Prayas JEE AIR 2025

WEEKLY TEST 01

DURATION : 180 Minutes | DATE : 14/07/2024 | M. MARKS : 300

ANSWER KEY

PHYSICS

1. (1)
2. (4)
3. (2)
4. (4)
5. (1)
6. (3)
7. (1)
8. (2)
9. (4)
10. (2)
11. (3)
12. (2)
13. (3)
14. (3)
15. (1)
16. (3)
17. (3)
18. (1)
19. (3)
20. (3)
21. (2)
22. (10)
23. (4)
24. (34)
25. (53)
26. (20)
27. (375)
28. (3)
29. (2)
30. (3)

CHEMISTRY

31. (3)
32. (2)
33. (1)
34. (2)
35. (1)
36. (4)
37. (2)
38. (3)
39. (2)
40. (3)
41. (1)
42. (3)
43. (4)
44. (3)
45. (1)
46. (4)
47. (2)
48. (3)
49. (2)
50. (4)
51. (33)
52. (25)
53. (43)
54. (21)
55. (31)
56. (144)
57. (34)
58. (16)
59. (535)
60. (20)

MATHEMATICS

61. (3)
62. (1)
63. (4)
64. (4)
65. (3)
66. (3)
67. (3)
68. (3)
69. (4)
70. (3)
71. (3)
72. (3)
73. (4)
74. (3)
75. (4)
76. (2)
77. (2)
78. (3)
79. (3)
80. (1)
81. (3)
82. (17)
83. (2)
84. (21)
85. (3)
86. (9)
87. (6)
88. (1)
89. (0)
90. (161)

SECTION-I (PHYSICS)

1. (1)

$$a_t = t \Rightarrow v = \frac{t^2}{2}$$

$$a_n = \frac{v^2}{R} = \frac{t^4}{4(2)} = \frac{t^4}{8}$$

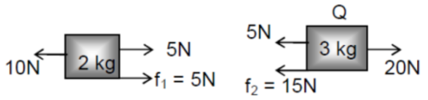
$$\because a_t = a_n$$

$$\Rightarrow t = \frac{t^4}{8}$$

$\therefore$ $t = 2$ second

2. (4)

$$\vec{f}_1 = 5N\hat{i}, \quad \vec{f}_2 = -15N\hat{i}, T = 5N$$



3. (2)

$$W_g + W_{sp} + W_{fr} = 0$$

$$\Rightarrow mgx_m - \frac{1}{2}kx_m^2 - \mu mgx_m = 0$$

$$\Rightarrow \frac{3mg}{4} = \frac{kx_m}{2}$$

Hence, $x_m = \dfrac{3mg}{2k}$

4. (4)

$$v_B = \sqrt{2\,gl\sin\theta}$$

$$v_C = \sqrt{2\,gl}$$

$$\because 2\sqrt{2\,gl\sin\theta} = \sqrt{2\,gl}$$

Hence, $\sin\theta = \dfrac{1}{4}$

5. (1)

Acceleration of each block is

$$a = \frac{4g - \mu(4g)}{4+4} = 4\,\text{m/s}^2$$

$$\therefore a_{CM} = 2\sqrt{2}\,\text{m/s}^2$$

6. (3)

Range $R = 12 + 4 = 16$ m

We know, $y = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

$$H = (12)\left(\frac{4}{3}\right)\left(1 - \frac{12}{16}\right)$$

= 4 m

7. (1)

$$\frac{1}{2}mv_0^2 = \frac{1}{2}k\left(\frac{l_0}{2}\right)^2 - \frac{1}{2}k\left(\frac{l_0}{4}\right)^2 \quad \Rightarrow k = \frac{16}{3}\frac{mv}{l_0^2}$$

8. (2)

$$t_0 = \frac{2u\sin\alpha}{g}$$

$$-H = D\tan\alpha - \frac{gD^2}{2\left(\dfrac{g^2t_0^2}{4\sin^2\alpha}\right)\cos^2\alpha}$$

$$= \frac{gt_0^2}{4D}\left[1 + \sqrt{1 + \frac{8H}{gt_0^2}}\right]$$

9. (4)

$$x = A(1 - e^{-\lambda t})$$

$$v = A\lambda e^{-\lambda t}$$

$$\frac{v}{A\lambda} + \frac{x}{A} = 1$$

10. (2)

$$v = \frac{dx}{dt} = \sqrt{x}$$

$$\Rightarrow \int_4^x \frac{dx}{\sqrt{x}} = \int_0^t dt$$

$$\Rightarrow x = \left(\frac{t}{2} + 2\right)^2$$

$$v^2 = x \text{ and } 2v\frac{dv}{dx} = 1$$

$$\Rightarrow a = \frac{vdv}{dx} = \frac{1}{2}$$

11. (3)

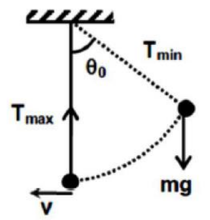
Using equation of Trajectory, we get

$$u = 15\sqrt{2}\ \text{m/s}$$

12. (2)

Using constraint : $a_A = 2a_B$ and writing equation of motion & solving we get the answer.

13. (3)



$T_{\min} = mg\cos\theta_0$ … (i)

$T_{\max} - mg = \dfrac{mv^2}{R}$ … (ii)

$\dfrac{1}{2}mv^2 = mg\left(R - R\cos\theta\right)\Big)$ … (iii)

$\dfrac{T_{\max}}{T_{\min}} = 4$

$\theta_0 = \cos^{-1}\left(\dfrac{1}{2}\right)$

14. **(3)**

As the vertical component of velocity of both the particles is same.

