

# Model-Experiments

March 30, 2022

```
[1]: import pandas as pd
import numpy as np
from sklearn.metrics import *
from sklearn.model_selection import train_test_split

import seaborn as sns
import matplotlib.pyplot as plt
import plotly.express as px
import plotly.graph_objects as go
```

## 1 Loading Datasets

```
[2]: # Loading datas
labels = ['Lagged', 'MA', 'WMA', 'MA-Lagged', 'WMA-Lagged'] # names of each
↳ datasets

def load_datasets():
    """
    Excel files for each dataset are read into a
    dataframe and stored in a dictionary for easy
    access and use
    """
    datasets = dict()
    for lb in labels:
        new_df = pd.read_excel(f"River-Data-{lb}.xlsx")
        new_df.drop(["Unnamed: 0"], axis=1, inplace=True)
        datasets[lb] = new_df

    return datasets

data = load_datasets() # a dataframe for each dataset is stored in a dictionary
↳ called data
```

## 2 Utility Functions

Standardising and Unstandardising Values

```
[3]: # Utility Functions
## Functions for standardising and unstandardising values
def standardise_columns(df, cols):
    """
    This function works with dataframes to standardise values
    in multiple columns to the range [0.1, 0.9]
    """
    subset_df = df[cols]
    subset_df = 0.8 * ((subset_df - subset_df.min()) / (subset_df.max() -
    ↪subset_df.min())) + 0.1
    return subset_df

def unstandardise_columns(df, cols, max_val, min_val):
    """
    This function works with numpy arrays to destandardise values
    in multiple columns
    """
    subset_df = df[cols]
    subset_df = ((subset_df - subset_df.min()) / 0.8) * (max_val - min_val) +
    ↪min_val
    return subset_df

def standardise_value(x, max_val, min_val):
    """
    This function works with numpy arrays to standardise values
    in multiple arrays to the range [0.1, 0.9]
    """
    return 0.8 * ((x - min_val)) / (max_val - min_val) + 0.1

def unstandardise_value(x, max_val, min_val):
    """
    This function works with numpy arrays to destandardise values
    in multiple arrays
    """
    return ((x - 0.1) / 0.8) * (max_val - min_val) + min_val
```

## Plotting

```
[4]: ## Plotting functions
def plot_correlation_matrix(corr_data, title, figsize=(16,6), mask=False):
    """
    Utility function for plotting a correlation heatmap of a given feature set
    """
    if mask:
        mask = np.triu(np.ones_like(corr_data, dtype=bool))
    plt.figure(figsize=figsize, dpi=500)
    heatmap = sns.heatmap(corr_data, vmin=-1, vmax=1, annot=True, mask=mask)
    heatmap.set_title(title)
```

```

plt.show()

def plot_predictions(preds_df, standardised=False):
    """
    Utility function for plotting model predictions against actual value
    """
    preds_col = "Predicted Values"
    vals_col = "Actual Values"
    if standardised:
        preds_col += " (Standardised)"
        vals_col += " (Standardised)"

    line_plt = px.line(preds_df, y=vals_col)
    scatter_plt = px.scatter(preds_df, y=preds_col,
    ↪color_discrete_sequence=["#ff0000"])

    go.Figure(line_plt.data + scatter_plt.data, layout={"title": "Actual vs_
    ↪Predicted Values"}).show()

```

### 3 ANN Class

```

[5]: # Basic ANN class for MLP models
class BasicAnn:
    def __init__(self, layers, max_st_val, min_st_val, activ_func="sigmoid"):
        self.layers = layers
        self.num_layers = len(layers)
        self.max_val = max_st_val
        self.min_val = min_st_val
        self.activ_func = activ_func

        weight_shapes = [(layers[i-1],layers[i]) for i in range(1, len(layers))]
        self.weights = {
            f"W{i+1}": np.random.standard_normal(s)/s[0]**0.5
            for i, s in enumerate(weight_shapes)
        } # weights are stored as matrices that are implemented as 2D numpy
    ↪arrays

        self.biases = {
            f"B{i+1}": np.random.randn(1,1)/1**0.5
            for i, l in enumerate(layers[1:])
        } # biases are also stored as matrices that are implemented as 2D numpy
    ↪arrays

    def activation(self, x):
        """
        Function to return value with the selected activation
        """

```

```

        if self.activ_func == "sigmoid":
            return 1/(1+np.exp(-x))
        elif self.activ_func == "tanh":
            return (np.exp(x)-np.exp(-x))/(np.exp(x)+np.exp(-x))
        elif self.activ_func == "relu":
            return x * (x > 0)
        elif self.activ_func == "linear":
            return x

def activation_deriv(self, a):
    """
    Function to return value with the derivative of the selected activation
    """
    if self.activ_func == "sigmoid":
        return a * (1 - a)
    elif self.activ_func == "tanh":
        return 1 - a**2
    elif self.activ_func == "relu":
        return 1 * (a > 0)
    elif self.activ_func == "linear":
        return np.ones(a.shape)

def train(self, features, targets, epochs=1000, learning_rate=0.1,
    ↪val_set=None):
    """
    Function will train the model using the standard backpropagation
    ↪algorithm
    and return a dataframe storing various error metrics for the model on
    ↪the
    training set and, possibly, a validation set if that is given
    """
    results = pd.DataFrame()
    real_targets = unstandardise_value(targets, self.max_val, self.min_val)
    num_targets = len(targets)

    for _ in range(epochs):
        # Forward pass
        activations = self.forward_pass(features)

        # Error calculation
        output_layer = activations[f"A{self.num_layers - 1}"]
        real_preds = unstandardise_value(output_layer, self.max_val, self.
    ↪min_val)
        error_data = { # storing error metrics for both standardised and
    ↪unstandardised data
            "mse": mean_squared_error(real_targets, real_preds),

```

```

        "rmse": mean_squared_error(real_targets, real_preds,
→squared=False),
        "mae": mean_absolute_error(real_targets, real_preds),
        "r_sqr": r2_score(real_targets, real_preds),
        "st_mse": mean_squared_error(targets, output_layer),
        "st_rmse": mean_squared_error(targets, output_layer,
→squared=False),
        "st_mae": mean_absolute_error(targets, output_layer),
        "st_r_sqr": r2_score(targets, output_layer)
    }

    if val_set:
        # if there is a validation set the prediction error of the model
        # on the validation set will be stored
        r, err = self.predict(val_set[0].to_numpy(), val_set[1].
→to_numpy())
        error_data.update({f"val_{col}": err[col][0] for col in err.
→columns})

    results = results.append(error_data, ignore_index=True)

    # Backward pass (backpropagation algorithm)
    deltas = self.compute_deltas(activations, targets, output_layer)
    self.update_weights(deltas, activations, features, num_targets,
→learning_rate)

    return results

def predict(self, test_inputs, st_actual_outputs, actual_outputs=None):
    """
    Runs a forward pass of the network with the newly configured weights
    and biases and returns a dataframe comparing the predicted values
    to actual values as well as a dataframe with various error metrics
    """
    # Forward pass
    activations = self.forward_pass(test_inputs)
    st_preds = activations[f"A{self.num_layers - 1}"]

    # Comparing predicted values with actual values
    if actual_outputs is None:
        actual_outputs = unstandardise_value(st_actual_outputs, self.
→max_val, self.min_val)

    preds = unstandardise_value(st_preds, self.max_val, self.min_val)

    results = pd.DataFrame(

```

```

        data={
            "Actual Values": actual_outputs.flatten(),
            "Predicted Values": preds.flatten(),
            "Actual Values (Standardised)": st_actual_outputs.flatten(),
            "Predicted Values (Standardised)": st_preds.flatten(),
        }
    )

    # Error calculation
    results["Absolute Error"] = abs(results["Actual Values"] -
↪results["Predicted Values"])
    st_absolute_err = abs(results["Actual Values (Standardised)"] -
↪results["Predicted Values (Standardised)"])
    results["Absolute Error (Standardised Values)"] = st_absolute_err

    error_metrics = pd.DataFrame(data={
        "mse": [mean_squared_error(actual_outputs, preds)],
        "rmse": [mean_squared_error(actual_outputs, preds, squared=False)],
        "mae": [mean_absolute_error(actual_outputs, preds)],
        "r_sqr": [r2_score(actual_outputs, preds)],
        "st_mse": [mean_squared_error(st_actual_outputs, st_preds)],
        "st_rmse": [mean_squared_error(st_actual_outputs, st_preds,
↪squared=False)],
        "st_mae": [mean_absolute_error(st_actual_outputs, st_preds)],
        "st_r_sqr": [r2_score(st_actual_outputs, st_preds)]
    })

    return results, error_metrics

def forward_pass(self, features):
    """
    Runs a forward pass of neural network through repeated
    multiplication of weights and bias matrices. Returns
    list of each activation layer including the output layer.
    """
    activation = self.activation(np.dot(features, self.weights["W1"]) +
↪self.biases["B1"].T)
    activations = {"A1": activation}
    for i in range(2, self.num_layers):
        activation = self.activation(np.dot(activation, self.
↪weights[f"W{i}"]) + self.biases[f"B{i}"].T)
        activations[f"A{i}"] = activation

    return activations

def compute_deltas(self, activations, targets, output_layer):
    """

```

```

Computes errors between layers for backproagation.
Returns a dictionary of lists which contain the errors
for each node in each layer.
"""
output_err = targets - output_layer
output_delta = output_err * self.activation_deriv(output_layer)
deltas = {"dw1": output_delta}

for i in range(self.num_layers - 1, 1, -1):
    dw = deltas[f"dw{self.num_layers - i}"]
    act = activations[f"A{i-1}"]
    w = self.weights[f"W{i}"]
    deltas[f"dw{self.num_layers - i + 1}"] = np.dot(dw, w.T) * self.
↪activation_deriv(act)

return deltas

def update_weights(self, deltas, activations, features, num_targets, l_rate):
    """
    Updates weights and biases according to given errors, activations
    and the chosen learning rate
    """
    delta = deltas[f"dw{self.num_layers - 1}"]
    self.weights["W1"] += l_rate * (np.dot(features.T, delta)) / num_targets
    self.biases["B1"] += l_rate * (np.dot(delta.T, np.ones((num_targets, 1)))) / num_targets
    ↪1)))) / num_targets

    for i in range(2, self.num_layers):
        act = activations[f"A{i-1}"]
        dw = deltas[f"dw{self.num_layers - i}"]
        self.weights[f"W{i}"] += l_rate * (np.dot(act.T, dw)) / num_targets
        self.biases[f"B{i}"] += l_rate * np.dot(dw.T, np.ones((num_targets, 1)))) / num_targets
    ↪1)))) / num_targets

```

### Build, Train and Test ANN Model

```

[6]: def build_train_test(feature_set, feature_cols, target_cols, layers=("auto", 1),
    ↪1), activ_func="linear", epochs=1000, l_rate=0.1):
    """
    Function to build, train and test MLP models
    """
    # Splitting and standardising datasets to create standardised and
    ↪unstandardised
    # training, validation and testing sets.
    train_val_set, test_set = train_test_split(feature_set, test_size=0.2)
    st_train_val_set = standardise_columns(train_val_set, train_val_set.columns)

```

```

st_test_set = standardise_columns(test_set, test_set.columns)

# Preparing features and targets for training and testing
features = st_train_val_set[feature_cols]
targets = st_train_val_set[target_cols]

X_train, X_val, y_train, y_val = train_test_split(features, targets,
→test_size=0.25)
X_test, y_test = st_test_set[feature_cols], st_test_set[target_cols]

# Getting standardisation values for targets
min_val = train_val_set[target_cols].min()[0]
max_val = train_val_set[target_cols].max()[0]

# Building model
if layers[0] == "auto":
    # if the size of the input layer is not specified
    # then it will be set to the number of predictors
    layers = (len(feature_cols),) + layers[1:]

ann = BasicAnn(layers, max_val, min_val, activ_func)

# Training model
training_results = ann.train(
    X_train.to_numpy(),
    y_train.to_numpy(),
    val_set=(X_val, y_val), # training with a validation set
    epochs=epochs,
    learning_rate=l_rate
)

# Predicting model
prediction_results = ann.predict(
    X_test.to_numpy(),
    y_test.to_numpy(),
    actual_outputs=test_set[target_cols].to_numpy()
)

predictions, error_metrics = prediction_results[0], prediction_results[1]

return {
    "training_results": training_results,
    "final_test_results": predictions,
    "error_metrics": error_metrics,
    "model": ann
}

```



## 4 Selecting Features/Predictors

### Building Feature Sets

```
[7]: # Function for building custom feature and target sets
def build_feature_set(*datasets):
    assert len(datasets) > 0, "No data sets entered"
    datasets = list(datasets)
    min_rows = min(d.shape[0] for d in datasets)

    for i, ds in enumerate(datasets):
        datasets[i] = ds.truncate(before=ds.shape[0]-min_rows).reset_index()
        datasets[i].drop(["index"], axis=1, inplace=True)

    merged_df = datasets[0].iloc[:, :2]
    for ds in datasets:
        merged_df = pd.concat([merged_df, ds.iloc[:, 2:]], axis=1)

    merged_cols = list(merged_df.columns)
    selected_cols = []

    for i in range(0, len(merged_cols), 2):
        format_str = f"{i+1}) {merged_cols[i]}"
        if i != len(merged_cols) - 1:
            second_part = f"{i+2}) {merged_cols[i+1]}"
            num_spaces = 50 - len(format_str)
            format_str += num_spaces*" " + second_part
        print(format_str)

    selected_indices = input("\nSelect columns: ")
    for index in selected_indices.split(","):
        if "-" in index:
            first_i, second_i = index.split("-")
            selected_cols += merged_cols[int(first_i) - 1: int(second_i)]
        else:
            selected_cols.append(merged_cols[int(index) - 1])

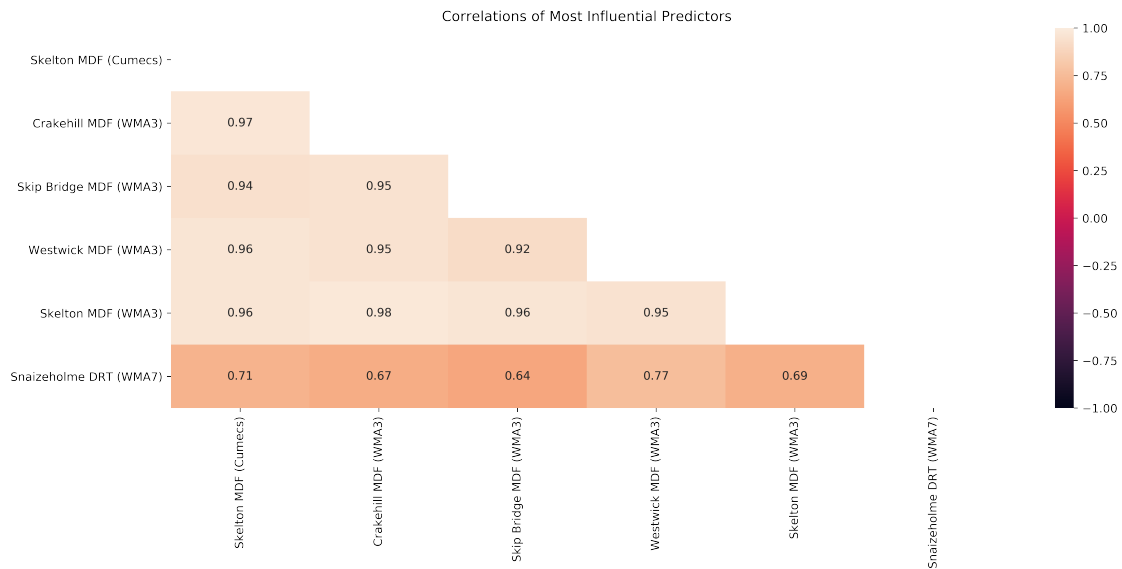
    return merged_df[selected_cols]
```

```
[8]: # 2,3-6,42
fs = build_feature_set(data['WMA'])
plot_correlation_matrix(fs.corr(), "Correlations of Most Influential_
↳Predictors", mask=True)
```

- |                         |                           |
|-------------------------|---------------------------|
| 1) Date                 | 2) Skelton MDF (Cumeecs)  |
| 3) Crakehill MDF (WMA3) | 4) Skip Bridge MDF (WMA3) |
| 5) Westwick MDF (WMA3)  | 6) Skelton MDF (WMA3)     |
| 7) Crakehill MDF (WMA4) | 8) Skip Bridge MDF (WMA4) |
| 9) Westwick MDF (WMA4)  | 10) Skelton MDF (WMA4)    |

