Game Theory Assignment 1
Fictitious Play

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Experiment Environment

The experiment environment is based on

- Windows 10 Education 22H2
- Python 3.12 with package
    - Numpy 1.26.0

Source Code Overview

Setup

The *FictitiousPlay* object will be constructed with *gameType* (which game does the user want to simulate by fictitious play model), *iterTime* (the number of game round), *numPrev* (the sum of prior belief, means how much rounds does two players already played before round 0), and *actionTh* (threshold for stopping the iteration if two players keep choosing the same best response).

```
class FictitiousPlay():
  def __init__(self, gameType, iterTime, numStrategy = 2, numPrev = 1000):
    self.gameType = gameType
    self.iterTime = iterTime
    # fictitious play is designed for games with 2 player
    self.numPlayer = 2
    # the threshold of repeated action
    # if two player repeat the same action for more than actionTh times, stop the iteration
    self.actionTh = 3000
    self.numStrategy = numStrategy
    self.numPrev = numPrev
    self.payoffMat = np.zeros([self.numPlayer * self.numStrategy, self.numStrategy])
    self.beliefMat = np.zeros([self.numPlayer, self.numStrategy])
    self.actionSet = np.zeros([self.numPlayer, iterTime + 1], dtype = np.uint8)
    self.payoffSet = np.zeros([self.numPlayer, iterTime, self.numStrategy])
```

After knowing the game type, *game_selection* function will setup the payoff matrix.

```
def game_selection(self):
  # set the payoff matrix of each player
  if self.gameType == "Q1":
      self.payoffMat[:self.numStrategy, :] = [[-1, 1], [0, 3]]
      self.payoffMat[self.numStrategy:, :] = [[-1, 1], [0, 3]]
```

The initial belief can be defined by programmer or assigned randomly.

```
def init_belief(self):
  # setup the prior belief of each player
  if self.numPrev == -1:
    # customize the initial belief
    self.beliefMat = np.array([[500, 500], [500, 500]])
  else:
    playTime = random.uniform(0, self.numPrev)
    self.beliefMat[0, :] = [playTime, (self.numPrev - playTime)]
    playTime = random.uniform(0, self.numPrev)
    self.beliefMat[1, :] = [playTime, (self.numPrev - playTime)]
```

The *payoff_cal* function will calculate the payoff by player's belief and payoff matrix.

```
def payoff_cal(self, belief, payoffMat):
  # calculate the payoff of player by belief
  payoff = np.zeros([self.numStrategy])
  # beliefNor = belief / np.sum(belief)
  beliefNor = belief
  for idx in range(self.numStrategy):
      payoff[idx] = np.sum(np.multiply(beliefNor, payoffMat[idx]))
  return payoff
```

For each iteration in *play_loop* function, the payoff of each player will be calculated to find the best response (randomly choose if the payoffs are same for each strategy), and the belief will be updated according to the other player's strategy. The iteration may be stopped due to the repeated selection of the same best response for two players.

```
def play_loop(self):
  pre_action1 = -1; pre_action2 = -1
  for iter in range(self.iterTime):
    #  calculate the payoff to find the best response
    self.payoffSet[0, iter, :] = self.payoff_cal(self.beliefMat[0,
 :], self.payoffMat[:2, :])
    self.payoffSet[1, iter, :] = self.payoff_cal(self.beliefMat[1,
 :], self.payoffMat[2:, :])

    self.logger(iter, self.actionSet[:, iter], self.beliefMat,
 self.payoffSet[:, iter, :])

    # same payoff for each strategy --> choose randomly
    if self.payoffSet[0, iter, 0] == self.payoffSet[0, iter, 1]:
      action1 = randint(0, 1)
    # different payoff--> choose best response
    else:
      action1 = int(np.argmax(self.payoffSet[0, iter, :]))
    self.actionSet[0, iter + 1] = action1
    if self.payoffSet[1, iter, 0] == self.payoffSet[1, iter, 1]:
      action2 = randint(0, 1)
    else:
      action2 = int(np.argmax(self.payoffSet[1, iter, :]))
    self.actionSet[1, iter + 1] = action2

    # update the other player's belief
    self.beliefMat[1, self.actionSet[0, iter + 1]] += 1
    self.beliefMat[0, self.actionSet[1, iter + 1]] += 1

    if action1 == pre_action1 and action2 == pre_action2:
      self.actionTh -= 1
      if self.actionTh == 0:
        # early stop with repeated best response
        self.logger(iter, self.actionSet[:, iter], self.beliefMat,
                    self.payoffSet[:, iter, :], last = True)
        break
    pre_action1, pre_action2 = action1, action2
```