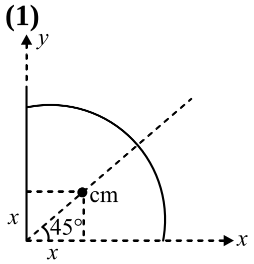
$\Rightarrow \vec{v}_{yO/O'} = 0$

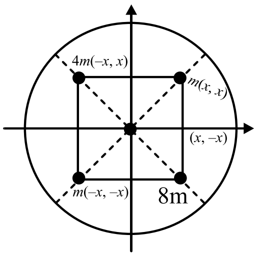
So the distance h between them will be constant with time and $\vec{v}_{xO/O'} = 2v\cos\alpha$

Total distance between the particles is minimum when horizontal distance between them is zero

$\Rightarrow t = \dfrac{d}{2v\cos\alpha}$

15. **(1)**





$x_{cm} = \dfrac{m(x) + 4m(-x) + 5m(-x) + 8m(x)}{m + 4m + 5m + 8m} = 0$

$y_{cm} = \dfrac{m(x) + 4m(x) + 5m(-x) + 8m(-x)}{18m}$

$= -\dfrac{8mx}{18m} = \dfrac{-4}{9}x = \dfrac{-4}{9} \times \left(\dfrac{4R}{3\pi}\right) = -\dfrac{16R}{27\pi}$

16. **(3)**

$A_r r_r = A_T r_T - A_{\text{cut}} r_{\text{cut}}$

$\left(4a^2 - \dfrac{\pi a^2}{2}\right) r_r = 4a^2 \times a - \dfrac{\pi a^2}{2}\dfrac{4a}{3\pi}$

$r_r = \left(0, \dfrac{20a}{3(8-\pi)}\right)$

17. **(3)**

$m_1 x_1 = x_2 x_2$

$\Rightarrow 10 \times 6 = 30 \times x_2$

$\Rightarrow x_2 = 2$ cm towards 10 kg block

18. **(1)**

$M_1 = \dfrac{4}{3}\pi R^3 \rho$

$M_2 = \dfrac{4}{3}\pi (1)^3 (-\rho)$

$X_{\text{com}} = \dfrac{M_1 X_1 + M_2 X_2}{M_1 + M_2}$

$\Rightarrow \dfrac{\left\lfloor \dfrac{4}{3}\pi R^3 \rho \right\rfloor 0 + \left\lfloor \dfrac{4}{3}\pi (1)^3 (-\rho) \right\rfloor [R-1]}{\dfrac{4}{3}\pi R^3 \rho + \dfrac{4}{3}\pi (1)^3 (-\rho)}$

$\Rightarrow \dfrac{(R-1)}{\left(R^3 - 1\right)} = (2 - R)$

$\dfrac{(R-1)}{(R-1)\left(R^2 + R + 1\right)} = 2 - R$

$\left(R^2 + R + 1\right)(2 - R) = 1$

19. **(3)**

Given $v = a_0 t$

$a_t = \dfrac{dv}{dt} = a_0$

Now, $v^2 = u^2 + 2a_t S$

$= 0 + 2a_0 \dfrac{\pi R}{2}$

$v^2 = \pi a_0 R$

So, $a_n = \dfrac{v^2}{R} = \pi a_0$

The angle between the velocity vector and the acceleration vector is

$\phi = \tan^{-1}\left(\dfrac{a_n}{a_t}\right) = \tan^{-1}\left(\dfrac{\pi a_0}{a_0}\right) = \tan^{-1}(\pi)$

20. **(3)**

$\dfrac{m}{2}g - T = \dfrac{m}{2}a$ … (i)

$T\cos 60^\circ = \dfrac{ma}{\cos 60^\circ}$ … (ii)

Solving (i) and (ii) acceleration of ring $= \dfrac{2g}{9}$

21. **(2)**

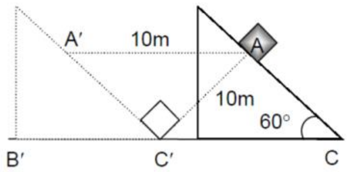
$\because y = \sin^{-1} x$ or $\sin y = x$

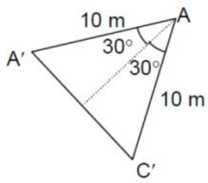
Or $\dfrac{d(\sin y)}{dx} = \dfrac{dx}{dx}$ or $\dfrac{d(\sin y)}{dy}\dfrac{dy}{dx} = 1$

Or $\cos y \dfrac{dy}{dx} = 1$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\cos y} = \dfrac{1}{\cos\dfrac{\pi}{3}} = \dfrac{1}{\dfrac{1}{2}} = 2$

22. **(10)**

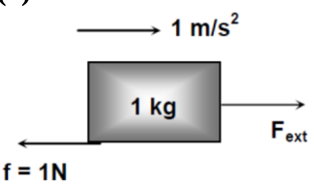




$AA' = AC' = 10$ m

$\Rightarrow A'C' = 20 \sin 30° = 10$ m

23. **(4)**



$F_{ext} = 2$ N

$v = (1)(2) = 2$ m/s

Hence, Power $P = Fv = 4$ watt

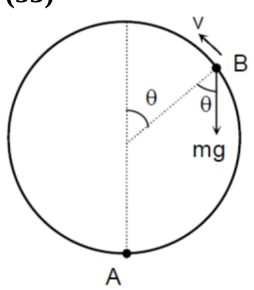
24. **(34)**

$$a_{CM} = 10 = \frac{2m(-2) + m(a)}{2m + m}$$

$$\Rightarrow a - 4 = 30$$

$$\Rightarrow a = 34 \text{ m/s}^2$$

25. **(53)**



When it leaves contact, $\text{mg}\cos\theta = \frac{\text{mv}}{\text{R}}$ … (i)