11) Crakehill MDF (WMA5)	12) Skip Bridge MDF (WMA5)
13) Westwick MDF (WMA5)	14) Skelton MDF (WMA5)
15) Crakehill MDF (WMA6)	16) Skip Bridge MDF (WMA6)
17) Westwick MDF (WMA6)	18) Skelton MDF (WMA6)
19) Crakehill MDF (WMA7)	20) Skip Bridge MDF (WMA7)
21) Westwick MDF (WMA7)	22) Skelton MDF (WMA7)
23) Arkengarthdale DRT (WMA3)	24) East Cowton DRT (WMA3)
25) Malham Tarn DRT (WMA3)	26) Snaizeholme DRT (WMA3)
27) Arkengarthdale DRT (WMA4)	28) East Cowton DRT (WMA4)
29) Malham Tarn DRT (WMA4)	30) Snaizeholme DRT (WMA4)
31) Arkengarthdale DRT (WMA5)	32) East Cowton DRT (WMA5)
33) Malham Tarn DRT (WMA5)	34) Snaizeholme DRT (WMA5)
35) Arkengarthdale DRT (WMA6)	36) East Cowton DRT (WMA6)
37) Malham Tarn DRT (WMA6)	38) Snaizeholme DRT (WMA6)
39) Arkengarthdale DRT (WMA7)	40) East Cowton DRT (WMA7)
41) Malham Tarn DRT (WMA7)	42) Snaizeholme DRT (WMA7)

Select columns: 2,3-6,42



## 5 Training and Network Selection

### Epochs

```
[9]: target_cols = [fs.columns[0]]
    feature_cols = list(fs.columns[1:])

    epoch_tests = dict()
    for i in range(1, 11):
        epoch_tests[f"Test-{i}"] = build_train_test(
```

```

        fs,
        feature_cols,
        target_cols,
        layers=("auto", 1),
        activ_func="linear",
        epochs=i*500
    )

```

- rmse -> root mean square error
- mae -> mean absolute error
- mse -> mean squared error
- r\_sqr -> R-Squared (Coefficient of Determination)
- val\_\* -> error metric on validation set
- st\_\* -> error metric on unstandardised values

```

[10]: # Testing number of epochs for training; between 800 and 1200 seems to be ideal
for i in range(1, 11):
    print(f"Model trained with {i*500} epochs", end=f"\n{'-'*100}\n")

    print("Final Training results")
    print(epoch_tests[f"Test-{i}"]["training_results"].iloc[-1, :4],
    ↪end=f"\n{'-'*100}\n")

    print("Final Validation results")
    print(epoch_tests[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
    ↪end=f"\n{'-'*100}\n")

    print("Test Set Results")
    print(epoch_tests[f"Test-{i}"]["error_metrics"].iloc[0],
    ↪end=f"\n{'-'*100}\n\n\n")

    ax = epoch_tests[f"Test-{i}"]["training_results"].plot(
        y=["rmse", "val_rmse"], title=f"Model Trained with {i*500} Epochs",
    )
    ax.set_xlabel("Epochs")
    ax.set_ylabel("Root Mean Squared Error (RMSE)")

```

Model trained with 500 epochs

```

-----
Final Training results
mae      14.250674
mse      608.270719
r_sqr     0.796728
rmse     24.663145

```

Name: 499, dtype: float64

-----  
-----  
Final Validation results

val\_mae        14.898253

val\_mse        760.187084

val\_r\_sqr      0.768658

val\_rmse       27.571490

Name: 499, dtype: float64

-----  
-----  
Test Set Results

mse            997.685762

rmse           31.586164

mae            19.087420

r\_sqr          0.691045

st\_mse         0.002737

st\_rmse        0.052316

st\_mae         0.031252

st\_r\_sqr       0.829853

Name: 0, dtype: float64

=====

Model trained with 1000 epochs

-----  
-----  
Final Training results

mae            7.462920

mse            198.739946

r\_sqr          0.940075

rmse           14.097516

Name: 999, dtype: float64

-----  
-----  
Final Validation results

val\_mae        8.262152

val\_mse        232.112890

val\_r\_sqr      0.911425

val\_rmse       15.235252

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse            932.481105

rmse           30.536554

```
mae          18.012795
r_sqr        0.681143
st_mse       0.001910
st_rmse      0.043706
st_mae       0.020994
st_r_sqr     0.914481
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with 1500 epochs

```
-----
-----
```

Final Training results

```
mae          12.254076
mse          401.123070
r_sqr        0.869136
rmse         20.028057
Name: 1499, dtype: float64
```

```
-----
-----
```

Final Validation results

```
val_mae      11.954071
val_mse      430.115301
val_r_sqr    0.877540
val_rmse     20.739221
Name: 1499, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse          749.105870
rmse         27.369799
mae          16.441220
r_sqr        0.730054
st_mse       0.001777
st_rmse      0.042156
st_mae       0.025847
st_r_sqr     0.871320
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with 2000 epochs

```
-----
```

```
-----  
Final Training results  
mae          6.324813  
mse          160.054810  
r_sqr        0.944095  
rmse         12.651277  
Name: 1999, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae       5.521292  
val_mse       113.716825  
val_r_sqr     0.966952  
val_rmse      10.663809  
Name: 1999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse           1581.427295  
rmse          39.767164  
mae           24.011278  
r_sqr         0.543158  
st_mse        0.001004  
st_rmse       0.031681  
st_mae        0.019146  
st_r_sqr      0.954978  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model trained with 2500 epochs

```
-----
```

```
-----  
Final Training results  
mae          8.060659  
mse          194.926005  
r_sqr        0.941976  
rmse         13.961590  
Name: 2499, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae       7.718837  
val_mse       153.990023  
val_r_sqr     0.937434  
val_rmse      12.409272
```

Name: 2499, dtype: float64

-----  
-----  
Test Set Results

mse	1143.020080
rmse	33.808580
mae	20.808783
r_sqr	0.611482
st_mse	0.001003
st_rmse	0.031676
st_mae	0.017815
st_r_sqr	0.946819

Name: 0, dtype: float64

=====

Model trained with 3000 epochs

-----  
-----  
Final Training results

mae	7.387638
mse	176.240466
r_sqr	0.949655
rmse	13.275559

Name: 2999, dtype: float64

-----  
-----  
Final Validation results

val_mae	7.344473
val_mse	149.975472
val_r_sqr	0.942712
val_rmse	12.246447

Name: 2999, dtype: float64

-----  
-----  
Test Set Results

mse	1030.928267
rmse	32.108072
mae	19.467383
r_sqr	0.563387
st_mse	0.001600
st_rmse	0.040000
st_mae	0.021656
st_r_sqr	0.927151

Name: 0, dtype: float64

=====

Model trained with 3500 epochs

-----

-----  
Final Training results

mae 9.188253  
mse 240.062707  
r\_sqr 0.916744  
rmse 15.493957  
Name: 3499, dtype: float64

-----

-----  
Final Validation results

val\_mae 9.977215  
val\_mse 284.290656  
val\_r\_sqr 0.913355  
val\_rmse 16.860921  
Name: 3499, dtype: float64

-----

-----  
Test Set Results

mse 715.415637  
rmse 26.747255  
mae 15.717038  
r\_sqr 0.798475  
st\_mse 0.002235  
st\_rmse 0.047271  
st\_mae 0.026956  
st\_r\_sqr 0.901951  
Name: 0, dtype: float64

=====

=====

Model trained with 4000 epochs

-----

-----  
Final Training results

mae 7.064710  
mse 169.929624  
r\_sqr 0.944214  
rmse 13.035706  
Name: 3999, dtype: float64

-----



```
-----  
Final Validation results  
val_mae      6.893713  
val_mse      181.037869  
val_r_sqr     0.946676  
val_rmse     13.455031  
Name: 3999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          825.084728  
rmse         28.724288  
mae          15.858095  
r_sqr        0.720780  
st_mse       0.001050  
st_rmse      0.032406  
st_mae       0.017812  
st_r_sqr     0.928742  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model trained with 4500 epochs

```
-----  
Final Training results  
mae          7.692935  
mse          190.144104  
r_sqr        0.932934  
rmse         13.789275  
Name: 4499, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae      7.696800  
val_mse      179.046700  
val_r_sqr     0.947344  
val_rmse     13.380833  
Name: 4499, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          195.611898  
rmse         13.986132  
mae          7.689333  
r_sqr        0.945380
```

```
st_mse      0.001680
st_rmse     0.040985
st_mae      0.022904
st_r_sqr    0.855593
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with 5000 epochs

```
-----
-----
```

Final Training results

```
mae      9.628797
mse     317.097017
r_sqr    0.900590
rmse    17.807218
Name: 4999, dtype: float64
```

