

# Appendix 4

## Implementation of MLP Algorithm and Development and Training of MLP Model

```
[1]: import pandas as pd
import numpy as np
from sklearn.metrics import *
from sklearn.model_selection import train_test_split

import seaborn as sns
import matplotlib.pyplot as plt
import plotly.express as px
import plotly.graph_objects as go
```

### 1 Loading Datasets

```
[2]: # Loading datas
labels = ['Lagged', 'MA', 'WMA', 'MA-Lagged', 'WMA-Lagged'] # names of each
↳ datasets

def load_datasets():
    """
    Excel files for each dataset are read into a
    dataframe and stored in a dictionary for easy
    access and use
    """
    datasets = dict()
    for lb in labels:
        new_df = pd.read_excel(f"River-Data-{lb}.xlsx")
        new_df.drop(["Unnamed: 0"], axis=1, inplace=True)
        datasets[lb] = new_df

    return datasets

data = load_datasets() # a dataframe for each dataset is stored in a dictionary
↳ called data
```

### 2 Utility Functions

Standardising and Unstandardising Values

```
[3]: # Utility Functions
## Functions for standardising and unstandardising values
def standardise_columns(df, cols):
    """
    This function works with dataframes to standardise values
    in multiple columns to the range [0.1, 0.9]
    """
    subset_df = df[cols]
    subset_df = 0.8 * ((subset_df - subset_df.min()) / (subset_df.max() -
    ↪subset_df.min())) + 0.1
    return subset_df

def unstandardise_columns(df, cols, max_val, min_val):
    """
    This function works with numpy arrays to destandardise values
    in multiple columns
    """
    subset_df = df[cols]
    subset_df = ((subset_df - subset_df.min()) / 0.8) * (max_val - min_val) +
    ↪min_val
    return subset_df

def standardise_value(x, max_val, min_val):
    """
    This function works with numpy arrays to standardise values
    in multiple arrays to the range [0.1, 0.9]
    """
    return 0.8 * ((x - min_val)) / (max_val - min_val) + 0.1

def unstandardise_value(x, max_val, min_val):
    """
    This function works with numpy arrays to destandardise values
    in multiple arrays
    """
    return ((x - 0.1) / 0.8) * (max_val - min_val) + min_val
```

## Plotting

```
[4]: ## Plotting functions
def plot_correlation_matrix(corr_data, title, figsize=(16,6), mask=False):
    """
    Utility function for plotting a correlation heatmap of a given feature set
    """
    if mask:
        mask = np.triu(np.ones_like(corr_data, dtype=bool))
    plt.figure(figsize=figsize, dpi=500)
    heatmap = sns.heatmap(corr_data, vmin=-1, vmax=1, annot=True, mask=mask)
    heatmap.set_title(title)
```

```

plt.show()

def plot_predictions(preds_df, standardised=False):
    """
    Utility function for plotting model predictions against actual value
    """
    preds_col = "Predicted Values"
    vals_col = "Actual Values"
    if standardised:
        preds_col += " (Standardised)"
        vals_col += " (Standardised)"

    line_plt = px.line(preds_df, y=vals_col)
    scatter_plt = px.scatter(preds_df, y=preds_col,
    ↪color_discrete_sequence=["#ff0000"])

    go.Figure(line_plt.data + scatter_plt.data, layout={"title": "Actual vs_
    ↪Predicted Values"}).show()

```

### 3 ANN Class

```

[5]: # Basic ANN class for MLP models
class BasicAnn:
    def __init__(self, layers, max_st_val, min_st_val, activ_func="sigmoid"):
        self.layers = layers
        self.num_layers = len(layers)
        self.max_val = max_st_val
        self.min_val = min_st_val
        self.activ_func = activ_func

        weight_shapes = [(layers[i-1],layers[i]) for i in range(1, len(layers))]
        self.weights = {
            f"W{i+1}": np.random.standard_normal(s)/s[0]**0.5
            for i, s in enumerate(weight_shapes)
        } # weights are stored as matrices that are implemented as 2D numpy
    ↪arrays

        self.biases = {
            f"B{i+1}": np.random.randn(1,1)/1**0.5
            for i, l in enumerate(layers[1:])
        } # biases are also stored as matrices that are implemented as 2D numpy
    ↪arrays

    def activation(self, x):
        """
        Function to return value with the selected activation
        """

```

```

        if self.activ_func == "sigmoid":
            return 1/(1+np.exp(-x))
        elif self.activ_func == "tanh":
            return (np.exp(x)-np.exp(-x))/(np.exp(x)+np.exp(-x))
        elif self.activ_func == "relu":
            return x * (x > 0)
        elif self.activ_func == "linear":
            return x

def activation_deriv(self, a):
    """
    Function to return value with the derivative of the selected activation
    """
    if self.activ_func == "sigmoid":
        return a * (1 - a)
    elif self.activ_func == "tanh":
        return 1 - a**2
    elif self.activ_func == "relu":
        return 1 * (a > 0)
    elif self.activ_func == "linear":
        return np.ones(a.shape)

def train(self, features, targets, epochs=1000, learning_rate=0.1,
    ↪val_set=None):
    """
    Function will train the model using the standard backpropagation
    ↪algorithm
    and return a dataframe storing various error metrics for the model on
    ↪the
    training set and, possibly, a validation set if that is given
    """
    results = pd.DataFrame()
    real_targets = unstandardise_value(targets, self.max_val, self.min_val)
    num_targets = len(targets)

    for _ in range(epochs):
        # Forward pass
        activations = self.forward_pass(features)

        # Error calculation
        output_layer = activations[f"A{self.num_layers - 1}"]
        real_preds = unstandardise_value(output_layer, self.max_val, self.
    ↪min_val)
        error_data = { # storing error metrics for both standardised and
    ↪unstandardised data
            "mse": mean_squared_error(real_targets, real_preds),

```

```

        "rmse": mean_squared_error(real_targets, real_preds,
→squared=False),
        "mae": mean_absolute_error(real_targets, real_preds),
        "r_sqr": r2_score(real_targets, real_preds),
        "st_mse": mean_squared_error(targets, output_layer),
        "st_rmse": mean_squared_error(targets, output_layer,
→squared=False),
        "st_mae": mean_absolute_error(targets, output_layer),
        "st_r_sqr": r2_score(targets, output_layer)
    }

    if val_set:
        # if there is a validation set the prediction error of the model
        # on the validation set will be stored
        r, err = self.predict(val_set[0].to_numpy(), val_set[1].
→to_numpy())
        error_data.update({f"val_{col}": err[col][0] for col in err.
→columns})

    results = results.append(error_data, ignore_index=True)

    # Backward pass (backpropagation algorithm)
    deltas = self.compute_deltas(activations, targets, output_layer)
    self.update_weights(deltas, activations, features, num_targets,
→learning_rate)

    return results

def predict(self, test_inputs, st_actual_outputs, actual_outputs=None):
    """
    Runs a forward pass of the network with the newly configured weights
    and biases and returns a dataframe comparing the predicted values
    to actual values as well as a dataframe with various error metrics
    """
    # Forward pass
    activations = self.forward_pass(test_inputs)
    st_preds = activations[f"A{self.num_layers - 1}"]

    # Comparing predicted values with actual values
    if actual_outputs is None:
        actual_outputs = unstandardise_value(st_actual_outputs, self.
→max_val, self.min_val)

    preds = unstandardise_value(st_preds, self.max_val, self.min_val)

    results = pd.DataFrame(

```

```

        data={
            "Actual Values": actual_outputs.flatten(),
            "Predicted Values": preds.flatten(),
            "Actual Values (Standardised)": st_actual_outputs.flatten(),
            "Predicted Values (Standardised)": st_preds.flatten(),
        }
    )

    # Error calculation
    results["Absolute Error"] = abs(results["Actual Values"] -
↪results["Predicted Values"])
    st_absolute_err = abs(results["Actual Values (Standardised)"] -
↪results["Predicted Values (Standardised)"])
    results["Absolute Error (Standardised Values)"] = st_absolute_err

    error_metrics = pd.DataFrame(data={
        "mse": [mean_squared_error(actual_outputs, preds)],
        "rmse": [mean_squared_error(actual_outputs, preds, squared=False)],
        "mae": [mean_absolute_error(actual_outputs, preds)],
        "r_sqr": [r2_score(actual_outputs, preds)],
        "st_mse": [mean_squared_error(st_actual_outputs, st_preds)],
        "st_rmse": [mean_squared_error(st_actual_outputs, st_preds,
↪squared=False)],
        "st_mae": [mean_absolute_error(st_actual_outputs, st_preds)],
        "st_r_sqr": [r2_score(st_actual_outputs, st_preds)]
    })

    return results, error_metrics

def forward_pass(self, features):
    """
    Runs a forward pass of neural network through repeated
    multiplication of weights and bias matrices. Returns
    list of each activation layer including the output layer.
    """
    activation = self.activation(np.dot(features, self.weights["W1"]) +
↪self.biases["B1"].T)
    activations = {"A1": activation}
    for i in range(2, self.num_layers):
        activation = self.activation(np.dot(activation, self.
↪weights[f"W{i}"]) + self.biases[f"B{i}"].T)
        activations[f"A{i}"] = activation

    return activations

def compute_deltas(self, activations, targets, output_layer):
    """

```

```

Computes errors between layers for backproagation.
Returns a dictionary of lists which contain the errors
for each node in each layer.
"""
output_err = targets - output_layer
output_delta = output_err * self.activation_deriv(output_layer)
deltas = {"dw1": output_delta}

for i in range(self.num_layers - 1, 1, -1):
    dw = deltas[f"dw{self.num_layers - i}"]
    act = activations[f"A{i-1}"]
    w = self.weights[f"W{i}"]
    deltas[f"dw{self.num_layers - i + 1}"] = np.dot(dw, w.T) * self.
↪activation_deriv(act)

return deltas

def update_weights(self, deltas, activations, features, num_targets, l_rate):
    """
    Updates weights and biases according to given errors, activations
    and the chosen learning rate
    """
    delta = deltas[f"dw{self.num_layers - 1}"]
    self.weights["W1"] += l_rate * (np.dot(features.T, delta)) / num_targets
    self.biases["B1"] += l_rate * (np.dot(delta.T, np.ones((num_targets, 1)))) / num_targets
    ↪1)))) / num_targets

    for i in range(2, self.num_layers):
        act = activations[f"A{i-1}"]
        dw = deltas[f"dw{self.num_layers - i}"]
        self.weights[f"W{i}"] += l_rate * (np.dot(act.T, dw)) / num_targets
        self.biases[f"B{i}"] += l_rate * np.dot(dw.T, np.ones((num_targets, 1)))) / num_targets
    ↪1)))) / num_targets

```

### Build, Train and Test ANN Model

```

[6]: def build_train_test(feature_set, feature_cols, target_cols, layers=("auto", 1),
    ↪1), activ_func="linear", epochs=1000, l_rate=0.1):
    """
    Function to build, train and test MLP models
    """
    # Splitting and standardising datasets to create standardised and
    ↪unstandardised
    # training, validation and testing sets.
    train_val_set, test_set = train_test_split(feature_set, test_size=0.2)
    st_train_val_set = standardise_columns(train_val_set, train_val_set.columns)

```

```

st_test_set = standardise_columns(test_set, test_set.columns)

# Preparing features and targets for training and testing
features = st_train_val_set[feature_cols]
targets = st_train_val_set[target_cols]

X_train, X_val, y_train, y_val = train_test_split(features, targets,
→test_size=0.25)
X_test, y_test = st_test_set[feature_cols], st_test_set[target_cols]

# Getting standardisation values for targets
min_val = train_val_set[target_cols].min()[0]
max_val = train_val_set[target_cols].max()[0]

# Building model
if layers[0] == "auto":
    # if the size of the input layer is not specified
    # then it will be set to the number of predictors
    layers = (len(feature_cols),) + layers[1:]

ann = BasicAnn(layers, max_val, min_val, activ_func)

# Training model
training_results = ann.train(
    X_train.to_numpy(),
    y_train.to_numpy(),
    val_set=(X_val, y_val), # training with a validation set
    epochs=epochs,
    learning_rate=l_rate
)

# Predicting model
prediction_results = ann.predict(
    X_test.to_numpy(),
    y_test.to_numpy(),
    actual_outputs=test_set[target_cols].to_numpy()
)

predictions, error_metrics = prediction_results[0], prediction_results[1]

return {
    "training_results": training_results,
    "final_test_results": predictions,
    "error_metrics": error_metrics,
    "model": ann
}

```



## 4 Selecting Features/Predictors

### Building Feature Sets

```
[7]: # Function for building custom feature and target sets
def build_feature_set(*datasets):
    assert len(datasets) > 0, "No data sets entered"
    datasets = list(datasets)
    min_rows = min(d.shape[0] for d in datasets)

    for i, ds in enumerate(datasets):
        datasets[i] = ds.truncate(before=ds.shape[0]-min_rows).reset_index()
        datasets[i].drop(["index"], axis=1, inplace=True)

    merged_df = datasets[0].iloc[:, :2]
    for ds in datasets:
        merged_df = pd.concat([merged_df, ds.iloc[:, 2:]], axis=1)

    merged_cols = list(merged_df.columns)
    selected_cols = []

    for i in range(0, len(merged_cols), 2):
        format_str = f"{i+1}) {merged_cols[i]}"
        if i != len(merged_cols) - 1:
            second_part = f"{i+2}) {merged_cols[i+1]}"
            num_spaces = 50 - len(format_str)
            format_str += num_spaces*" " + second_part
        print(format_str)

    selected_indices = input("\nSelect columns: ")
    for index in selected_indices.split(","):
        if "-" in index:
            first_i, second_i = index.split("-")
            selected_cols += merged_cols[int(first_i) - 1: int(second_i)]
        else:
            selected_cols.append(merged_cols[int(index) - 1])

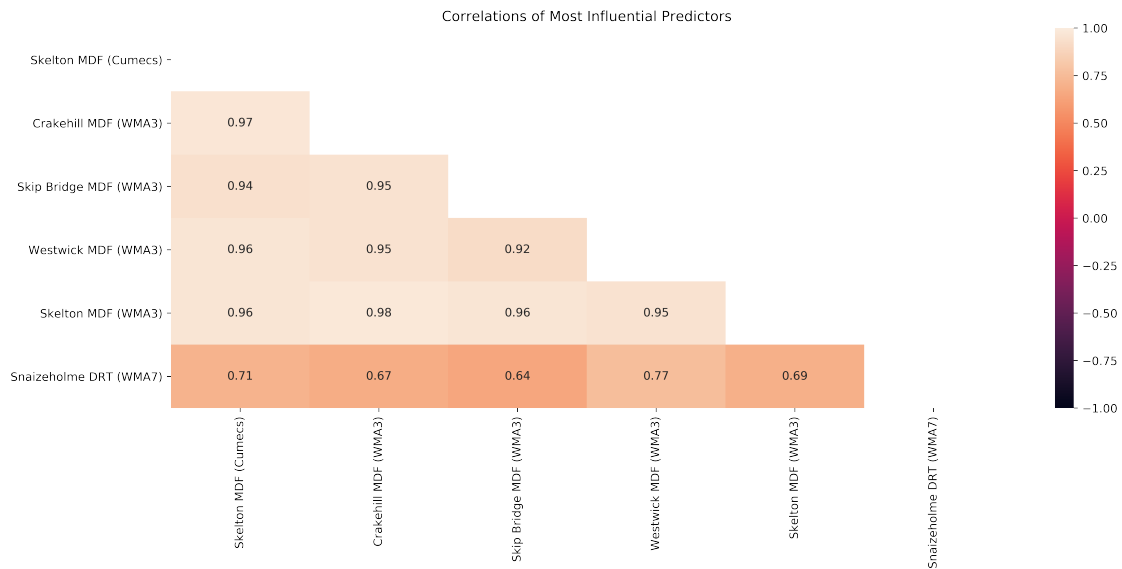
    return merged_df[selected_cols]
```

```
[8]: # 2,3-6,42
fs = build_feature_set(data['WMA'])
plot_correlation_matrix(fs.corr(), "Correlations of Most Influential_
↳Predictors", mask=True)
```

- |                         |                           |
|-------------------------|---------------------------|
| 1) Date                 | 2) Skelton MDF (Cumeecs)  |
| 3) Crakehill MDF (WMA3) | 4) Skip Bridge MDF (WMA3) |
| 5) Westwick MDF (WMA3)  | 6) Skelton MDF (WMA3)     |
| 7) Crakehill MDF (WMA4) | 8) Skip Bridge MDF (WMA4) |
| 9) Westwick MDF (WMA4)  | 10) Skelton MDF (WMA4)    |

|                               |                            |
|-------------------------------|----------------------------|
| 11) Crakehill MDF (WMA5)      | 12) Skip Bridge MDF (WMA5) |
| 13) Westwick MDF (WMA5)       | 14) Skelton MDF (WMA5)     |
| 15) Crakehill MDF (WMA6)      | 16) Skip Bridge MDF (WMA6) |
| 17) Westwick MDF (WMA6)       | 18) Skelton MDF (WMA6)     |
| 19) Crakehill MDF (WMA7)      | 20) Skip Bridge MDF (WMA7) |
| 21) Westwick MDF (WMA7)       | 22) Skelton MDF (WMA7)     |
| 23) Arkengarthdale DRT (WMA3) | 24) East Cowton DRT (WMA3) |
| 25) Malham Tarn DRT (WMA3)    | 26) Snaizeholme DRT (WMA3) |
| 27) Arkengarthdale DRT (WMA4) | 28) East Cowton DRT (WMA4) |
| 29) Malham Tarn DRT (WMA4)    | 30) Snaizeholme DRT (WMA4) |
| 31) Arkengarthdale DRT (WMA5) | 32) East Cowton DRT (WMA5) |
| 33) Malham Tarn DRT (WMA5)    | 34) Snaizeholme DRT (WMA5) |
| 35) Arkengarthdale DRT (WMA6) | 36) East Cowton DRT (WMA6) |
| 37) Malham Tarn DRT (WMA6)    | 38) Snaizeholme DRT (WMA6) |
| 39) Arkengarthdale DRT (WMA7) | 40) East Cowton DRT (WMA7) |
| 41) Malham Tarn DRT (WMA7)    | 42) Snaizeholme DRT (WMA7) |

Select columns: 2,3-6,42



## 5 Training and Network Selection

### Epochs

```
[9]: target_cols = [fs.columns[0]]
    feature_cols = list(fs.columns[1:])

    epoch_tests = dict()
    for i in range(1, 11):
        epoch_tests[f"Test-{i}"] = build_train_test(
```

```

        fs,
        feature_cols,
        target_cols,
        layers=("auto", 1),
        activ_func="linear",
        epochs=i*500
    )

```

- rmse -> root mean square error
- mae -> mean absolute error
- mse -> mean squared error
- r\_sqr -> R-Squared (Coefficient of Determination)
- val\_\* -> error metric on validation set
- st\_\* -> error metric on unstandardised values

```

[10]: # Testing number of epochs for training; between 800 and 1200 seems to be ideal
for i in range(1, 11):
    print(f"Model trained with {i*500} epochs", end=f"\n{'-'*100}\n")

    print("Final Training results")
    print(epoch_tests[f"Test-{i}"]["training_results"].iloc[-1, :4],
    ↪end=f"\n{'-'*100}\n")

    print("Final Validation results")
    print(epoch_tests[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
    ↪end=f"\n{'-'*100}\n")

    print("Test Set Results")
    print(epoch_tests[f"Test-{i}"]["error_metrics"].iloc[0],
    ↪end=f"\n{'-'*100}\n\n\n")

    ax = epoch_tests[f"Test-{i}"]["training_results"].plot(
        y=["rmse", "val_rmse"], title=f"Model Trained with {i*500} Epochs",
    )
    ax.set_xlabel("Epochs")
    ax.set_ylabel("Root Mean Squared Error (RMSE)")

```

Model trained with 500 epochs

```

-----
Final Training results
mae      11.115425
mse      298.617903
r_sqr     0.907014
rmse     17.280564