From conservation of mechanical energy at A and B,

$$\frac{1}{2}m\left(\sqrt{\frac{95}{25}Rg}\right)^2$$

$$= mgR(1+\cos\theta) + \frac{mv^2}{2} \quad \ldots \text{(i)}$$

From (i) and (ii)

$$\cos\theta = \frac{3}{5}$$

$$\Rightarrow \theta = 53^\circ$$

26. **(20)**

$W = \mu mgx = 20$ J

27. **(375)**

Centre of mass of the system stops moving after both the blocks stop. Left block stops at $t = 10$ s and the right block stops at $t = 5$ s

$$v_{cm} = \frac{(20)(2)-(10)(2)}{2+2} = 5 \text{ m/s}$$

Displacement of centre of mass in 5 sec $S_1 = 25$ m

Retardation of centre of mass between $t = 5$ to $t = 10$ s is $a_{CM} = -1 \text{ m/s}^2$ Further displacement of centre of mass before it stops

$$S_2 = (5)(5) - \frac{1}{2}(1)(5)^2 = 12.5 \text{ m}$$

Hence, total displacement of centre of mass of the system $= 37.50$ m $= 3750$ cm

28. **(3)**

For minimum value of $y(0.75\sin\theta + \cos\theta)$ should be maximum.

$$\therefore \frac{d}{d\theta}(0.75\sin\theta + \cos\theta) = 0$$

Or, $0.75\cos\theta - \sin\theta = 0$

Or, $\tan\theta = 0.75 = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$

and $\cos\theta = \frac{4}{5}$

$$\therefore y_{\min} = \frac{15}{4\left(0.75 \times \frac{3}{5} + \frac{4}{5}\right)}$$

$$= \frac{15}{4\left(\frac{9}{20} + \frac{4}{5}\right)} = \frac{15}{4\left(\frac{9+16}{20}\right)}$$

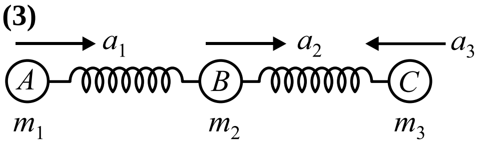
$$= \frac{15 \times 20}{4 \times 25} = 3$$

29. **(2)**

$$\frac{dy}{dt} = 18x\frac{dx}{dt} = 6x$$

$$a_y = \frac{d^2y}{dt^2} = 6\frac{dx}{dt} = 2 \text{ m/s}^2$$

30. **(3)**



The acceleration of center of mass of the system

$$a_{cm} = \frac{|m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3|}{m_1 + m_2 + m_3}$$

$\Rightarrow$ The net force acting on the system

$$= |m_1\vec{a}_1 + \vec{m}_2a_2 + m_3\vec{a}_3|$$

$$\Rightarrow F_{net} = (m_1a_1 + m_2a_2 - m_3a_3)$$

$$= \left|(1)(1) + (2)(2) - \frac{1}{2}(4)\right| \text{N} = 3 \text{ N.}$$

SECTION-II (CHEMISTRY)

31. (3)

Molarity = $\dfrac{(w/w)\times d\times 10}{\text{Molar mass of solute}}$

$=\dfrac{12.8\times 1.313\times 10}{40}$

$\because \dfrac{\text{mol of solute}}{\text{Vol}}=\dfrac{12.8\times 1.13\times 10}{40}$ vol = 1.38

32. (2)

Let the mass of CH_4 in the mixture is x gm.

$\therefore \left[\dfrac{x}{16}+\dfrac{10-x}{28}\times 2\right]44=29$

on solving x = 6.182 g

Now the amount of water formed

$=\left(\dfrac{6.182\times 2}{16}+\dfrac{3.818}{28}\times 2\right)\times 18=18.81g$

33. (1)

(a) In 100 mL (140 g) solution mass of solute 100 mL (140 g) = 70

$=\dfrac{70}{140}\times 46=23$ g

(b) $10M=\dfrac{\text{Mass of solute}/46}{\dfrac{50}{1g000}}$

Mass of solute = 23 g

(c) 100 g solution contain 25g of solute mass of solute $=\dfrac{25}{100}\times 50=12.5g$

(d) $5M=\dfrac{\text{Mass of solute}/46}{46/1000}$

Mass of solute = 10.58 g

34. (2)

$FeSO_4$

1 mole of SO_4^{2-} combines with 1 mole Fe^{2+}

In $Fe_2(SO_4)_3$

3 mole of SO_4^{2-} combines with = 2 mole Fe^{3+}

1 mole of SO_4^{2-} combines with = $\dfrac{2}{3}$ mole Fe^{3+}

Ratio = $\dfrac{Fe^{2+}}{Fe^{3+}}=\dfrac{1}{\frac{2}{3}}=\dfrac{3}{2}$

35. (1)

$A\rightarrow S$; $B\rightarrow Q$; $C\rightarrow R$; $D\rightarrow P$

36. (4)

$3CN^-+7NO_3^-+10H^+\rightarrow 3CO_2+10NO+5H_2O$

$\therefore$ a = 3 ; b = 7, c = 10

37. (2)

$I_2+Na_2S_2O_3\rightarrow I^-+S_4O_6^{2-}$

Let x mL of I_2 react with hypo

Meq of I_2 = meq of Hypo

$xN = 15\times 0.4 \Rightarrow xN = 6$ …. (i)

Meq of H_2SO_4 used by base = 10 × 0.3 × 2 = 6

Meq of NaOH used by I_2 = (30 – 6)

(150 – x) N = 24 …. (ii)

$\dfrac{150-x}{x}=4\Rightarrow 5x=150$

x = 30 mL

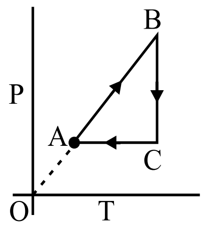
30N = 6

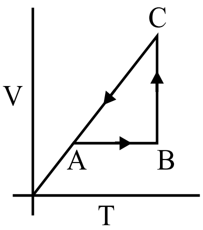
$N=\dfrac{1}{5}$ N = M × n – factor

$\dfrac{1}{5}=M\times 2$

$M=\dfrac{1}{10}=0.1$

38. (3)





AB = isochoric heating

BC = isothermal expansion

CA = isobaric cooling

39. (2)