```
-----
-----
```

Final Validation results

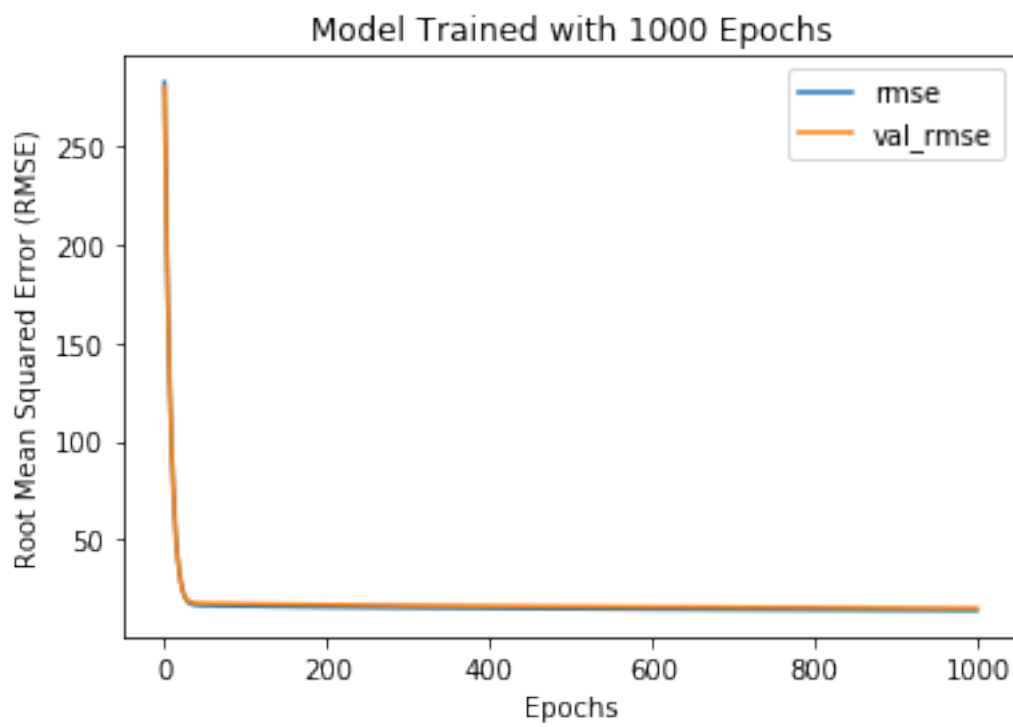
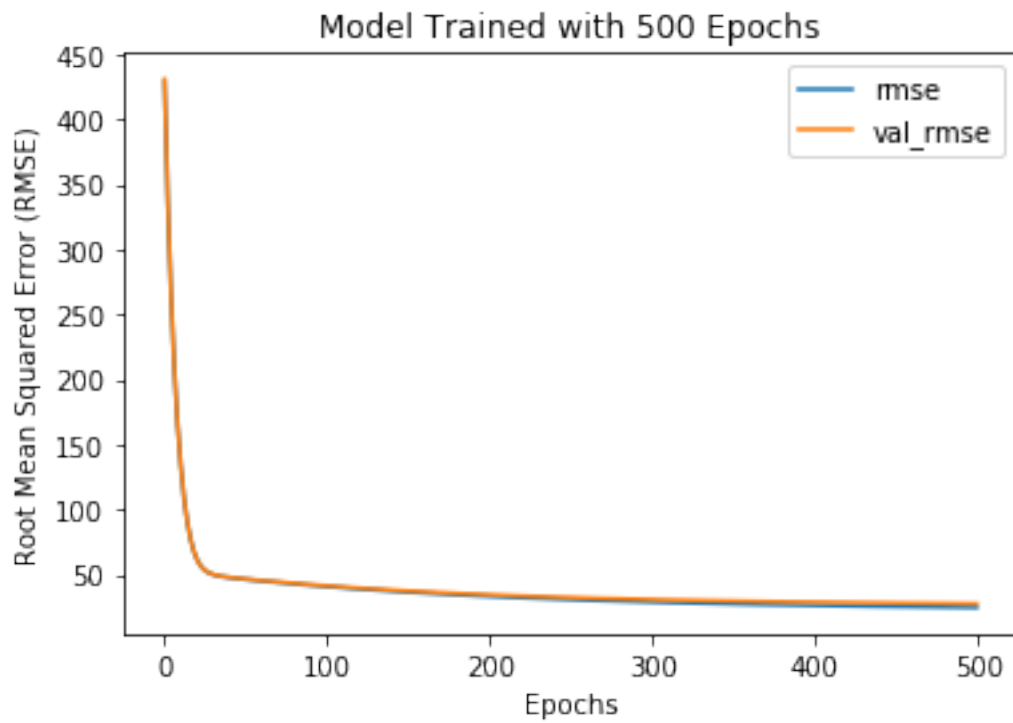
```
val_mae    10.011215
val_mse    323.438232
val_r_sqr   0.904837
val_rmse   17.984389
Name: 4999, dtype: float64
```

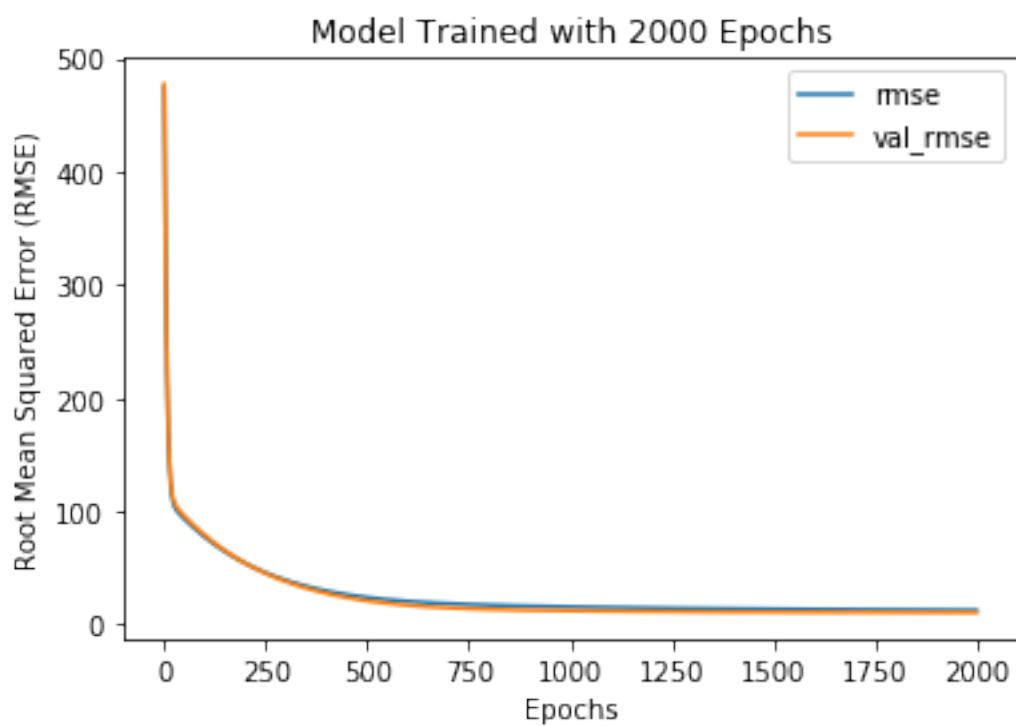
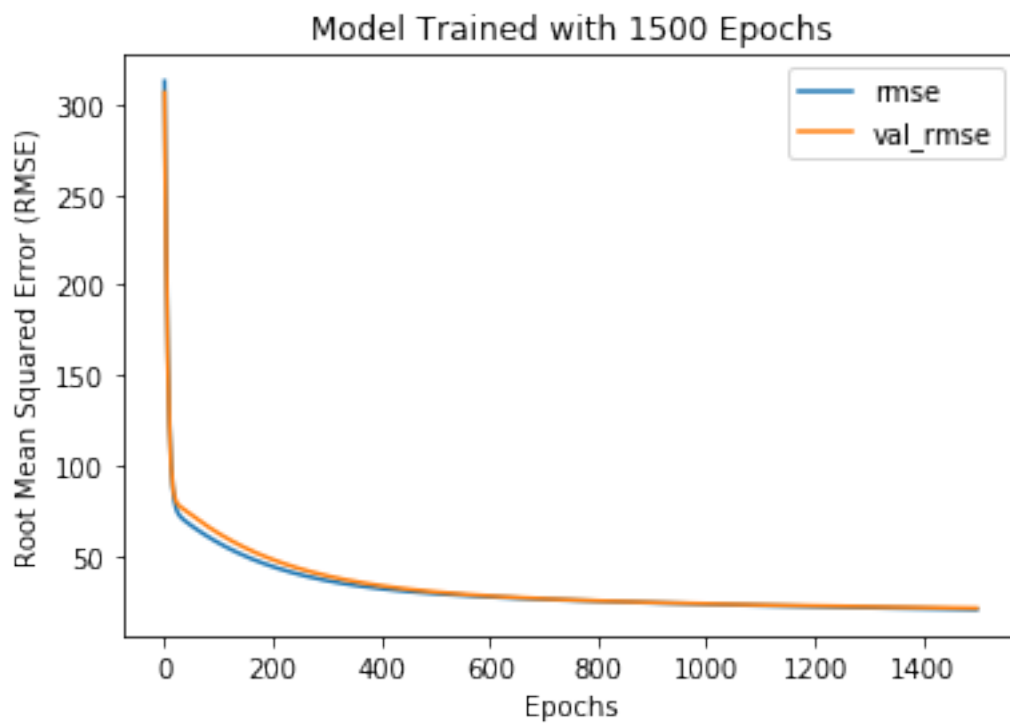
```
-----
-----
```

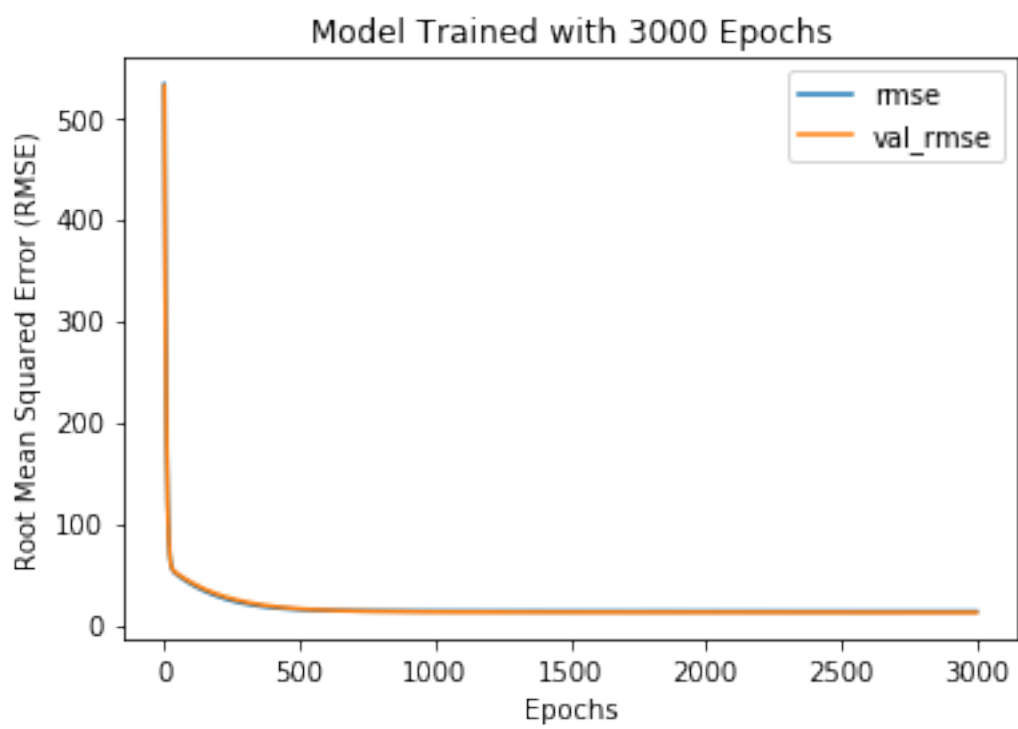
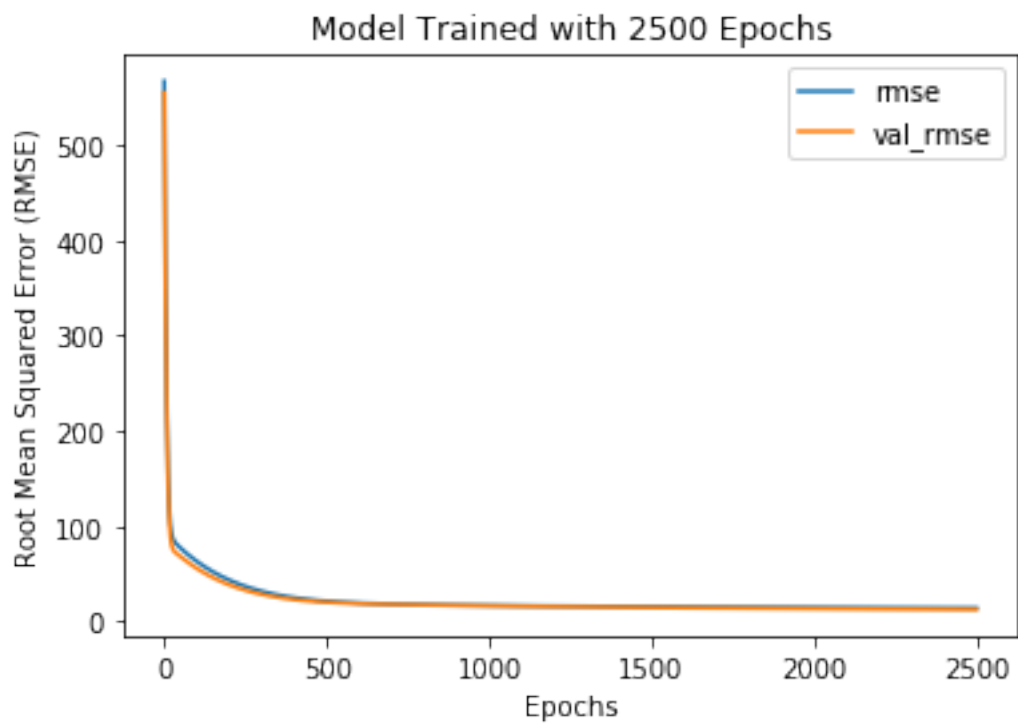
Test Set Results

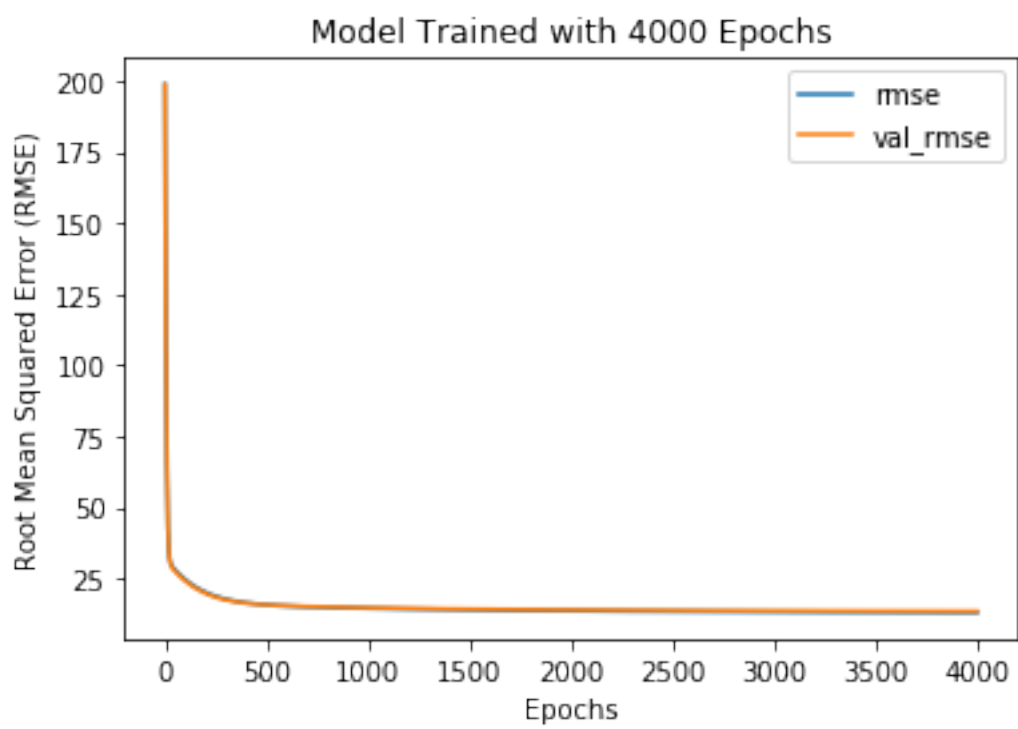
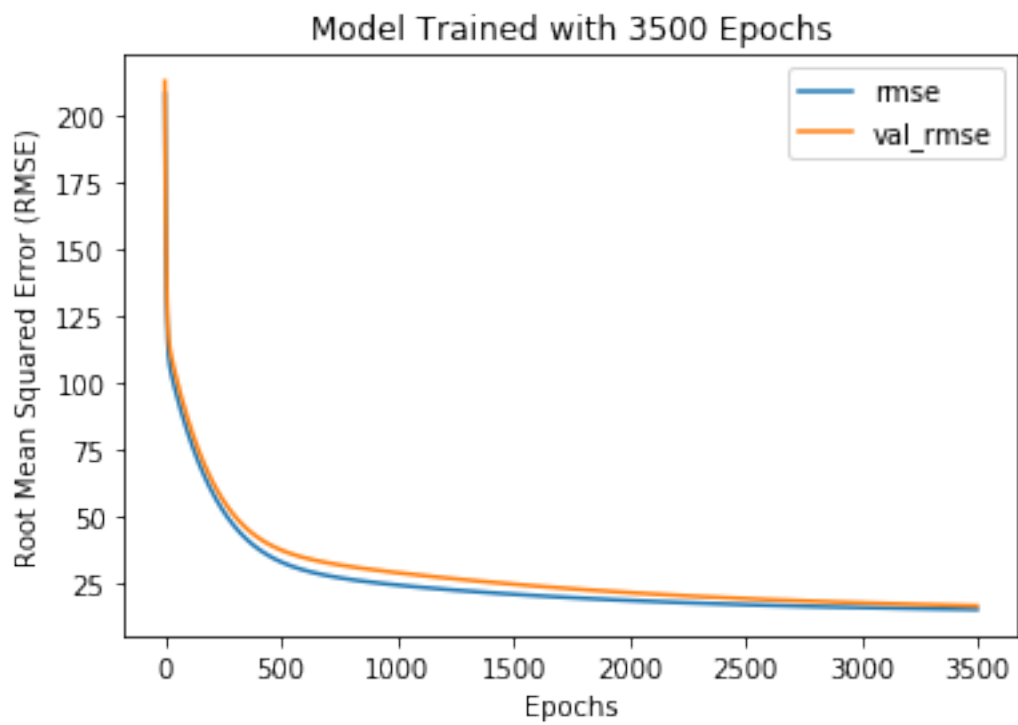
```
mse      270.961722
rmse     16.460915
mae       9.025331
r_sqr     0.892656
st_mse    0.002518
st_rmse   0.050182
st_mae    0.027726
st_r_sqr  0.799610
Name: 0, dtype: float64
```

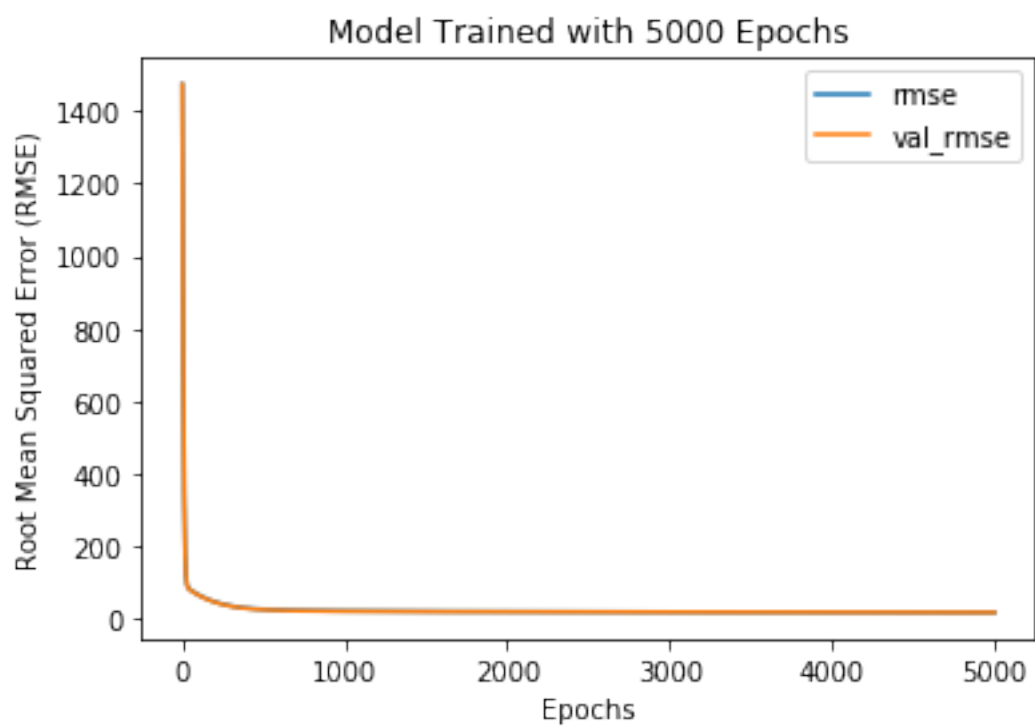
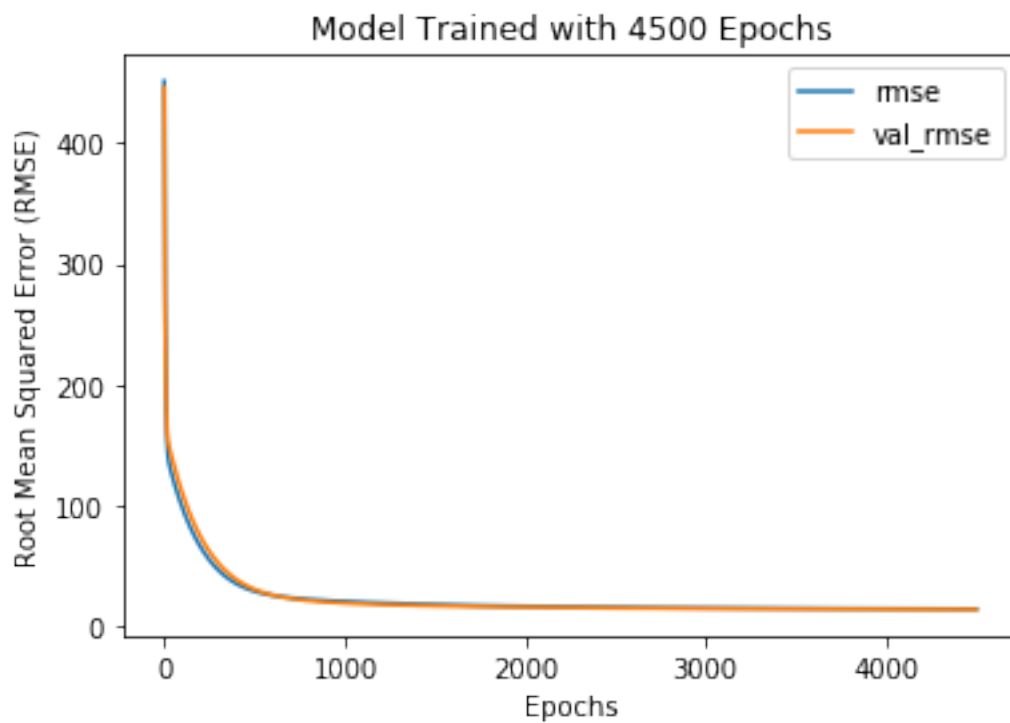
```
=====
=====
```











- Models seem to overfit after about 800 to 1200 epochs
- Suitable number for training is around 1000

## Learning Rate

```
[11]: l_rate_tests = dict()
      for i in range(1, 11):
          l_rate_tests[f"Test-{i}"] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", 1),
              activ_func="linear",
              epochs=1000,
              l_rate=0.1*i
          )

[12]: for i in range(1,11):
      print(f"Model Trained with Learning Rate of {0.1*i}", end=f"\n{'-'*100}\n")

      print("Final Training results")
      print(l_rate_tests[f"Test-{i}"]["training_results"].iloc[-1, :4],
            ↪end=f"\n{'-'*100}\n")

      print("Final Validation results")
      print(l_rate_tests[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
            ↪end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(l_rate_tests[f"Test-{i}"]["error_metrics"].iloc[0],
            ↪end=f"\n{'-'*100}\n\n\n")

      ax = l_rate_tests[f"Test-{i}"]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with Learning Rate of_
            ↪{i*0.1}",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Learning Rate of 0.1

```
-----
Final Training results
mae      8.646140
mse      225.832283
r_sqr     0.932451
rmse     15.027717
Name: 999, dtype: float64
-----
```



```
-----  
Final Validation results  
val_mae      8.206920  
val_mse      226.594284  
val_r_sqr     0.911532  
val_rmse     15.053049  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          499.063453  
rmse         22.339728  
mae          13.266776  
r_sqr        0.827178  
st_mse       0.001469  
st_rmse      0.038327  
st_mae       0.021745  
st_r_sqr     0.920706  
Name: 0, dtype: float64  
=====
```