```

Name: 499, dtype: float64

-----  
-----  
Final Validation results

val\_mae        11.824059

val\_mse        355.111826

val\_r\_sqr      0.899109

val\_rmse       18.844411

Name: 499, dtype: float64

-----  
-----  
Test Set Results

mse            540.297189

rmse           23.244294

mae            18.232164

r\_sqr          0.766672

st\_mse         0.001970

st\_rmse        0.044380

st\_mae         0.025539

st\_r\_sqr       0.888120

Name: 0, dtype: float64

=====

Model trained with 1000 epochs

-----  
-----  
Final Training results

mae            8.688907

mse            212.143011

r\_sqr          0.937055

rmse           14.565130

Name: 999, dtype: float64

-----  
-----  
Final Validation results

val\_mae        8.612176

val\_mse        224.111494

val\_r\_sqr      0.928040

val\_rmse       14.970354

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse            263.936692

rmse           16.246129

```
mae          9.433544
r_sqr        0.880873
st_mse       0.000677
st_rmse      0.026011
st_mae       0.016009
st_r_sqr     0.938698
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with 1500 epochs

```
-----
-----
```

Final Training results

```
mae          7.725300
mse         180.929892
r_sqr        0.939766
rmse        13.451018
Name: 1499, dtype: float64
```

```
-----
-----
```

Final Validation results

```
val_mae       8.400230
val_mse      230.727938
val_r_sqr     0.930766
val_rmse     15.189731
Name: 1499, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse          763.629961
rmse         27.633855
mae          17.428569
r_sqr        0.756392
st_mse       0.001362
st_rmse      0.036908
st_mae       0.019721
st_r_sqr     0.933314
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with 2000 epochs

```
-----
```

```
-----  
Final Training results  
mae          9.859058  
mse          323.622713  
r_sqr         0.903529  
rmse         17.989517  
Name: 1999, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae       9.298527  
val_mse       262.621712  
val_r_sqr     0.897150  
val_rmse      16.205607  
Name: 1999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          1534.300343  
rmse         39.170146  
mae          23.837348  
r_sqr         0.465289  
st_mse        0.001491  
st_rmse       0.038611  
st_mae        0.023668  
st_r_sqr      0.919180  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model trained with 2500 epochs

```
-----
```

```
-----  
Final Training results  
mae          10.612285  
mse          311.711768  
r_sqr         0.905703  
rmse         17.655361  
Name: 2499, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae       9.807566  
val_mse       261.335346  
val_r_sqr     0.908631  
val_rmse      16.165870
```

Name: 2499, dtype: float64

-----  
-----  
Test Set Results

|          |             |
|----------|-------------|
| mse      | 2619.121607 |
| rmse     | 51.177354   |
| mae      | 32.500738   |
| r_sqr    | 0.035265    |
| st_mse   | 0.003658    |
| st_rmse  | 0.060480    |
| st_mae   | 0.037223    |
| st_r_sqr | 0.766383    |

Name: 0, dtype: float64

=====

Model trained with 3000 epochs

-----  
-----  
Final Training results

|       |            |
|-------|------------|
| mae   | 6.129363   |
| mse   | 124.789594 |
| r_sqr | 0.960619   |
| rmse  | 11.170926  |

Name: 2999, dtype: float64

-----  
-----  
Final Validation results

|           |            |
|-----------|------------|
| val_mae   | 6.326957   |
| val_mse   | 124.882068 |
| val_r_sqr | 0.952840   |
| val_rmse  | 11.175065  |

Name: 2999, dtype: float64

-----  
-----  
Test Set Results

|          |            |
|----------|------------|
| mse      | 648.318969 |
| rmse     | 25.462109  |
| mae      | 14.113753  |
| r_sqr    | 0.802339   |
| st_mse   | 0.001672   |
| st_rmse  | 0.040895   |
| st_mae   | 0.024533   |
| st_r_sqr | 0.926110   |

Name: 0, dtype: float64

=====

Model trained with 3500 epochs

-----

Final Training results

mae 8.736078  
mse 250.379080  
r\_sqr 0.920063  
rmse 15.823371  
Name: 3499, dtype: float64

-----

Final Validation results

val\_mae 9.033819  
val\_mse 256.454614  
val\_r\_sqr 0.921544  
val\_rmse 16.014200  
Name: 3499, dtype: float64

-----

Test Set Results

mse 313.532026  
rmse 17.706836  
mae 10.328859  
r\_sqr 0.888670  
st\_mse 0.004751  
st\_rmse 0.068925  
st\_mae 0.038536  
st\_r\_sqr 0.740614  
Name: 0, dtype: float64

=====

=====

Model trained with 4000 epochs

-----

Final Training results

mae 6.670029  
mse 157.370618  
r\_sqr 0.949555  
rmse 12.544745  
Name: 3999, dtype: float64

-----



```
-----  
Final Validation results  
val_mae      6.234223  
val_mse      178.456109  
val_r_sqr    0.935996  
val_rmse     13.358747  
Name: 3999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          212.698633  
rmse         14.584191  
mae          7.794754  
r_sqr        0.936354  
st_mse       0.000564  
st_rmse      0.023739  
st_mae       0.013208  
st_r_sqr     0.948000  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model trained with 4500 epochs

```
-----  
Final Training results  
mae          6.500001  
mse          155.093720  
r_sqr        0.946919  
rmse         12.453663  
Name: 4499, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae      6.386406  
val_mse      131.872036  
val_r_sqr    0.959946  
val_rmse     11.483555  
Name: 4499, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          932.030656  
rmse         30.529177  
mae          17.138363  
r_sqr        0.725804
```

```
st_mse      0.001296
st_rmse     0.036001
st_mae      0.020815
st_r_sqr    0.940711
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with 5000 epochs

-----  
Final Training results

```
mae      8.351984
mse     199.404995
r_sqr    0.934940
rmse     14.121083
Name: 4999, dtype: float64
```

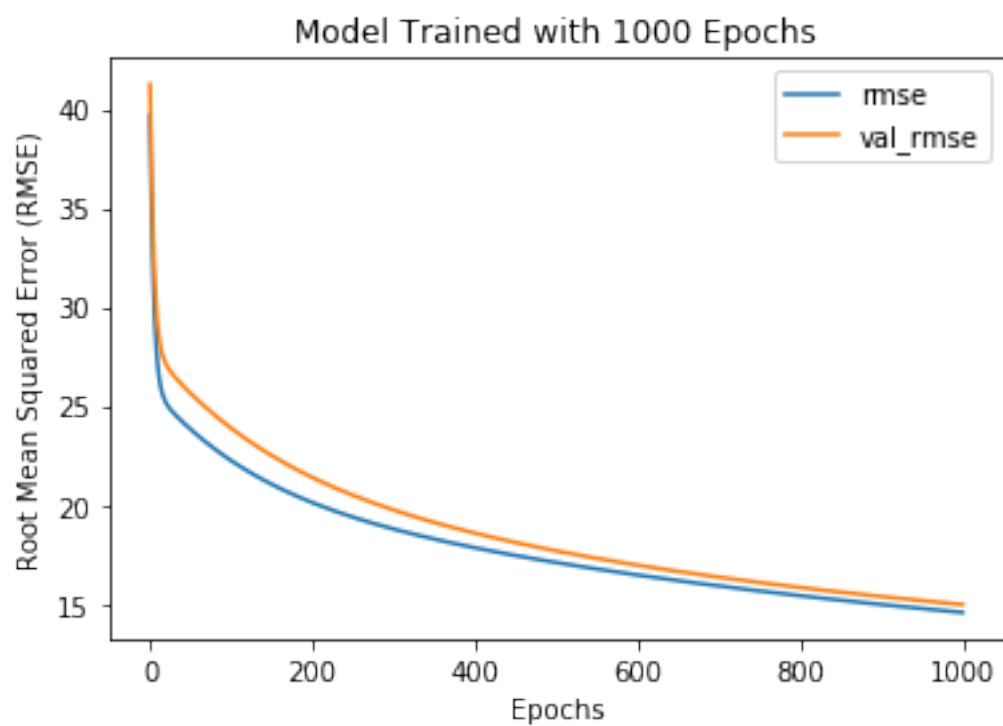
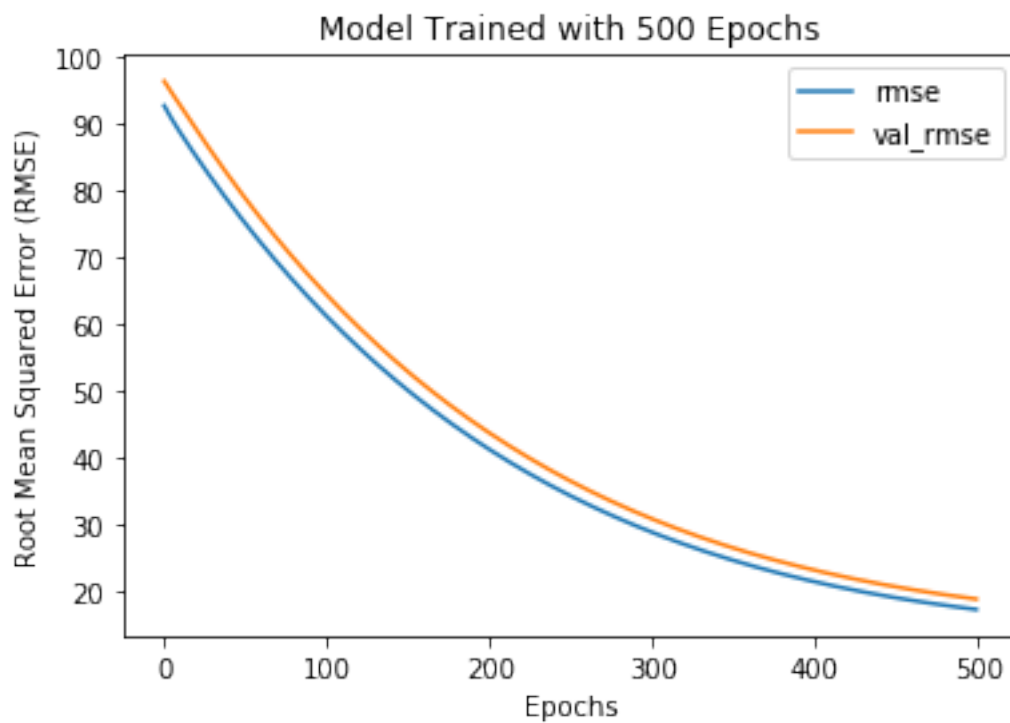
-----  
Final Validation results

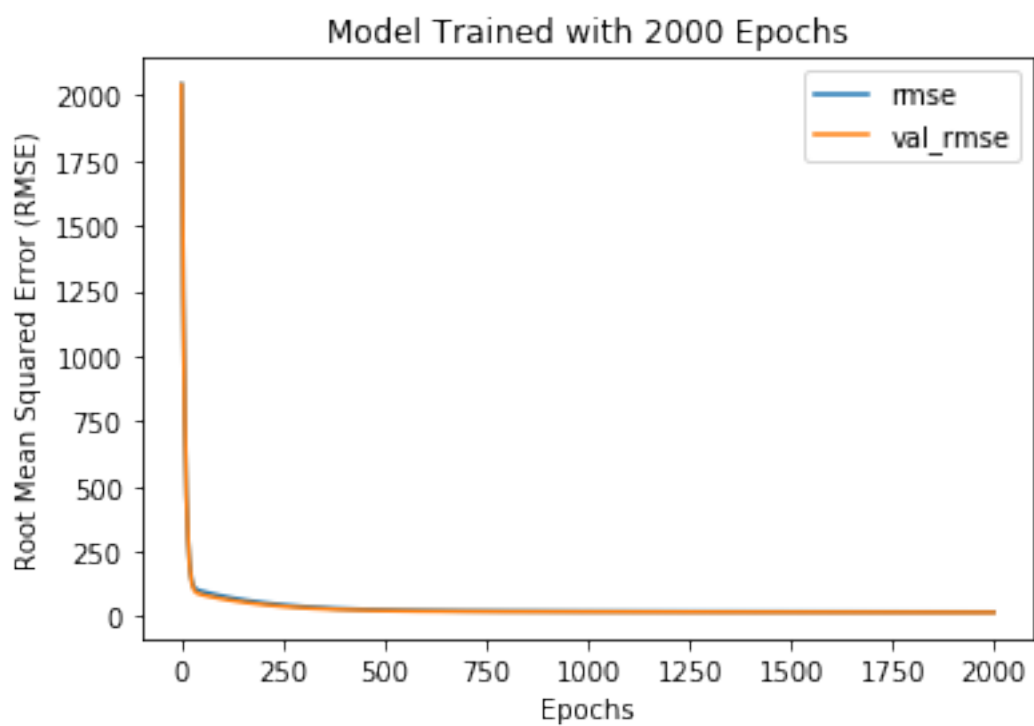
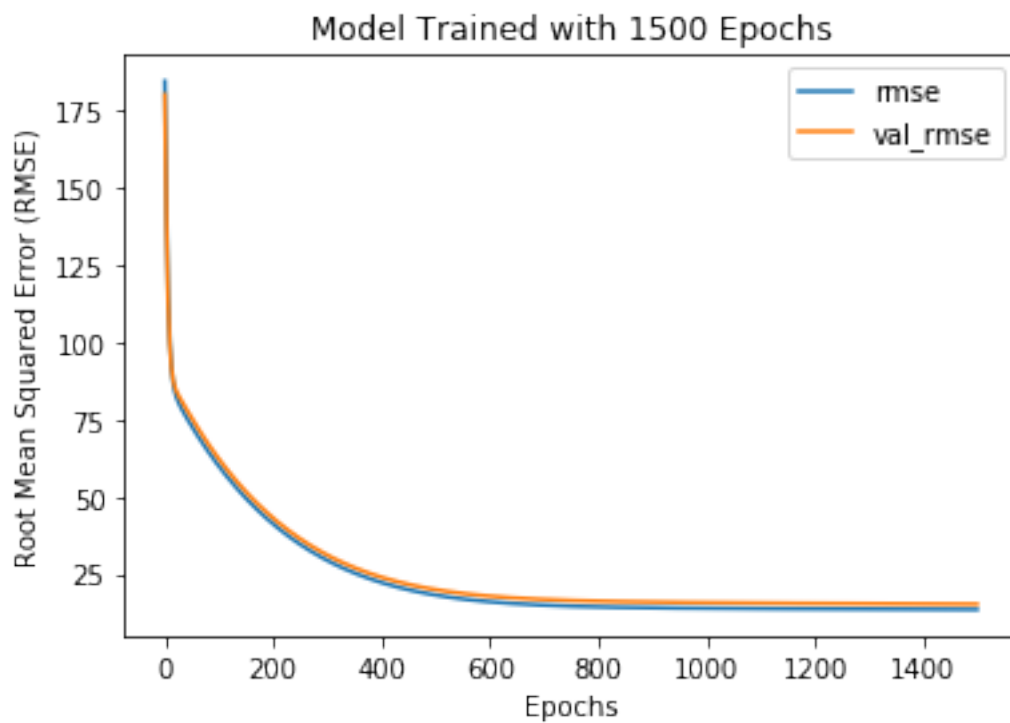
```
val_mae    8.269012
val_mse   176.502604
val_r_sqr  0.949772
val_rmse   13.285428
Name: 4999, dtype: float64
```

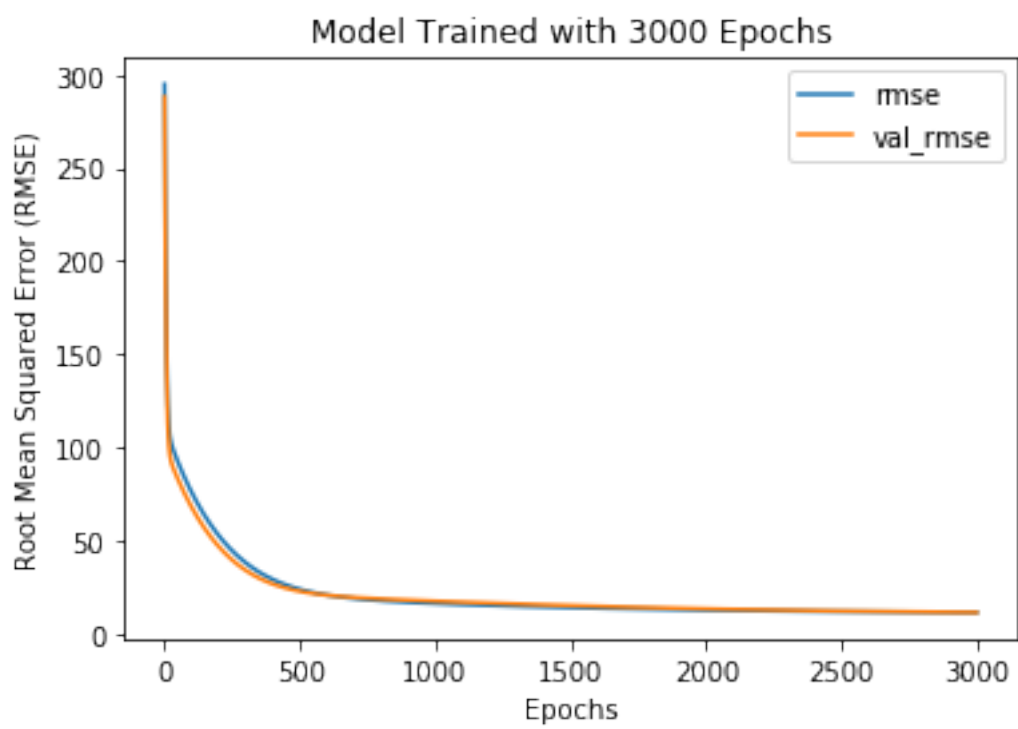
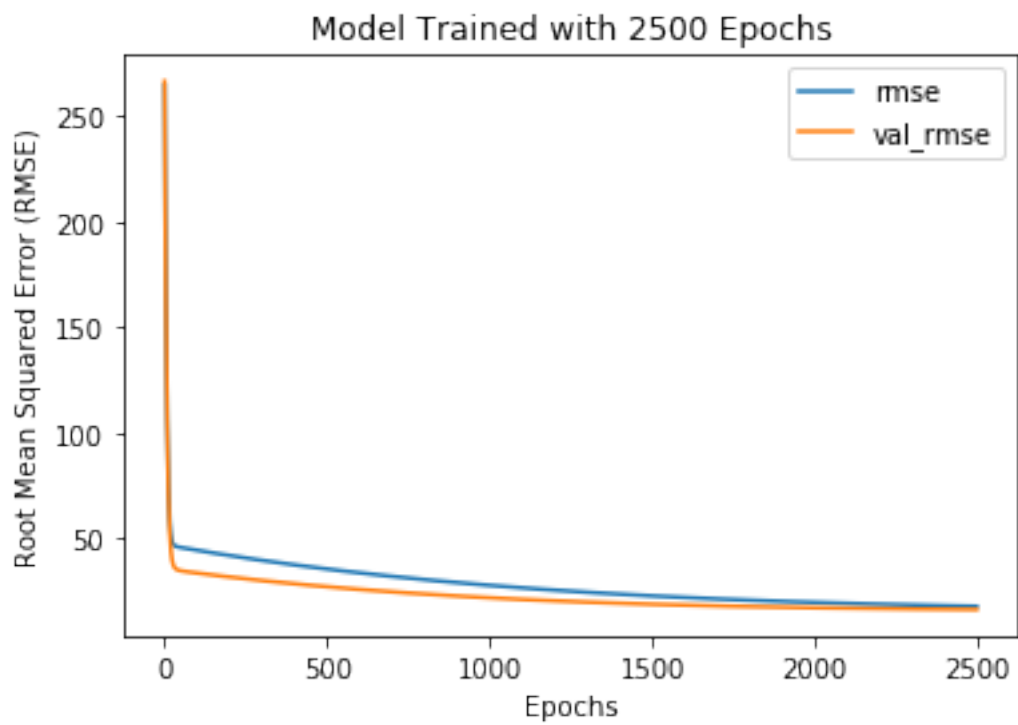
-----  
Test Set Results

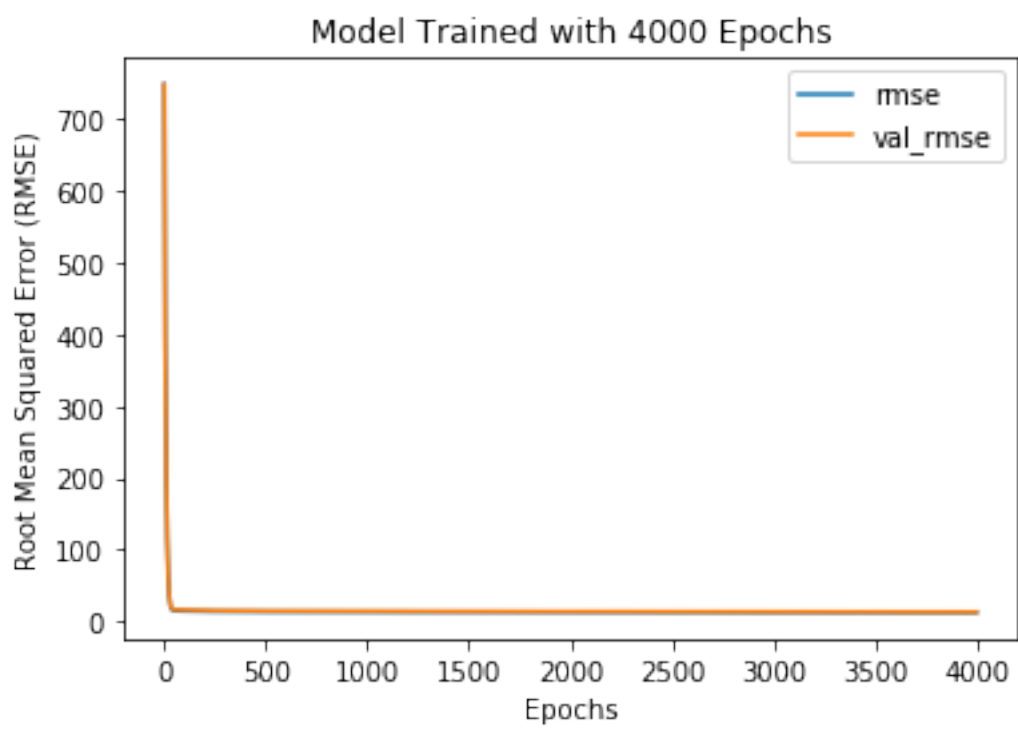
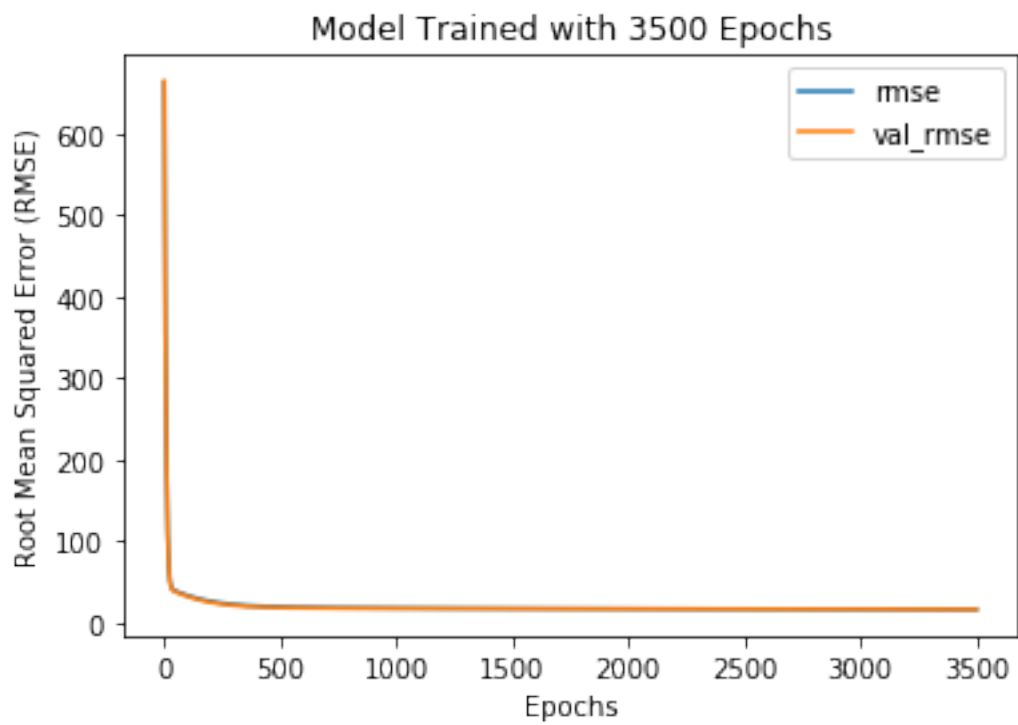
```
mse      830.270766
rmse     28.814419
mae      15.929263
r_sqr    0.702218
st_mse    0.005381
st_rmse   0.073353
st_mae    0.039558
st_r_sqr  0.794596
Name: 0, dtype: float64
```

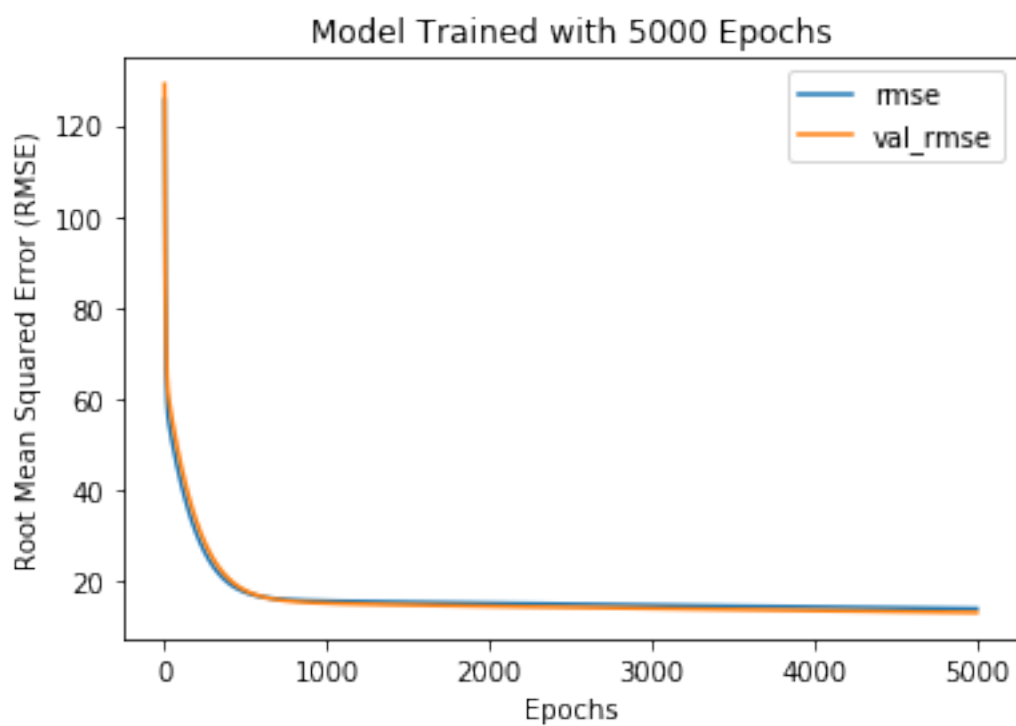
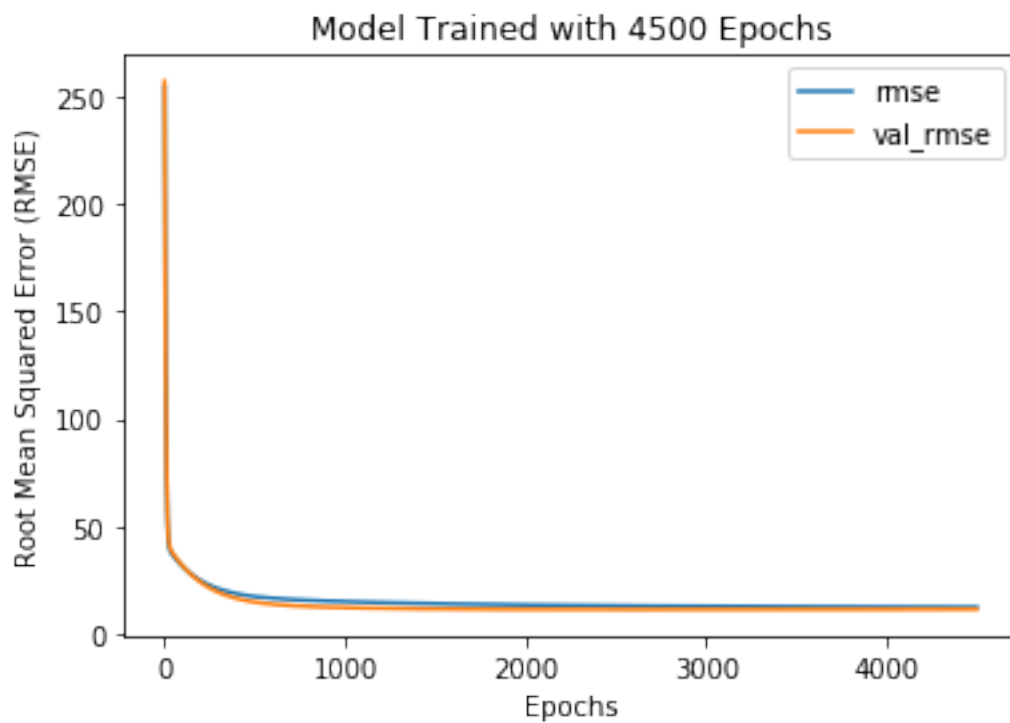
```
=====
=====
```











- Models seem to overfit after about 800 to 1200 epochs
- Suitable number for training is around 1000

## Learning Rate