Theory Based

40. (3)

Appling Hess's law,

ΔH = [3 (110.5) – 282.9 + 2 (–285.8) +3 (–74.8)]

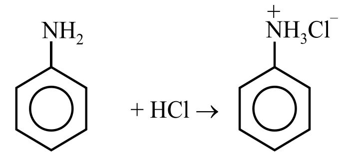
= – 747.4

41. (1)

$h=\sqrt{\dfrac{K_h}{C}}$ $h=\sqrt{\dfrac{K_w}{K_a\times C}}$

$h=\sqrt{\dfrac{10^{-14}\times 80}{1.3\times 10^{-9}\times 1}}$ $h\%=2.48\%$

42. (3)



At. Initial	5mmol	2.5mmol	
At. Final	2.5mmol	0	2.5mmol

$$8 = pK_a + \log\left[\frac{2.5}{2.5}\right] \Rightarrow pK_a = 8, pK_b = 6$$

$$\text{Now } pH = \frac{1}{2}(14 - 6 - \log(10^{-2}))$$

$$= \frac{1}{2}(10) = 5$$

43. (4)

$$K_{sp} = [Cd^{2+}][S^{-2}]$$

$$K_{a_1} \times K_{a_2} = \frac{[H^+]^2[S^{2-}]}{[H_2S]} \Rightarrow [S^{-2}] = 1.1 \times 10^{-21} M$$

$\therefore$ $6.33 \times 10^{-15} \times 6.33 \times 10^{-15} = [Cd^{2+}]\,[1.1 \times 10^{-21}]$

$$[Cd^{2+}] = 3.643 \times 10^{-8} M$$

44. (3)

As the container is rigid and closed, so volume will be constant and we know that addition of inert gas at constant volume does not affect the equilibrium

45. (1)

Pseudo unimolecular reactions are in presence of one or more reactants in excess. Usually such reactions are carried out in solvents, which are themselves one of the reactants.

46. (4)

$$\text{Rate} = -\frac{1}{2}\frac{d[N_2O_5]}{dt} = +\frac{1}{4}\frac{d[NO_2]}{dt}$$

$$= +\frac{d[O_2]}{dt}$$

$$\text{Here } \frac{d[NO_2]}{dt} = 0.0076 \text{ mole / lit / sec}$$

(a) Rate of appearance of O_2

$= \frac{1}{4} \times$ rate of appearance of NO_2

$$\frac{d[O_2]}{dt} = \frac{1}{4} \times \frac{d[NO_2]}{dt}$$

$$= \frac{1}{4} \times 0.0076$$

$$= 0.0019 \text{ mole / lit / sec}$$

(b) Rate of disappearance of N_2O_5

$= \frac{1}{2} \times$ rate of appearance of NO_2

$$-\frac{d[N_2O_5]}{dt} = \frac{1}{2} \times \frac{d[NO_2]}{dt}$$

$$\frac{d[N_2O_5]}{dt} = -\frac{1}{2} \times 0.0076$$

$$= -0.0038 \text{ mole / lit / sec}$$

(c) Rate of reaction $= \frac{1}{4} \times \frac{d[NO_2]}{dt}$

$$= \frac{1}{4} \times 0.0076 = 0.0019 \text{ mol / lit / sec}$$

47. (2)

$$i \times \frac{1.2}{58.5} \times 10 \times RT = \frac{7.2}{180} \times 10 \times RT$$

i = 1.95

48. (3)

Theory Based

49. (2)

$$X_A = 2/5 = 0.4, X_B = 0.6$$

$$P_A = X_A P_A^o$$

= 0.4 × 120 = 48 mm

$$P_B = X_B P_B^o$$

= 0.6 × 180 = 108 mm

So, total vapour pressure

= 48 + 108 = 156 mm.

50. (4)

$$\pi = \frac{w}{M} RT$$

$$M = \frac{0.082 \times 300 \times 50}{4.11} = 300$$

This is the apparent molecule weight.

$$i = \frac{M_{nor}}{M_{apparent}} = \frac{180}{300} = \frac{3}{5} = 0.6$$

$$\text{As } i = 1 + \alpha(1/n - 1)$$

here n = 2 α = degree of dissociation

So 0.6 = 1 + α (–1/2)

½ α = 0.4

α = 0.8 or 80%

51. (33)

$$M = \frac{(27/98)}{100/1.2} \times 1000 = 3.3$$

52. (25)

$$X_{ethanol} = \frac{46/46}{46/46 + 54/18} = 0.25$$

53. **(43)**

$KMnO_4 + X^{+n} \rightarrow XO_3^- + Mn^{+2}$

1.61×10^{-3} mole 2.63×10^{-3} mole

Eq. of $KMnO_4$ = Eq. of X^{+n}

$1.61 \times 10^{-3} \times 5 = 2.63 \times 10^{-3} \times (5 - n)$

$$n = 2 \Rightarrow 56 = \frac{M}{2} + 35.5 \quad M = 41$$

54. **(21)**

$$\Delta S = \frac{nC_PdT}{T}$$

$$= 1\times\int\left(\frac{25.5}{T} + 13.6\times10^{-3} - 42.5\times10^{-7}T\right)dT$$

$$= 2.303\times25.5\log 2 + 13.6\times10^{-3}\times300 - 42.5$$

$$\times10^{-7}\frac{[(600)^2 - (300)^2]}{2}$$

$\Delta S = 21.356 \text{ J K}^{-1} \text{ mol}^{-1}$

55. **(31)**

Process is reversible adiabatic

$T_1 = 298.15K \qquad V_2 = 2V_1$

$T_2 = 248.44 \text{ K} \qquad PV^{\gamma} = K \quad PV = nRT$

$$\frac{T}{V}V^{\gamma} = K,\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\left(\frac{298.15}{248.44}\right) = 2^{\gamma-1}$$

