Individual Coursework Submission Form

Specialist Masters Programme

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Module Code: SMM065	
Module Title: Contingencies (Subject CM1(2))	
Lecturer: Douglas Wright	**Submission Date: 13/03/2024**

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1)

i) To start, we must use a suitable radix, which was given as 10,000 from the AM92 Table. From this, we can calculate l_x for any x using the equation $l_{x+1} = l_x(1-q_x)$. This can be read as 'the number of people still alive at age $x+1$ equals the number of people alive at age x surviving to $x+1$ with probability $(1-q_x) = p_x$. We can stretch this formula down to get l_x for $x = 17,18,\ldots,120$. More complex however, is to find $l_{[x]}$. We are aware that the selection period is 2 years, meaning that if someone were selected at age x, their mortality (per year) would be $q_{[x]} \rightarrow\ q_{[x]+1} \rightarrow q_{[x]+2} =\ q_{x+2}$. Note that in '*unit 1 – the life table (notes, students)'* we are given a recursive formula to solve this. Let $s = 2$ define the selection period, hence (for simplicity) we have: $l_{[x]+(s-1)} = l_{[x]+1} = \frac{l_{x+2}}{1-q_{[x]+1}}$ and $l_{[x]} = \frac{l_{[x]+1}}{1-q_{[x]}} = \frac{1}{1-q_{[x]}} * \frac{l_{x+2}}{1-q_{[x]+1}}$. The only trouble here is to find values of $q_{[x]+1}$, which isn't explicitly stated in the table. The way to overcome this is to notice that, for example, if a person was aged 18, $q_{[x]} = q_{[18]}$ and $q_{[x-1]+1} = q_{[17]+1}$, so if we were to use the age $x - 1 = 18 \rightarrow x = 19$, we would obtain $q_{[18]+1}$ as desired. This means that all we must do is use one age higher for the $q_{[x-1]+1}$ column to obtain $q_{[x]+1}$. This allows us to use the $l_{[x]}$ formula as required. Finally, we can manipulate this to get values for $l_{[x-1]+1} = \frac{1}{1-q_{[x-1]+1}} * \frac{l_{x+2}}{1-q_{[x-1]+2}} = \frac{1}{1-q_{[x-1]+1}} * \frac{l_{x+2}}{1-q_{x+1}}$ as required.

ii) ${}_{10}p_{[50]} = \frac{l_{60}}{l_{[50]}} = \frac{9287.2164}{9706.0977} = 0.9568$ and ${}_{10}p_{50} = \frac{l_{60}}{l_{50}} = \frac{9287.2164}{9712.0728} = 0.9563$ are the values calculated using the spreadsheet. Using the AM92 table, the values are the same.

iii) Using the curtate expectation of life, we sum up all the l_x's (from age $x+1$ to final age 120) and divide by that current l_x. The values compared to the AM92 table are the exact same. Please note that all values from the spreadsheet will have the same number of decimal places as their corresponding AM92 values.

iv) We do the same as in iii, but we must make small adjustments. Firstly, $e_{[x]} = \frac{l_{[x]+1}+l_{x+2}+\ldots}{l_{[x]}}$, so using age $x+1$, we can obtain $l_{[x-1]+1} = l_{[x+1-1]+1} = l_{[x]+1}$, hence being able to calculate $e_{[x]}$. We also know $e_{[x-1]+1} = \frac{l_{x+1}+l_{x+2}+\ldots}{l_{[x-1]+1}}$ as our selection period is only 2 years.

v) scrolling down the spreadsheet leads to the 10% reduction table. This has been accomplished by multiplying each q_x, $q_{[x-1]+1}$, and $q_{[x]}$ by 0.9. All other formulae have been kept the same, using the updated q values accordingly. Now ${}_{10}p_{[50]} = \frac{l_{60}}{l_{[50]}} = \frac{9356.2538}{9735.1105} = 0.9611$, and ${}_{10}p_{50} = \frac{l_{60}}{l_{50}} = \frac{9356.2538}{9740.5028} = 0.9606$

vi) originally, $e_{50} = 29.565$, and with the 10% reduction, $e_{50} = 30.528$. since e_x represents the expected future lifetime for a person of exact age x, we see that with the reduction, a person of age 50 will have a longer expected future lifetime, a percentage increase of $\left(\frac{30.528-29.565}{29.565}\right) \times 100 = 3.2573\%$

2)

i) We define the commutation functions:

- $D_x = v^x \times l_x$, where $v = \frac{1}{1+i}$, i being the interest rate (currently 4%)
- $D_{[x]} = v^x \times l_{[x]}$
- $D_{[x-1]+1} = v^x \times l_{[x-1]+1}$
- $C_x = v^{x+1} \times d_x$, where $d_x = l_x - l_{x+1}$
- $C_{[x]} = v^{x+1} \times d_{[x]}$, where $d_{[x]} = l_{[x]} - l_{[x]+1}$
- $C_{[x-1]+1} = v^{x+1} \times d_{[x-1]+1}$, where $d_{[x-1]+1} = l_{[x-1]+1} - l_{x+1}$

