$

$\lim_{x\to\infty}\frac{f(n)}{g(n)} = \text{constant} > 0$; if $f(n) \in \Theta(g(n))$ True (?) It can be $\subseteq$



f) $\log_2 \sqrt{n}$ and $(\log_2 n)^2$ are of the same asymptotical order.

$$\lim_{x\to\infty}\frac{\log_2\sqrt{x}}{(\log_2 x)^2} = \text{should be } 1 \Rightarrow \lim_{x\to\infty}\frac{\frac{1}{2}\cdot\log_2 x}{(\log_2 x)^2} \Rightarrow \frac{1}{2}\lim_{x\to\infty}\frac{1}{\underbrace{\log_2 x}_{\infty}} \Rightarrow 0$$

so we can not say they are of the same asmptotical order.

since limit is constant

False

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2) $\underbrace{10^n > 2^n}_{1} \underbrace{> 8^{\log n}}_{2} \underbrace{= n^3 >}_{3} \underbrace{n^2 \log n}_{4} \underbrace{> n^2}_{5} \underbrace{> \sqrt{n}}_{6} \underbrace{> \log n}_{7}$

1) $10^n > 2^n$

$$\lim_{x\to\infty} \frac{2^x}{10^x} = \lim_{x\to\infty} \frac{1}{5^x} = \frac{1}{\infty}$$

$= 0;\ f(n) \in o(g(n))$

which provides $10^n > 2^n$

2) $2^n > 8^{\log n}$

$$\lim_{x\to\infty} \frac{8^{\log x} \to x^3}{2^x} = \lim_{x\to\infty} \frac{x^3}{2^x} \overset{\text{L'hopitals}}{=} \lim_{x\to\infty} \frac{3x^2}{2^x \ln(2)} \Rightarrow$$

$$\lim_{x\to\infty} \left(\frac{6x}{\ln^2(2) \cdot 2^x}\right) \Rightarrow \lim_{x\to\infty} \left(\frac{6}{\ln^3 2 \cdot 2^x}\right) =$$

$$\frac{6}{\ln^3(2)} \cdot \underbrace{\lim_{x\to\infty} \left(\frac{1}{2^x}\right)}_{0} = 0;\ f(n) \in o(g(n))$$

which provides $2^n > 8^{\log n}$

3) $8^{\log n} = n^3$ property of logaritmic expression

$n^{(\log_2 8)\,3} = n^3$

4) $n^3 > n^2 \log n$

$$\lim_{x\to\infty} \frac{x^2 \log_2 x}{x^3} \Rightarrow \lim_{x\to\infty} \frac{\log_2 x}{x} \overset{\text{l hop..}}{=} \lim_{x\to\infty} \left(\frac{\frac{1}{x \ln 2}}{1}\right) \Rightarrow$$

$$\frac{1}{\ln 2} \cdot \underbrace{\lim_{x\to\infty} \frac{1}{x}}_{0} = 0;\ f(n) \in o(g(n))$$

which proved $n^3 > n^2 \log n$

5) $n^2 \log n > n^2$

$$\lim_{x\to\infty} \frac{n^2}{n^2 \log n} \Rightarrow \lim_{x\to\infty} \frac{1}{\log n} = 0$$

$0;\ f(n) \in o(g(n))$ proves $n^2 \log n > n^2$

6) $n^2 > \sqrt{n}$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{x^2} \Rightarrow 0;\ f(n) \in o(g(n))$$

which proves $n^2 > \sqrt{n}$

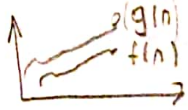
7) $\sqrt{x} > \log n$

$$\lim_{x\to\infty} \frac{\log_2 x}{\sqrt{x}} \Rightarrow \lim_{x\to\infty} \left(\frac{\frac{1}{x \ln 2}}{\frac{1}{2\sqrt{x}}}\right) \Rightarrow$$

$$\lim_{x\to\infty} \left(\frac{2}{\ln 2 \sqrt{x}}\right) = \frac{2}{\ln 2} \cdot \underbrace{\lim_{x\to\infty} \frac{1}{\sqrt{x}}}_{0} = 0$$

$0;\ f(n) \in o(g(n))$

PS: I assume bigger growth rate functions as $g(n)$ and smaller ones $f(n)$ so if we $\lim_{x\to\infty} \frac{f(n)}{g(n)} = 0$ it provides $f(n) \in o(g(n))$



3)

a)

```
void f(int my-array[]){
        for(int i=0; i<sizeofArray; i++){
            if(my-array[i]<first element){
                    second_element=first-element;
                    first-element= my-array[i];
            }
            else if(my-array[i]<second_element){
                    if(my-array[i]!=first_element){
                            Second_element= my_array[i];
                    }
            }
        }
}
```

Each time when it enters to the loop it will repeat count of input size (sizeofArray), since there isnot any parameter to change behaviour of loop we can say it will run with $\theta(n)$ complexity because for worst, best, it runs for same complexity

b)

```
void f(int n){
        int count=0;
        for(int i=2; i<=n; i++){
            if(i%2==0){
                    count++;
            }
            else{
                    i=(i-1)i
            }
        }
}
```

→ We can say that loop increases $i^2 \cdot i$ so its considered as $O(\log(\log n))$ from given explanation below:

$2, 2^k, (2)^{kk}, 2^{kkk} = 2^{k^2} \ldots 2^{k^{\log_2 k(\log(n))}}$ until the n it repeats so we can say

$n = 2^{k \log k(\log n))}$

$2^{\log_2(n)} = 2^{k \log k(\log(n))}$ so we can say that loop repeats $\log k(\log(n))$ times thus the complexity will be $O(\log(\log n))$

4) a) $\sum_{i=1}^{n} i^2 \log i$

$\downarrow$ nondecreasing ✓

$$\int_0^n x^2 \log x \, dx \leq f(n) \leq \int_1^{n+1} x^2 \log x \, dx$$

$$\frac{1}{3}x^3\log_2(x) - \underbrace{\int \frac{x^2}{3\ln 2} dx} \leq f(n) \leq \frac{1}{3}x^3\log_2 x - \int \frac{x^2}{3\ln 2} dx$$

$$\frac{1}{3}x^3\log_2 x - \frac{x^3}{9\ln 2} + c \Big|_0^n \leq f(n) \leq \frac{1}{3}x^3\log_2 x - \frac{x^3}{9\ln 2} + c \Big|_1^{n+1}$$

$$\underbrace{\frac{1}{3}n^3\log_2 n - \frac{n^3}{9\ln 2}}_{n^3\log n} \leq f(n) \leq \underbrace{\frac{1}{3}(n+1)^3\log_2(n+1) - \frac{(n+1)^3}{9\ln 2}}_{n^3\log n} - \left(0 - \frac{1}{9\ln 2}\right)$$

$$\leq f(n) \leq n^3\log n$$

$$= \Theta(n^3\log n)$$

b) $\sum_{i=1}^{n} i^3$

$\downarrow$ non-decreasing ✓

$$\int_0^n x^3 dx \leq f(n) \leq \int_1^{n+1} x^3 dx$$

$$\frac{x^4}{4}\Big|_0^n \leq f(n) \leq \Big|_1^{n+1} \frac{x^4}{4}$$

$$\overbrace{\frac{n^4}{4}}^{n^4} \leq f(n) \leq \overbrace{\frac{(n+1)^4}{4}}^{n^4} - \frac{1}{4}$$

$$n^4 \leq f(n) \leq n^4 \quad \Theta(n^4)$$

c) $\sum_{i=1}^{n} 1/(2\sqrt{i})$

$\downarrow$ nondecreasing No! / non-increasing ✓

$$\int_1^{n+1} x \, dx \leq f(n) \leq \int_0^n x \, dx$$

$$\int_1^{n+1} \frac{1}{2\sqrt{x}} dx \leq f(n) \leq \int_0^n \frac{1}{2\sqrt{x}} dx$$

$$\sqrt{x} + c \Big|_1^{n+1} \leq f(n) \leq \sqrt{x} + c \Big|_0^n$$

$$\underbrace{\sqrt{n+1}}_{\sqrt{n}} \leq f(n) \leq \underbrace{\sqrt{n}}_{\sqrt{n}} = \Theta(\sqrt{n})$$

d) $\sum_{i=1}^{n} 1/i$

$\downarrow$ non-decreasing No! / non-increasing ✓

$$\int_1^{n+1} \frac{1}{x} dx \leq f(n) \leq \int_0^n \frac{1}{x} dx$$

$$\ln x \Big|_1^{n+1} \leq f(n) \leq \ln x \Big|_0^n$$

$$\underbrace{\ln(n+1) - \ln 1} \leq f(n) \leq \ln n - \ln 0$$

$$\infty$$

So this integration method works for finding a lower bound but not for upper bound

$$1 + \sum_{i=1}^{n} 1/i \Rightarrow f(n) \leq 1 + \int_1^n g(x) \, dx$$

$$f(n) \leq 1 + \ln(x) \Big|_1^n$$

$$f(n) \leq \underline{1 + \ln(x) + 1} \quad \underline{\Theta(\ln n)}$$

$$f(n) \leq \ln(x) + 1$$

$$\ln(n+1) \leq f(n) \leq \ln(n) + 1$$

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5)

```
function Linear search (L[1:n], x)
    for i=1 to n do
        if (L[i]= x) then   }
            return(i)       }  comparison
        end if
    end for
    return ∅
end
```

The linear search algorithm starts beginning of the list and scan it until finding the element

Bestcase: bestcase occurs $B(n) = 1 \in O(1)$

Worst case: If x L[n] or x does not occur in the list $W(n) = n \in O(n)$

Average case: Assume that the prob of a successful search is p. Assume that x can equally likely be found in any position (uniform probability dist) (classical assumption)

If we say the elements are distinct then given explanation proves that

$$= \sum_{p=1}^{W(n)} i \cdot p_i \quad \rightarrow W(n) \text{ worstcase}$$

$$\left[\sum_{i=1}^{n-1} i \cdot \frac{p}{n}\right] + n \cdot \left(\frac{p}{n} + (1-p)\right)$$

$$= \frac{p}{n} \cdot \frac{n(n-1)}{2} + p + n(1-p) = \frac{p(n-1)}{2} + p + n - np$$

$$\frac{pn}{2} - \frac{p}{2} + p + n - np = \frac{p}{2} + n - \frac{pn}{2}$$

$$= \left(1 - \frac{p}{2}\right)n + \frac{p}{2} = n \in \Theta(n)$$

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HW#1

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