log (298.15) – log (248.44) = $(\gamma - 1)$ log 2

$$(\gamma - 1) = \frac{0.08}{0.3}$$

$$C_{V,m} = 8.3\times\frac{0.3}{0.08} \qquad C_{V,m} = 31.125$$

56. **(144)**

$2HI \rightleftharpoons H_2 + I_2$

1–0.8 0.4 0.4

$$K_C = \frac{0.4\times0.4}{(0.2)^2} = 4$$

Let x mol of H_2 & I_2 react

$$4 = \frac{(0.135 - x)^2}{(2x)^2}$$

On solving

x = 0.027

$I_2 + 2Na_2S_2O_3 \rightarrow 2NaI + Na_2S_4O_6$

(0.135 – x) 1.5 M

If V L of hypo is used

(0.135 – x) 2 = 1.5 V

$$V = \frac{(0.135 - 0.027)\times2}{1.5} = 0.144L = 144mL$$

57. **(34)**

Let equilibrium concentration of C be a M.

$$K_C = \frac{(4^2)(a)}{3} = \frac{16a}{3}\text{.....(i)}$$

On doubling volume, all concentrations are halved and equilibrium shifts forward

$A \rightleftharpoons 2B + C$

3/2–x 2+2x a/2+x

Given: 2x +2 = 3 ⇒ x = ½

$$K_C = \frac{(3)^2\left(\frac{a}{2}+\frac{1}{2}\right)}{\left(\frac{3}{2}-\frac{1}{2}\right)} \qquad \text{…. (ii)}$$

From (1) & (2),

32a = 27a + 27

5a = 27

a = 5.4

$$K_C = \frac{9(3.2)}{(1)} = 28.8$$

58. **(16)**

$$t = \frac{2.303}{k}\log_{10}\frac{a}{a-x}$$

If $t = t/2, x = a/2$

$$t_{1/2} = \frac{2.303}{k}\log_{10}\frac{a}{a - a/2} \qquad \text{...(i)}$$

If $t = t_{99\%}, x = 99a/100$

$$t_{99\%} = \frac{2.303}{k}\log_{10}\frac{a}{a - 99a/100}$$

From equation (i) and (ii)

$$t_{99\%} = \frac{\log_{10}100}{\log_{10}2}\times t_{1/2}$$

$$= \frac{2}{0.3010}\times2.1 = 13.95 \text{ hour}$$

Mole of N_2O formed

$$= \frac{99}{100}\times\text{mole of } NH_2NO_2 \text{ taken}$$

$$= \frac{99}{100}\times\frac{6.2}{62} = 0.099$$

Thus, volume of N_2O formed at STP

= 0.099 × 22.4

= 2.217 litre

59. **(535)**

	$2P \rightarrow$	$4Q$	$+ R + S(l)$	
t = 0	P_0	0	0	
t = 30 min	$P_0 - x$	2x	$\frac{x}{2}$	V.P. = 25
t = 60 min	$P_0 - y$	2y	$\frac{y}{2}$	V.P. = 25
t = ∞	0	$2P_0$	$\frac{P_0}{2}$	V.P. = 25

From question, $2P_0 + \frac{P_0}{2} + 25 = 625 \Rightarrow P_0 = 240$

and $(P_0 - x) + 2x + \frac{x}{2} + 25 = 445 \Rightarrow x = 120$

Now, $\frac{1}{30} \cdot \ln \frac{P_0}{P_0 - x} = \frac{1}{60} \cdot \ln \frac{P_0}{P_0 - y} \Rightarrow y = 180$

$\therefore P_{60} = (P_0 - y) + 2y + \frac{y}{2} + 25$

= 240 + 3 × 90 + 25 = 535 mm Hg

60. **(20)**

Given:

$\pi = 4.92$ atm; T = 27 + 273 = 300 K

V = 1 litre

n_1 = mole of non-electrolyte $= \frac{X}{200}$.

n_2 = more of NaCl = 0.05

According to $\pi V = nRT$

$\pi V = n_1 RT + n_2(1+\alpha)RT$

$\pi V = [n_1 + n_2(1+\alpha)]RT$

Here $\alpha = 1$ given for NaCl

$$4.92 \times 1 = \left[\left(\frac{X}{200}\right) + 0.05 \times 2\right] \times 0.082 \times 300$$

X= 20 g.

SECTION-III (MATHEMATICS)

61. **(3)**

We have for $\cos^{-1}(1-x) \geq 0 \Rightarrow -1 \leq (1-x) \leq 1$

$\Rightarrow -2 \leq -x \leq 0$

$\Rightarrow 0 \leq x \leq 2$ …(1)

also $10 \cdot 3^{x-2} - 9^{x-1} - 1 > 0$

$10 \cdot 3^x - 9^x - 9 > 0$

$10 \cdot 3^x - 3^{2x} - 9 > 0$

$3^{2x} - 10 \cdot 3^x + 9 < 0$

$(3^x - 1)(3^x - 9) < 0$

$1 < 3^x < 9$

$\Rightarrow \quad 0 < x < 2$ …(2)

from (1) and (2)

$0 < x < 2$

62. **(1)**

$$f(x) = \frac{e^x \ln 5^{(x^2+2)} \cdot (x-2)(x-5)}{(2x-3)(x-4)}$$

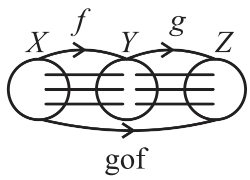
Note that at x = 3/2 and x = 4 function is not defined and in open interval (3/2, 4) function is continuous.

$$\lim_{x \to \frac{3}{2}^+} = \frac{(+ve)(-ve)(-ve)}{(+ve)(-ve)} \to -\infty$$

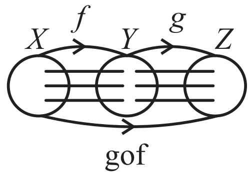
$$\lim_{x \to 4^-} = \frac{(+ve)(+ve)(-ve)}{(+ve)(-ve)} \to \infty$$

In the open interval (3/2, 4) the function is continuous and takes up all real values from (–∞, ∞) Hence range of the function is (–∞, ∞) or R.