Model Trained with Learning Rate of 0.2

```
-----  
Final Training results  
mae          8.149623  
mse          183.333082  
r_sqr        0.936561  
rmse         13.540055  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation results  
val_mae      8.724627  
val_mse      259.048363  
val_r_sqr     0.930949  
val_rmse     16.094979  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          279.405541  
rmse         16.715428  
mae          10.350494  
r_sqr        0.909043
```

```
st_mse      0.001193
st_rmse     0.034541
st_mae      0.019907
st_r_sqr    0.922097
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Learning Rate of 0.30000000000000004

```
-----
-----
```

Final Training results

```
mae      8.875316
mse     232.164985
r_sqr    0.916766
rmse    15.236961
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation results

```
val_mae    9.317005
val_mse   264.603169
val_r_sqr  0.936175
val_rmse   16.266627
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse      550.630922
rmse     23.465526
mae      13.402733
r_sqr    0.815074
st_mse    0.004784
st_rmse   0.069170
st_mae    0.037795
st_r_sqr  0.807651
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Learning Rate of 0.4

```
-----
-----
```

Final Training results

```
mae          9.358519
mse          268.980714
r_sqr        0.909705
rmse         16.400632
Name: 999, dtype: float64
```

---

Final Validation results

```
val_mae       8.922762
val_mse       244.808540
val_r_sqr     0.919457
val_rmse      15.646359
Name: 999, dtype: float64
```

---

Test Set Results

```
mse          469.365558
rmse         21.664846
mae          12.242115
r_sqr        0.866366
st_mse        0.001745
st_rmse       0.041778
st_mae        0.021946
st_r_sqr      0.913637
Name: 0, dtype: float64
```

---

```
=====
=====
```

Model Trained with Learning Rate of 0.5

---

Final Training results

```
mae          6.682250
mse          136.031159
r_sqr        0.951552
rmse         11.663240
Name: 999, dtype: float64
```

---

Final Validation results

```
val_mae       6.567995
val_mse       155.812608
val_r_sqr     0.956987
val_rmse      12.482492
Name: 999, dtype: float64
```

---

```

-----
Test Set Results
mse          204.102515
rmse         14.286445
mae          8.496443
r_sqr        0.940618
st_mse       0.001763
st_rmse      0.041987
st_mae       0.022318
st_r_sqr     0.896972
Name: 0, dtype: float64
=====
=====

```

Model Trained with Learning Rate of 0.6000000000000001

```

-----
Final Training results
mae          8.614596
mse         221.458964
r_sqr        0.932705
rmse        14.881497
Name: 999, dtype: float64
-----

```

```

-----
Final Validation results
val_mae      8.635414
val_mse     211.222346
val_r_sqr    0.924020
val_rmse    14.533490
Name: 999, dtype: float64
-----

```

```

-----
Test Set Results
mse          268.766229
rmse         16.394091
mae          9.765973
r_sqr        0.905441
st_mse       0.000744
st_rmse      0.027267
st_mae       0.015875
st_r_sqr     0.947569
Name: 0, dtype: float64
=====
=====

```

Model Trained with Learning Rate of 0.7000000000000001

Final Training results

mae 5.532851  
mse 117.335432  
r\_sqr 0.962092  
rmse 10.832148  
Name: 999, dtype: float64

Final Validation results

val\_mae 6.086643  
val\_mse 151.433658  
val\_r\_sqr 0.955680  
val\_rmse 12.305838  
Name: 999, dtype: float64

Test Set Results

mse 2144.682506  
rmse 46.310717  
mae 27.081051  
r\_sqr 0.227825  
st\_mse 0.001620  
st\_rmse 0.040247  
st\_mae 0.021826  
st\_r\_sqr 0.910596  
Name: 0, dtype: float64

Model Trained with Learning Rate of 0.8

Final Training results

mae 5.954080  
mse 127.160886  
r\_sqr 0.956463  
rmse 11.276564  
Name: 999, dtype: float64

Final Validation results

```
val_mae      7.456241
val_mse      202.591333
val_r_sqr     0.945969
val_rmse     14.233458
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          916.333690
rmse         30.271004
mae          18.496051
r_sqr        0.689704
st_mse       0.000795
st_rmse      0.028189
st_mae       0.015935
st_r_sqr     0.958190
Name: 0, dtype: float64
```

=====

Model Trained with Learning Rate of 0.9

-----  
-----  
Final Training results

```
mae          6.346123
mse         135.253986
r_sqr        0.955358
rmse        11.629875
Name: 999, dtype: float64
```

-----  
-----  
Final Validation results

```
val_mae      6.373476
val_mse     119.421225
val_r_sqr    0.956759
val_rmse    10.928002
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          328.133564
rmse         18.114457
mae          10.229089
r_sqr        0.909651
st_mse       0.000777
st_rmse      0.027875
```

```
st_mae      0.014975
st_r_sqr     0.957077
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Learning Rate of 1.0

```
-----
-----
```

Final Training results

```
mae      7.569875
mse     191.786131
r_sqr    0.938449
rmse     13.848687
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation results

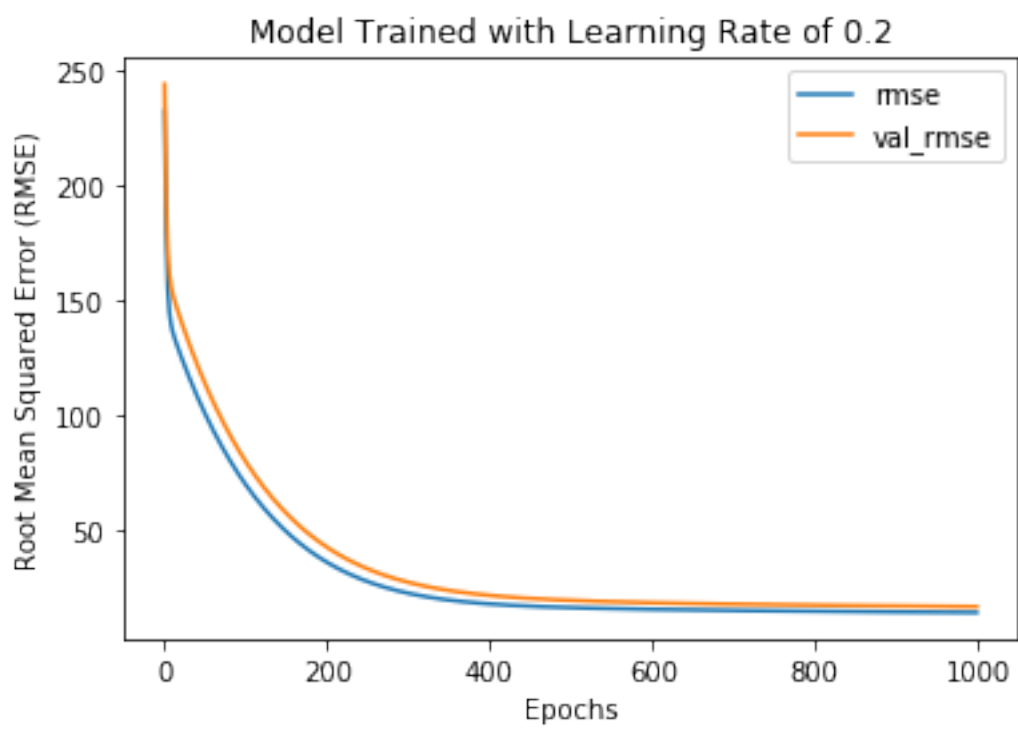
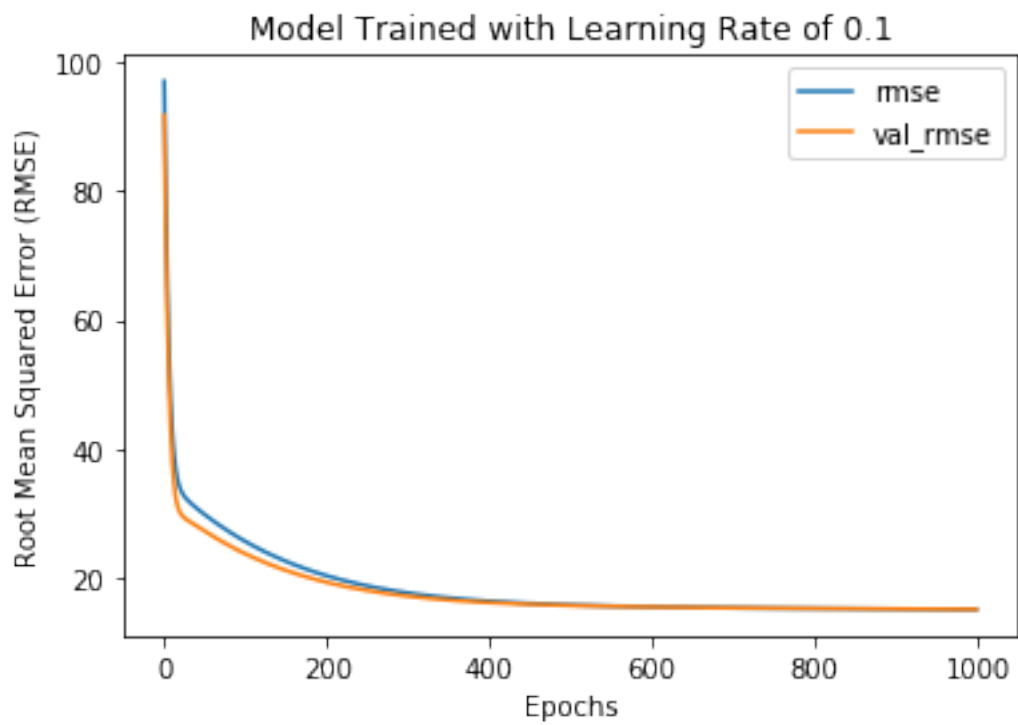
```
val_mae    7.170502
val_mse    190.116785
val_r_sqr   0.933657
val_rmse    13.788284
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

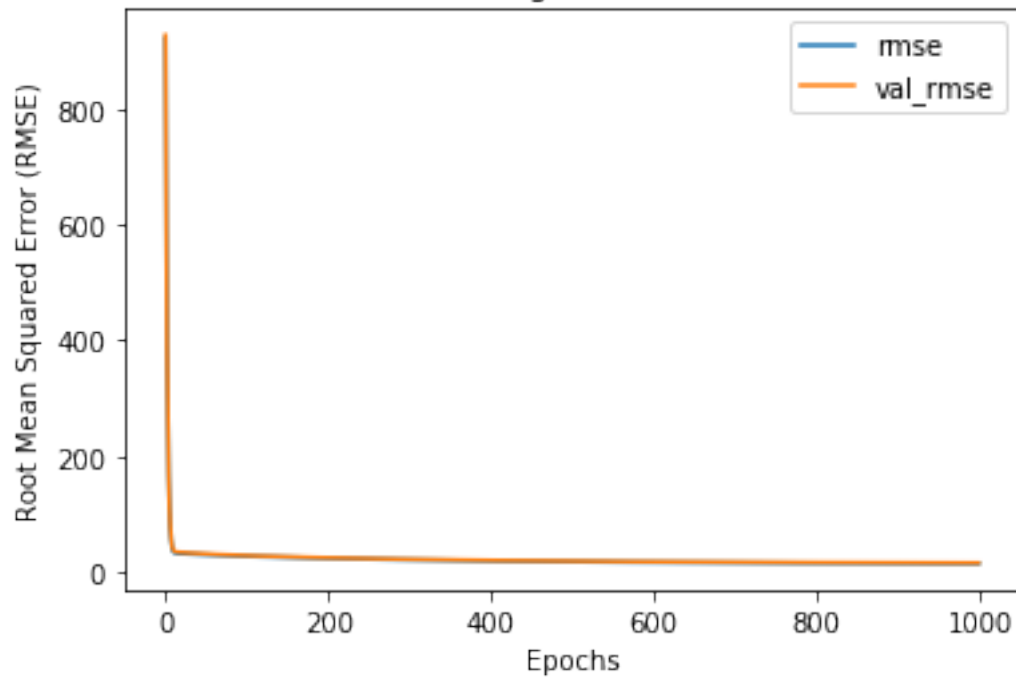
```
mse      581.593532
rmse     24.116250
mae      14.473857
r_sqr     0.822551
st_mse     0.000960
st_rmse    0.030981
st_mae     0.016611
st_r_sqr    0.949105
Name: 0, dtype: float64
```

```
=====
=====
```