```
[11]: l_rate_tests = dict()
      for i in range(1, 11):
          l_rate_tests[f"Test-{i}"] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", 1),
              activ_func="linear",
              epochs=1000,
              l_rate=0.1*i
          )

[12]: for i in range(1,11):
      print(f"Model Trained with Learning Rate of {0.1*i}", end=f"\n{'-'*100}\n")

      print("Final Training results")
      print(l_rate_tests[f"Test-{i}"]["training_results"].iloc[-1, :4],
            ↪end=f"\n{'-'*100}\n")

      print("Final Validation results")
      print(l_rate_tests[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
            ↪end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(l_rate_tests[f"Test-{i}"]["error_metrics"].iloc[0],
            ↪end=f"\n{'-'*100}\n\n\n")

      ax = l_rate_tests[f"Test-{i}"]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with Learning Rate of
            ↪{i*0.1}",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Learning Rate of 0.1

```
-----
Final Training results
mae      6.728895
mse     144.544700
r_sqr     0.954585
rmse     12.022674
Name: 999, dtype: float64
-----
```



```
-----  
Final Validation results  
val_mae      6.827042  
val_mse      154.903146  
val_r_sqr     0.951899  
val_rmse     12.446009  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          166.056780  
rmse         12.886302  
mae          6.526054  
r_sqr        0.938928  
st_mse       0.001043  
st_rmse      0.032299  
st_mae       0.015748  
st_r_sqr     0.923022  
Name: 0, dtype: float64  
=====
```

Model Trained with Learning Rate of 0.2

```
-----  
Final Training results  
mae          9.262914  
mse          288.251083  
r_sqr        0.913986  
rmse         16.977959  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae      8.271171  
val_mse      240.247581  
val_r_sqr     0.909595  
val_rmse     15.499922  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          361.894289  
rmse         19.023519  
mae          9.305752  
r_sqr        0.867981
```

```
st_mse      0.004561
st_rmse     0.067532
st_mae      0.035913
st_r_sqr    0.800938
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Learning Rate of 0.30000000000000004

```
-----
-----
```

Final Training results

```
mae      8.066564
mse     204.581414
r_sqr    0.930061
rmse    14.303196
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation results

```
val_mae    8.145104
val_mse   229.466800
val_r_sqr  0.916561
val_rmse   15.148162
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse      236.679537
rmse     15.384393
mae       9.438774
r_sqr    0.939067
st_mse    0.001687
st_rmse   0.041069
st_mae    0.024278
st_r_sqr  0.912746
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Learning Rate of 0.4

```
-----
-----
```

Final Training results

```
mae      8.032468
mse     223.892823
r_sqr    0.926797
rmse    14.963049
Name: 999, dtype: float64
```

---

Final Validation results

```
val_mae      8.067513
val_mse     192.114710
val_r_sqr    0.934863
val_rmse    13.860545
Name: 999, dtype: float64
```

---

Test Set Results

```
mse      359.787579
rmse     18.968067
mae     11.404283
r_sqr    0.892779
st_mse    0.001621
st_rmse   0.040257
st_mae    0.021636
st_r_sqr  0.916093
Name: 0, dtype: float64
```

---

```
=====
=====
```

Model Trained with Learning Rate of 0.5

---

Final Training results

```
mae      6.481272
mse     137.914126
r_sqr    0.955400
rmse     11.743685
Name: 999, dtype: float64
```

---

Final Validation results

```
val_mae      6.547086
val_mse     172.664087
val_r_sqr    0.950973
val_rmse    13.140171
Name: 999, dtype: float64
```

---

```

-----
Test Set Results
mse          1857.773818
rmse         43.101900
mae          26.614465
r_sqr        0.310309
st_mse       0.001140
st_rmse      0.033761
st_mae       0.019458
st_r_sqr     0.938681
Name: 0, dtype: float64
=====
=====

```

Model Trained with Learning Rate of 0.6000000000000001

```

-----
Final Training results
mae          6.137523
mse         131.414319
r_sqr        0.959792
rmse        11.463608
Name: 999, dtype: float64
-----

```

```

-----
Final Validation results
val_mae      5.821425
val_mse     132.483197
val_r_sqr    0.956994
val_rmse    11.510135
Name: 999, dtype: float64
-----

```

```

-----
Test Set Results
mse          1153.281994
rmse         33.960006
mae          20.625440
r_sqr        0.555230
st_mse       0.000767
st_rmse      0.027698
st_mae       0.015266
st_r_sqr     0.957287
Name: 0, dtype: float64
=====
=====

```

Model Trained with Learning Rate of 0.7000000000000001

Final Training results

mae 5.179923  
mse 104.984511  
r\_sqr 0.965446  
rmse 10.246195  
Name: 999, dtype: float64

Final Validation results

val\_mae 5.386293  
val\_mse 120.189784  
val\_r\_sqr 0.949562  
val\_rmse 10.963110  
Name: 999, dtype: float64

Test Set Results

mse 777.561693  
rmse 27.884793  
mae 14.325743  
r\_sqr 0.804767  
st\_mse 0.000993  
st\_rmse 0.031512  
st\_mae 0.014685  
st\_r\_sqr 0.950056  
Name: 0, dtype: float64

Model Trained with Learning Rate of 0.8

Final Training results

mae 6.580505  
mse 152.342796  
r\_sqr 0.947733  
rmse 12.342722  
Name: 999, dtype: float64

Final Validation results

```
val_mae      7.899153
val_mse      215.759098
val_r_sqr     0.940280
val_rmse     14.688741
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          99.381783
rmse         9.969041
mae          5.665376
r_sqr        0.967669
st_mse       0.000953
st_rmse      0.030868
st_mae       0.015169
st_r_sqr     0.937715
Name: 0, dtype: float64
```

=====

Model Trained with Learning Rate of 0.9

-----  
-----  
Final Training results

```
mae          6.368586
mse         154.791285
r_sqr        0.953307
rmse        12.441515
Name: 999, dtype: float64
```

-----  
-----  
Final Validation results

```
val_mae      6.191001
val_mse     124.999111
val_r_sqr     0.952075
val_rmse     11.180300
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          353.851397
rmse         18.810938
mae          10.752358
r_sqr        0.879272
st_mse       0.000580
st_rmse      0.024089
```

```
st_mae      0.012475
st_r_sqr     0.960218
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Learning Rate of 1.0

```
-----
-----
```

Final Training results

```
mae      6.446859
mse     141.742626
r_sqr    0.955973
rmse     11.905571
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation results

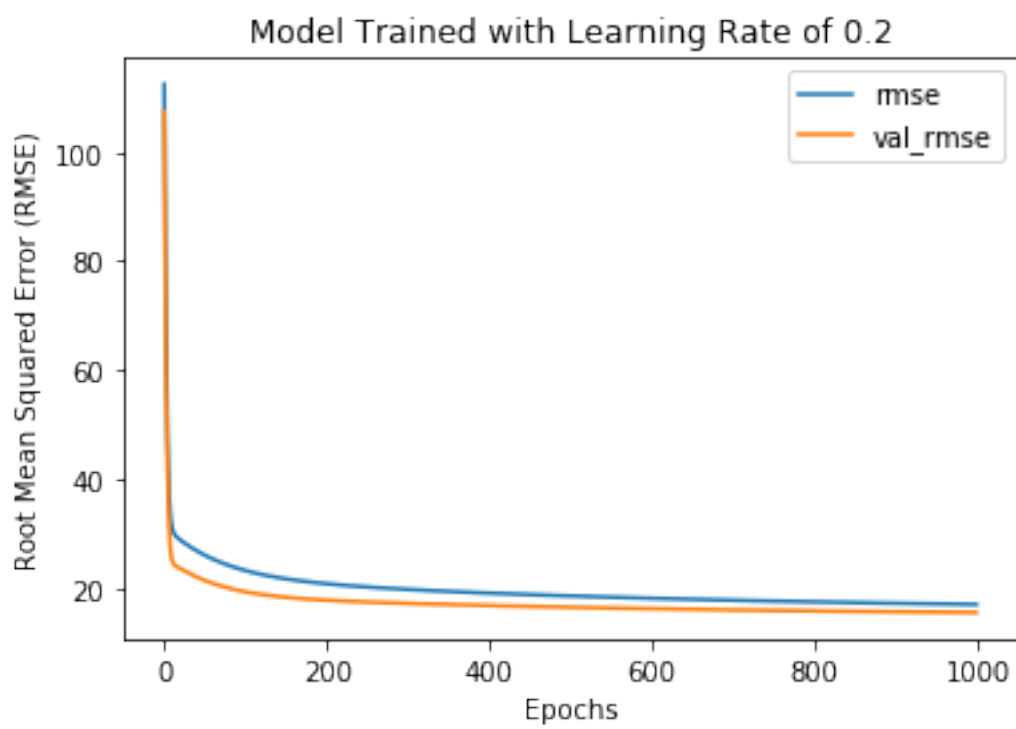
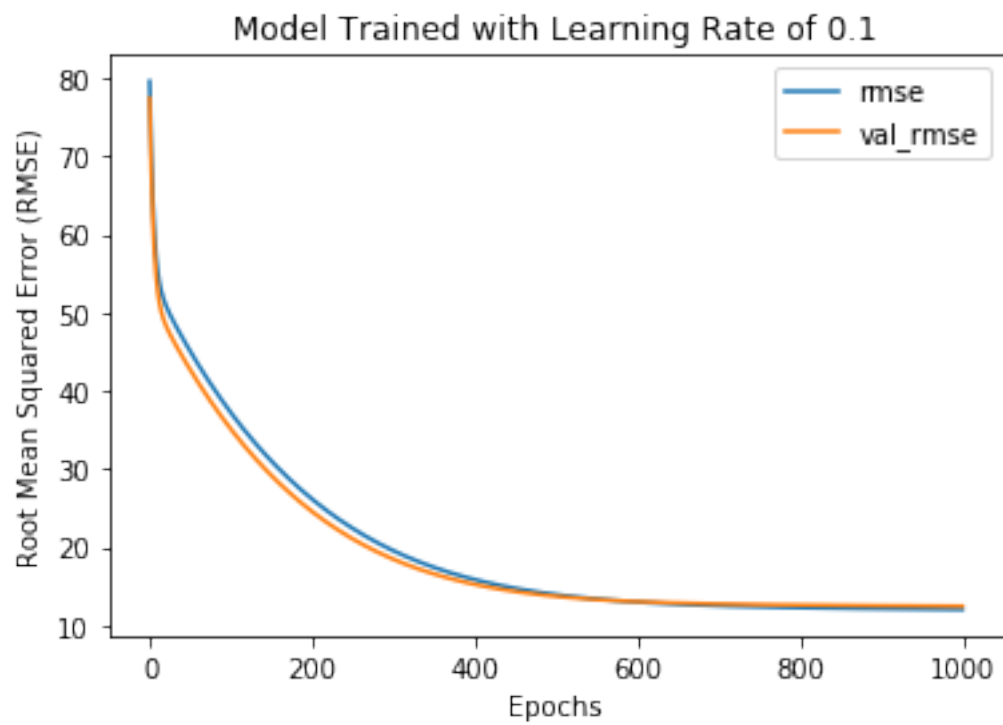
```
val_mae    7.205870
val_mse    183.563748
val_r_sqr   0.938170
val_rmse    13.548570
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