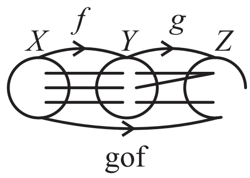
63. **(4)**



$\Rightarrow$ (A) is correct;



$\Rightarrow$ (B) is correct;



$\Rightarrow$ (C) is also correct;

64. **(4)**

I. $f(x) = x$ and $g(x) = -x$ or $f(x) = x$ and $g(x) = -x^3$

II. $f(x) = x$ and $g(x) = x^3$

III. $f(x) = \sin x$ which is odd but not one-one or $f(x) = x^2 \sin x$ which is odd but many one.

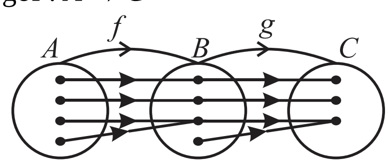
65. **(3)**

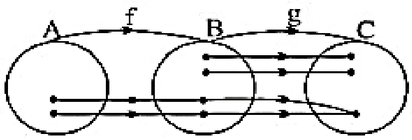
(1) We have

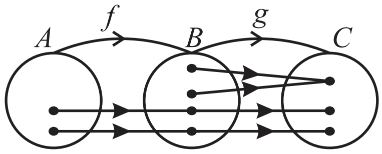
$f : A \rightarrow B$,

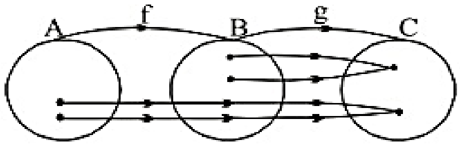
$g : B \rightarrow C$

and gof : $A \rightarrow C$

(A) 

(B) 

(C) 

(D) 

clearly gof is one-one but g is many one function

66. **(3)**
conceptual

67. **(3)**

Statement-1: $A = \{(x, y)\ \hat{I}\ R \times R: y - x$ is an integer$\}$

(a) **Reflexive**
xRx : $(x - x)$ is an integer
i.e., True
$\therefore$ Reflexive
(b) **Symmetric**
xRy : $(x - y\})$ is an integer
$\Rightarrow$ $-(y - x)$ is an integer
$\Rightarrow$ $(y - x)$ is an integer.
$\Rightarrow$ yRx $\therefore$ Symmetric
(c) **Transitive**
xRy and yRz
$\Rightarrow$ $(x - y)$ is an integer and $(y - z)$ is an integer.
$\Rightarrow$ $(x - y) + (y - z)$ is an integer
$\Rightarrow$ $(x - z)$ is an integer
$\Rightarrow$ xRz $\therefore$ Transitive
$\Rightarrow$ Equivalence Relation

$B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$
If $\alpha = 1$
$xRy : x = y$ (To check equivalence)
(a) **Reflexive**
$xRx : x = x$ (True)
$\therefore$ Reflexive
(b) Symmetric
$xRy : x = y \Rightarrow y = x \Rightarrow yRx$
$\therefore$ Symmetric
(c) Transitive
xRy and yRz
$\Rightarrow$ $x = y$ and $y = z$
$\Rightarrow$ $x = z$
$\Rightarrow$ $xR \Rightarrow$ Equivalence
$\therefore$ Both are true but Statement-2 is not correct explanation of Statement-1.

Note : Statement-2 is not equivalence for $\alpha \in R - \{1\}$.

68. **(3)**
Since, $A \cap B = A \cap C$ and $A \cup B = A \cup C$
$\Rightarrow B = C$.

69. **(4)**
$AB = A \Rightarrow A^2 = A \Rightarrow A^n = A$
and $BA = B \Rightarrow B^2 = B \Rightarrow B^n = B$
Now $(A^{2010} + B^{2010})^{2011} = (A + B)^{2011}$
$(A + B)^2 = A^2 + B^2 + AB + BA$
$= 2(A + B)$
$(A + B)^k = 2^k (A + B)$
Hence, the correct answer is (4).

70. **(3)**
$A^5(AB^2) = A^5BA$
$\Rightarrow \quad B^2 = A^5BA$
$\Rightarrow \quad B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$
$\Rightarrow \quad B^4 = A^4BA^2$
$\Rightarrow \quad B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2$
$= A^4(A^5BA)A^2$
$\Rightarrow \quad B^8 = A^3BA^3$
$\Rightarrow \quad B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^3$
$\Rightarrow \quad B^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4$
$= A^2(A^5BA)A^4 = ABA^5$
$\Rightarrow \quad B^{64} = (ABA^5)(ABA^5) = AB^2A^5$
$= A(A^5BA)A^5 = B$
$\Rightarrow \quad B^{63} = I$
Hence, the correct answer is (3).

71. **(3)**
We have $2b = a + c$ and a, p, b and b, q, c are in A.P

$$\Rightarrow \quad p = \frac{a+b}{2}, q = \frac{b+c}{2}$$

Again, $p' = \sqrt{ab}$ and $q' = \sqrt{bc}$

$$\therefore \quad p^2 - q^2 = \frac{(a+b)^2 - (b+c)^2}{4}$$

$$= \frac{(a-c)(a+c+2b)}{4} = (a-c)b = p'^2 - q'^2$$

Hence, the correct answer is (3).

72. **(3)**

If tan θ + cot θ are roots of the equation $x^2+2x+1=0$ then tan θ + cot θ = –2 then tan θ = – 1 and cot θ = –1 then the least value of $x^2+\tan x+\cot\theta$

$\Rightarrow x^2-x-1$

$\Rightarrow\left(x^2-x+\frac{1}{4}\right)-\frac{5}{4}$

$\Rightarrow\left(x-\frac{1}{2}\right)^2+\left(-\frac{5}{4}\right)$

∴ least value of function is $-\frac{5}{4}$.