Questions

Q1. One pure-strategy Nash Equilibrium

The following block shows the result of Q1 by fictitious play, for each iteration the program will print out the action, belief and payoff of each player. When the game end (or converge), the strategy distribution will be shown, which can be use to find the mixed or pure strategy of the game (**the strategy distribution only considers the actions after round 0, which means the prior belief is ignored**).

```
                          Round 0
belief of player1: [264.93 735.07], player2: [231.05 768.95]
payoff of player1: [ 470.15 2205.22], player2: [ 537.9  2306.86]
==================================================================
                          Round 1
action of player1: 1, palyer2: 1
belief of player1: [264.93 736.07], player2: [231.05 769.95]
payoff of player1: [ 471.15 2208.22], player2: [ 538.9  2309.86]
==================================================================
.
.
==================================================================
                          Round 3000
action of player1: 1, palyer2: 1
belief of player1: [ 264.93 3736.07], player2: [ 231.05 3769.95]
payoff of player1: [ 3470.15 11205.22], player2: [ 3537.9 11306.86]
==================================================================
strategy distribution of player1: [0. 1.], player2: [0. 1.]
```

The pure-strategy NE (r_2, c_2) can be found by fictitious paly. For player1, the best response will always be r_2 not matter what strategy player2 chooses, and vice versa.

Q2. Two or more pure-strategy NE

```
                          Round 0
belief of player1: [637.08 362.92], player2: [551.77 448.23]
payoff of player1: [1637.08 1088.77], player2: [1551.77 1344.7 ]
==================================================================
                          Round 1
action of player1: 0, palyer2: 0
belief of player1: [638.08 362.92], player2: [552.77 448.23]
payoff of player1: [1639.08 1088.77], player2: [1553.77 1344.7 ]
==================================================================
.
.
==================================================================
                          Round 3000
action of player1: 0, palyer2: 0
belief of player1: [3638.08  362.92], player2: [3552.77  448.23]
payoff of player1: [7637.08 1088.77], player2: [7551.77 1344.7 ]
==================================================================
strategy distribution of player1: [1. 0.], player2: [1. 0.]
```

```
                         Round 0
belief of player1: [297.91 702.09], player2: [487.34 512.66]
payoff of player1: [1297.91 2106.28], player2: [1487.34 1537.97]
==================================================================
                         Round 1
action of player1: 1, palyer2: 1
belief of player1: [297.91 703.09], player2: [487.34 513.66]
payoff of player1: [1298.91 2109.28], player2: [1488.34 1540.97]
==================================================================
.
.
==================================================================
                         Round 3000
action of player1: 1, palyer2: 1
belief of player1: [ 297.91 3703.09], player2: [ 487.34 3513.66]
payoff of player1: [ 4297.91 11106.28], player2: [ 4487.34
  10537.97]
==================================================================
strategy distribution of player1: [0. 1.], player2: [0. 1.]
```

The pure-strategy NE (r_1, c_1) and (r_2, c_2) can be found by fictitious paly. No matter what the initial belief is, at some point the game will lead the belief of both players to $(x, y), x = y$, and the result will have two situation:

1. Once the belief of both player are $(x, y), x > y$, player1 will keep choosing r_1 rather than r_2, and player2 will keep choosing c_1 rather than c_2, which result in pure-strategy NE (r_1, c_1).
2. Once the belief of both player are $(x, y), x < y$, player1 will keep choosing r_2 rather than r_1, and player2 will keep choosing c_2 rather than c_1, which result in pure-strategy NE (r_2, c_2).