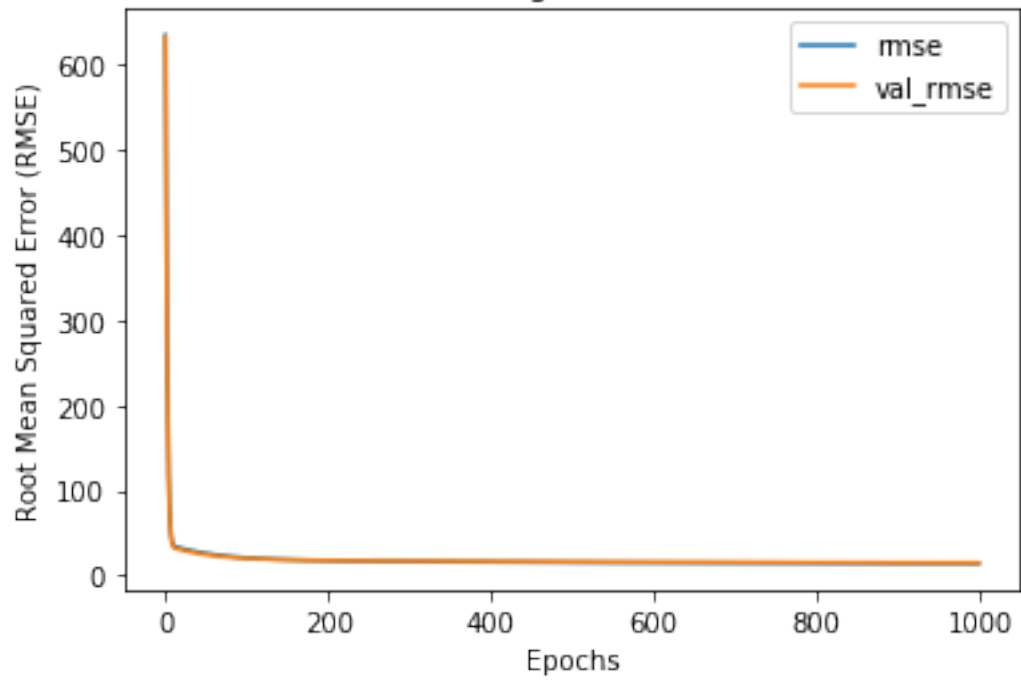
```
mse      173.410666
rmse     13.168548
mae       7.166303
r_sqr     0.939510
st_mse     0.000876
st_rmse    0.029593
st_mae     0.015108
st_r_sqr   0.938673
Name: 0, dtype: float64
```

```
=====
=====
```

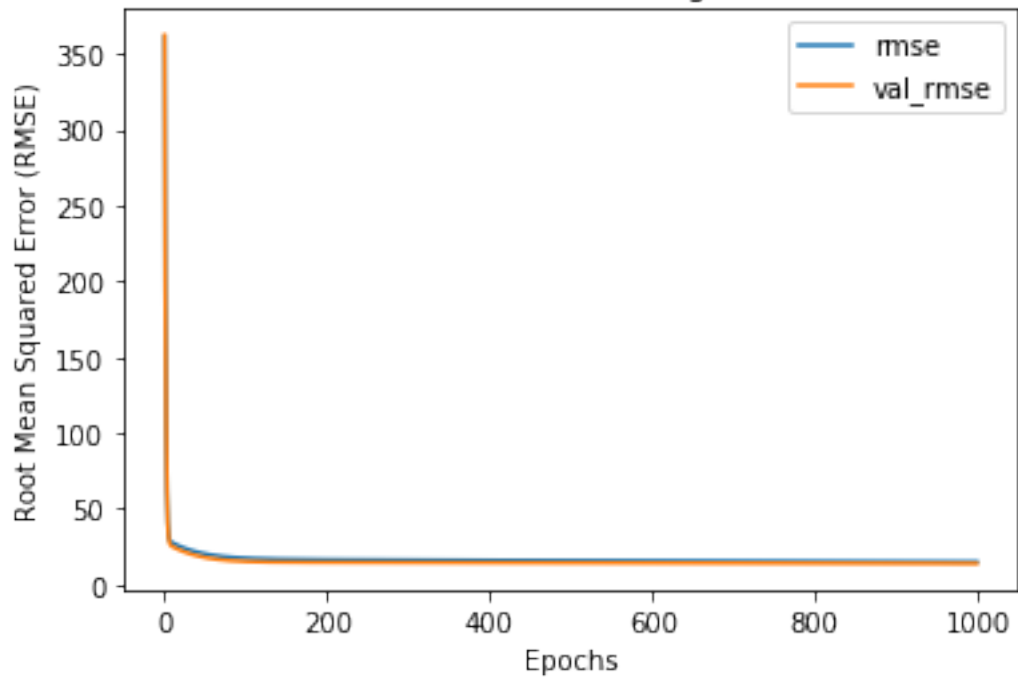


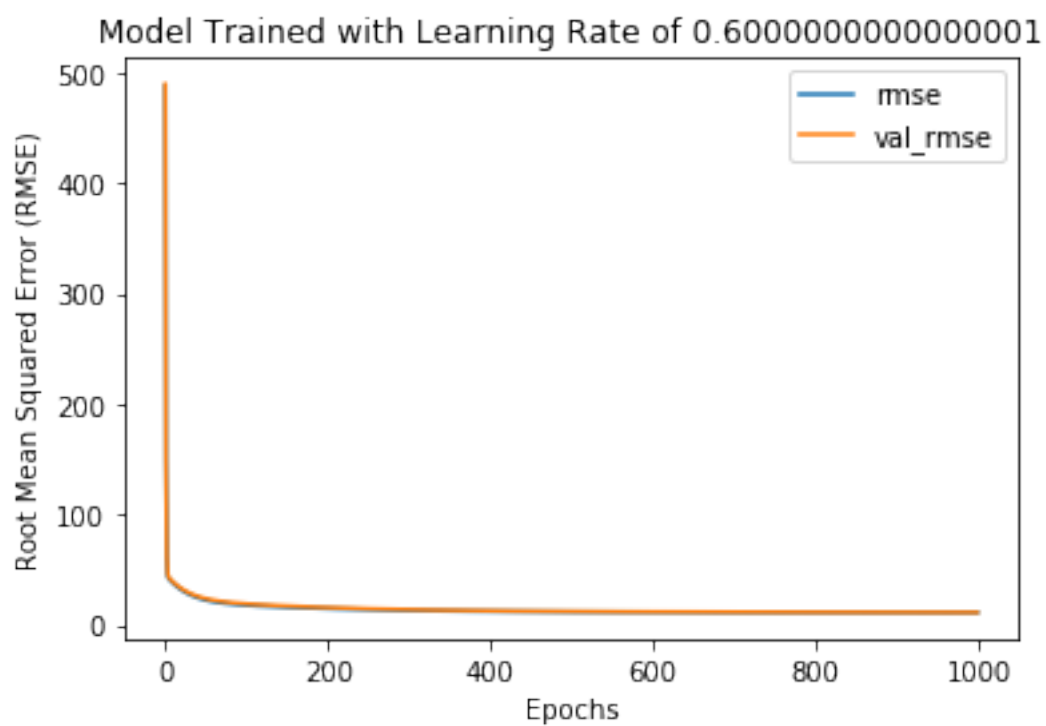
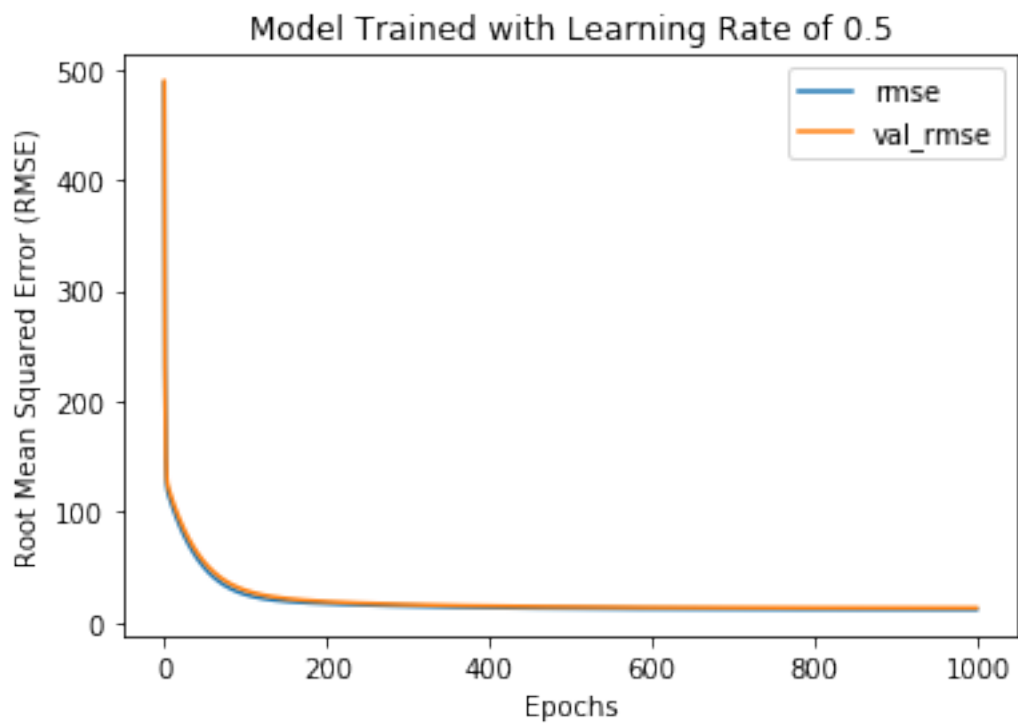


Model Trained with Learning Rate of 0.30000000000000004

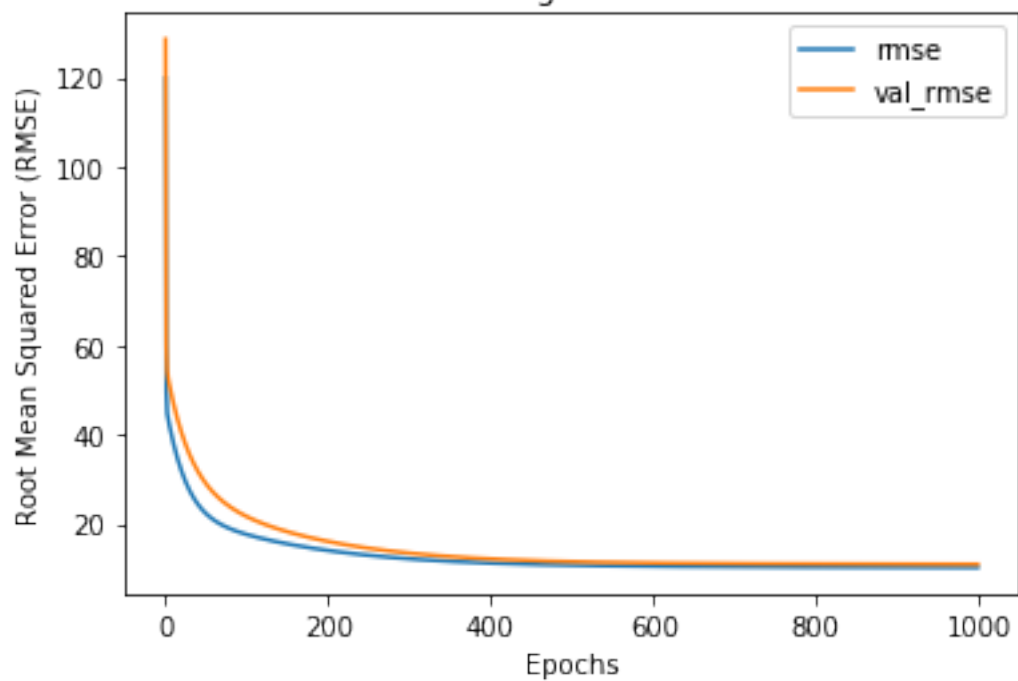


Model Trained with Learning Rate of 0.4

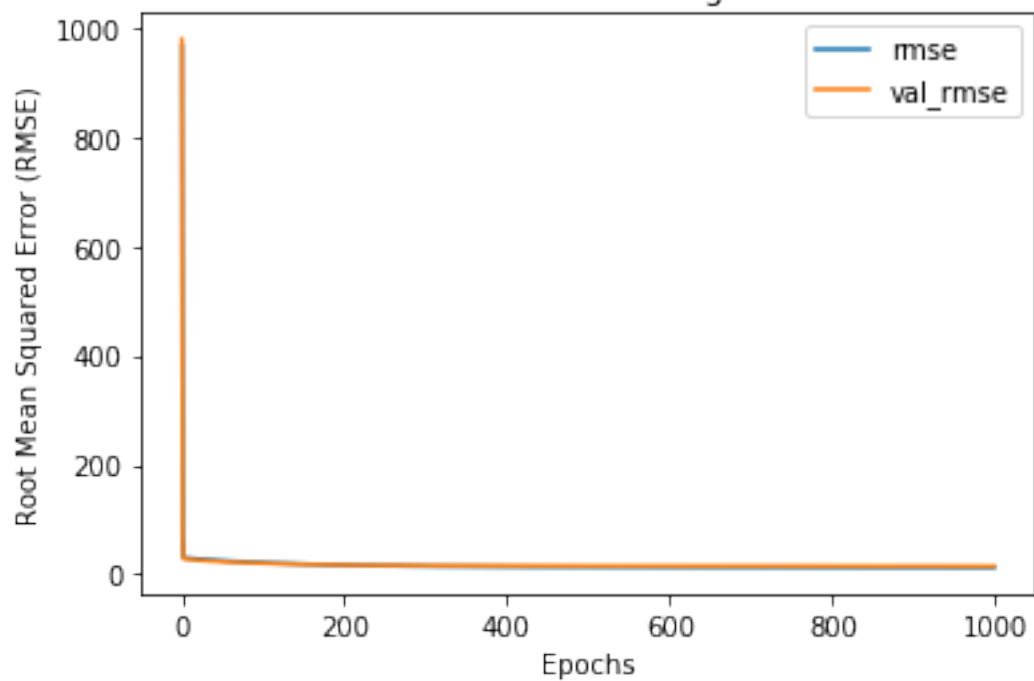


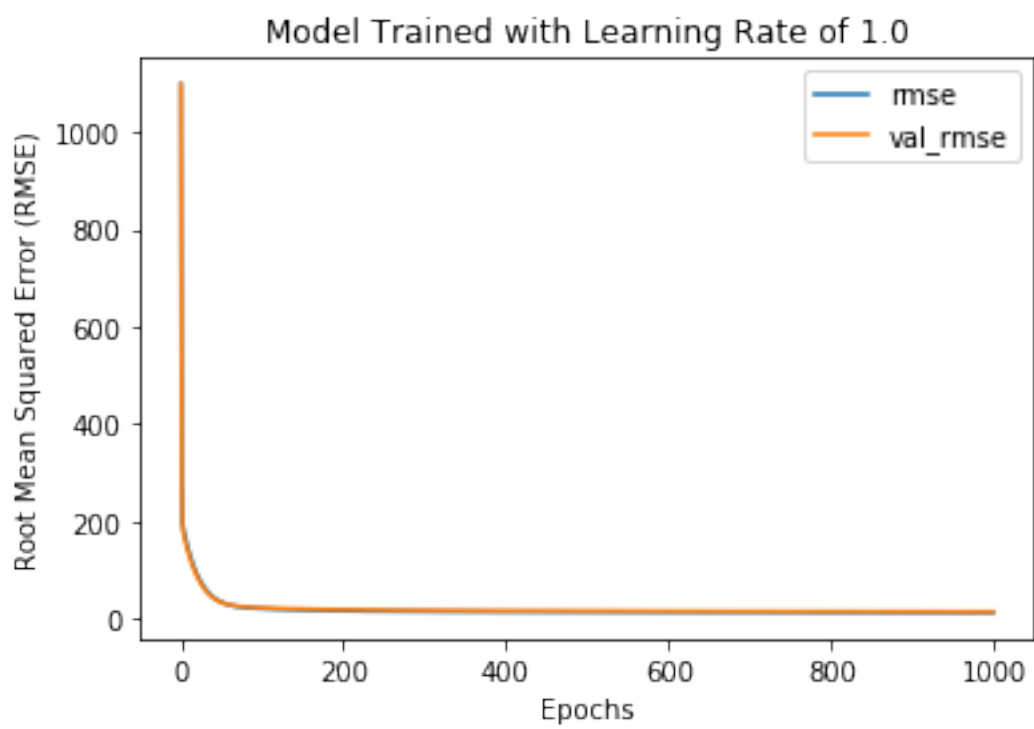
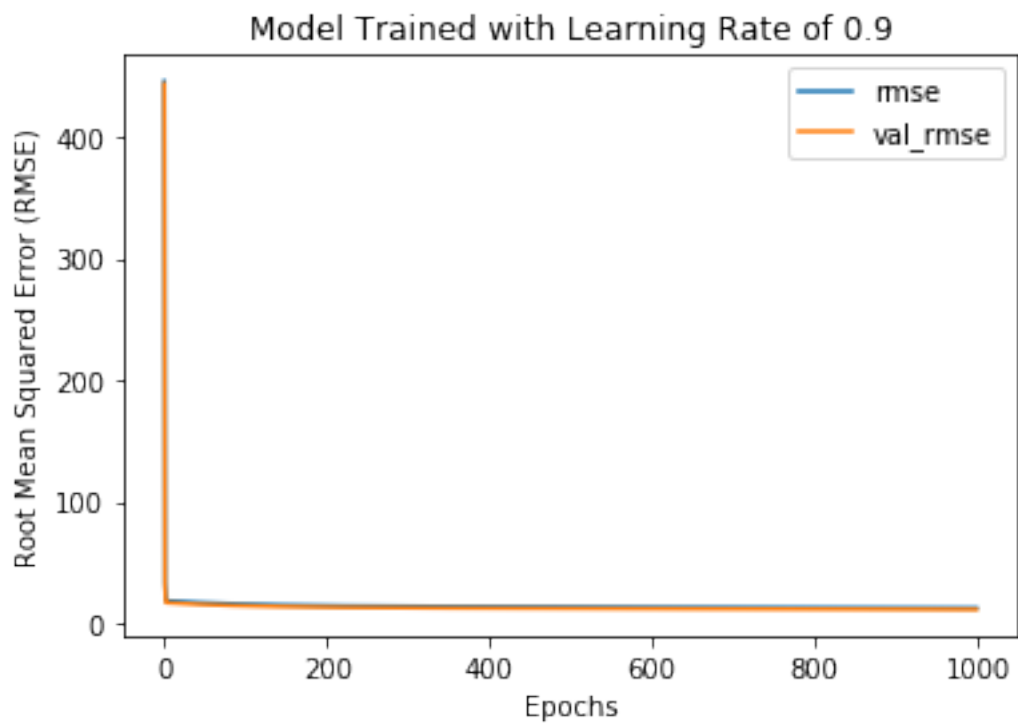


Model Trained with Learning Rate of 0.7000000000000001



Model Trained with Learning Rate of 0.8





- learning rate of 0.3 appears to offer most consistent results between training, validation and testing sets