Referring to the AM92 Mortality table, the values are equal.

ii) We define the commutation functions:

- $N_x = \sum_{k=o}^{\infty} D_{x+k} = D_x + D_{x+1} + D_{x+2} + \cdots$ up to D_{120}
- $N_{[x]} = \sum_{k=o}^{\infty} D_{[x]+k} = D_{[x]} + D_{[x]+1} + D_{x+2} + \cdots$ up to D_{120}
- $N_{[x-1]+1} = \sum_{k=o}^{\infty} D_{[x-1]+1+k} = D_{[x-1]+1} + D_{x+1} + D_{x+2} + \cdots$ up to D_{120}
- $S_x = \sum_{k=o}^{\infty} N_{x+k} = N_x + N_{x+1} + N_{x+2} + \cdots$ up to N_{120}
- $S_{[x]} = \sum_{k=o}^{\infty} N_{[x]+k} = N_{[x]} + N_{[x]+1} + N_{x+2} + \cdots$ up to N_{120}
- $S_{[x-1]+1} = \sum_{k=o}^{\infty} N_{[x-1]+1+k} = N_{[x-1]+1} + N_{x+1} + N_{x+2} + \cdots$ up to N_{120}
- $M_x = \sum_{k=o}^{\infty} C_{x+k} = C_x + C_{x+1} + C_{x+2} + \cdots$ up to C_{120}
- $M_{[x]} = \sum_{k=o}^{\infty} C_{[x]+k} = C_{[x]} + C_{[x]+1} + C_{x+2} + \cdots$ up to C_{120}
- $M_{[x-1]+1} = \sum_{k=o}^{\infty} C_{[x-1]+1+k} = C_{[x-1]+1} + C_{x+1} + C_{x+2} + \cdots$ up to C_{120}
- $R_x = \sum_{k=o}^{\infty} M_{x+k} = M_x + M_{x+1} + M_{x+2} + \cdots$ up to M_{120}
- $R_{[x]} = \sum_{k=o}^{\infty} M_{[x]+k} = M_{[x]} + M_{[x]+1} + M_{x+2} + \cdots$ up to M_{120}
- $R_{[x-1]+1} = \sum_{k=o}^{\infty} M_{[x-1]+1+k} = M_{[x-1]+1} + M_{x+1} + M_{x+2} + \cdots$ up to M_{120}

Referring to the AM92 Mortality table, the values are equal.

3)

i) We are trying to find the value of $P_{45} = \frac{100000 \times A_{45}}{\ddot{a}_{45}}$. By the commutation functions presented, $A_x = \frac{M_x}{D_x}$ and $\ddot{a}_x = \frac{N_x}{D_x}$. We can then calculate $P_{45} = 1588.45$, which is the level annual premium paid.

ii) We start with the 20-year term assurance. Here, $P^1_{45:\overline{20}|} = \frac{100000 \times A^1_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|}}$. By the commutation functions presented, $A^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x}$, and $\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x}$. We can then calculate $P^1_{45:\overline{20}|} = 439.67$, which is the level annual premium paid.

Next is the 20-year endowment assurance. Here, $P_{45:\overset{1}{\overline{20}|}} = \frac{100000 \times A_{45:\overset{1}{\overline{20}|}}}{\ddot{a}_{45:\overline{20}|}}$. By the commutation functions presented, $A_{x:\overset{1}{\overline{n}|}} = \frac{D_{x+n}}{D_x}$, and we can use the same value for $\ddot{a}_{x:\overline{n}|}$ as in our term assurance above. This gives us $P_{45:\overset{1}{\overline{20}|}} = 3155.88$, which is the level annual premium paid.

iii) The premium for the term assurance is lower compared to the whole life assurance because it only covers a specific duration. With term assurance, the insurer only needs to cover the risk of death during the chosen term. On the other hand, whole life assurance is in place for the entire lifetime of the insured. This lifelong coverage contributes to higher premiums. The premiums for endowment assurance are typically higher than those for whole life assurance due to the combined benefits of insurance protection and a guaranteed lump sum payout at the policy's maturity.

4)

i) with the change of i from 3.5% to 5% we obtain $P_{45} = 1249.71$, $P^{1}_{45:\overline{20}|} = 410.83$, and $P_{45:\overset{1}{\overline{20}|}} = 2655.30$

ii) Insurance companies invest the premiums they receive to generate returns. A higher interest rate means they can earn more from their investments, which allows insurers to pay lower premiums.

iii) The percentage reduction for the whole of life assurance is $\frac{1588.45-1249.71}{1588.45} \times 100 = 21.3252\%$. The percentage reduction for the 20-year term assurance is $\frac{439.67-41\ \ .83}{439.67} \times 100 = 6.5595\%$. The percentage reduction for the 20-year endowment assurance is $\frac{3155.88-2655.30}{3155.88} \times 100 = 15.8618\%$.

iv) Since the premiums for term assurance are primarily driven by mortality risk, changes in interest rates have less impact on premiums. Therefore, the relative change in premiums for term assurance is lowest because interest rates have minimal influence on its cost structure. The premiums for whole of life assurance are higher because they not only cover mortality risk but also accumulate cash value over time. When interest rates change, the investment component of whole of life assurance is directly affected. Therefore, the relative change in premiums for whole of life assurance is highest. Endowment assurance falls between as it shares characteristics of both assurances.

