Individual Coursework Submission Form

Specialist Masters Programme

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Question 1

Part i)

Given an interest rate of i = 2.25% per annum payable quarterly, we have $(1+\frac{i^{(4)}}{4})^4 = (1+i)$. This can be rearranged to give us the effective quarterly interest rate of $\frac{i^{(4)}}{4} = (1+i)^{\frac{1}{4}} - 1 = 0.005578$. Since we are working in quarter years, we will consider the payments of 100 quarter years instead of 25 years. The basis of these level payments is that $R =$ [Capital + Interest] is consistent each year, and that the PV of outgoing payments = PV of income. Hence $300000 = R(V + V^2 + \cdots + V^{100}) = Ra_{\overline{100}|}$ so that $R = \frac{300000}{a_{\overline{100}|}}$. To calculate $a_{\overline{100}|}$, we must find the discount rate V (at $\frac{i^{(4)}}{4}$%) which is equal to $\frac{1}{1+\frac{i^{(4)}}{4}} = 0.99445$. Compounding this rate to 100 gives us $V^{100} = 0.57335$, which we can use to find the annuity $a_{\overline{100}|} = \frac{1-V^{100}}{(\frac{i^{(4)}}{4})} = 76.48654$. Therefore, each installment will be £3922.26 per quarter.

Part ii)

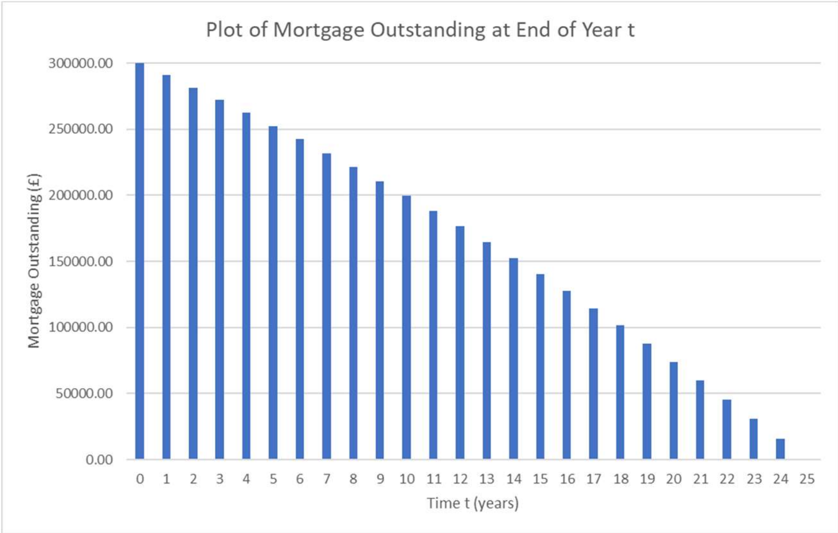
The outstanding mortgage at the end of year 1 is equivalent to the outstanding mortgage at the end of the 4th quarter. Notice that in the 4th quarter, the due amount was £290929.20.

Part iii)

For any nth quarter, the interest payable is 2.25% of the (n-1) quarter of capital outstanding. The capital payments will be the difference between the interest payable and the level rate, so we can then use every 4th payment period's capital outstanding for the table:

Year t	Mortgage oustanding at end of year t	Year t	Mortgage oustanding at end of year t
0	300000.00	13	164770.10
1	290929.20	14	152656.63
2	281654.31	15	140270.61
3	272170.74	16	127605.90
4	262473.78	17	114656.23
5	252558.65	18	101415.20
6	242420.42	19	87876.25
7	232054.08	20	74032.67
8	221454.50	21	59877.61
9	210616.43	22	45404.05
10	199534.51	23	30604.85
11	188203.24	24	15472.66
12	176617.01	25	0.00

Part iv)



The graph looks quite linear in shape, constantly decreasing. This means that the capital repaid by the borrower increases steadily over time, until everything is paid off at the end of the 25th year.