Q3: Two or more pure-strategy NE (Conti.)

```
                          Round 0
belief of player1: [179.43 820.57], player2: [741.07 258.93]
payoff of player1: [179.43   0.  ], player2: [741.07   0.  ]
==================================================================
                          Round 1
action of player1: 0, palyer2: 0
belief of player1: [180.43 820.57], player2: [742.07 258.93]
payoff of player1: [180.43   0.  ], player2: [742.07   0.  ]
==================================================================
.
.
==================================================================
                          Round 3000
action of player1: 0, palyer2: 0
belief of player1: [3180.43  820.57], player2: [3742.07  258.93]
payoff of player1: [3179.43    0.  ], player2: [3741.07    0.  ]
==================================================================
strategy distribution of player1: [1. 0.], player2: [1. 0.]
```

Only one pure-strategy NE (r_1, c_1) can be found by fictitious paly. Defining $(b_{c1}, b_{c2}), b_{c1} > 0, b_{c2} > 0$ as player1's initial belief, $(b_{r1}, b_{r2}), b_{r1} > 0, b_{r2} > 0$ as player2's initial belief, the payoff of r_1 (c_1) will always be larger than r_2 (c_2), so the result will only converge to NE (r_1, c_1).

Q4: Mixed-Strategy Nash Equilibrium

```
                          Round 0
belief of player1: [336.21 663.79], player2: [664.74 335.26]
payoff of player1: [1327.58  672.42], player2: [ 664.74 1341.03]
====================================================================
                          Round 1
action of player1: 0, palyer2: 1
belief of player1: [336.21 664.79], player2: [665.74 335.26]
payoff of player1: [1329.58  672.42], player2: [ 665.74 1341.03]
====================================================================
.
.
====================================================================
                          Round 4999
action of player1: 0, palyer2: 1
belief of player1: [2864.21 3134.79], player2: [4647.74 1351.26]
payoff of player1: [6269.58 5728.42], player2: [4647.74 5405.03]
====================================================================
strategy distribution of player1: [0.8 0.2], player2: [0.51 0.49]
```

The mixed-strategy NE $P(r_1) = \frac{4}{5}, P(r_2) = \frac{1}{5}, P(c_1) = \frac{1}{2}, P(c_2) = \frac{1}{2}$ can be found by fictitious paly. The best response of two players can be analysis by four situation:

1. belief $b_{c1} > b_{c2}, b_{r1} > b_{r2}$, **player1 will choose r_2 as best response**, player2 will choose c_1, which lead to situation 2
2. belief $b_{c1} > b_{c2}, b_{r1} < b_{r2}$, player1 will choose r_2 as best response, **player2 will choose** c_2, which lead to situation 3
3. belief $b_{c1} < b_{c2}, b_{r1} < b_{r2}$, **player1 will choose r_1 as best response**, player2 will choose c_2, which lead to situation 4
4. belief $b_{c1} < b_{c2}, b_{r1} > b_{r2}$, player1 will choose r_1 as best response, **player2 will choose** c_1, which lead to back to situation 1

With large number of iteration and the best response cycle mentioned above, the mixed-strategy NE can be found by fictitious play.

Q5: Best-reply path

```
                          Round 0
belief of player1: [846.93 153.07], player2: [768.67 231.33]
payoff of player1: [153.07 846.93], player2: [768.67 231.33]
====================================================================
                          Round 1
action of player1: 1, palyer2: 0
belief of player1: [847.93 153.07], player2: [768.67 232.33]
payoff of player1: [153.07 847.93], player2: [768.67 232.33]
====================================================================
.
.
====================================================================
                          Round 4999
action of player1: 1, palyer2: 0
belief of player1: [3380.93 2618.07], player2: [3235.67 2763.33]
payoff of player1: [2618.07 3380.93], player2: [3235.67 2763.33]
====================================================================
strategy distribution of player1: [0.49 0.51], player2: [0.51 0.49]
```

The mixed-strategy NE $P(r_1) = \frac{1}{2}, P(r_2) = \frac{1}{2}, P(c_1) = \frac{1}{2}, P(c_2) = \frac{1}{2}$ can be found by fictitious paly. The best response of two player can be analysis by four situation:

1. belief $b_{c1} > b_{c2}, b_{r1} > b_{r2}$, **player1 will choose r_2 as best response**, player2 will choose c_1, which lead to situation 2
2. belief $b_{c1} > b_{c2}, b_{r1} < b_{r2}$, player1 will choose r_2 as best response, **player2 will choose c_2**, which lead to situation 3
3. belief $b_{c1} < b_{c2}, b_{r1} < b_{r2}$, **player1 will choose r_1 as best response**, player2 will choose c_2, which lead to situation 4
4. belief $b_{c1} < b_{c2}, b_{r1} > b_{r2}$, player1 will choose r_1 as best response, **player2 will choose c_1**, which lead to back to situation 1

With large number of iteration and the best response cycle mentioned above, the mixed-strategy NE can be found by fictitious play.

Q6: Pure-Coordination Game

```
                           Round 0
belief of player1: [859.82 140.18], player2: [814.87 185.13]
payoff of player1: [8598.24 1401.76], player2: [8148.69 1851.31]
=================================================================
                           Round 1
action of player1: 0, palyer2: 0
belief of player1: [860.82 140.18], player2: [815.87 185.13]
payoff of player1: [8608.24 1401.76], player2: [8158.69 1851.31]
=================================================================
.
.
=================================================================
                           Round 4999
action of player1: 0, palyer2: 0
belief of player1: [5858.82  140.18], player2: [5813.87  185.13]
payoff of player1: [58588.24  1401.76], player2: [58138.69
  1851.31]
=================================================================
strategy distribution of player1: [1. 0.], player2: [1. 0.]
```

```
                           Round 0
belief of player1: [224.77 775.23], player2: [ 57.43 942.57]
payoff of player1: [2247.67 7752.33], player2: [ 574.34 9425.66]
=================================================================
                           Round 1
action of player1: 1, palyer2: 1
belief of player1: [224.77 776.23], player2: [ 57.43 943.57]
payoff of player1: [2247.67 7762.33], player2: [ 574.34 9435.66]
=================================================================
.
.
=================================================================
                           Round 4999
action of player1: 1, palyer2: 1
belief of player1: [ 224.77 5774.23], player2: [  57.43 5941.57]
payoff of player1: [ 2247.67 57742.33], player2: [  574.34
  59415.66]
=================================================================
strategy distribution of player1: [0. 1.], player2: [0. 1.]
```

If the difference of initial payoff is larger than 10 ($|payoff_{r1} - payoff_{r2}| > 10$ and $|payoff_{c1} - payoff_{c2}| > 10$), the pure-strategy NE (r_1, c_1) and (r_2, c_2) can be found by fictitious paly. The result will have two situation:

1. Once the belief of both players are $(x,y), x > y$, player1 will keep choosing r_1 rather than r_2, and player2 will keep choosing c_1 rather than c_2, which result in pure-strategy NE (r_1, c_1).

2. Once the belief of both players are $(x,y), x < y$, player1 will keep choosing r_2 rather than r_1, and player2 will keep choosing c_2 rather than c_1, which result in pure-strategy NE (r_2, c_2).

```
                         Round 0
belief of player1: [500.01 499.99], player2: [499.96 500.04]
payoff of player1: [5000.1 4999.9], player2: [4999.6 5000.4]
===================================================================
                         Round 1
action of player1: 0, palyer2: 1
belief of player1: [500.01 500.99], player2: [500.96 500.04]
payoff of player1: [5000.1 5009.9], player2: [5009.6 5000.4]
===================================================================
.
.
===================================================================
                         Round 4999
action of player1: 0, palyer2: 1
belief of player1: [2999.01 2999.99], player2: [2999.96 2999.04]
payoff of player1: [29990.1 29999.9], player2: [29999.6 29990.4]
===================================================================
strategy distribution of player1: [0.5 0.5], player2: [0.5 0.5]
```

If the difference of payoff is always lower than 10 ($|payoff_{r1} - payoff_{r2}| < 10$ and $|payoff_{c1} - payoff_{c2}| < 10$) during the iteration and strategy distribution of two player is $b_{r1} > b_{r2}, b_{c1} < b_{c2}$ or $b_{r1} < b_{r2}, b_{c1} > b_{c2}$, the mixed-strategy NE $P(r_1) = \frac{1}{2}, P(r_2) = \frac{1}{2}, P(c_1) = \frac{1}{2}, P(c_2) = \frac{1}{2}$ can be found by fictitious paly. Since the best response of each iteration will be led to the cycle between (r_1, c_2) and (r_2, c_1).