### Activation Functions

```
[15]: activ_tests = dict()
      activations = ("sigmoid", "tanh", "relu", "linear")

      for func in activations:
          activ_tests[func] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", 1),
              activ_func=func,
              epochs=1000,
              l_rate=0.3
          )

[16]: for a in activations:
      print(f"Model Trained with {a.capitalize()} Activation Function",
      ↪end=f"\n{'-'*100}\n")

      print("Final Training Results")
      print(activ_tests[a]["training_results"].iloc[-1, :4], end=f"\n{'-'*100}\n")

      print("Final Validation Results")
      print(activ_tests[a]["training_results"].iloc[-1, 8:12],
      ↪end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(activ_tests[a]["error_metrics"].iloc[0], end=f"\n{'-'*100}\n\n\n\n")

      ax = activ_tests[a]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with {a.capitalize()}
      ↪Activation Function",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Sigmoid Activation Function

```
-----
-----
Final Training Results
mae      38.514325
mse      2535.839810
r_sqr     0.099437
rmse      50.357123
Name: 999, dtype: float64
```

```
-----  
-----  
Final Validation Results  
val_mae      40.178245  
val_mse      3115.092457  
val_r_sqr     0.116448  
val_rmse     55.813013  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          3205.065470  
rmse         56.613298  
mae          41.138513  
r_sqr        0.081623  
st_mse       0.010746  
st_rmse      0.103662  
st_mae       0.082285  
st_r_sqr     0.050099  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model Trained with Tanh Activation Function

```
-----
```

```
-----  
Final Training Results  
mae          8.854036  
mse          230.046785  
r_sqr        0.928153  
rmse         15.167293  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae      7.966150  
val_mse      175.057784  
val_r_sqr     0.927396  
val_rmse     13.230940  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          278.640078  
rmse         16.692516  
mae          9.667961
```

```
r_sqr      0.919583
st_mse     0.001965
st_rmse    0.044330
st_mae     0.029884
st_r_sqr   0.825110
Name: 0, dtype: float64
```

```
=====
=====
```

#### Model Trained with Relu Activation Function

```
-----
-----
```

##### Final Training Results

```
mae      100.179389
mse      13609.240910
r_sqr    -2.808559
rmse     116.658651
Name: 999, dtype: float64
```

```
-----
-----
```

##### Final Validation Results

```
val_mae   99.653643
val_mse   12710.176655
val_r_sqr -3.573111
val_rmse  112.739419
Name: 999, dtype: float64
```

```
-----
-----
```

##### Test Set Results

```
mse      10807.651568
rmse     103.959856
mae       93.997537
r_sqr    -4.480235
st_mse    0.053553
st_rmse   0.231416
st_mae    0.199983
st_r_sqr  -2.949367
Name: 0, dtype: float64
```

```
=====
=====
```

#### Model Trained with Linear Activation Function

```
-----
-----
```

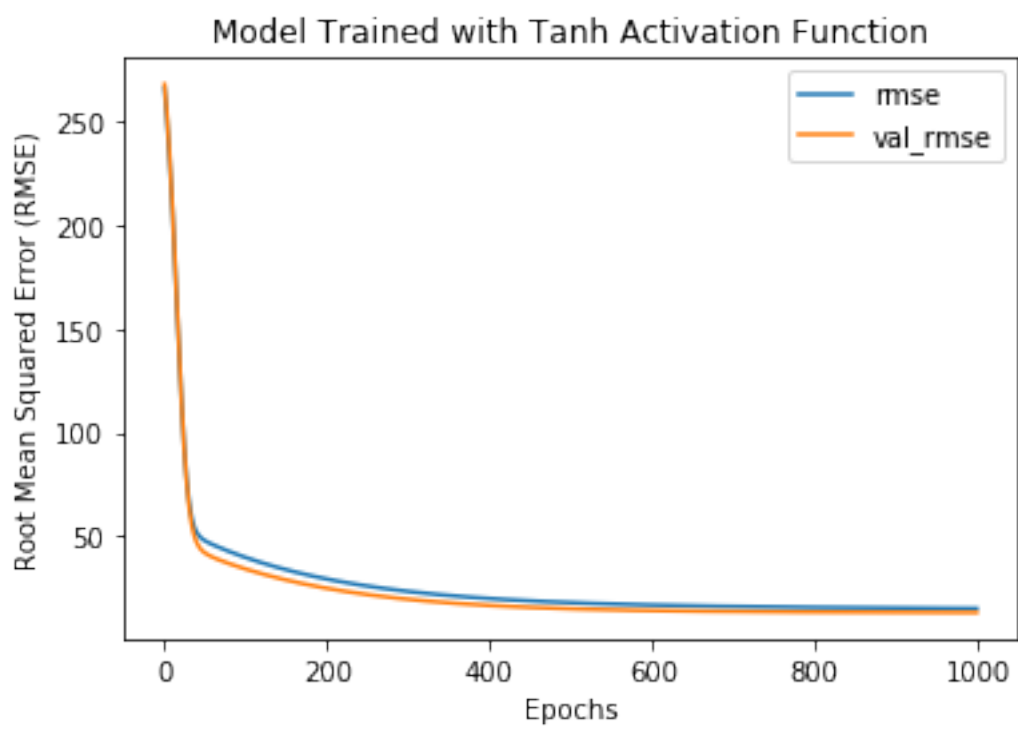
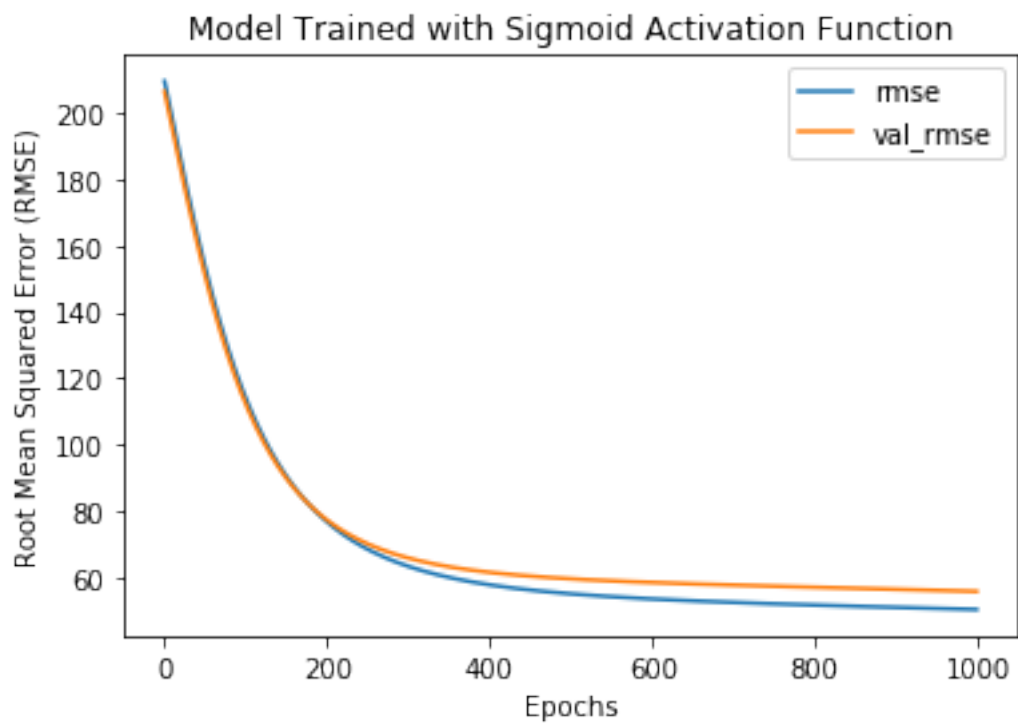
Final Training Results  
mae 12.242387  
mse 389.890987  
r\_sqr 0.868005  
rmse 19.745657  
Name: 999, dtype: float64

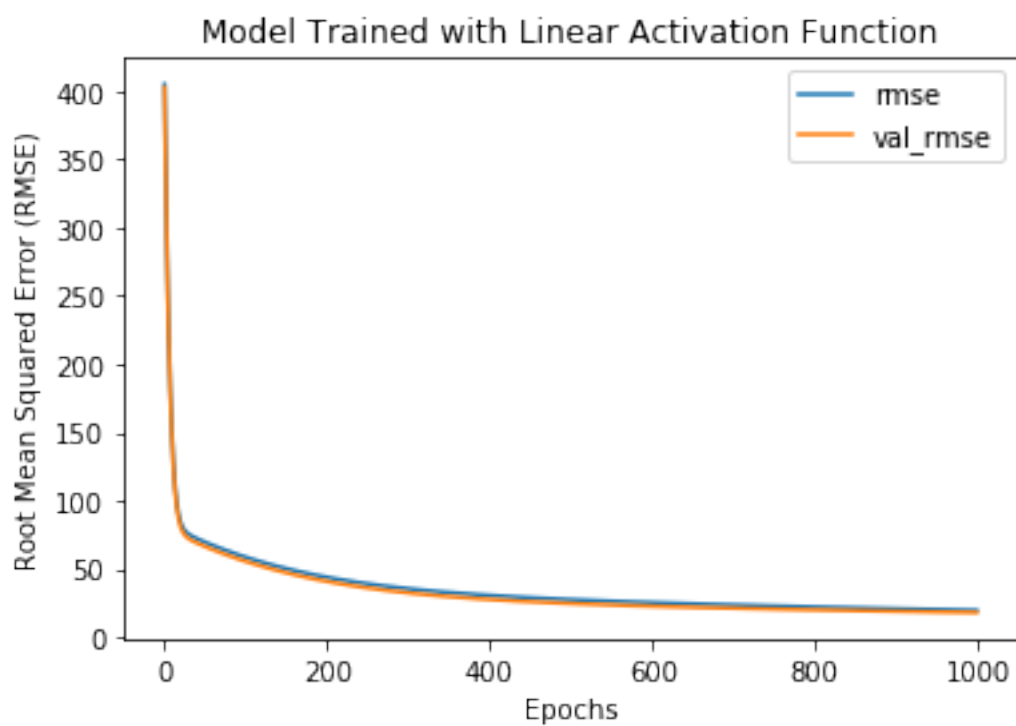
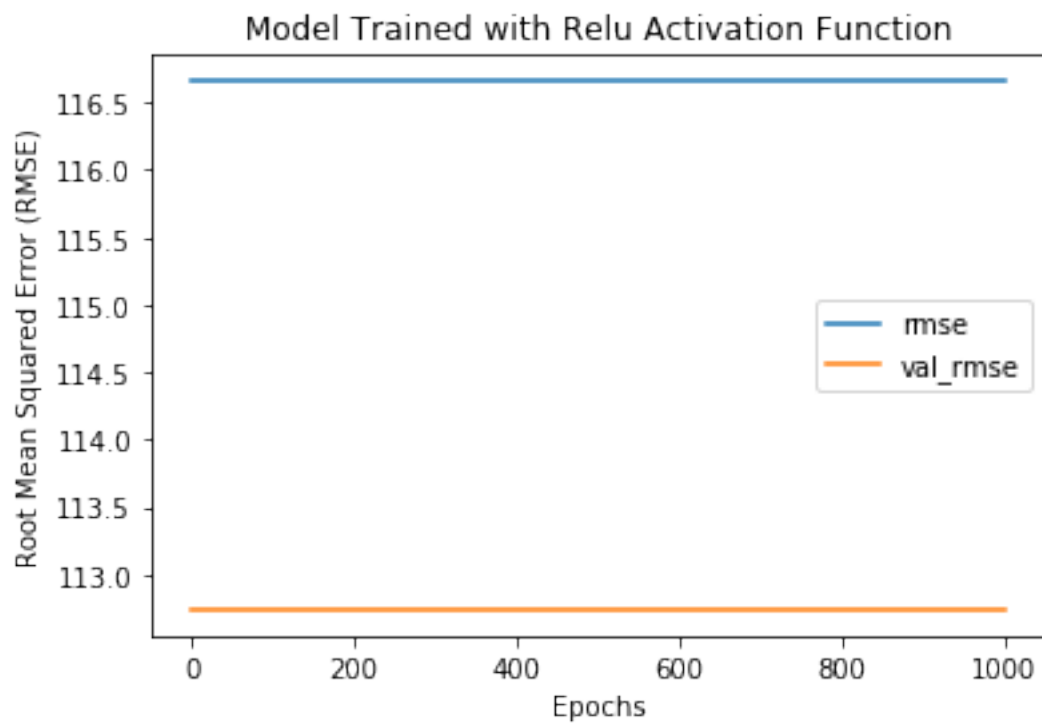
-----  
Final Validation Results  
val\_mae 11.112784  
val\_mse 330.478029  
val\_r\_sqr 0.895496  
val\_rmse 18.179055  
Name: 999, dtype: float64

-----  
Test Set Results  
mse 496.884198  
rmse 22.290899  
mae 13.422733  
r\_sqr 0.856823  
st\_mse 0.001640  
st\_rmse 0.040497  
st\_mae 0.023602  
st\_r\_sqr 0.854173  
Name: 0, dtype: float64

=====







- tanh is quite clearly the best performing activation function

## Hidden Layers

```
[23]: layer_tests = dict()
for i in range(1, 11):
    layer_tests[f"Test-{i}"] = build_train_test(
        fs,
        feature_cols,
        target_cols,
        layers=("auto", i, 1),
        activ_func="tanh",
        epochs=1000,
        l_rate=0.3
    )
```

```
[24]: for i in range(1,11):
        print(f"Model trained with Single Hidden Layer of Size {i}",
        ↪end=f"\n{'-'*100}\n")

        print("Final Training Results")
        print(layer_tests[f"Test-{i}"]["training_results"].iloc[-1, :4],
        ↪end=f"\n{'-'*100}\n")

        print("Final Validation Results")
        print(layer_tests[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
        ↪end=f"\n{'-'*100}\n")

        print("Test Set Results")
        print(layer_tests[f"Test-{i}"]["error_metrics"].iloc[0],
        ↪end=f"\n{'-'*100}\n\n\n\n")

        ax = layer_tests[f"Test-{i}"]["training_results"].plot(
            y=["rmse", "val_rmse"], title=f"Model trained with Single Hidden Layer
            ↪of Size {i}",
        )
        ax.set_xlabel("Epochs")
        ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model trained with Single Hidden Layer of Size 1

-----

Final Training Results

|       |            |
|-------|------------|
| mae   | 6.537448   |
| mse   | 141.683314 |
| r_sqr | 0.953481   |
| rmse  | 11.903080  |

Name: 999, dtype: float64

-----  
-----  
Final Validation Results

val\_mae            6.708576

val\_mse           143.885159

val\_r\_sqr          0.946122

val\_rmse           11.995214

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse                727.067186

rmse               26.964183

mae                15.432374

r\_sqr              0.802375

st\_mse             0.001012

st\_rmse            0.031812

st\_mae             0.017118

st\_r\_sqr           0.957242

Name: 0, dtype: float64

=====

Model trained with Single Hidden Layer of Size 2

-----  
-----  
Final Training Results

mae                10.205298

mse                298.325655

r\_sqr              0.904494

rmse               17.272106

Name: 999, dtype: float64

-----  
-----  
Final Validation Results

val\_mae            9.514489

val\_mse            230.076913

val\_r\_sqr          0.907843

val\_rmse           15.168286

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse                510.334071

rmse               22.590575

```
mae          13.883413
r_sqr         0.858922
st_mse        0.003538
st_rmse       0.059479
st_mae        0.027689
st_r_sqr      0.849668
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with Single Hidden Layer of Size 3

```
-----
-----
```

Final Training Results

```
mae          7.705780
mse         184.419617
r_sqr        0.939006
rmse        13.580118
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

```
val_mae       8.703655
val_mse       217.514316
val_r_sqr     0.925730
val_rmse     14.748367
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse          865.644607
rmse         29.421839
mae          20.407450
r_sqr        0.752135
st_mse       0.001487
st_rmse      0.038567
st_mae       0.021021
st_r_sqr     0.933960
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with Single Hidden Layer of Size 4

```
-----
```

```
-----  
Final Training Results  
mae      11.749247  
mse      349.502596  
r_sqr     0.892044  
rmse     18.694989  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae    9.944671  
val_mse   190.068800  
val_r_sqr   0.905531  
val_rmse   13.786544  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse      428.229721  
rmse     20.693712  
mae      14.541870  
r_sqr     0.885195  
st_mse     0.002819  
st_rmse    0.053096  
st_mae     0.028576  
st_r_sqr   0.868651  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model trained with Single Hidden Layer of Size 5

```
-----
```

```
-----  
Final Training Results  
mae      10.956204  
mse      323.253611  
r_sqr     0.891739  
rmse     17.979255  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae    11.432056  
val_mse   330.366397  
val_r_sqr   0.903258  
val_rmse   18.175984
```

Name: 999, dtype: float64

-----  
-----  
Test Set Results

|          |            |
|----------|------------|
| mse      | 583.194248 |
| rmse     | 24.149415  |
| mae      | 15.859290  |
| r_sqr    | 0.812844   |
| st_mse   | 0.002409   |
| st_rmse  | 0.049082   |
| st_mae   | 0.025178   |
| st_r_sqr | 0.881684   |

Name: 0, dtype: float64

=====

Model trained with Single Hidden Layer of Size 6

-----  
-----  
Final Training Results

|       |            |
|-------|------------|
| mae   | 8.474634   |
| mse   | 217.399014 |
| r_sqr | 0.917411   |
| rmse  | 14.744457  |

Name: 999, dtype: float64

-----  
-----  
Final Validation Results

|           |            |
|-----------|------------|
| val_mae   | 9.297501   |
| val_mse   | 301.599923 |
| val_r_sqr | 0.904126   |
| val_rmse  | 17.366632  |

Name: 999, dtype: float64

-----  
-----  
Test Set Results

|          |            |
|----------|------------|
| mse      | 427.127333 |
| rmse     | 20.667059  |
| mae      | 10.664607  |
| r_sqr    | 0.903359   |
| st_mse   | 0.001479   |
| st_rmse  | 0.038454   |
| st_mae   | 0.023261   |
| st_r_sqr | 0.896758   |

Name: 0, dtype: float64

=====

=====

Model trained with Single Hidden Layer of Size 7

-----

-----  
Final Training Results

mae 7.878499  
mse 185.417026  
r\_sqr 0.942938  
rmse 13.616792  
Name: 999, dtype: float64

-----

-----  
Final Validation Results

val\_mae 8.807907  
val\_mse 220.582351  
val\_r\_sqr 0.922844  
val\_rmse 14.852015  
Name: 999, dtype: float64

-----

-----  
Test Set Results

mse 644.390302  
rmse 25.384844  
mae 15.620218  
r\_sqr 0.776858  
st\_mse 0.002876  
st\_rmse 0.053625  
st\_mae 0.027910  
st\_r\_sqr 0.880527  
Name: 0, dtype: float64

=====

=====

Model trained with Single Hidden Layer of Size 8

-----

-----  
Final Training Results

mae 9.326841  
mse 261.368740  
r\_sqr 0.922007  
rmse 16.166903  
Name: 999, dtype: float64

-----



```
-----  
Final Validation Results  
val_mae      9.086521  
val_mse     220.420668  
val_r_sqr    0.929665  
val_rmse     14.846571  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          459.131447  
rmse         21.427353  
mae          12.533972  
r_sqr        0.799316  
st_mse       0.003604  
st_rmse      0.060036  
st_mae       0.032903  
st_r_sqr     0.816063  
Name: 0, dtype: float64  
=====
```

Model trained with Single Hidden Layer of Size 9

```
-----  
Final Training Results  
mae          8.846222  
mse         208.756315  
r_sqr        0.933647  
rmse        14.448402  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae      8.027564  
val_mse     162.109535  
val_r_sqr    0.940558  
val_rmse     12.732224  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          329.562140  
rmse         18.153846  
mae          12.064198  
r_sqr        0.900492
```

```
st_mse      0.001648
st_rmse     0.040592
st_mae      0.021139
st_r_sqr    0.900036
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with Single Hidden Layer of Size 10

```
-----
-----
```