5)

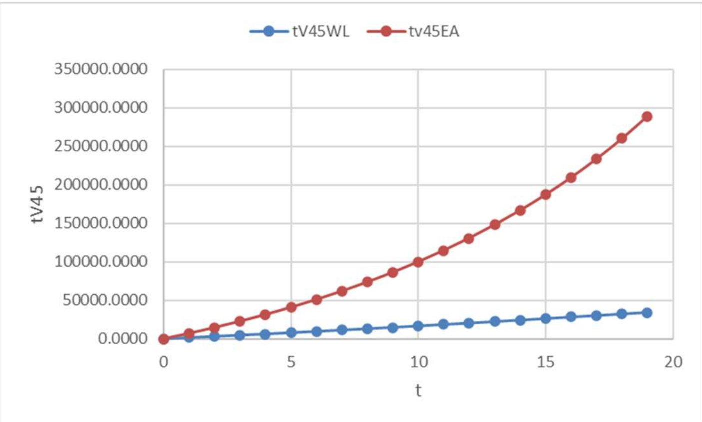
i) From unit 3, ${}_tV_x^{WL} = 100000 \times \left(1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}\right)$ will be used to calculate the premium reserve for a whole life assurance.

WL
0.0000
1501.3759
3042.3350
4622.6230
6241.8248
7899.5679
9594.9934
11327.4132
13095.6264
14898.4754
16734.5348
18602.1529
20499.4142
22424.2671
24374.4191
26347.3200
28340.2984
30350.4120
32374.5240
34409.2472

ii)

iii) ${}_tV_x^{EA} = 100000 \times \left(1 - \frac{\ddot{a}_{x+t:\overline{n-t}|}}{\ddot{a}_{x:\overline{n}|}}\right)$ is used to calculate the premium reserve for an endowment assurance for n years.

EA
0.0000
7009.3582
14553.1004
22671.9602
31410.5330
40817.9550
50948.1645
61861.0764
73623.0807
86308.3643
100000.0000
114791.4736
130788.4910
148111.0954
166896.2638
187301.1233
209506.6004
233722.2883
260192.0843
289201.4579

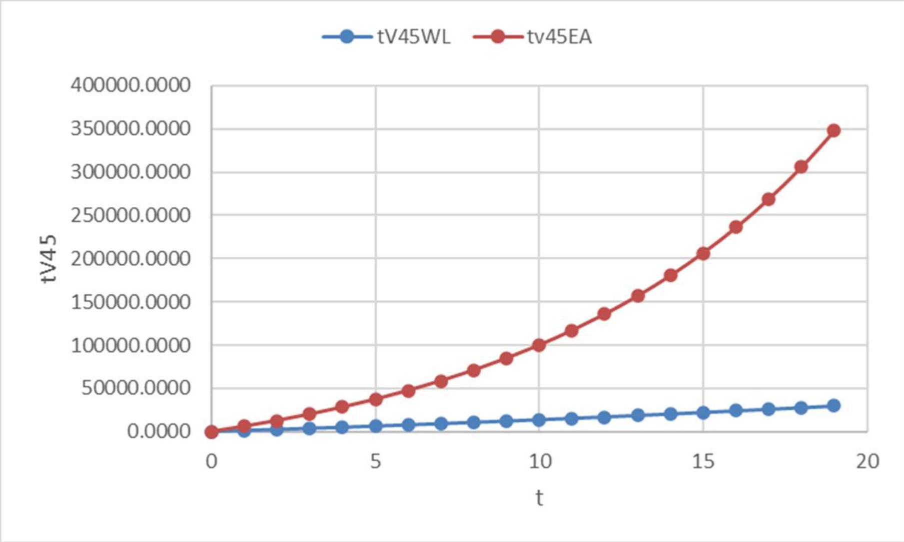


iv)

6)

i)

WL	EA
0.0000	0.0000
1169.3752	6046.9610
2383.6724	12738.6647
3643.3834	20145.3049
4948.8520	28345.3904
6300.4760	37427.0727
7698.1689	47489.1408
9142.0194	58643.0943
10631.5932	71014.6568
12166.4921	84746.3432
13746.0330	100000.0000
15369.2821	116960.0832
17035.0092	135837.5720
18741.8130	156874.6251
20488.0055	180350.2660
22271.5870	206587.3904
24090.3814	235961.0138
25941.8729	268908.9040
27823.2776	305944.3044
29731.4775	347672.1378



ii)