Part v)

The total interest payable is the sum of all interest payments per quarter. Referencing the payment period table, it is the sum of all the interest payments, which is £92225.85.

Part vi)

The borrower, at the end of 11 years (start of 12th) owes £176617.01. Then, after the 1st quarter of that year, they will have £173679.95 outstanding on their mortgage.

Question 2

Part i)

a) Using the same method as 1i), we obtain our $\frac{i^{(4)}}{4} = 0.020016$, $V = 0.98729$, $V^{100} = 0.27826$, $a_{\overline{100}|} = 56.06095$, to give us $R = £5351.32$.

b) Using the same method as 1i), we obtain our $\frac{i^{(4)}}{4} = 0.012874$, $V = 0.98038$, $V^{100} = 0.13782$, $a_{\overline{100}|} = 43.07474$, to give us $R = £6964.64$.

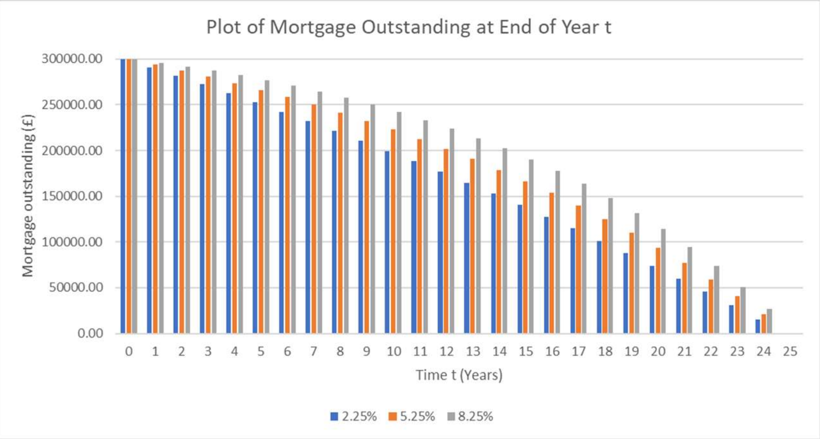
Part ii)

The formula to find the percentage change is $100(\frac{R_{new}-R_{old}}{R_{old}})$. For 5.25%, $100\left(\frac{5351.32-39\ \ .26}{3922.26}\right) = 36.43461\%$ increase. for 8.25%, $100\left(\frac{6964.64-3922.26}{3922.26}\right) = 77.56706\%$ increase.

Part iii)

Using the same method as 1iii), we obtain

Year t	Mortgage Outstanding at end of year t		
	2.25%	5.25%	8.25%
0	300000.00	300000.00	300000.00
1	290929.20	293927.80	296043.80
2	281654.31	287536.82	291761.22
3	272170.74	280810.30	287125.32
4	262473.78	273730.64	282106.96
5	252558.65	266279.31	276674.59
6	242420.42	258436.77	270794.04
7	232054.08	250182.51	264428.35
8	221454.50	241494.89	257537.49
9	210616.43	232351.18	250078.14
10	199534.51	222727.42	242003.38
11	188203.24	212598.41	233262.46
12	176617.01	201937.63	223800.42
13	164770.10	190717.15	213557.76
14	152656.63	178907.61	202470.07
15	140270.61	166478.06	190467.65
16	127605.90	153395.96	177475.04
17	114656.23	139627.05	163410.53
18	101415.20	125135.28	148185.70
19	87876.25	109882.68	131704.82
20	74032.67	93829.32	113864.27
21	59877.61	76933.17	94551.88
22	45404.05	59149.96	73646.21
23	30604.85	40433.14	51015.82
24	15472.66	20733.68	26518.43
25	0.00	0.00	0.00



Comparing all 3 interest rates, we see that the higher the rate becomes, the initial capital repayments are less.

However towards the end, the capital payments become much greater. This is evident by the linearity of the 2.25%

but the non-linearity of the higher rates.