Final Training Results

```
mae      8.088137
mse     202.259429
r_sqr    0.930396
rmse    14.221794
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

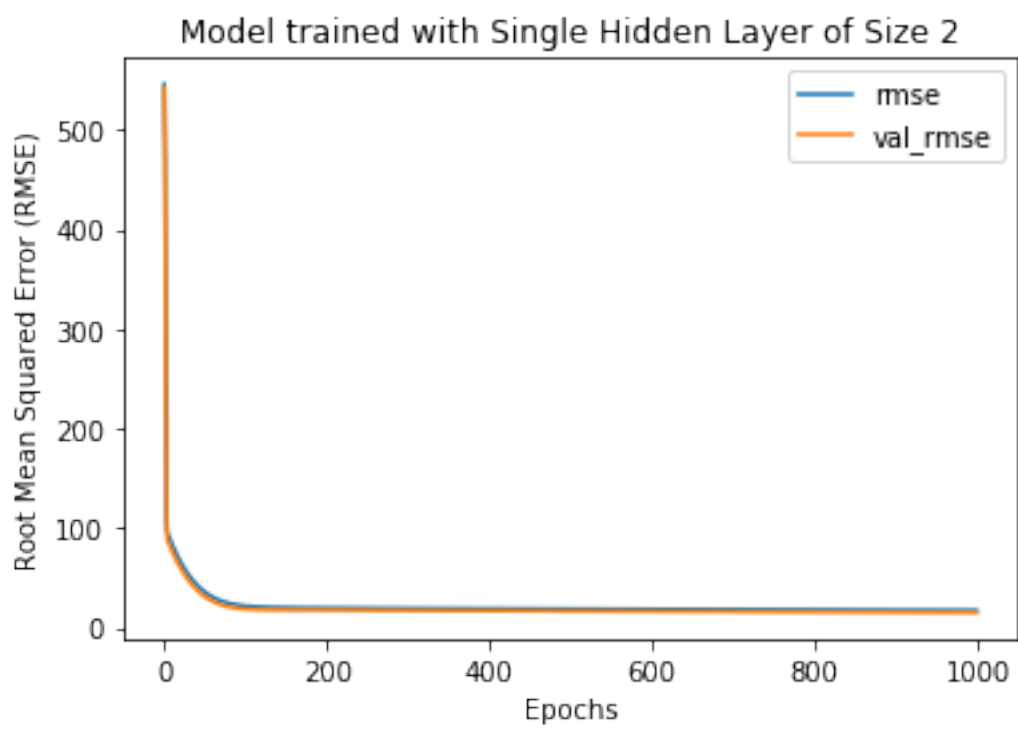
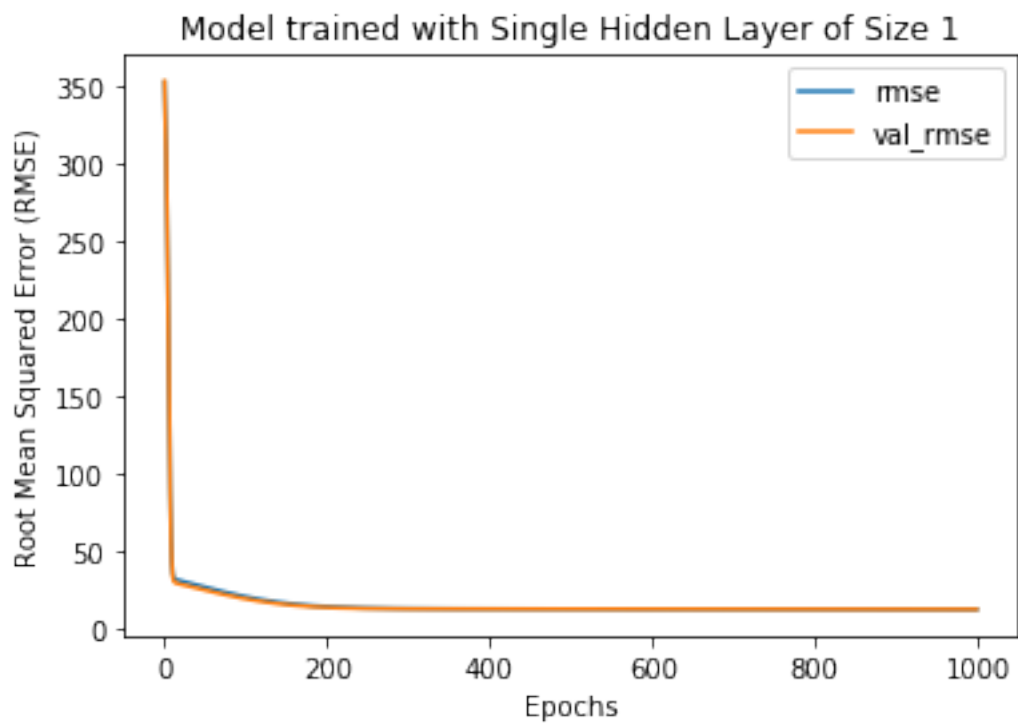
```
val_mae    8.665697
val_mse   191.862063
val_r_sqr   0.951003
val_rmse   13.851428
Name: 999, dtype: float64
```

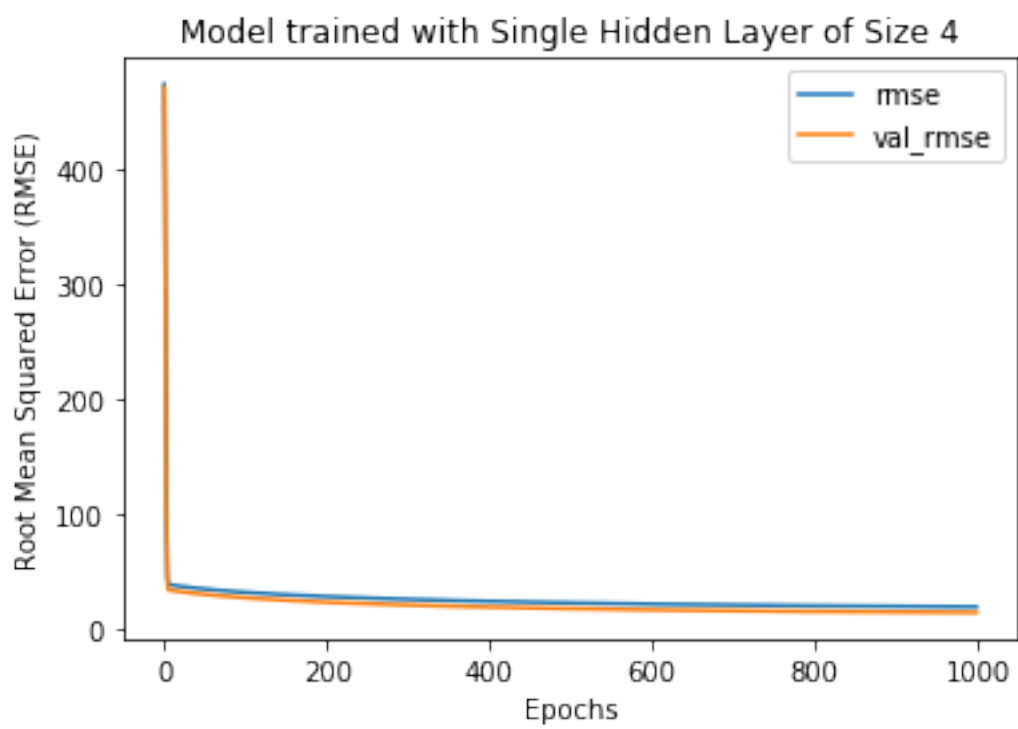
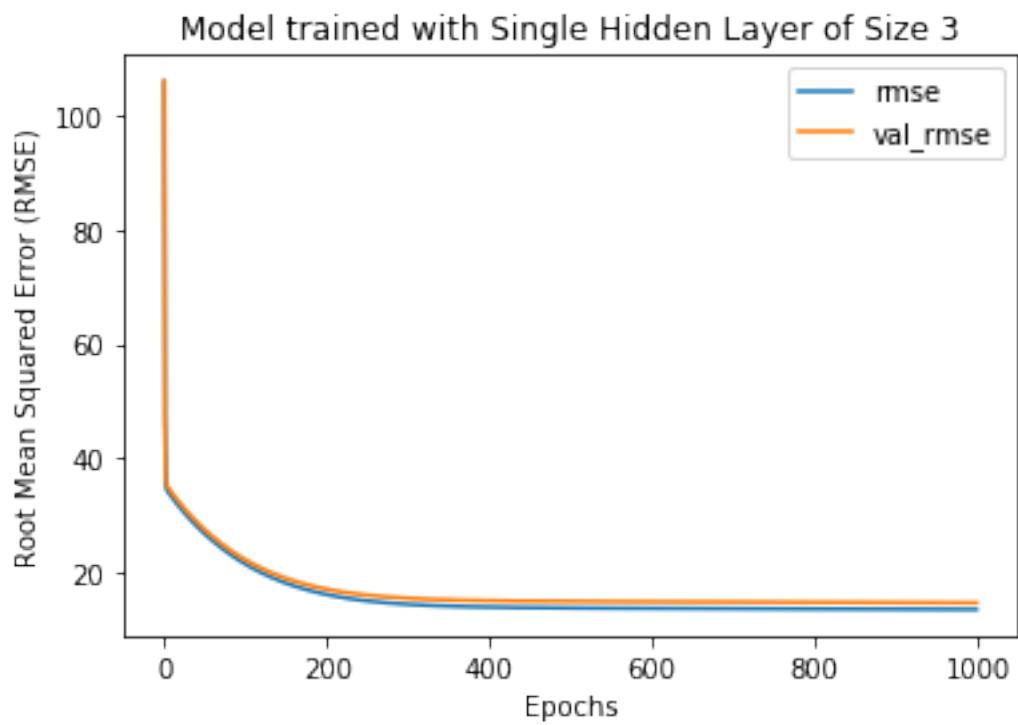
```
-----
-----
```

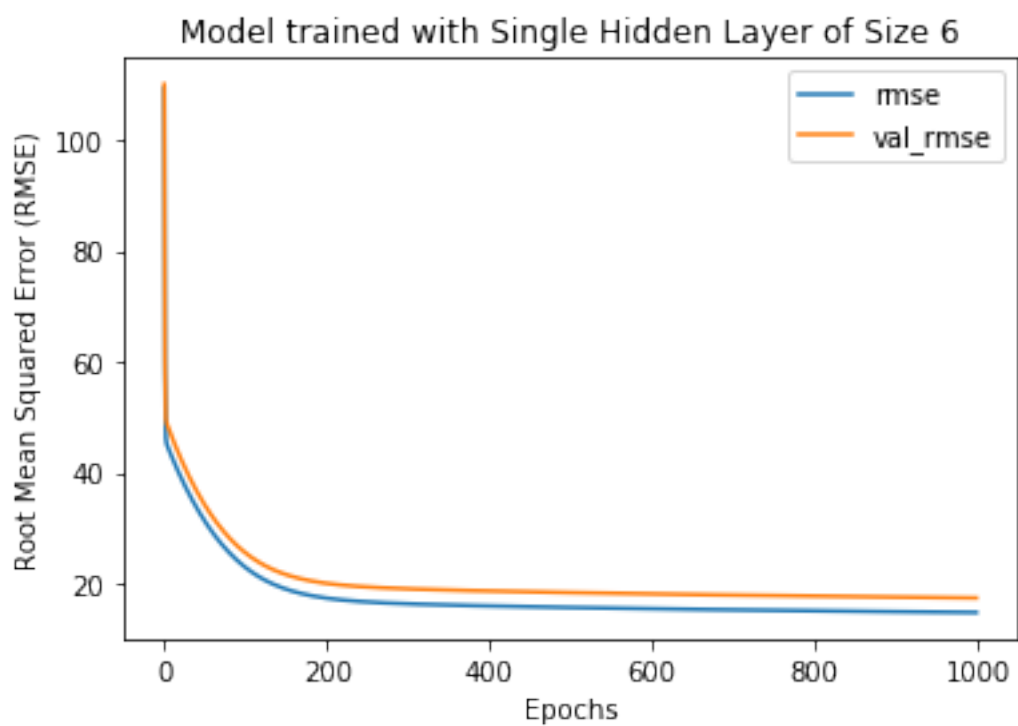
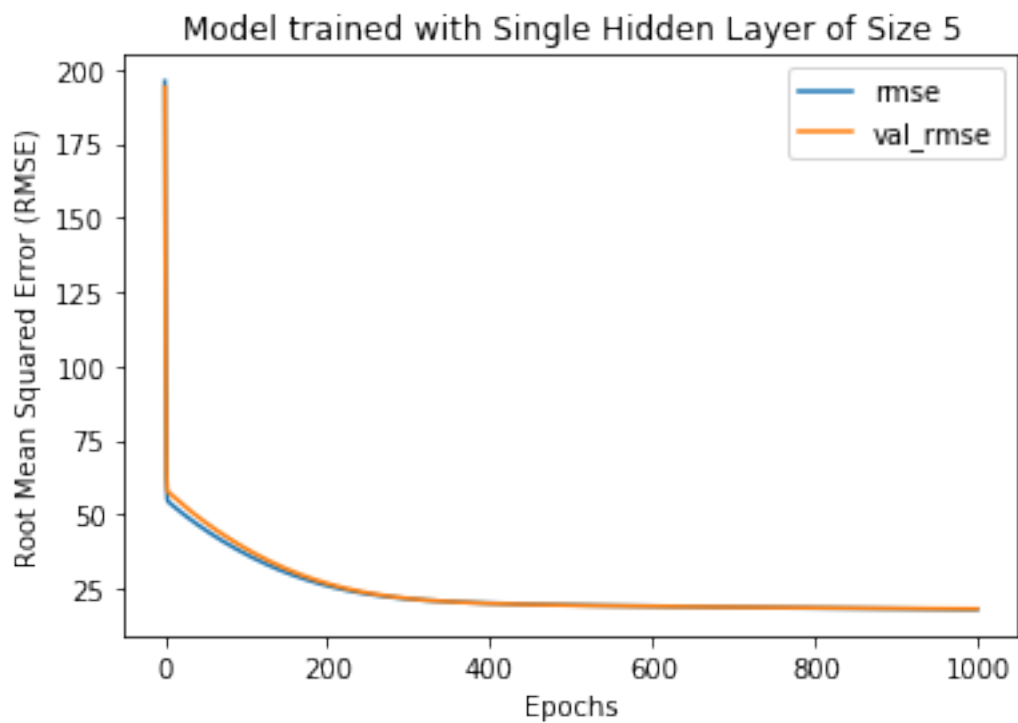
Test Set Results

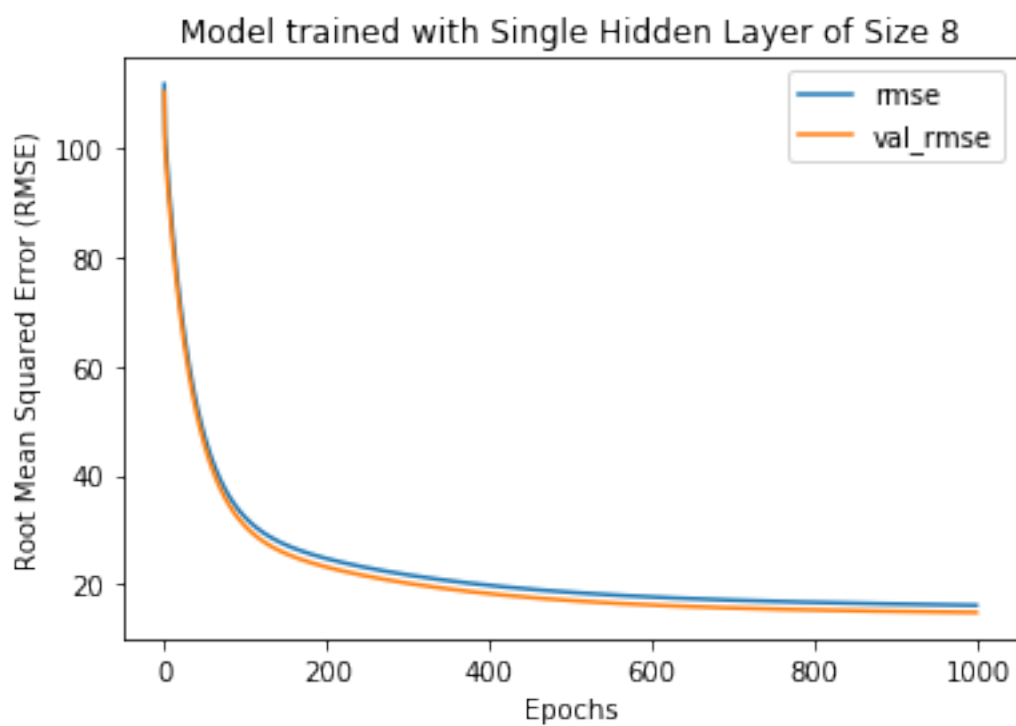
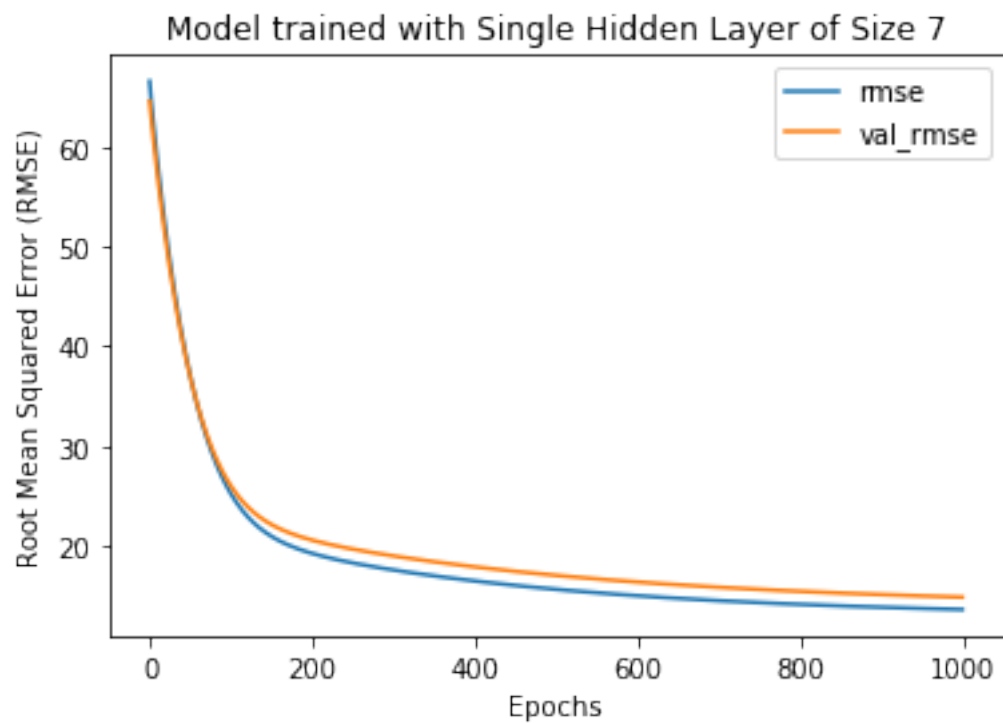
```
mse      178.686358
rmse     13.367362
mae       8.343952
r_sqr     0.936293
st_mse    0.001677
st_rmse   0.040955
st_mae    0.022716
st_r_sqr  0.880053
Name: 0, dtype: float64
```

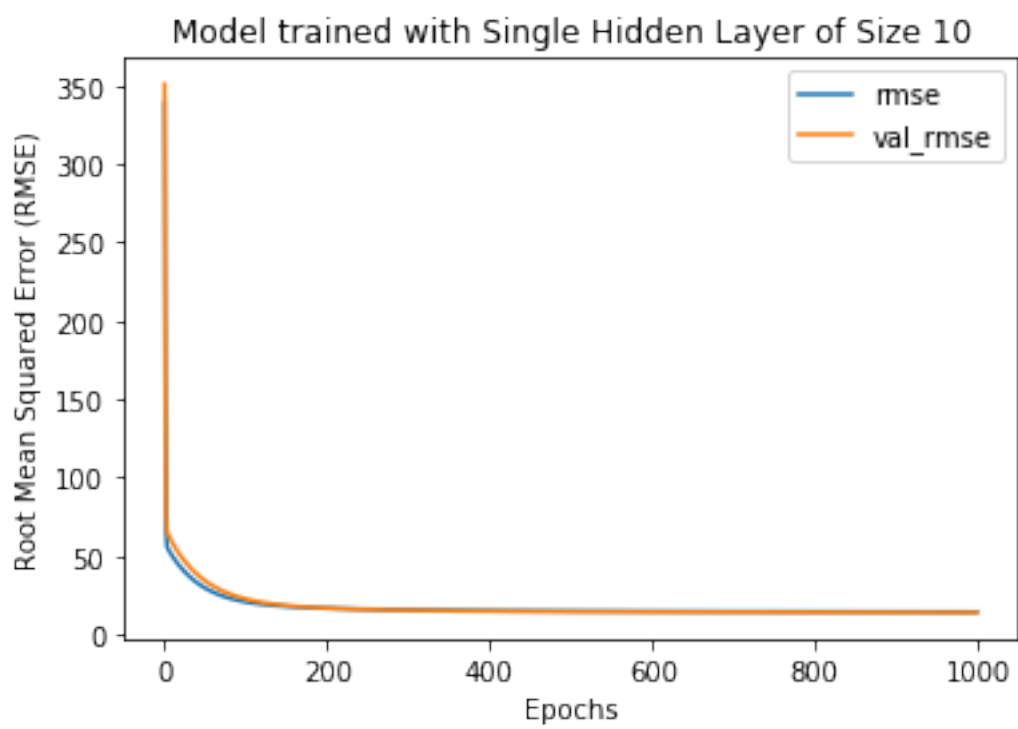
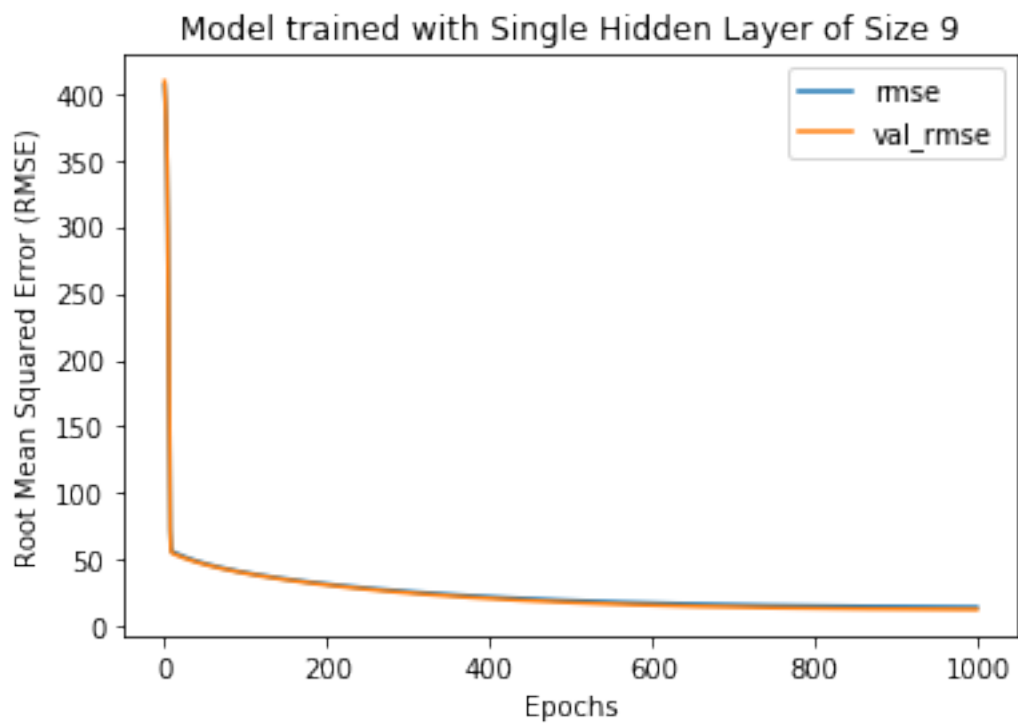
```
=====
=====
```