Q7: Anti-Coordination game

```
                         Round 0
belief of player1: [179.25 820.75], player2: [559.01 440.99]
payoff of player1: [820.75 179.25], player2: [440.99 559.01]
===================================================================
                         Round 1
action of player1: 0, palyer2: 1
belief of player1: [179.25 821.75], player2: [560.01 440.99]
payoff of player1: [821.75 179.25], player2: [440.99 560.01]
===================================================================
.
.
===================================================================
                         Round 4999
action of player1: 0, palyer2: 1
belief of player1: [ 179.25 5819.75], player2: [5558.01  440.99]
payoff of player1: [5819.75  179.25], player2: [ 440.99 5558.01]
===================================================================
strategy distribution of player1: [1. 0.], player2: [0. 1.]
```

```
                          Round 0
belief of player1: [963.11  36.89], player2: [165.47 834.53]
payoff of player1: [ 36.89 963.11], player2: [834.53 165.47]
=================================================================
                          Round 1
action of player1: 1, palyer2: 0
belief of player1: [964.11  36.89], player2: [165.47 835.53]
payoff of player1: [ 36.89 964.11], player2: [835.53 165.47]
=================================================================
.
.
=================================================================
                          Round 4999
action of player1: 1, palyer2: 0
belief of player1: [5962.11   36.89], player2: [ 165.47 5833.53]
payoff of player1: [  36.89 5962.11], player2: [5833.53  165.47]
=================================================================
strategy distribution of player1: [0. 1.], player2: [1. 0.]
```

If the difference of initial payoff is larger than 1 ($|payoff_{r1} - payoff_{r2}| > 1$ and $|payoff_{c1} - payoff_{c2}| > 1$) The pure-strategy NE (r_1, c_2) and (r_2, c_1) can be found by fictitious paly. The result will have two situation:

1. Once the belief $b_{r1} > b_{r2}, b_{c1} < b_{c2}$, player1 will keep choosing r_1 rather than r_2, and player2 will keep choosing c_2 rather than c_1, which result in pure-strategy NE (r_1, c_2).
2. Once the belief $b_{r1} < b_{r2}, b_{c1} > b_{c2}$, player1 will keep choosing r_2 rather than r_1, and player2 will keep choosing c_1 rather than c_2, which result in pure-strategy NE (r_2, c_1).

```
                          Round 0
belief of player1: [500.01 499.99], player2: [500.01 499.99]
payoff of player1: [499.99 500.01], player2: [499.99 500.01]
=================================================================
                          Round 1
action of player1: 1, palyer2: 1
belief of player1: [500.01 500.99], player2: [500.01 500.99]
payoff of player1: [500.99 500.01], player2: [500.99 500.01]
=================================================================
.
.
=================================================================
                          Round 4999
action of player1: 1, palyer2: 1
belief of player1: [2999.01 2999.99], player2: [2999.01 2999.99]
payoff of player1: [2999.99 2999.01], player2: [2999.99 2999.01]
=================================================================
strategy distribution of player1: [0.5 0.5], player2: [0.5 0.5]
```

If the difference of initial payoff is always lower than 1 ($|payoff_{r1} - payoff_{r2}| < 1$ and $|payoff_{c1} - payoff_{c2}| < 1$) during iteration and strategy distribution of two player is $b_{r1} > b_{r2}, b_{c1} > b_{c2}$ or $b_{r1} < b_{r2}, b_{c1} < b_{c2}$, the mixed-strategy NE $P(r_1) = \frac{1}{2}, P(r_2) = \frac{1}{2}, P(c_1) = \frac{1}{2}, P(c_2) = \frac{1}{2}$ can be found by fictitious paly. Since the best response of each iteration will be lead to the cycle between (r_1, c_1) and (r_2, c_2).