73. **(4)**

$x+y+z=4$ and $x^2+y^2+z^2=6$

then $xy+yz+zx=5$

∴ $y+z=4-x$

$y\,z=5-x(4-x)$

then if y and z are roots of any quadratic then

$f(t)=t^2-(4-x)t+5-x(4-x)$

If t is real then $D\geq 0$

$\Rightarrow(4-x)^2-4(5-x(4-x))\geq 0$

$\Rightarrow 16+x^2-8x-20+4x(4-x)\geq 0$

$\Rightarrow 16+x^2-8x-20+16x-4x^2\geq 0$

$\Rightarrow -3x^2+8x-4\geq 0$

$\Rightarrow -3x^2-8x+4\leq 0$

$x\in\left\lfloor\frac{2}{3},2\right\rfloor$

74. **(3)**

Let $2^x=t$ then

$t^2+(k-3)t+(k-4)=0$

$\Rightarrow t=\frac{-(k-3)\pm\sqrt{(k-3)^2-4(k-4)}}{2}$

$\Rightarrow t=\frac{-(k-3)\pm(k-5)}{2}$

$\Rightarrow t=\frac{-2k+8}{2}$

$\Rightarrow -k+4$

∴ x is non positive then $\mathrm{x}\leq 0$

$\therefore 0<2^x\leq 1$

$\Rightarrow 0<-k+4\leq 1$

$-3\leq k<4$

∴ largest integral value of k is 3.

75. **(4)**

$f(x)=(2+b+b^2)x^2+2\sqrt{2}(2b+1)x+8$

Min value of

$f(x)=\frac{-D}{4a}=\frac{-[8(2b+1)^2-32(2+b+b^2)]}{4(b^2+b+2)}$

$m(b)=\frac{56}{4(b^2+b+2)}$

Maximum value of $m(b)$ is obtain when minimum value of b^2+b+2 is obtain minimum value

$b^2+b+2=\left(\frac{-1}{2}\right)^2+\left(\frac{-1}{2}\right)+2$

$=\frac{1}{4}-\frac{1}{2}+2=\frac{7}{8}$

Maximum value of $m(b)=\frac{56}{4\times\frac{7}{4}}=8$.

76. **(2)**

If roots are real and distinct

then $\Delta>0$

∴ For equation $x^2+6x+a=0$

then $36-4a>0$

or $a<9$ …(1)

$\alpha-\beta\leq 4$

$\Rightarrow 0<(\alpha-\beta)^2\leq 16$

$\Rightarrow 0<(\alpha-\beta)^2-4\alpha\beta\leq 16$

$\Rightarrow 0<36-4a\leq 16$

$5\leq a<9$

77. **(2)**

$R_2\to R_2-R_3$

$$\begin{vmatrix}\alpha^2 & \beta^2 & \gamma^2\\ \alpha & \beta & \gamma\\ (\alpha-1)^2 & (\beta-1)^2 & (\gamma-1)^2\end{vmatrix}=0$$

Apply $R_3\to R_3-R_1$

$$\begin{vmatrix}\alpha^2 & \beta^2 & \gamma^2\\ \alpha & \beta & \gamma^2\\ 1-2\alpha & 1-2\beta & 1-2\gamma\end{vmatrix}=0$$

$R_3\to R_3+2R_2$

$$\begin{vmatrix}\alpha^2 & \beta^2 & \gamma^2\\ \alpha & \beta & \gamma\\ 1 & 1 & 1\end{vmatrix}=0$$

$\Rightarrow(\beta-\alpha)(\gamma-\alpha)(\beta-\gamma)=0$

Hence, the correct answer is (2).

78. (3)

$\Delta \cdot \Delta^2 = 64$ ($\because$ Δ and its adjoint)

$\Rightarrow \quad \Delta^3 = 64$

$\Rightarrow \quad \Delta = 4$

G.E. = $[(2 \times 3 \times 5) + (3 \times 5 \times 4)]\Delta$

$= (30 + 60)\Delta = 90(4) = 360$

($\because$ splitting into 8 determinants only 2 will survive)

Hence, the correct answer is (3).

79. (3)

$a_r = \frac{20}{r}, r = 1, 2, 3, \ldots 9$

The value of determinant is 50/21.

80. (1)

We have

$$A = \begin{pmatrix} 0 & \sin\alpha & \sin\alpha \cdot \sin\beta \\ -\sin\alpha & 0 & \cos\alpha \cdot \cos\beta \\ -\sin\alpha \cdot \sin\beta & -\cos\alpha \cdot \cos\beta & 0 \end{pmatrix}$$

As, matrix A is skew-symmetric matrix of odd order

$\Rightarrow |A| = 0 \Rightarrow$ A is a singular matrix

$\therefore \quad A^{-1}$ does not exists.

$\Rightarrow |A|$ is independents of α and β.

Hence, the correct answer is (1).

81. (3)

$$\log(\log_3 10) = \log\left(\frac{1}{\log_{10} 3}\right) = -\log_{10}(\log_{10} 3)$$

Given $f(-\log_{10}(\log_{10} 3)) = 5$

Now $f(x) = a\sin x + bx^{1/3} + 4$

$f(-x) = -a\sin x - bx^{1/3} + 4$

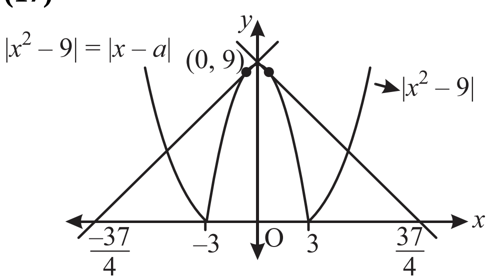
$f(x) + f(-x) = 8$

$f(\log_{10}(\log_{10} 3)) + f(-\log_{10}(\log_{10} 3)) = 8$

$f(\log_{10}(\log_{10} 3)) + 5 = 8$

$f(\log_{10}(\log_{10} 3)) = 3$

82. (17)



For tangency, $x^2 - 9 = x - a$

$\Rightarrow x^2 - x + a - 9 = 0$

Put $D = 0 \Rightarrow 1 - 4a + 36 = 0 \Rightarrow a = \frac{37}{4}$

$a = \frac{-37}{4}$

$\therefore$ For 4 distinct solution,

$$a \in \left(\frac{-37}{4}, -3\right) \cup (-3, 3) \cup \left(3, \frac{37}{4}\right)$$

Hence, number of integers are 17.