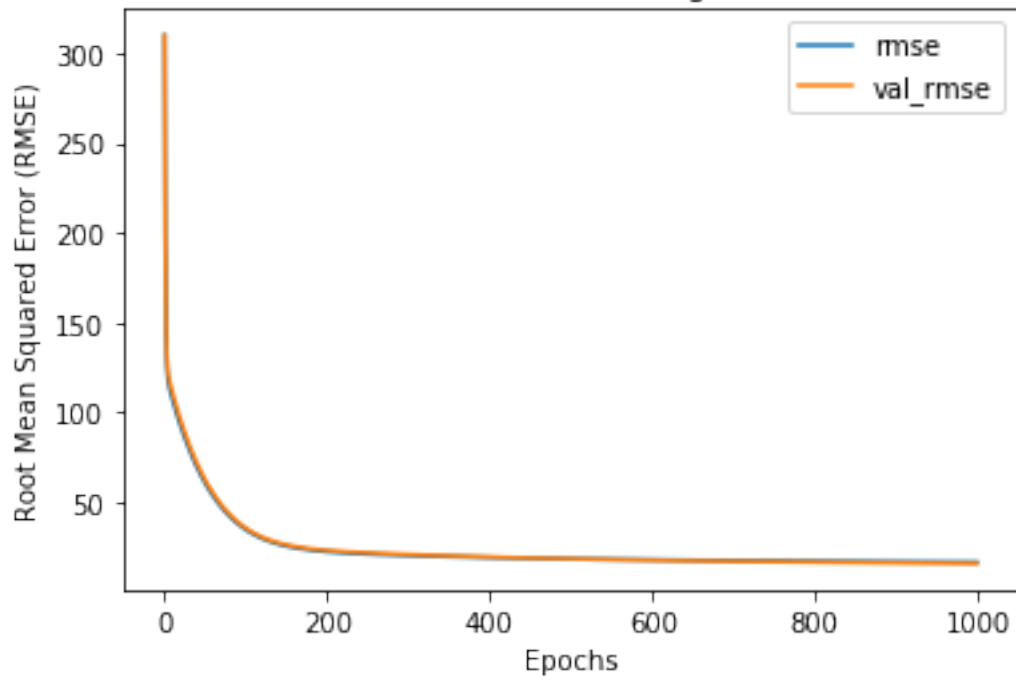


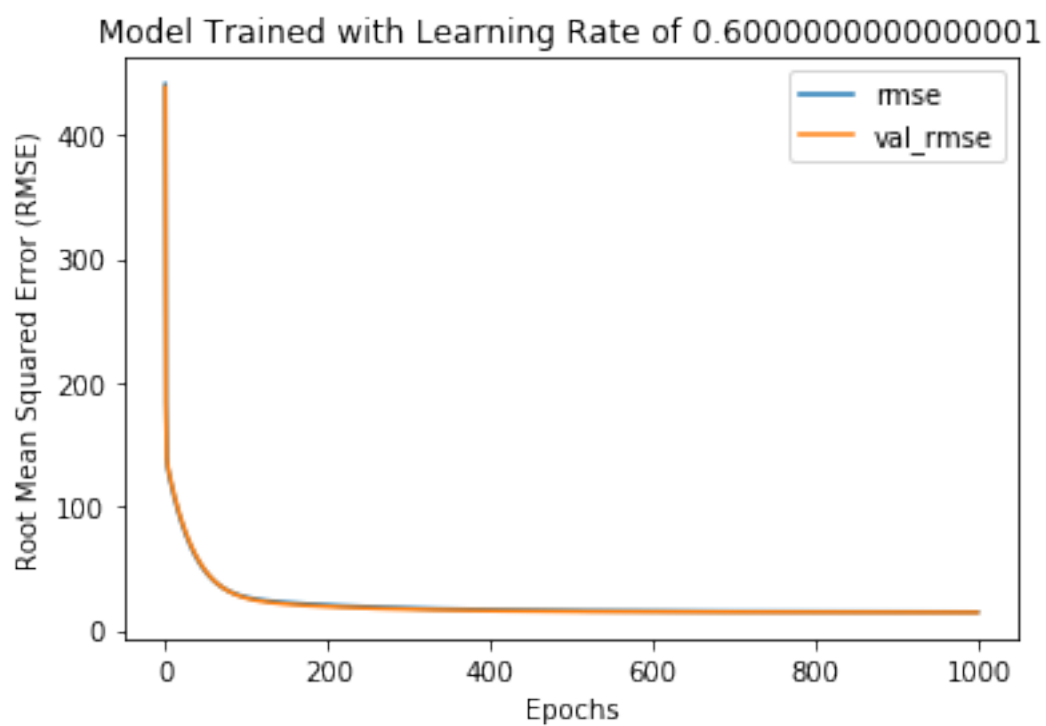
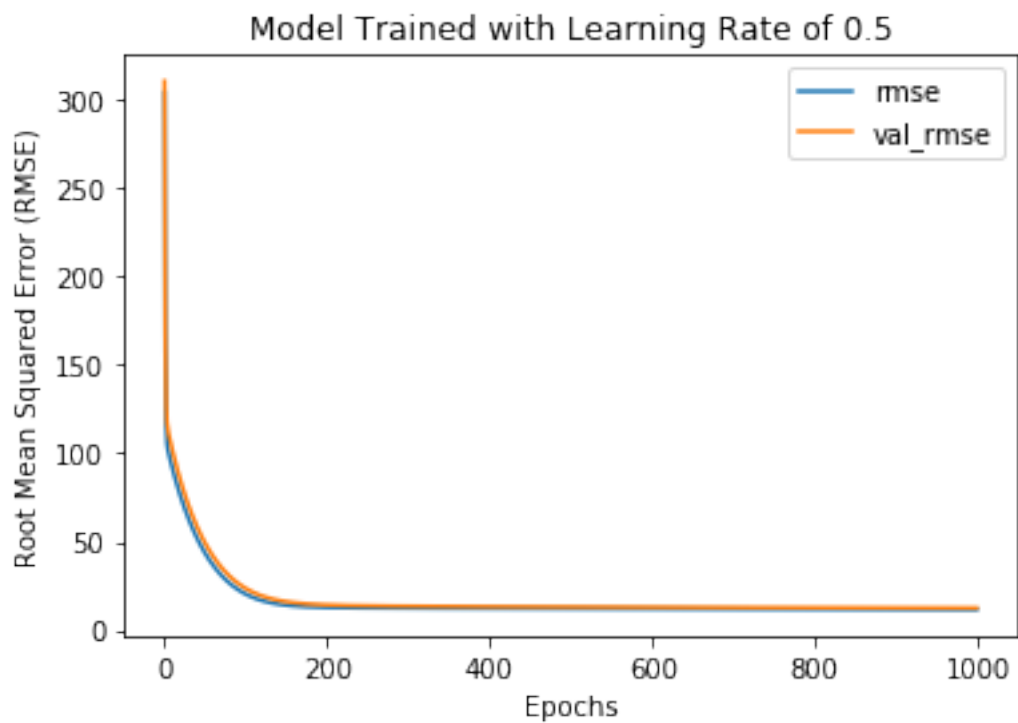


Model Trained with Learning Rate of 0.30000000000000004

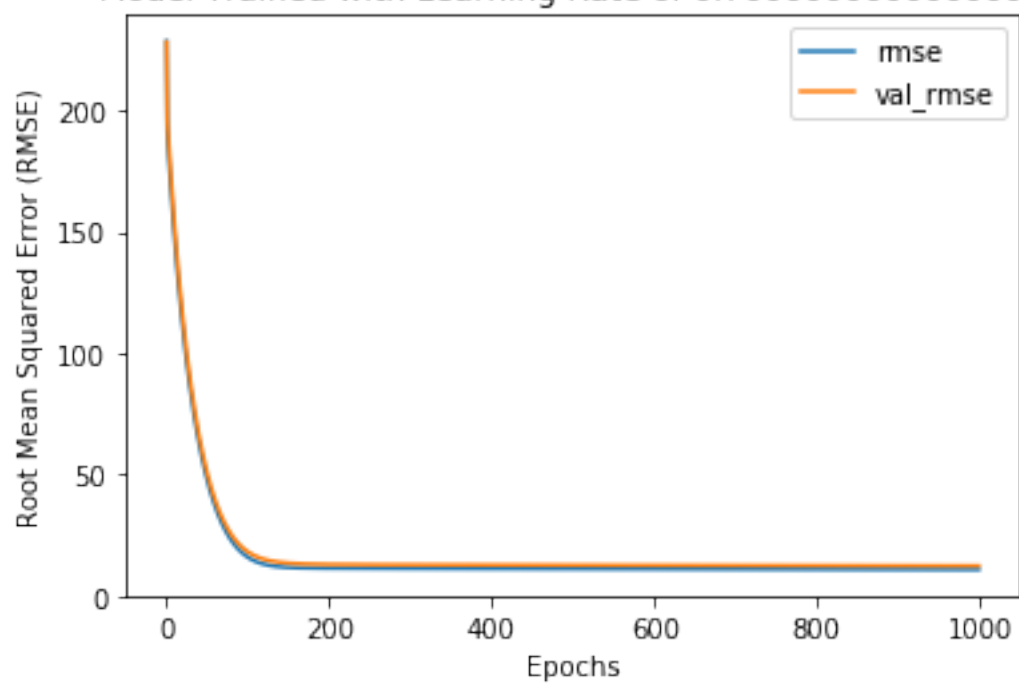


Model Trained with Learning Rate of 0.4

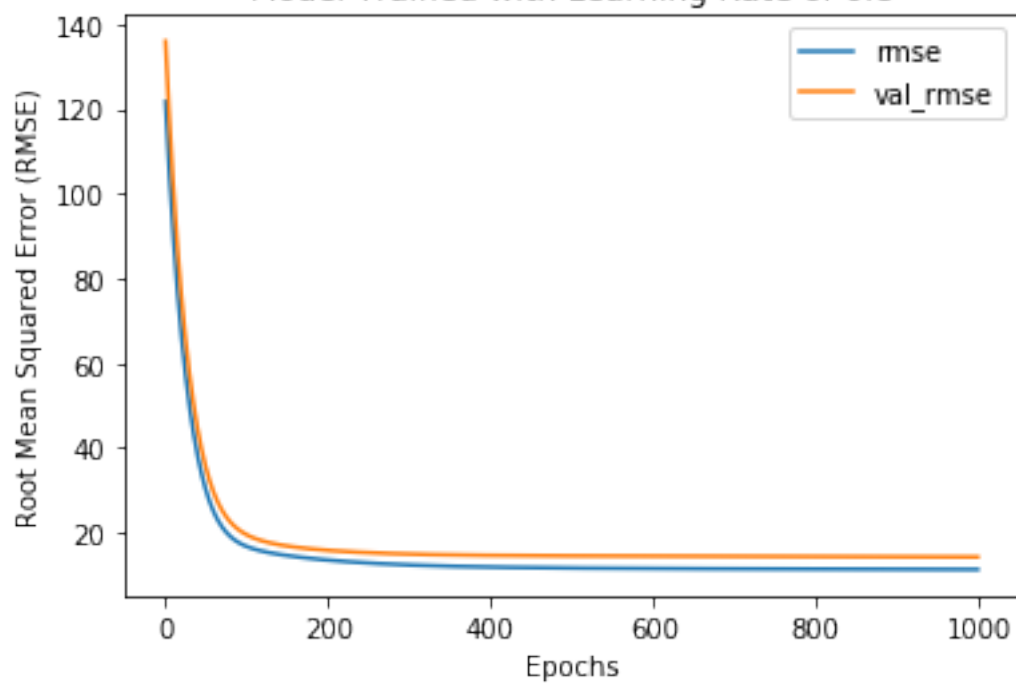


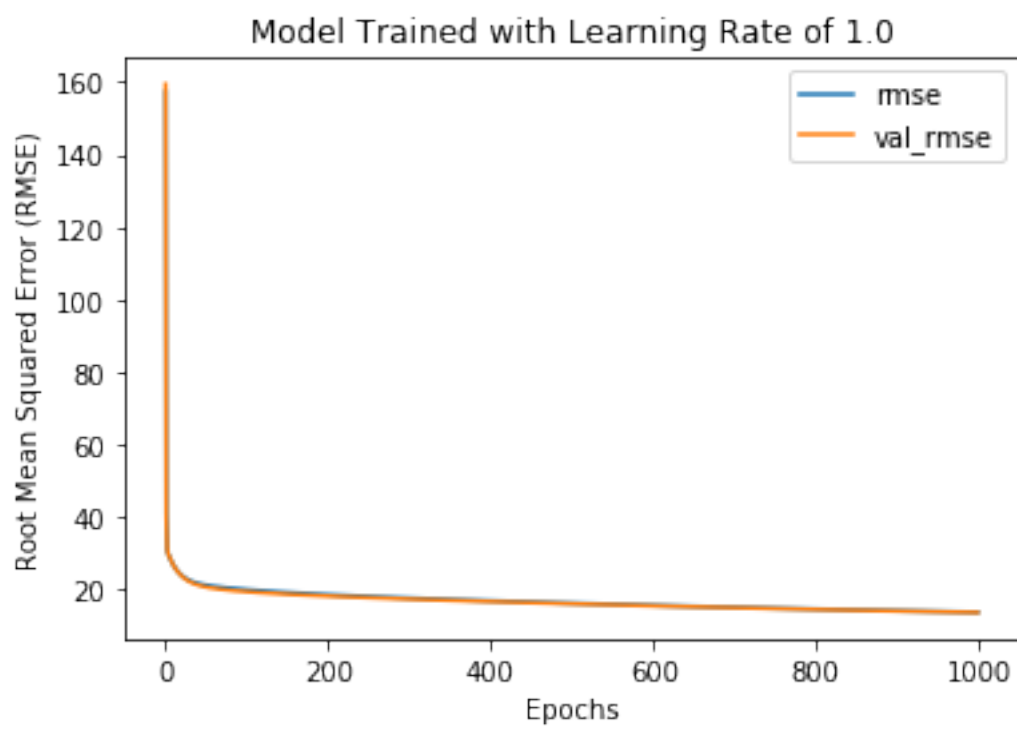
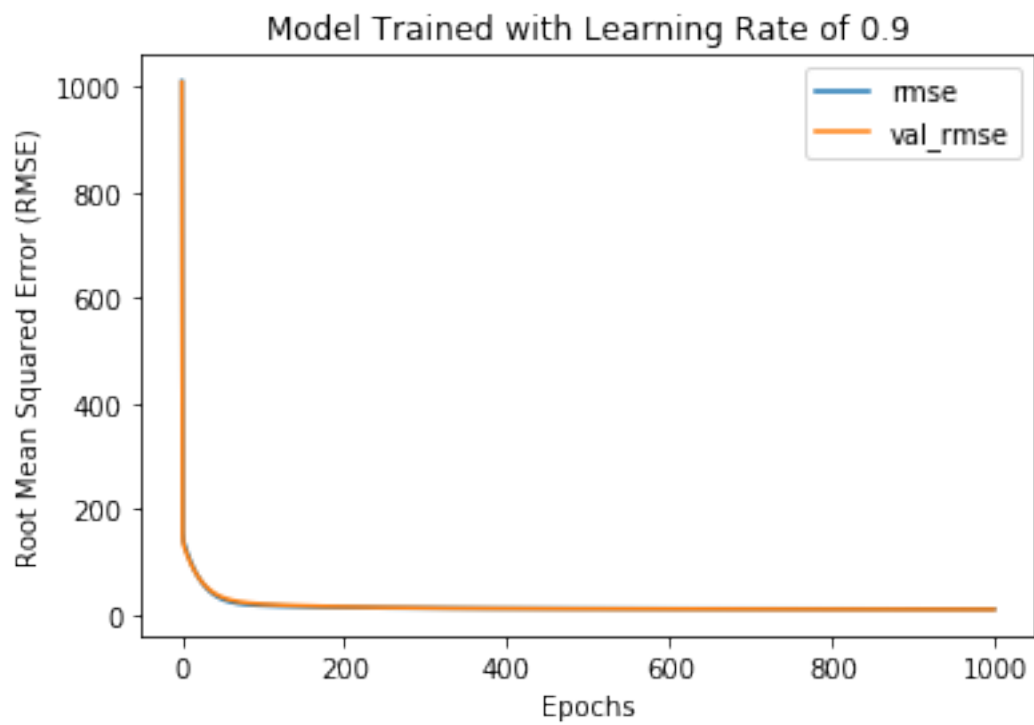


Model Trained with Learning Rate of 0.7000000000000001



Model Trained with Learning Rate of 0.8





- learning rate of 0.3 appears to offer most consistent results between training, validation and testing sets

### Activation Functions

```
[13]: activ_tests = dict()
      activations = ("sigmoid", "tanh", "relu", "linear")

      for func in activations:
          activ_tests[func] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", 1),
              activ_func=func,
              epochs=1000,
              l_rate=0.3
          )

[14]: for a in activations:
      print(f"Model Trained with {a.capitalize()} Activation Function",
        ↪end=f"\n{'-'*100}\n")

      print("Final Training Results")
      print(activ_tests[a]["training_results"].iloc[-1, :4], end=f"\n{'-'*100}\n")

      print("Final Validation Results")
      print(activ_tests[a]["training_results"].iloc[-1, 8:12],
        ↪end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(activ_tests[a]["error_metrics"].iloc[0], end=f"\n{'-'*100}\n\n\n\n")

      ax = activ_tests[a]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with {a.capitalize()}
        ↪Activation Function",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Sigmoid Activation Function

```
-----
-----
Final Training Results
mae      37.054013
mse      2444.546711
r_sqr     0.245836
rmse      49.442357
Name: 999, dtype: float64
```

```

-----
Final Validation Results
val_mae      36.524513
val_mse      2135.781825
val_r_sqr     0.215808
val_rmse     46.214520
Name: 999, dtype: float64
-----

```

```

-----
Test Set Results
mse          2216.249955
rmse         47.077064
mae          36.780372
r_sqr        0.267296
st_mse       0.011574
st_rmse      0.107584
st_mae       0.075433
st_r_sqr     0.231381
Name: 0, dtype: float64
=====
=====

```

#### Model Trained with Tanh Activation Function

```

-----
Final Training Results
mae          7.083939
mse          161.562093
r_sqr        0.946840
rmse         12.710708
Name: 999, dtype: float64
-----

```

```

-----
Final Validation Results
val_mae      7.491849
val_mse      206.925613
val_r_sqr     0.942838
val_rmse     14.384909
Name: 999, dtype: float64
-----

```

```

-----
Test Set Results
mse          1363.293353
rmse         36.922803
mae          24.791772

```

```
r_sqr      0.504479
st_mse     0.001226
st_rmse    0.035017
st_mae     0.020053
st_r_sqr   0.935344
Name: 0, dtype: float64
```

```
=====
=====
```

#### Model Trained with Relu Activation Function

```
-----
```

##### Final Training Results

```
mae      8.207435
mse     212.462493
r_sqr    0.930997
rmse    14.576093
Name: 999, dtype: float64
```

```
-----
```

##### Final Validation Results

```
val_mae    9.290627
val_mse   254.122270
val_r_sqr  0.934525
val_rmse   15.941213
Name: 999, dtype: float64
```

```
-----
```

##### Test Set Results

```
mse     1365.702469
rmse     36.955412
mae     24.426133
r_sqr    0.420647
st_mse    0.000955
st_rmse   0.030899
st_mae    0.019378
st_r_sqr  0.937721
Name: 0, dtype: float64
```