```
[28]: layer_tests_2 = dict()
      for i in range(1, 11):
          layer_tests_2[f"Test-{i}"] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", 10, i, 1),
              activ_func="tanh",
              epochs=1000,
              l_rate=0.3
          )
```

```
[29]: for i in range(1,11):
      print(f"Model Trained with Hidden Layer Configuration (10, {i})",
            end=f"\n{'-'*100}\n")

      print("Final Training Results")
      print(layer_tests_2[f"Test-{i}"]["training_results"].iloc[-1, :4],
            end=f"\n{'-'*100}\n")

      print("Final Validation Results")
      print(layer_tests_2[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
            end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(layer_tests_2[f"Test-{i}"]["error_metrics"].iloc[0],
            end=f"\n{'-'*100}\n\n\n")

      ax = layer_tests_2[f"Test-{i}"]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with Hidden Layer
            Configuration (10, {i})",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Hidden Layer Configuration (10, 1)

-----

Final Training Results

|                           |            |
|---------------------------|------------|
| mae                       | 11.039224  |
| mse                       | 372.824255 |
| r_sqr                     | 0.892309   |
| rmse                      | 19.308658  |
| Name: 999, dtype: float64 |            |

-----

Final Validation Results



```
val_mae      8.767553
val_mse     197.875779
val_r_sqr    0.918034
val_rmse     14.066833
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          327.410711
rmse         18.094494
mae          11.472224
r_sqr        0.877149
st_mse       0.006786
st_rmse      0.082379
st_mae       0.046483
st_r_sqr     0.703177
Name: 0, dtype: float64
```

=====

Model Trained with Hidden Layer Configuration (10, 2)

-----  
-----  
Final Training Results

```
mae          14.960459
mse          528.579588
r_sqr        0.827110
rmse         22.990859
Name: 999, dtype: float64
```

-----  
-----  
Final Validation Results

```
val_mae      13.888633
val_mse     505.515482
val_r_sqr    0.845042
val_rmse     22.483671
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          930.783480
rmse         30.508744
mae          22.436335
r_sqr        0.695578
st_mse       0.003913
st_rmse      0.062555
```

```
st_mae      0.035549
st_r_sqr     0.801065
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 3)

```
-----
-----
```

Final Training Results

```
mae      8.733926
mse     228.135558
r_sqr    0.933564
rmse    15.104157
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

```
val_mae    7.949196
val_mse   168.438061
val_r_sqr   0.932911
val_rmse   12.978369
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse      1792.981943
rmse     42.343617
mae      29.433090
r_sqr     0.327214
st_mse     0.002311
st_rmse    0.048068
st_mae     0.030563
st_r_sqr    0.865453
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 4)

```
-----
-----
```

Final Training Results

```
mae      10.329634
mse     297.055861
```

```
r_sqr      0.904130
rmse      17.235309
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      10.720939
val_mse      273.030036
val_r_sqr     0.916427
val_rmse     16.523621
Name: 999, dtype: float64
```

---

Test Set Results

```
mse      297.127479
rmse     17.237386
mae      11.082555
r_sqr     0.898321
st_mse     0.001642
st_rmse    0.040518
st_mae     0.022925
st_r_sqr    0.887155
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (10, 5)

---

Final Training Results

```
mae      6.650090
mse     154.343884
r_sqr     0.948926
rmse     12.423521
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      7.309605
val_mse     178.053204
val_r_sqr     0.945661
val_rmse     13.343658
Name: 999, dtype: float64
```

---

Test Set Results

```
mse          301.371642
rmse         17.360059
mae          11.063077
r_sqr        0.904170
st_mse       0.001250
st_rmse      0.035358
st_mae       0.016522
st_r_sqr     0.931113
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 6)

```
-----
Final Training Results
mae          9.374567
mse         238.607777
r_sqr        0.927337
rmse        15.446934
Name: 999, dtype: float64
```

```
-----
Final Validation Results
val_mae       9.233305
val_mse      277.627647
val_r_sqr     0.925936
val_rmse     16.662162
Name: 999, dtype: float64
```

```
-----
Test Set Results
mse          2038.522945
rmse         45.150005
mae          32.770927
r_sqr       -0.095314
st_mse       0.002843
st_rmse      0.053322
st_mae       0.028133
st_r_sqr     0.887762
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 7)

Final Training Results

mae 6.788405  
mse 155.399767  
r\_sqr 0.943998  
rmse 12.465944  
Name: 999, dtype: float64

Final Validation Results

val\_mae 7.665292  
val\_mse 240.926042  
val\_r\_sqr 0.936648  
val\_rmse 15.521792  
Name: 999, dtype: float64

Test Set Results

mse 505.007571  
rmse 22.472373  
mae 14.885314  
r\_sqr 0.849342  
st\_mse 0.001550  
st\_rmse 0.039370  
st\_mae 0.019088  
st\_r\_sqr 0.919637  
Name: 0, dtype: float64

Model Trained with Hidden Layer Configuration (10, 8)

Final Training Results

mae 9.593583  
mse 264.748185  
r\_sqr 0.911685  
rmse 16.271084  
Name: 999, dtype: float64

Final Validation Results

val\_mae 9.723533  
val\_mse 261.125326

```
val_r_sqr      0.918077
val_rmse       16.159373
Name: 999, dtype: float64
```

---

Test Set Results

```
mse           257.575178
rmse          16.049149
mae           9.836957
r_sqr         0.922273
st_mse        0.002280
st_rmse       0.047754
st_mae        0.024605
st_r_sqr      0.861724
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (10, 9)

---

Final Training Results

```
mae           7.164161
mse          176.956075
r_sqr         0.938995
rmse          13.302484
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae       9.147647
val_mse      305.478118
val_r_sqr     0.910433
val_rmse     17.477932
Name: 999, dtype: float64
```

---

Test Set Results