83. (2)

$c_1 \to c_1 + c_2$

$$\begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 2 + 4\sin 4\theta = 0 \Rightarrow \sin 4\theta = \frac{-1}{2}$

Hence, the correct answer is (2).

84. (21)

The integer $m\gamma = \gamma(111) = \gamma.3.37$ with γ equal to the final digit of β. Now 37 must divide one of $(\alpha\beta)$ and $(\delta\beta)$ say $(\alpha\beta)$. Therefore $\alpha\beta$ = 37 or 74.

$\alpha\beta = 74$ is not possible because

$(\alpha\beta)(\delta\beta) \geq 74.14 = 1036$

$\Rightarrow \alpha\beta = 37, \delta\beta = 27, \gamma = 9$

$\Rightarrow$ trace of matrix A = 3 + 7 + 9 + 2 = 21.

85. (3)

conceptual

86. (9)

A.M. $\geq$ H.M.

$$\Rightarrow \left(\sum x_i\right)\left(\sum \frac{1}{x_i}\right) \geq 3^2 = 9$$

Hence, the correct answer is (6).

87. (6)

General term

$$(t_r^2) = 1 + \frac{1}{n^2} + \frac{1}{(n+1)^2} = \frac{n^2(n+1)^2 + (n+1)^2 + 2}{n^2(n+1)^2}$$

$$\Rightarrow \quad t_r^2 = \frac{n^4 + 2n^3 + 3n^2 + 2n + 1}{n^2(n+1)^2}$$

$$= \left\{ \frac{n^2 + n + 1}{n(n+1)} \right\}^2$$

$$\therefore \quad t_r = 1 + \frac{1}{n(n+1)'};$$

Required sum

$$= \sum_{r=1}^{999} \left(1 + \frac{1}{r} - \frac{1}{r+1} \right) = 999 + 1 - \frac{1}{1000} = \frac{10^6 - 1}{10^3}$$

$k = 6$

Hence, the correct answer is (6).

88. (1)

$\tan A + \tan B + \tan C$

$= \tan A \tan B \tan C = 6$

$\therefore \quad \cot A \cot B \cot C = 1/6$

89. (0)

From the third relation, we get

$$\cos\theta\cos\phi + \sin\theta\sin\phi = \sin\beta\sin\gamma$$

$$\Rightarrow \quad \sin^2\theta\sin^2\phi = (\cos\theta\cos\phi - \sin\beta\sin\gamma)^2$$

$$\Rightarrow \left(1 - \frac{\sin^2\beta}{\sin^2\alpha}\right)\left(1 - \frac{\sin^2\gamma}{\sin^2\alpha}\right) = \left(\frac{\sin\beta\sin\gamma}{\sin^2\alpha} - \sin\beta\sin\gamma\right)^2$$

$$\Rightarrow \quad (\sin^2\alpha - \sin^2\beta)(\sin^2\alpha - \sin^2\gamma) = \sin^2\beta\sin^2\gamma(1 - \sin^2\alpha)^2$$

$$\Rightarrow \quad \sin^4\alpha(1 - \sin^2\beta\sin^2\gamma) - \sin^2\alpha(\sin^2\beta + \sin^2\gamma - 2\sin^2\beta\sin^2\gamma) = 0$$

$$\therefore \quad \sin^2\alpha = \frac{\sin^2\beta + \sin^2\gamma - 2\sin^2\beta\sin^2\gamma}{1 - \sin^2\beta\sin^2\gamma}$$

$$\text{and } \cos^2\alpha = \frac{1 - \sin^2\beta - \sin^2\gamma + \sin^2\beta\sin^2\gamma}{1 - \sin^2\beta\sin^2\gamma}$$

$$\Rightarrow \quad \tan^2\alpha = \frac{\sin^2\beta - \sin^2\beta\sin^2\gamma + \sin^2\gamma - \sin^2\beta\sin^2\gamma}{\cos^2\beta - \sin^2\gamma(1 - \sin^2\beta)}$$

$$= \frac{\sin^2\beta\cos^2\gamma + \cos^2\beta\sin^2\gamma}{\cos^2\beta\cos^2\gamma}$$

$$= \tan^2\beta + \tan^2\gamma$$

$$\Rightarrow \tan^2\alpha - \tan^2\beta - \tan^2\gamma = 0$$

90. (161)

$$\sin x\sin y + 3\cos y + 4\sin y\cos x = \sqrt{26}$$

$$\therefore \quad 3\cos y + (\sin x + 4\cos x)\sin y = \sqrt{26}$$

$$\therefore \quad 3\cos y + (\sin x + 4\cos x)\sin y$$

$$\le \sqrt{9 + (\sin x + 4\cos x)^2}$$

$$\le \sqrt{9 + 1 + 16}$$

$$= \sqrt{26}$$

$$\therefore \quad \sin x\sin y + 3\cos y + 4\sin y\cos x = \sqrt{26}$$

$$\Rightarrow \sin x\sin y = \frac{\cos y}{3} = \frac{\sin y\cos x}{4}$$

$$\Rightarrow 3\tan y = \operatorname{cosec} x \text{ and } \tan x = 1/4$$

$$\Rightarrow 9\tan^2 y = \operatorname{cosec}^2 x = (1 + \cot^2 x) = 17$$

$$\Rightarrow \tan^2 x + \cot^2 y = \frac{1}{16} + \frac{9}{17}$$