```
=====
=====
```

#### Model Trained with Linear Activation Function

```
-----
```

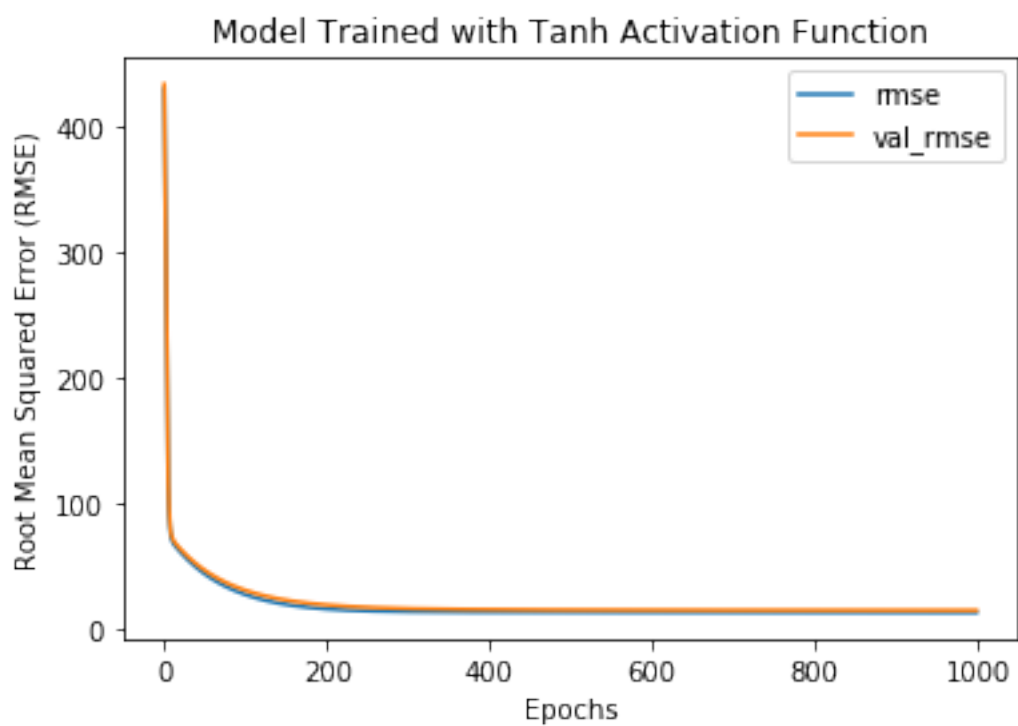
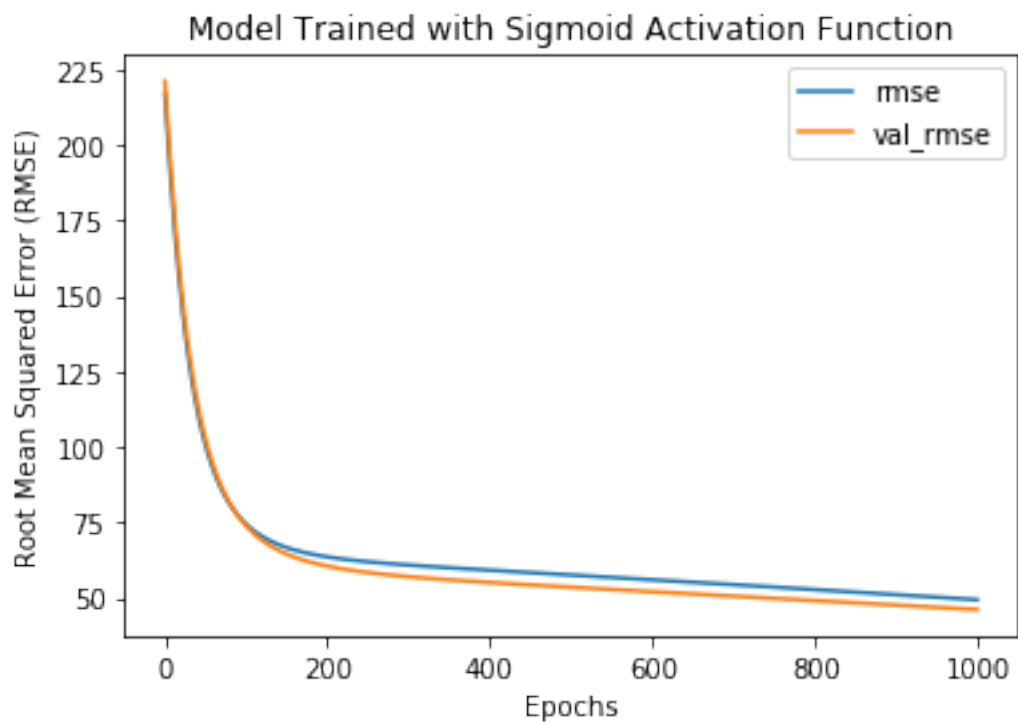
Final Training Results  
mae 7.397667  
mse 183.284561  
r\_sqr 0.941414  
rmse 13.538263  
Name: 999, dtype: float64

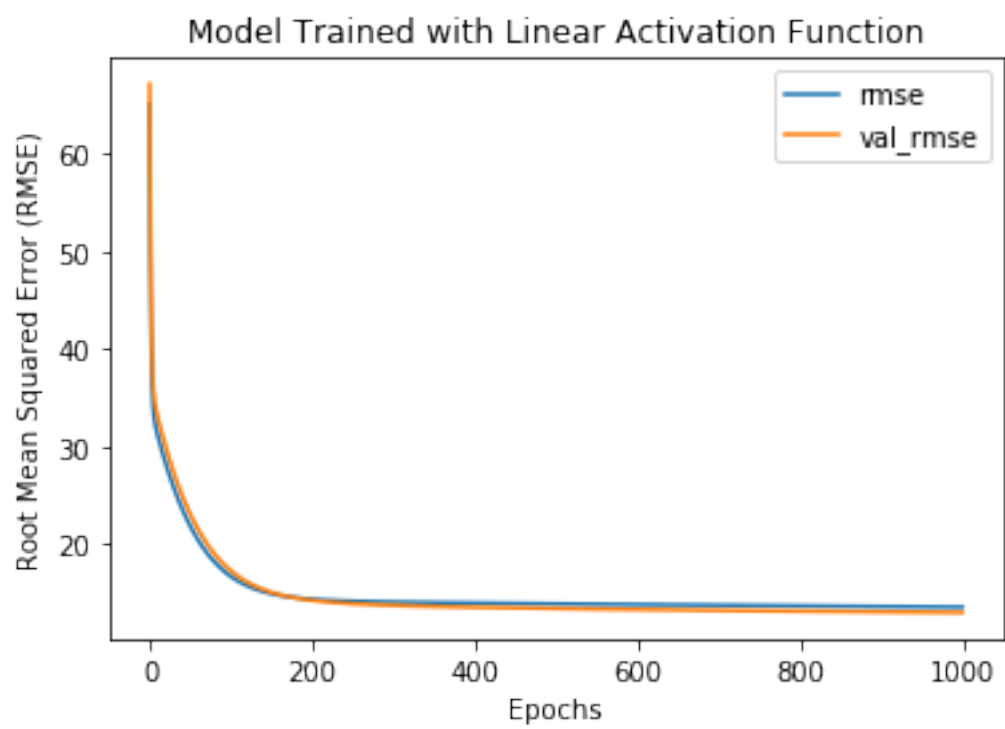
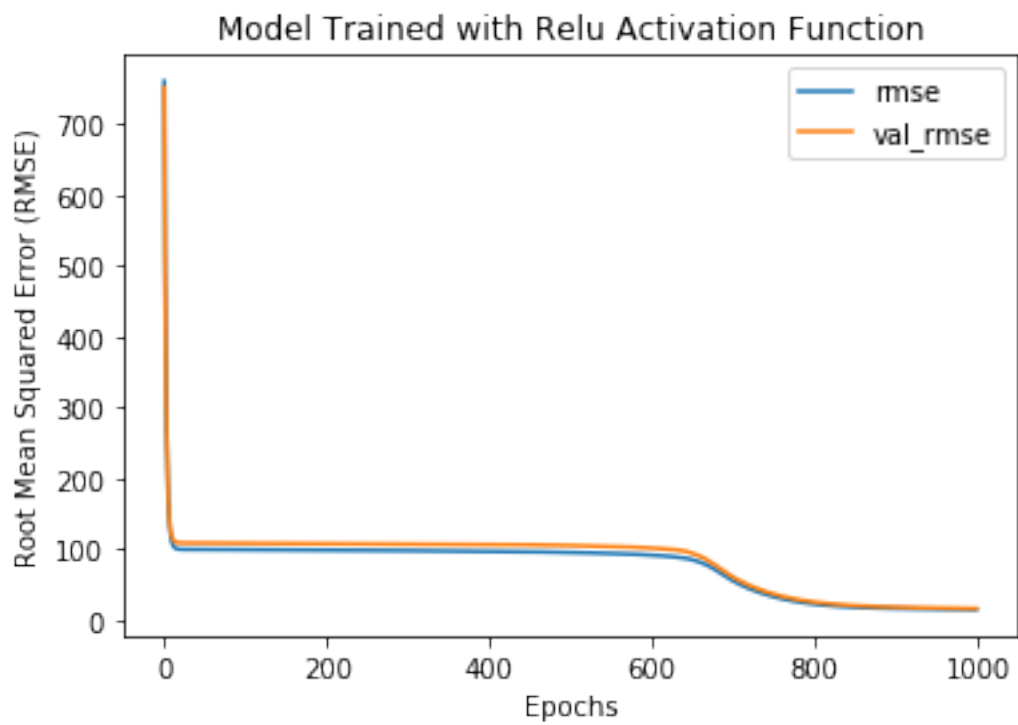
-----  
Final Validation Results  
val\_mae 6.809972  
val\_mse 168.663900  
val\_r\_sqr 0.952675  
val\_rmse 12.987067  
Name: 999, dtype: float64

-----  
Test Set Results  
mse 1335.219034  
rmse 36.540649  
mae 22.382932  
r\_sqr 0.472678  
st\_mse 0.001089  
st\_rmse 0.032999  
st\_mae 0.018397  
st\_r\_sqr 0.937636  
Name: 0, dtype: float64

=====







- tanh is quite clearly the best performing activation function

## Hidden Layers

```
[15]: layer_tests = dict()
for i in range(1, 11):
    layer_tests[f"Test-{i}"] = build_train_test(
        fs,
        feature_cols,
        target_cols,
        layers=("auto", i, 1),
        activ_func="tanh",
        epochs=1000,
        l_rate=0.3
    )
```

```
[16]: for i in range(1,11):
        print(f"Model trained with Single Hidden Layer of Size {i}",
        ↪end=f"\n{'-'*100}\n")

        print("Final Training Results")
        print(layer_tests[f"Test-{i}"]["training_results"].iloc[-1, :4],
        ↪end=f"\n{'-'*100}\n")

        print("Final Validation Results")
        print(layer_tests[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
        ↪end=f"\n{'-'*100}\n")

        print("Test Set Results")
        print(layer_tests[f"Test-{i}"]["error_metrics"].iloc[0],
        ↪end=f"\n{'-'*100}\n\n\n\n")

        ax = layer_tests[f"Test-{i}"]["training_results"].plot(
            y=["rmse", "val_rmse"], title=f"Model trained with Single Hidden Layer
        ↪of Size {i}",
        )
        ax.set_xlabel("Epochs")
        ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model trained with Single Hidden Layer of Size 1

-----

Final Training Results

mae	8.064012
mse	192.208285
r_sqr	0.941583
rmse	13.863920

Name: 999, dtype: float64

-----  
-----  
Final Validation Results

val\_mae           7.717680

val\_mse           171.990340

val\_r\_sqr          0.943096

val\_rmse           13.114509

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse               847.342010

rmse               29.109140

mae                19.468714

r\_sqr              0.671788

st\_mse             0.001094

st\_rmse            0.033083

st\_mae             0.018382

st\_r\_sqr           0.944212

Name: 0, dtype: float64

=====

Model trained with Single Hidden Layer of Size 2

-----  
-----  
Final Training Results

mae                8.915232

mse               220.109533

r\_sqr              0.927152

rmse               14.836089

Name: 999, dtype: float64

-----  
-----  
Final Validation Results

val\_mae            9.058116

val\_mse           215.478264

val\_r\_sqr          0.942478

val\_rmse           14.679178

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse               233.035666

rmse               15.265506

```
mae          10.703701
r_sqr        0.912676
st_mse       0.001095
st_rmse      0.033083
st_mae       0.017417
st_r_sqr     0.917615
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with Single Hidden Layer of Size 3

```
-----
-----
```

Final Training Results

```
mae          9.254691
mse         270.616416
r_sqr        0.905137
rmse        16.450423
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

```
val_mae      9.867702
val_mse     257.389892
val_r_sqr    0.928529
val_rmse    16.043375
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse         1078.781547
rmse        32.844810
mae         22.570404
r_sqr       0.671273
st_mse      0.002443
st_rmse     0.049426
st_mae      0.025979
st_r_sqr    0.883922
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with Single Hidden Layer of Size 4

```
-----
```

```
-----  
Final Training Results  
mae      8.600294  
mse     222.283558  
r_sqr    0.929792  
rmse     14.909177  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae   8.790956  
val_mse  229.241784  
val_r_sqr  0.919376  
val_rmse  15.140733  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse     1726.179383  
rmse    41.547315  
mae     27.559650  
r_sqr    0.452181  
st_mse   0.001560  
st_rmse  0.039501  
st_mae   0.023094  
st_r_sqr  0.922811  
Name: 0, dtype: float64  
=====
```

```
=====
```

Model trained with Single Hidden Layer of Size 5

```
-----
```

```
-----  
Final Training Results  
mae     11.540708  
mse    368.044396  
r_sqr    0.870646  
rmse    19.184483  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae   14.414949  
val_mse  672.346771  
val_r_sqr  0.827337  
val_rmse  25.929650
```

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse	528.112732
rmse	22.980703
mae	14.177557
r_sqr	0.826392
st_mse	0.008521
st_rmse	0.092310
st_mae	0.051561
st_r_sqr	0.664568

Name: 0, dtype: float64

=====

Model trained with Single Hidden Layer of Size 6

-----  
-----  
Final Training Results

mae	8.790562
mse	238.483448
r_sqr	0.916710
rmse	15.442909

Name: 999, dtype: float64

-----  
-----  
Final Validation Results

val_mae	9.148302
val_mse	263.229903
val_r_sqr	0.922821
val_rmse	16.224361

Name: 999, dtype: float64

-----  
-----  
Test Set Results

mse	537.106556
rmse	23.175559
mae	13.916583
r_sqr	0.845319
st_mse	0.002910
st_rmse	0.053947
st_mae	0.026422
st_r_sqr	0.871343

Name: 0, dtype: float64

=====

=====

Model trained with Single Hidden Layer of Size 7

-----

Final Training Results

mae 9.686895  
mse 282.799530  
r\_sqr 0.916667  
rmse 16.816644  
Name: 999, dtype: float64

-----

Final Validation Results

val\_mae 9.006291  
val\_mse 248.341253  
val\_r\_sqr 0.910603  
val\_rmse 15.758847  
Name: 999, dtype: float64

-----

Test Set Results

mse 478.347390  
rmse 21.871154  
mae 13.226941  
r\_sqr 0.809252  
st\_mse 0.001348  
st\_rmse 0.036720  
st\_mae 0.019616  
st\_r\_sqr 0.906809  
Name: 0, dtype: float64

=====

=====

Model trained with Single Hidden Layer of Size 8

-----

Final Training Results

mae 8.573392  
mse 212.627106  
r\_sqr 0.930075  
rmse 14.581739  
Name: 999, dtype: float64

-----



```
-----  
Final Validation Results  
val_mae      8.780433  
val_mse      240.458554  
val_r_sqr     0.931281  
val_rmse     15.506726  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          364.947716  
rmse         19.103605  
mae          10.540391  
r_sqr        0.872627  
st_mse       0.000673  
st_rmse      0.025940  
st_mae       0.016634  
st_r_sqr     0.927550  
Name: 0, dtype: float64  
=====
```

Model trained with Single Hidden Layer of Size 9

```
-----  
Final Training Results  
mae          10.862719  
mse          283.080187  
r_sqr        0.915056  
rmse         16.824987  
Name: 999, dtype: float64  
-----
```

```
-----  
Final Validation Results  
val_mae      10.174926  
val_mse      228.511120  
val_r_sqr     0.903925  
val_rmse     15.116584  
Name: 999, dtype: float64  
-----
```

```
-----  
Test Set Results  
mse          1065.580731  
rmse         32.643234  
mae          22.909868  
r_sqr        0.655941
```

```
st_mse      0.002602
st_rmse     0.051005
st_mae      0.028064
st_r_sqr    0.889588
Name: 0, dtype: float64
```

```
=====
=====
```

Model trained with Single Hidden Layer of Size 10

-----  
Final Training Results