Q8: Battle of the Sexes

```
                        Round 0
belief of player1: [505.52 494.48], player2: [887.14 112.86]
payoff of player1: [1516.57  988.96], player2: [1774.27  338.59]
=================================================================
                        Round 1
action of player1: 0, palyer2: 0
belief of player1: [506.52 494.48], player2: [888.14 112.86]
payoff of player1: [1519.57  988.96], player2: [1776.27  338.59]
=================================================================
.
.
=================================================================
                        Round 4999
action of player1: 0, palyer2: 0
belief of player1: [5504.52  494.48], player2: [5886.14  112.86]
payoff of player1: [16513.57   988.96], player2: [11772.27  338.59]
=================================================================
strategy distribution of player1: [1. 0.], player2: [1. 0.]
```

```
                        Round 0
belief of player1: [  9.52 990.48], player2: [ 53.65 946.35]
payoff of player1: [  28.57 1980.95], player2: [ 107.29 2839.06]
=================================================================
                        Round 1
action of player1: 1, palyer2: 1
belief of player1: [  9.52 991.48], player2: [ 53.65 947.35]
payoff of player1: [  28.57 1982.95], player2: [ 107.29 2842.06]
=================================================================
.
.
=================================================================
                        Round 4999
action of player1: 1, palyer2: 1
belief of player1: [   9.52 5989.48], player2: [  53.65 5945.35]
payoff of player1: [   28.57 11978.95], player2: [   107.29  17836.06]
=================================================================
strategy distribution of player1: [0. 1.], player2: [0. 1.]
```

If the difference of initial payoff is larger than 2 ($|payoff_{r1} - payoff_{r2}| > 1$ and $|payoff_{c1} - payoff_{c2}| > 1$) The pure-strategy NE (r_1, c_1) and (r_2, c_2) can be found by fictitious paly. The result will have two situation:

1. Once the belief $b_{r1} > b_{r2}, b_{c1} > b_{c2}$, player1 will keep choosing r_1 rather than r_2, and player2 will keep choosing c_1 rather than c_2, which result in pure-strategy NE (r_1, c_1).

2. Once the belief $b_{r1} < b_{r2}, b_{c1} < b_{c2}$, player1 will keep choosing r_2 rather than r_1, and player2 will keep choosing c_2 rather than c_1, which result in pure-strategy NE (r_2, c_2).

```
                    Round 0
belief of player1: [399.99 600.01], player2: [600.01 399.99]
payoff of player1: [1199.97 1200.02], player2: [1200.02 1199.97]
==================================================================
                    Round 1
action of player1: 1, palyer2: 0
belief of player1: [400.99 600.01], player2: [600.01 400.99]
payoff of player1: [1202.97 1200.02], player2: [1200.02 1202.97]
==================================================================
                    Round 4999
action of player1: 1, palyer2: 0
belief of player1: [2399.99 3599.01], player2: [3599.01 2399.99]
payoff of player1: [7199.97 7198.02], player2: [7198.02 7199.97]
==================================================================
strategy distribution of player1: [0.6 0.4], player2: [0.4 0.6]
```

If the difference of initial payoff is lower than 2 ($|payoff_{r1} - payoff_{r2}| < 2$ and $|payoff_{c1} - payoff_{c2}| < 2$) during iteration and strategy distribution of two player is $b_{r1} > b_{r2}, b_{c1} < b_{c2}$ or $b_{r1} < b_{r2}, b_{c1} > b_{c2}$, the mixed-strategy NE $P(r_1) = \frac{3}{5}, P(r_2) = \frac{2}{5}, P(c_1) = \frac{2}{5}, P(c_2) = \frac{3}{5}$ can be found by fictitious paly. Since the best response of each iteration will be lead to the cycle between (r_1, c_2) and (r_2, c_1).