```
mse           470.744577
rmse          21.696649
mae          12.374243
r_sqr         0.858922
st_mse        0.005060
st_rmse       0.071134
st_mae        0.038185
st_r_sqr      0.818139
```

Name: 0, dtype: float64

=====

Model Trained with Hidden Layer Configuration (10, 10)

-----

Final Training Results

mae 8.182189

mse 216.348942

r\_sqr 0.923964

rmse 14.708805

Name: 999, dtype: float64

-----

Final Validation Results

val\_mae 9.491528

val\_mse 295.626709

val\_r\_sqr 0.916262

val\_rmse 17.193799

Name: 999, dtype: float64

-----

Test Set Results

mse 418.041829

rmse 20.446071

mae 10.285933

r\_sqr 0.877130

st\_mse 0.000812

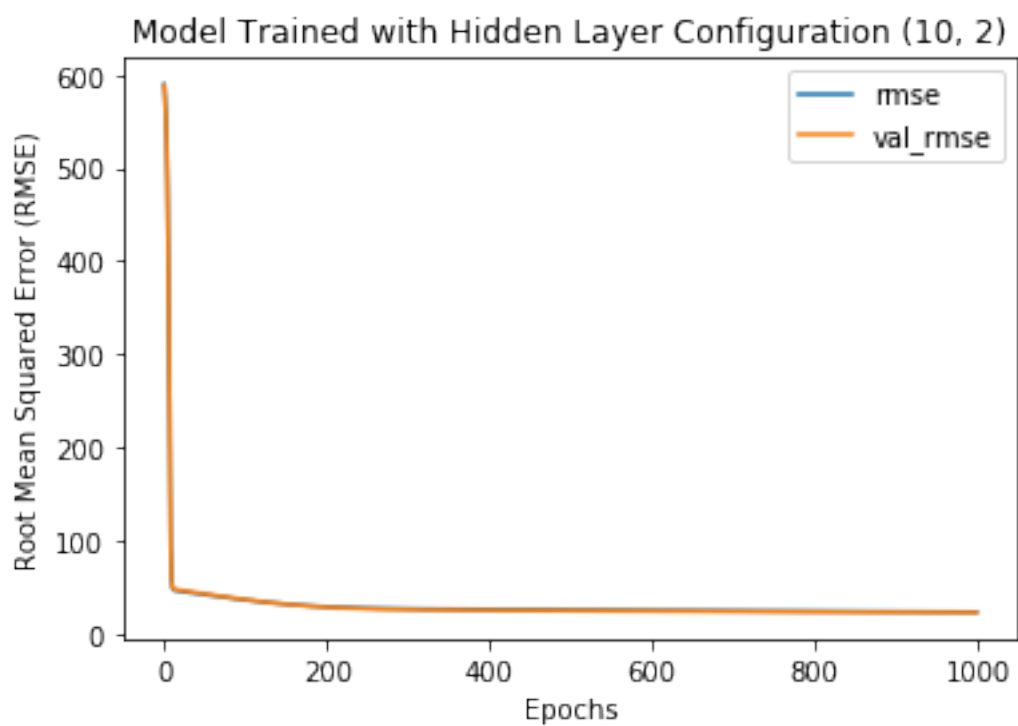
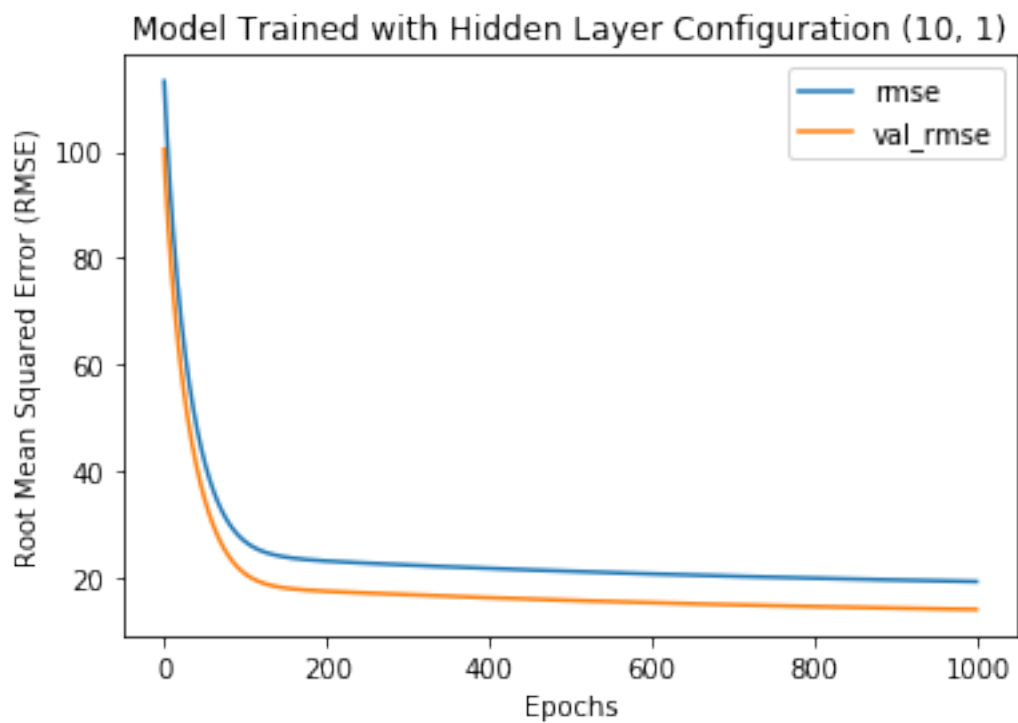
st\_rmse 0.028500

st\_mae 0.015443

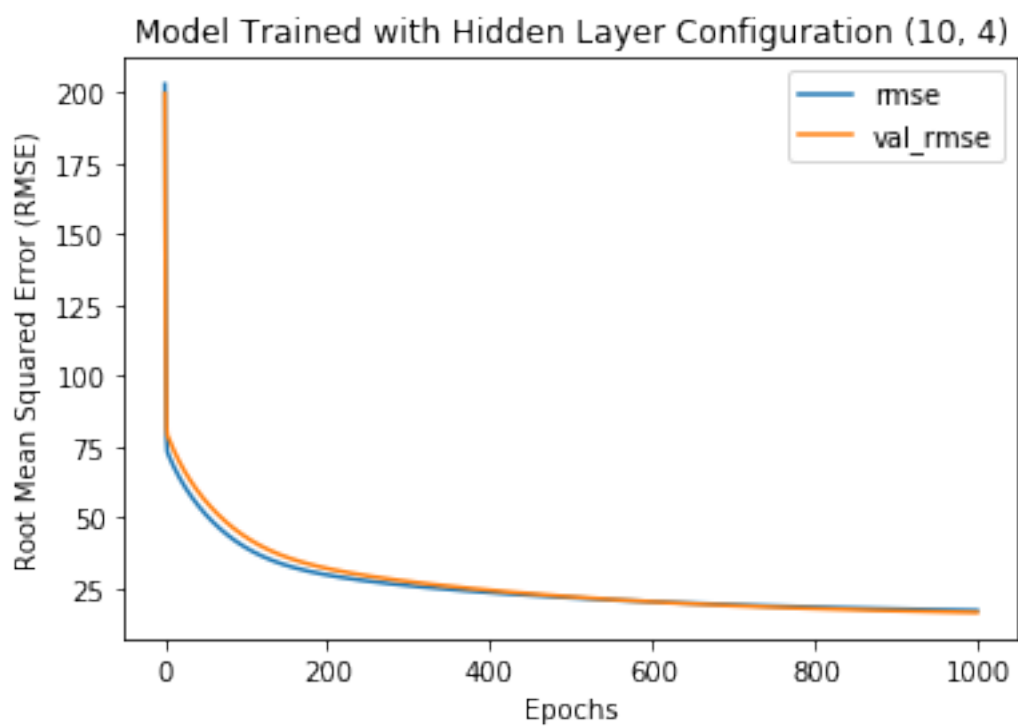
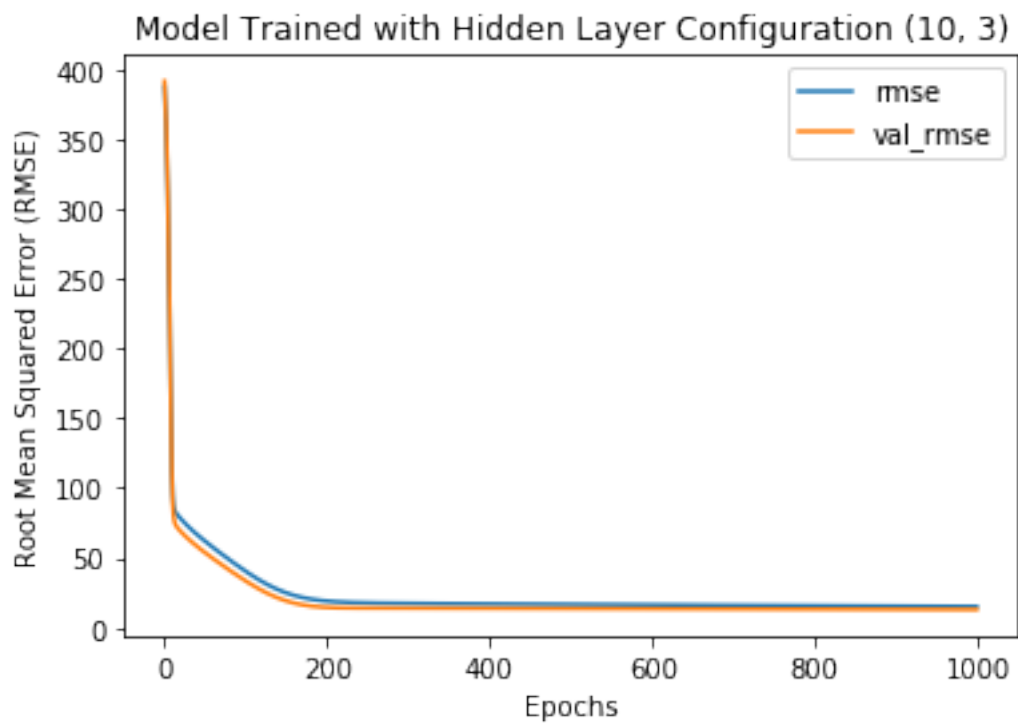
st\_r\_sqr 0.926349

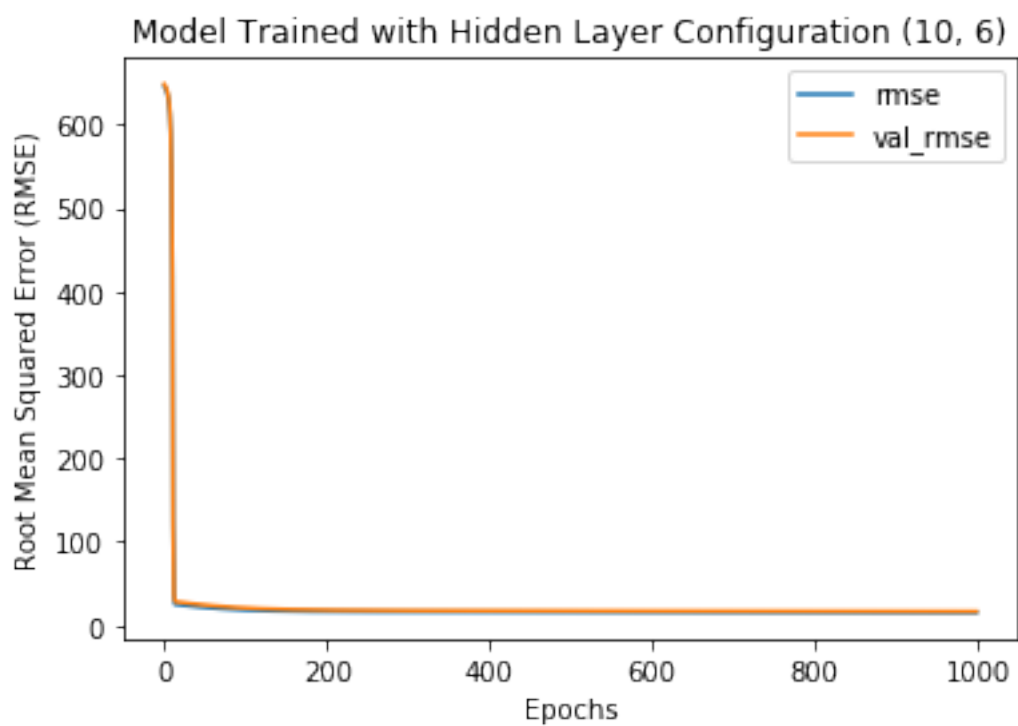
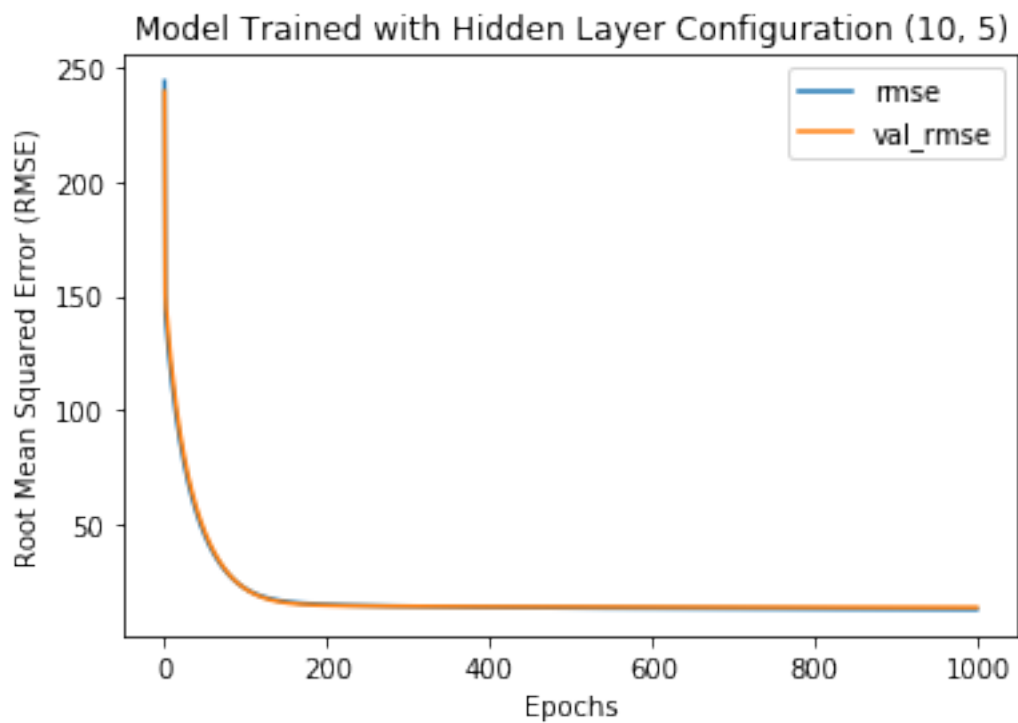
Name: 0, dtype: float64

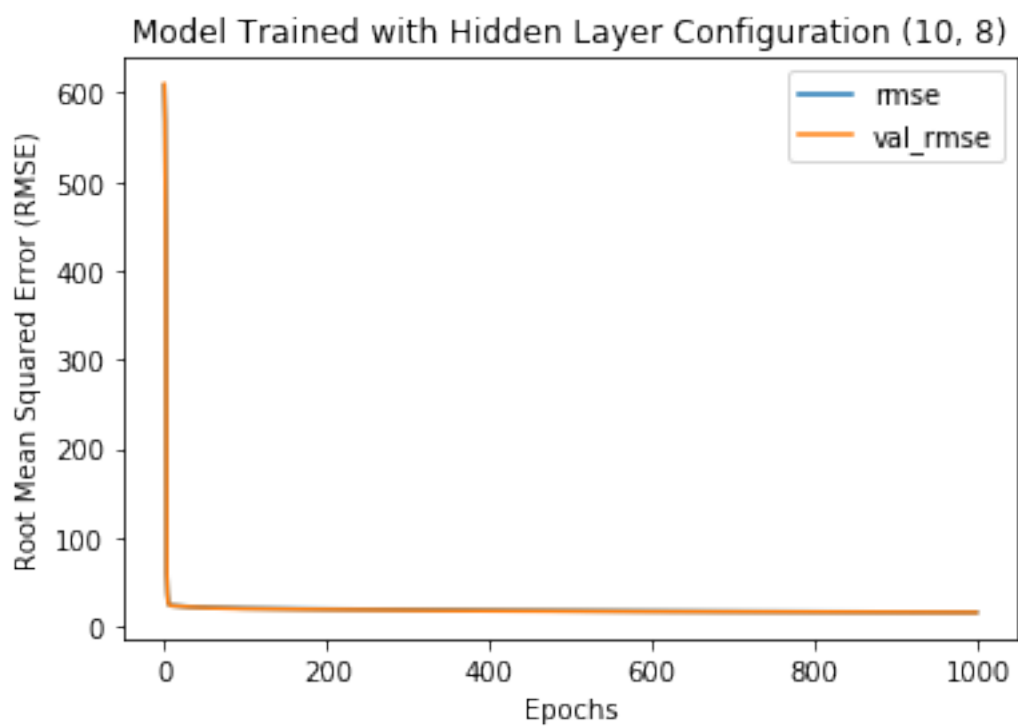
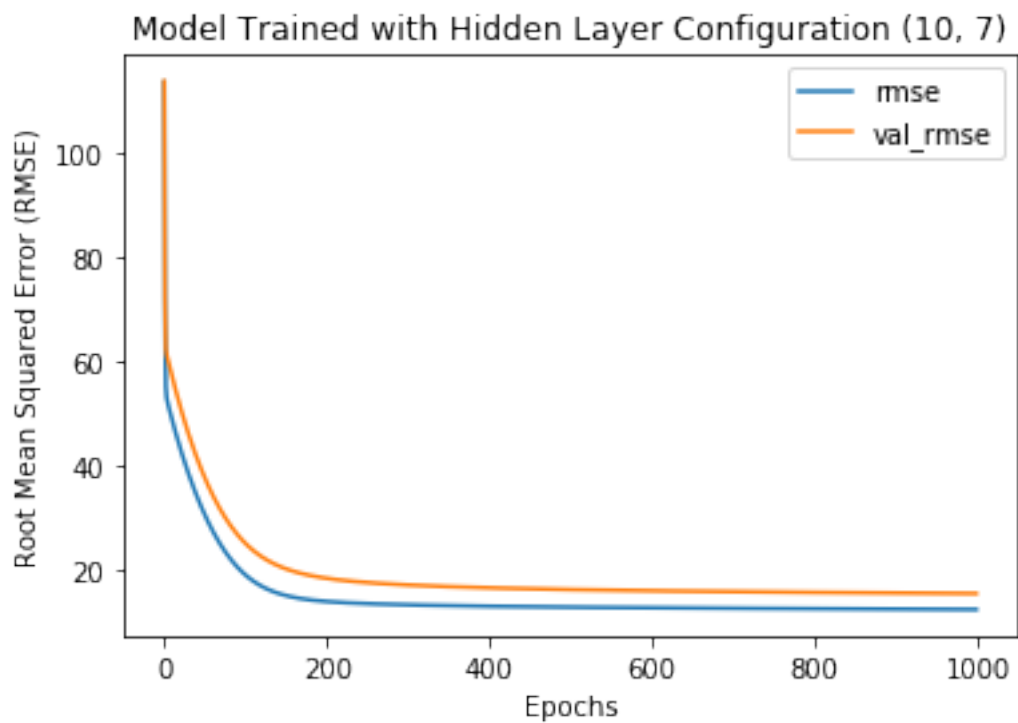
=====

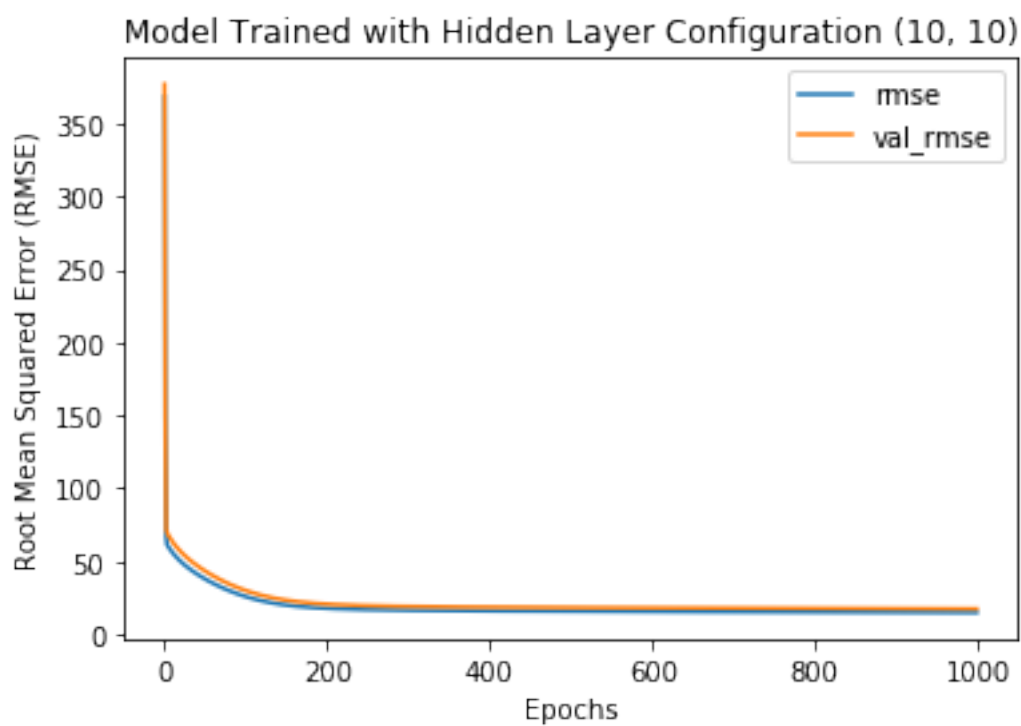
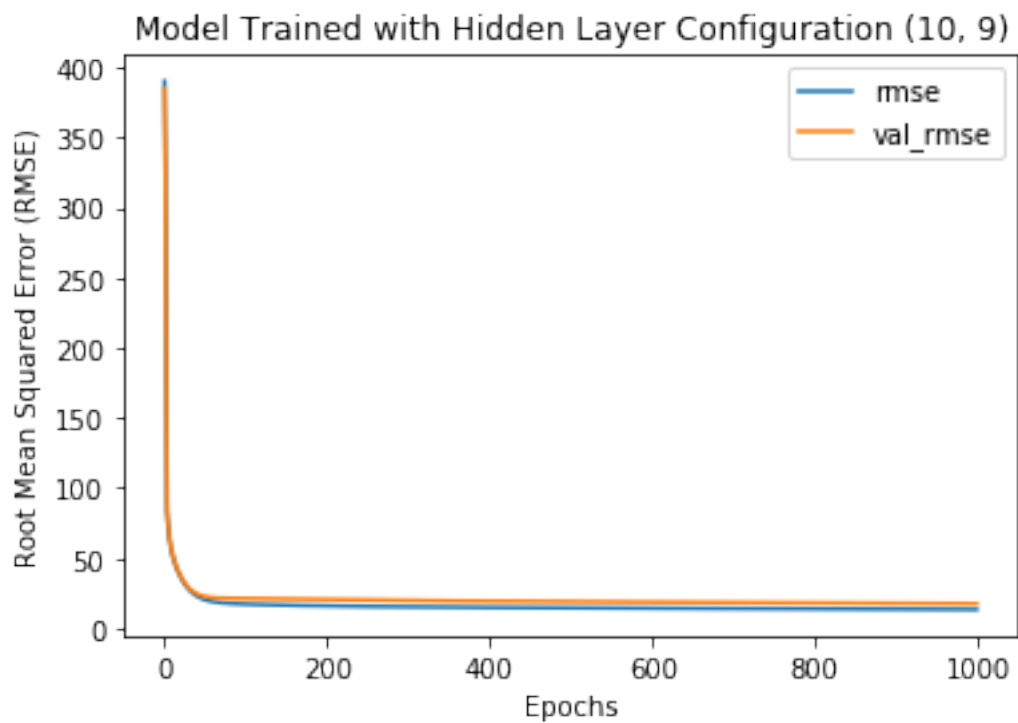












```
[32]: layer_tests_3 = dict()
      for i in range(1, 11):
          layer_tests_3[f"Test-{i}"] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", i, 10, 1),
              activ_func="tanh",
              epochs=1000,
              l_rate=0.3
          )
```

```
[33]: for i in range(1,11):
      print(f"Model Trained with Hidden Layer Configuration ({i}, 10)",
            end=f"\n{'-'*100}\n")

      print("Final Training Results")
      print(layer_tests_3[f"Test-{i}"]["training_results"].iloc[-1, :4],
            end=f"\n{'-'*100}\n")

      print("Final Validation Results")
      print(layer_tests_3[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
            end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(layer_tests_3[f"Test-{i}"]["error_metrics"].iloc[0],
            end=f"\n{'-'*100}\n\n\n")

      ax = layer_tests_3[f"Test-{i}"]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with Hidden Layer
            Configuration ({i}, 10)",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Hidden Layer Configuration (1, 10)

-----

Final Training Results

|                           |             |
|---------------------------|-------------|
| mae                       | 40.675349   |
| mse                       | 3289.605540 |
| r_sqr                     | 0.000141    |
| rmse                      | 57.355083   |
| Name: 999, dtype: float64 |             |

-----

Final Validation Results

```
val_mae      40.452444
val_mse      3191.763601
val_r_sqr    -0.000552
val_rmse     56.495695
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          2442.108068
rmse         49.417690
mae          36.684681
r_sqr        -0.005315
st_mse       0.024735
st_rmse      0.157273
st_mae       0.094491
st_r_sqr     -0.084921
Name: 0, dtype: float64
```

=====

Model Trained with Hidden Layer Configuration (2, 10)

-----  
-----  
Final Training Results

```
mae          9.205754
mse          279.591917
r_sqr        0.907125
rmse         16.721002
Name: 999, dtype: float64
```

-----  
-----  
Final Validation Results

```
val_mae      10.477098
val_mse      355.636679
val_r_sqr    0.903670
val_rmse     18.858332
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          948.689127
rmse         30.800798
mae          19.482716
r_sqr        0.655228
st_mse       0.002617
st_rmse      0.051161
```

```
st_mae      0.023390
st_r_sqr     0.886004
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (3, 10)

```
-----
-----
```

Final Training Results

```
mae      29.776235
mse     1000.587635
r_sqr     0.651198
rmse     31.632067
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

```
val_mae     30.532874
val_mse     1100.822412
val_r_sqr    0.716457
val_rmse     33.178644
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse      1227.786313
rmse     35.039782
mae      30.576602
r_sqr     0.589698
st_mse     0.003323
st_rmse     0.057648
st_mae     0.050783
st_r_sqr    0.658818
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (4, 10)

```
-----
-----
```

Final Training Results

```
mae      9.533718
mse     299.505396
```

```
r_sqr      0.901843
rmse      17.306224
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      8.655035
val_mse     241.227884
val_r_sqr    0.908075
val_rmse     15.531513
Name: 999, dtype: float64
```

---

Test Set Results

```
mse      545.763014
rmse     23.361571
mae      12.153721
r_sqr     0.852945
st_mse    0.001061
st_rmse   0.032575
st_mae    0.018374
st_r_sqr  0.911921
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (5, 10)

---

Final Training Results

```
mae      8.727363
mse     216.310404
r_sqr    0.928333
rmse     14.707495
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      9.774685
val_mse     322.415189
val_r_sqr    0.907832
val_rmse     17.955924
Name: 999, dtype: float64
```

---

Test Set Results



```
mse          487.227778
rmse         22.073237
mae          14.595240
r_sqr        0.834330
st_mse       0.002324
st_rmse      0.048203
st_mae       0.023174
st_r_sqr     0.878759
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (6, 10)

```
-----
Final Training Results
mae          6.903358
mse         159.462782
r_sqr        0.952458
rmse         12.627857
Name: 999, dtype: float64
```

```
-----
Final Validation Results
val_mae       6.846378
val_mse      169.339608
val_r_sqr     0.950305
val_rmse     13.013055
Name: 999, dtype: float64
```

```
-----
Test Set Results
mse          2610.194233
rmse         51.090060
mae          35.145469
r_sqr       -0.311430
st_mse       0.002085
st_rmse      0.045664
st_mae       0.022646
st_r_sqr     0.922044
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (7, 10)

Final Training Results

mae 7.294836  
mse 166.755323  
r\_sqr 0.948493  
rmse 12.913378  
Name: 999, dtype: float64

Final Validation Results

val\_mae 7.466487  
val\_mse 188.788827  
val\_r\_sqr 0.936254  
val\_rmse 13.740045  
Name: 999, dtype: float64

Test Set Results

mse 246.147520  
rmse 15.689089  
mae 7.769502  
r\_sqr 0.912560  
st\_mse 0.001219  
st\_rmse 0.034917  
st\_mae 0.019874  
st\_r\_sqr 0.866350  
Name: 0, dtype: float64

Model Trained with Hidden Layer Configuration (8, 10)

Final Training Results

mae 6.053548  
mse 124.757734  
r\_sqr 0.961100  
rmse 11.169500  
Name: 999, dtype: float64

Final Validation Results

val\_mae 5.847862  
val\_mse 102.754911

```
val_r_sqr      0.961203
val_rmse       10.136810
Name: 999, dtype: float64
```

---

Test Set Results

```
mse           2323.555325
rmse          48.203271
mae           34.414942
r_sqr         0.279108
st_mse        0.002943
st_rmse       0.054247
st_mae        0.036393
st_r_sqr      0.857782
Name: 0, dtype: float64
```

---

---

Model Trained with Hidden Layer Configuration (9, 10)

---

Final Training Results

```
mae           8.399457
mse          218.070847
r_sqr        0.933390
rmse         14.767222
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae       7.389402
val_mse      157.882046
val_r_sqr     0.941587
val_rmse     12.565112
Name: 999, dtype: float64
```

---

Test Set Results

```
mse           706.22259
rmse          26.574843
mae           16.900812
r_sqr         0.761802
st_mse        0.001696
st_rmse       0.041186
st_mae        0.021709
st_r_sqr      0.910816
```

Name: 0, dtype: float64

=====

Model Trained with Hidden Layer Configuration (10, 10)

-----

Final Training Results

mae 8.578838

mse 254.414124

r\_sqr 0.913281

rmse 15.950364

Name: 999, dtype: float64

-----

Final Validation Results

val\_mae 8.812246

val\_mse 340.880065

val\_r\_sqr 0.923956

val\_rmse 18.462938

Name: 999, dtype: float64

-----

Test Set Results

mse 225.380567

rmse 15.012680

mae 9.438959

r\_sqr 0.896025

st\_mse 0.005652

st\_rmse 0.075180

st\_mae 0.042396

st\_r\_sqr 0.721736

Name: 0, dtype: float64

=====