```
mae      7.434438
mse     182.382977
r_sqr    0.944810
rmse    13.504924
Name: 999, dtype: float64
```

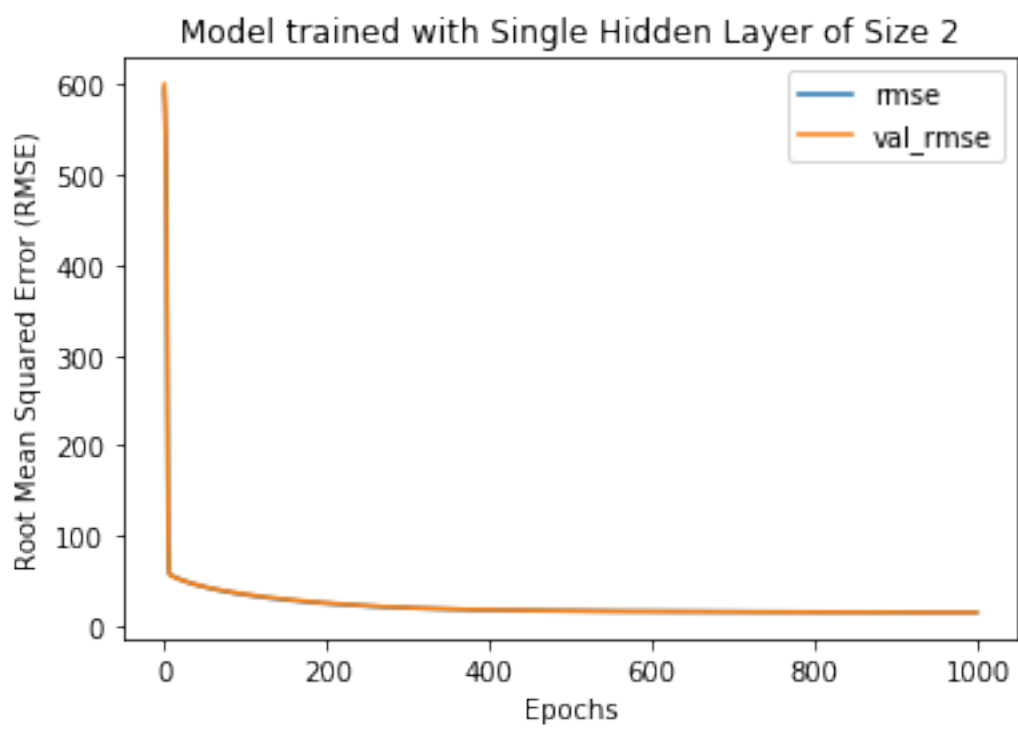
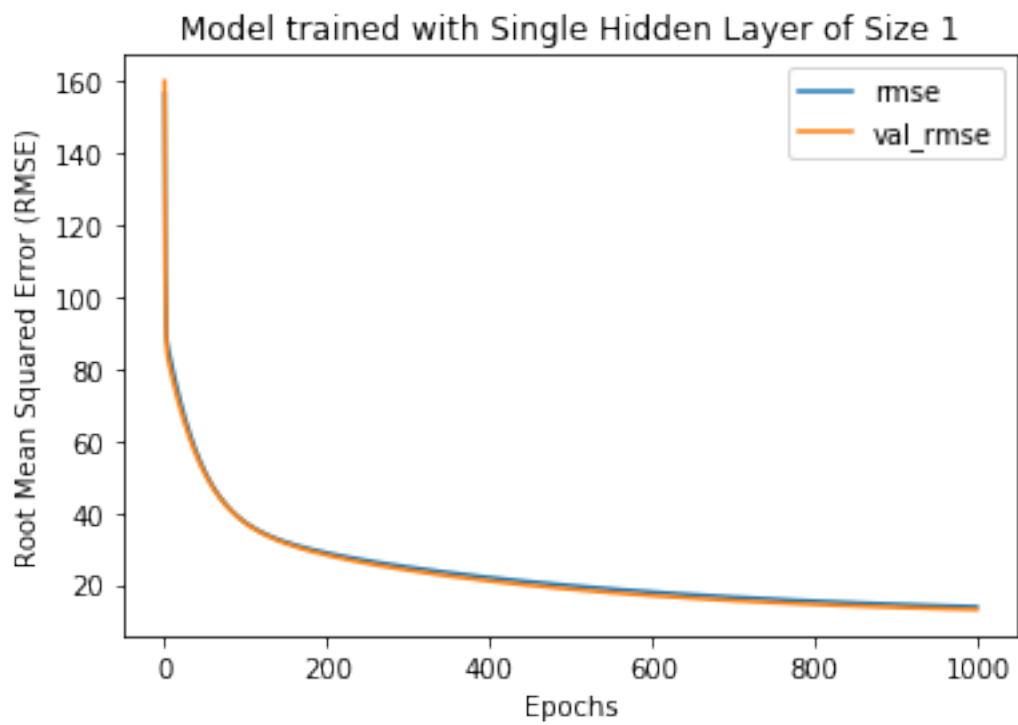
-----  
Final Validation Results

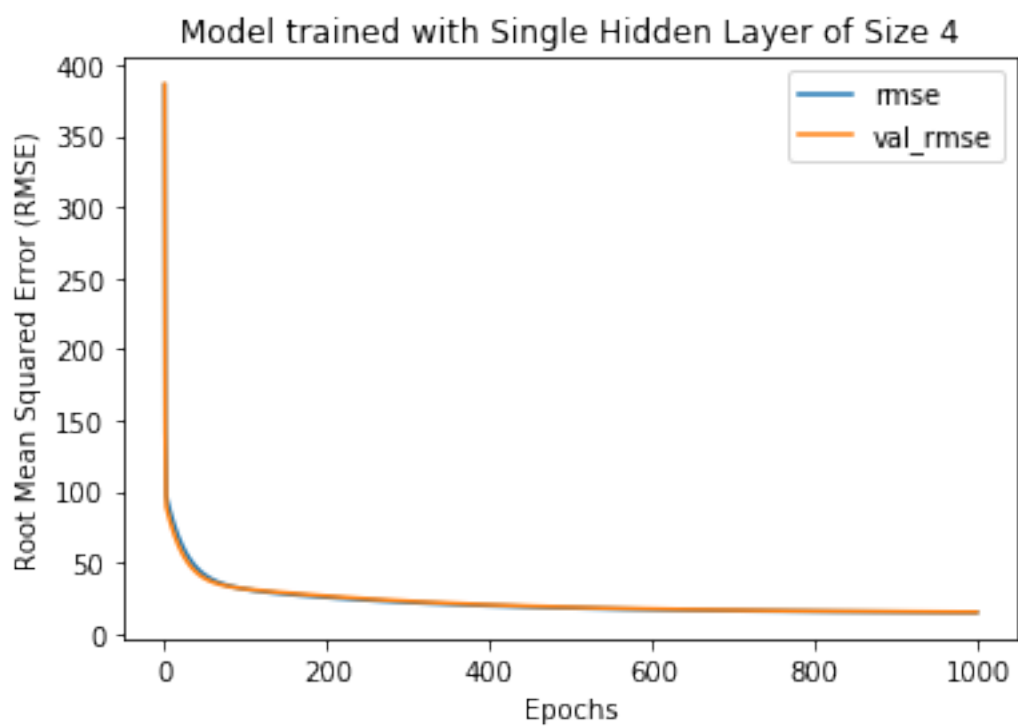
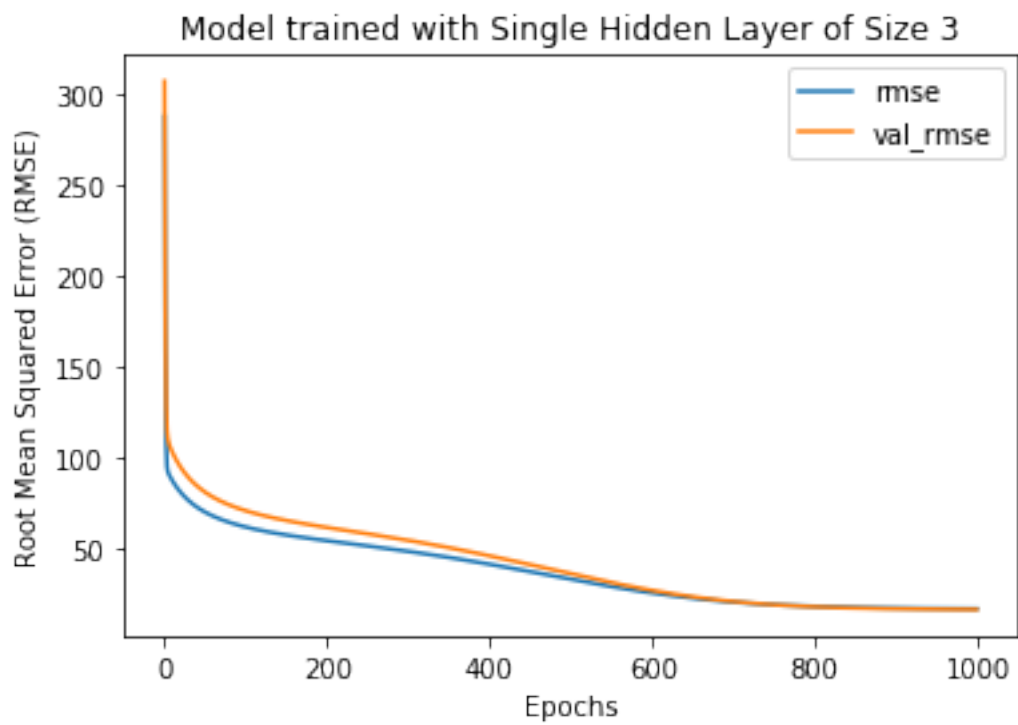
```
val_mae    8.809503
val_mse   265.871295
val_r_sqr  0.901530
val_rmse   16.305560
Name: 999, dtype: float64
```

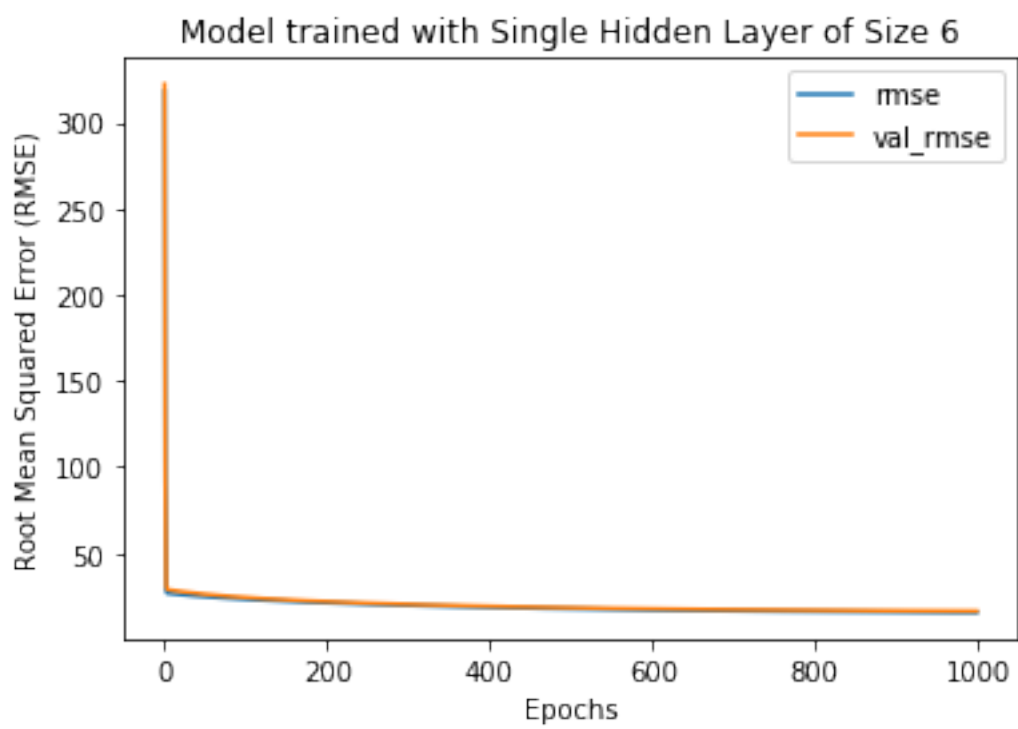
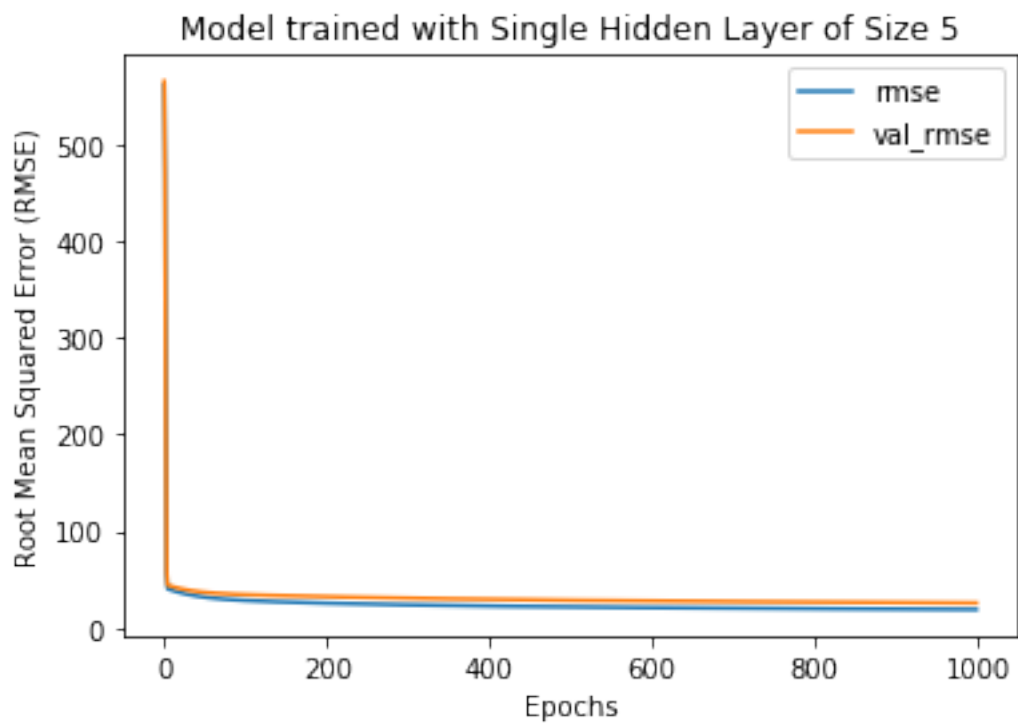
-----  
Test Set Results

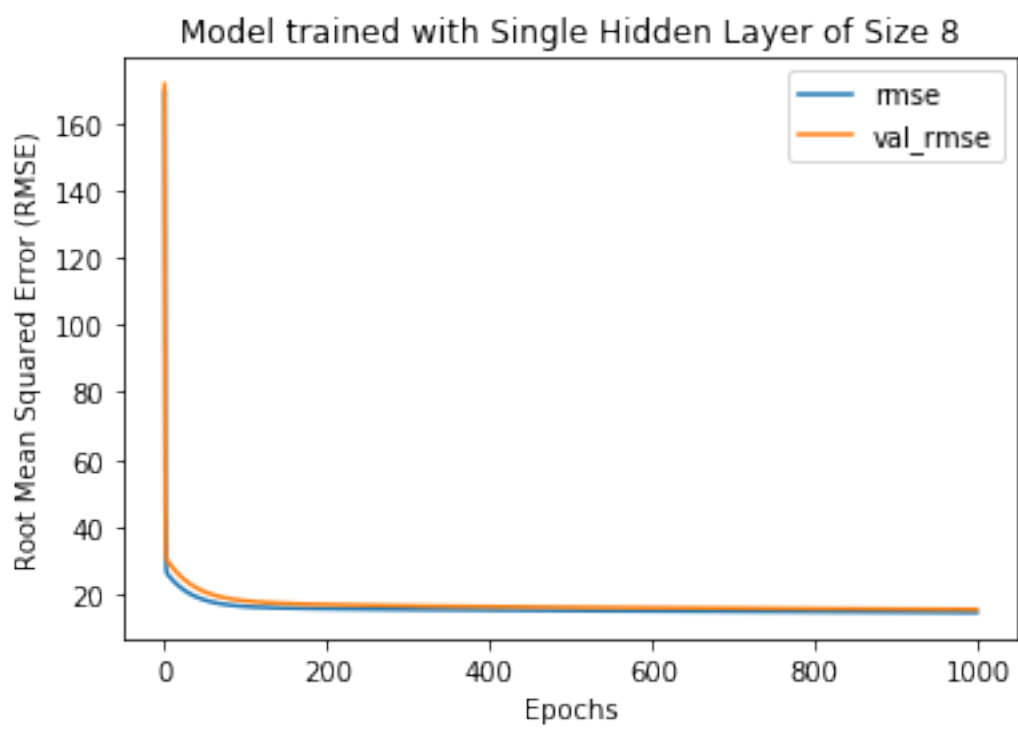
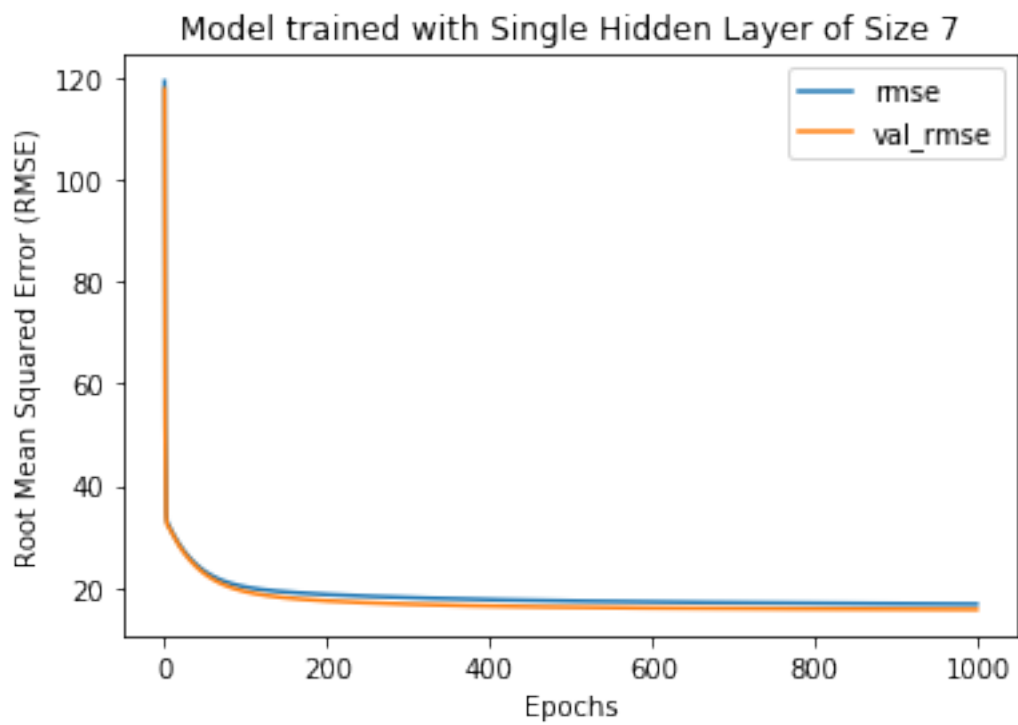
```
mse      1304.304852
rmse     36.115161
mae      24.295704
r_sqr    0.547036
st_mse    0.001535
st_rmse   0.039184
st_mae    0.023707
st_r_sqr  0.907362
Name: 0, dtype: float64
```

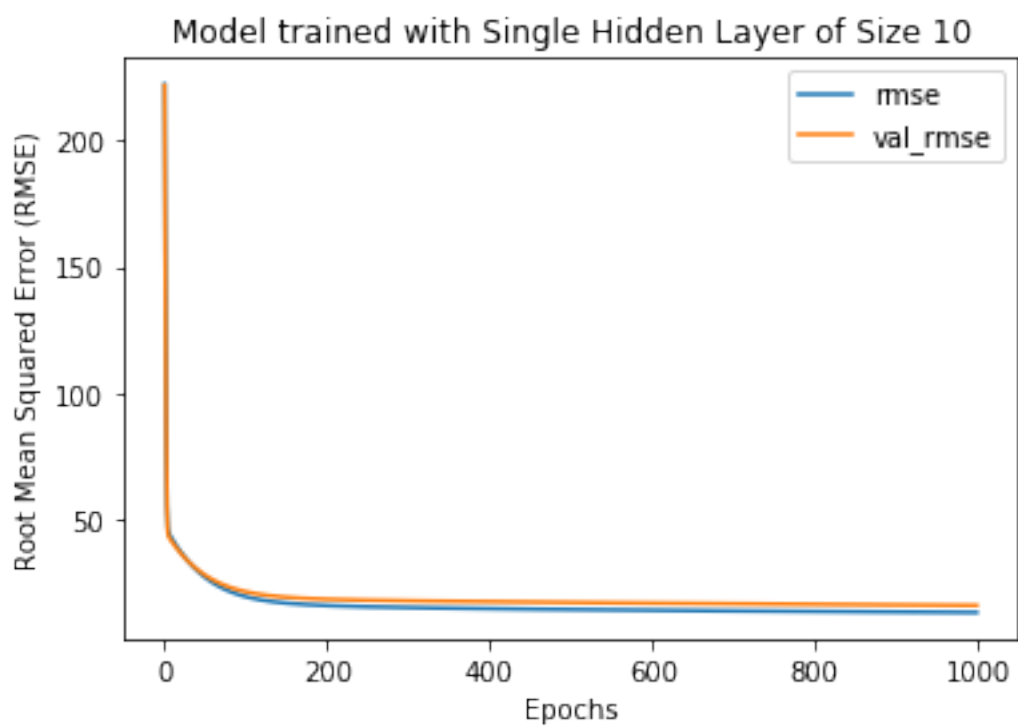
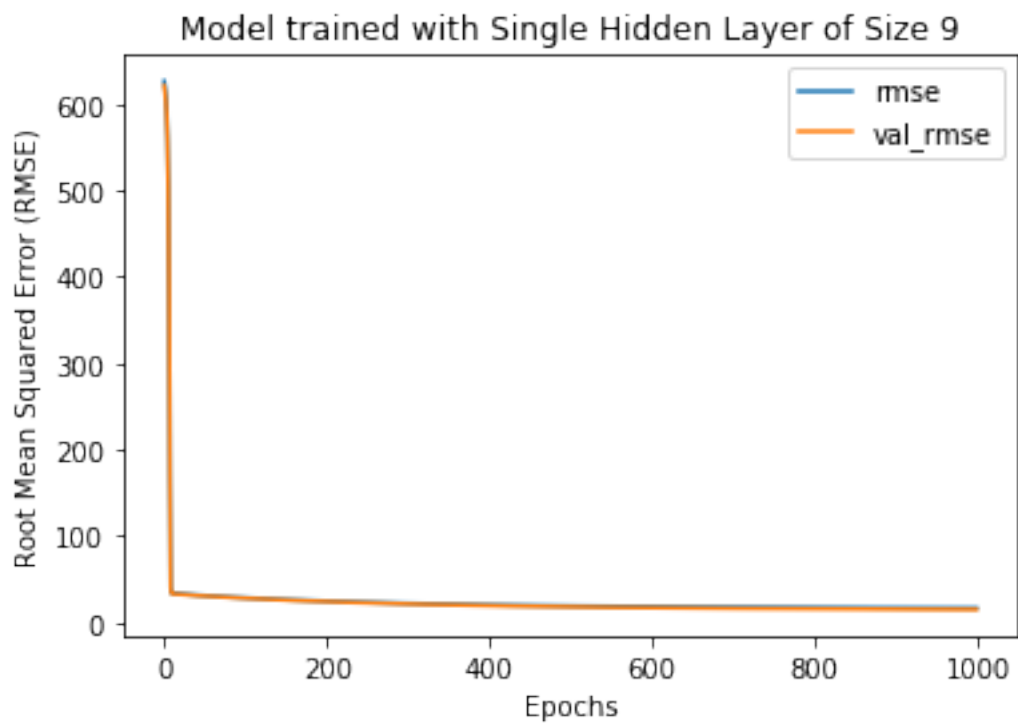
```
=====
=====
```