Q9: Stag Hunt Game

```
                      Round 0
belief of player1: [642.99 357.01], player2: [903.66  96.34]
payoff of player1: [1928.96 1642.99], player2: [2710.97 1903.66]
==================================================================
                      Round 1
action of player1: 0, palyer2: 0
belief of player1: [643.99 357.01], player2: [904.66  96.34]
payoff of player1: [1931.96 1644.99], player2: [2713.97 1905.66]
==================================================================
.
.
==================================================================
                      Round 4999
action of player1: 0, palyer2: 0
belief of player1: [5641.99  357.01], player2: [5902.66   96.34]
payoff of player1: [16925.96 11640.99], player2: [17707.97
 11901.66]
==================================================================
strategy distribution of player1: [1. 0.], player2: [1. 0.]
```

```
                      Round 0
belief of player1: [234.27 765.73], player2: [144.81 855.19]
payoff of player1: [ 702.8  1234.27], player2: [ 434.42 1144.81]
==================================================================
                      Round 1
action of player1: 1, palyer2: 1
belief of player1: [234.27 766.73], player2: [144.81 856.19]
payoff of player1: [ 702.8  1235.27], player2: [ 434.42 1145.81]
==================================================================
.
.
==================================================================
                      Round 4999
action of player1: 1, palyer2: 1
belief of player1: [ 234.27 5764.73], player2: [ 144.81 5854.19]
payoff of player1: [ 702.8  6233.27], player2: [ 434.42 6143.81]
==================================================================
strategy distribution of player1: [0. 1.], player2: [0. 1.]
```

If the difference of initial payoff is larger than 1 ($|payoff_{r1} - payoff_{r2}| > 1$ and $|payoff_{c1} - payoff_{c2}| > 1$) The pure-strategy NE (r_1, c_1) and (r_2, c_2) can be found by fictitious paly. The result will have two situation:

1. Once the belief $b_{r1} > b_{r2}, b_{c1} > b_{c2}$, player1 will keep choosing r_1 rather than r_2, and player2 will keep choosing c_1 rather than c_2, which result in pure-strategy NE (r_1, c_1).
2. Once the belief $b_{r1} < b_{r2}, b_{c1} < b_{c2}$, player1 will keep choosing r_2 rather than r_1, and player2 will keep choosing c_2 rather than c_1, which result in pure-strategy NE (r_2, c_2).

```
                     Round 0
belief of player1: [500.01 499.99], player2: [499.99 500.01]
payoff of player1: [1500.03 1500.01], player2: [1499.97 1499.99]
===================================================================
                     Round 1
action of player1: 0, palyer2: 1
belief of player1: [500.01 500.99], player2: [500.99 500.01]
payoff of player1: [1500.03 1501.01], player2: [1502.97 1501.99]
===================================================================
.
.
===================================================================
                     Round 4999
action of player1: 0, palyer2: 1
belief of player1: [2999.01 2999.99], player2: [2999.99 2999.01]
payoff of player1: [8997.03 8998.01], player2: [8999.97 8998.99]
===================================================================
strategy distribution of player1: [0.5 0.5], player2: [0.5 0.5]
```

If the difference of initial payoff is lower than 1 ($|payoff_{r1} - payoff_{r2}| < 1$ and $|payoff_{c1} - payoff_{c2}| < 1$) during iteration and strategy distribution of two player is $b_{r1} > b_{r2}, b_{c1} < b_{c2}$ or $b_{r1} < b_{r2}, b_{c1} > b_{c2}$, the mixed-strategy NE $P(r_1) = \frac{1}{2}, P(r_2) = \frac{1}{2}, P(c_1) = \frac{1}{2}, P(c_2) = \frac{1}{2}$ can be found by fictitious paly. Since the best response of each iteration will be lead to the cycle between (r_1, c_2) and (r_2, c_1).

Q10: Observation and Conclusion

1. Fictitious play may cannot find some pure-strategy NE. Take Q3's game matrix for example, fictitious play cannot find the NE (r_2, c_2) since the action of two player will always converge to (r_1, c_1).
2. Fictitious play may cannot find some mixed-strategy NE. If the user has no prior knowledge to setup the initial belief for specific mixed-strategy NE, the fictitious play may not converge to mixed-strategy NE.