```
[17]: layer_tests_2 = dict()
      for i in range(1, 11):
          layer_tests_2[f"Test-{i}"] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", 10, i, 1),
              activ_func="tanh",
              epochs=1000,
              l_rate=0.3
          )
```

```
[18]: for i in range(1,11):
      print(f"Model Trained with Hidden Layer Configuration (10, {i})",
            end=f"\n{'-'*100}\n")

      print("Final Training Results")
      print(layer_tests_2[f"Test-{i}"]["training_results"].iloc[-1, :4],
            end=f"\n{'-'*100}\n")

      print("Final Validation Results")
      print(layer_tests_2[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
            end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(layer_tests_2[f"Test-{i}"]["error_metrics"].iloc[0],
            end=f"\n{'-'*100}\n\n\n")

      ax = layer_tests_2[f"Test-{i}"]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with Hidden Layer
            Configuration (10, {i})",
      )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Hidden Layer Configuration (10, 1)

-----

Final Training Results

mae	9.292601
mse	417.818817
r_sqr	0.862269
rmse	20.440617
Name: 999, dtype: float64	

-----

Final Validation Results



```
val_mae      9.735302
val_mse      376.200705
val_r_sqr    0.878966
val_rmse     19.395894
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          407.018132
rmse         20.174690
mae          13.710399
r_sqr        0.875703
st_mse       0.003585
st_rmse      0.059873
st_mae       0.026779
st_r_sqr     0.810105
Name: 0, dtype: float64
```

=====

Model Trained with Hidden Layer Configuration (10, 2)

-----  
-----  
Final Training Results

```
mae          6.076114
mse         140.742164
r_sqr        0.959537
rmse         11.863480
Name: 999, dtype: float64
```

-----  
-----  
Final Validation Results

```
val_mae      6.630507
val_mse     157.705785
val_r_sqr    0.938995
val_rmse     12.558096
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          2379.356258
rmse         48.778646
mae          33.361029
r_sqr        0.039664
st_mse       0.001525
st_rmse      0.039051
```

```
st_mae      0.021237
st_r_sqr     0.937248
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 3)

```
-----
-----
```

Final Training Results

```
mae      8.889117
mse     235.792069
r_sqr    0.926761
rmse    15.355522
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

```
val_mae    8.498321
val_mse   221.075697
val_r_sqr   0.924965
val_rmse   14.868614
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse      562.700580
rmse     23.721311
mae     15.513718
r_sqr    0.805169
st_mse    0.002758
st_rmse   0.052520
st_mae    0.026391
st_r_sqr  0.874321
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 4)

```
-----
-----
```

Final Training Results

```
mae      6.909384
mse     157.518853
```

```
r_sqr      0.946891
rmse       12.550651
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      6.185888
val_mse     113.961663
val_r_sqr    0.957834
val_rmse     10.675283
Name: 999, dtype: float64
```

---

Test Set Results

```
mse         219.828390
rmse        14.826611
mae          8.826869
r_sqr        0.943506
st_mse       0.001560
st_rmse      0.039499
st_mae       0.017900
st_r_sqr     0.919833
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (10, 5)

---

Final Training Results

```
mae         10.522868
mse         311.979397
r_sqr        0.907433
rmse        17.662939
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      9.899300
val_mse     245.319428
val_r_sqr    0.902181
val_rmse     15.662676
Name: 999, dtype: float64
```

---

Test Set Results

```
mse          537.153056
rmse         23.176563
mae          15.734755
r_sqr        0.812915
st_mse       0.001906
st_rmse      0.043662
st_mae       0.023878
st_r_sqr     0.866824
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (10, 6)

```
-----
Final Training Results
mae          5.748496
mse         113.480440
r_sqr        0.958104
rmse         10.652720
Name: 999, dtype: float64
```

```
-----
Final Validation Results
val_mae       7.020809
val_mse      193.100435
val_r_sqr     0.944236
val_rmse     13.896058
Name: 999, dtype: float64
```

```
-----
Test Set Results
mse          376.260625
rmse         19.397439
mae          9.536395
r_sqr        0.902956
st_mse       0.000709
st_rmse      0.026622
st_mae       0.014719
st_r_sqr     0.943632
Name: 0, dtype: float64
```

```
=====
=====
```

#### Model Trained with Hidden Layer Configuration (10, 7)

##### Final Training Results

```
mae      6.601837
mse     131.016762
r_sqr    0.957874
rmse     11.446255
Name: 999, dtype: float64
```

##### Final Validation Results

```
val_mae    7.153569
val_mse   138.887926
val_r_sqr  0.946894
val_rmse   11.785072
Name: 999, dtype: float64
```

##### Test Set Results

```
mse      375.751609
rmse     19.384313
mae      12.805029
r_sqr    0.893769
st_mse    0.001086
st_rmse   0.032958
st_mae    0.018602
st_r_sqr  0.938426
Name: 0, dtype: float64
```

#### Model Trained with Hidden Layer Configuration (10, 8)

##### Final Training Results

```
mae      7.958540
mse     197.719732
r_sqr    0.925305
rmse     14.061285
Name: 999, dtype: float64
```

##### Final Validation Results

```
val_mae    8.296390
val_mse   239.692717
```

```
val_r_sqr      0.920791
val_rmse       15.482013
Name: 999, dtype: float64
```

---

Test Set Results

```
mse           371.719131
rmse          19.280019
mae           11.847606
r_sqr         0.917240
st_mse        0.001597
st_rmse       0.039967
st_mae        0.020562
st_r_sqr      0.928691
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (10, 9)

---

Final Training Results

```
mae           6.954281
mse          151.877066
r_sqr         0.949522
rmse          12.323841
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae       7.531540
val_mse      145.484447
val_r_sqr     0.957469
val_rmse     12.061693
Name: 999, dtype: float64
```

---

Test Set Results

```
mse          1538.547901
rmse          39.224328
mae           27.420260
r_sqr         0.494127
st_mse        0.001627
st_rmse       0.040332
st_mae        0.024097
st_r_sqr      0.922437
```

Name: 0, dtype: float64

=====

Model Trained with Hidden Layer Configuration (10, 10)

-----

Final Training Results

mae 9.935704

mse 316.371832

r\_sqr 0.902427

rmse 17.786844

Name: 999, dtype: float64

-----

Final Validation Results

val\_mae 9.591323

val\_mse 259.981100

val\_r\_sqr 0.921527

val\_rmse 16.123929

Name: 999, dtype: float64

-----

Test Set Results

mse 348.213726

rmse 18.660486

mae 11.980118

r\_sqr 0.857762

st\_mse 0.008356

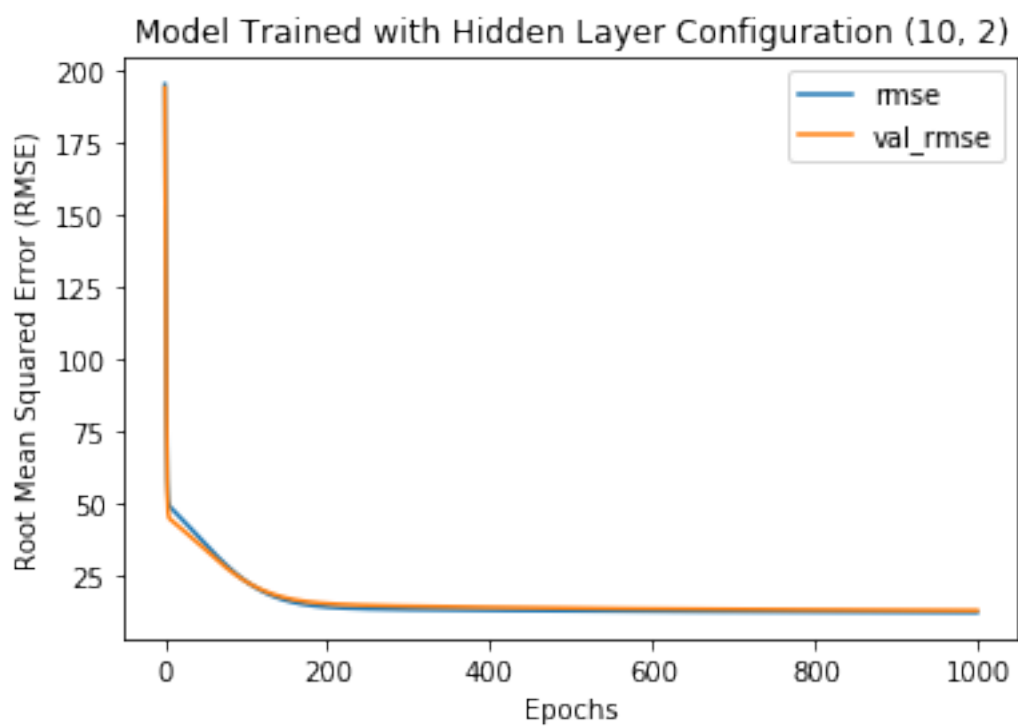
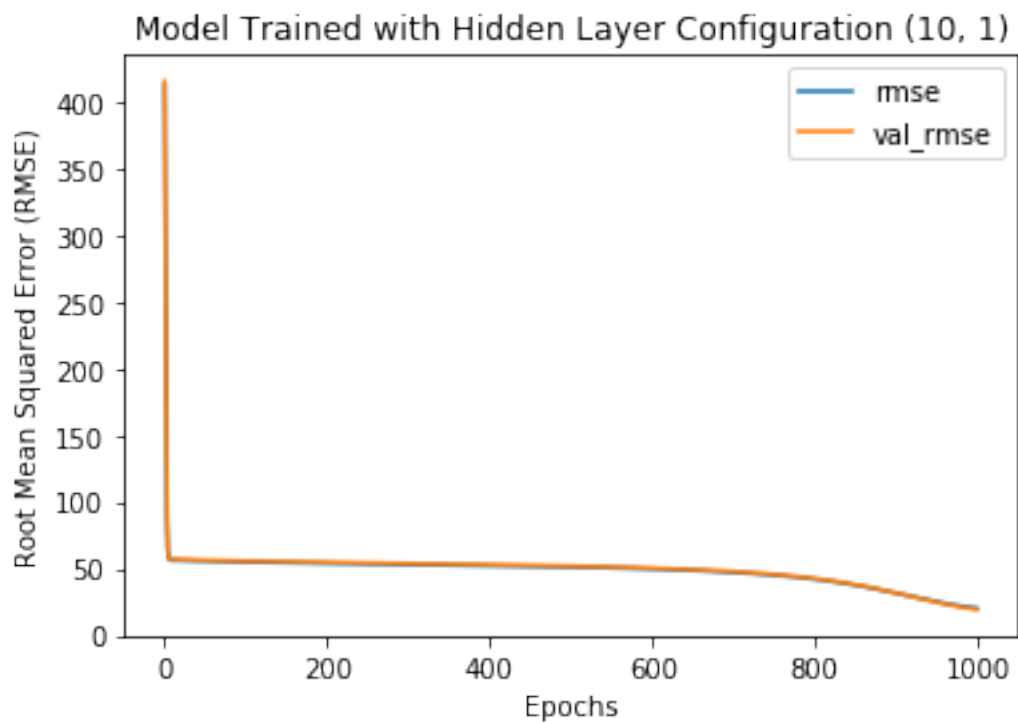
st\_rmse 0.091413

st\_mae 0.047380

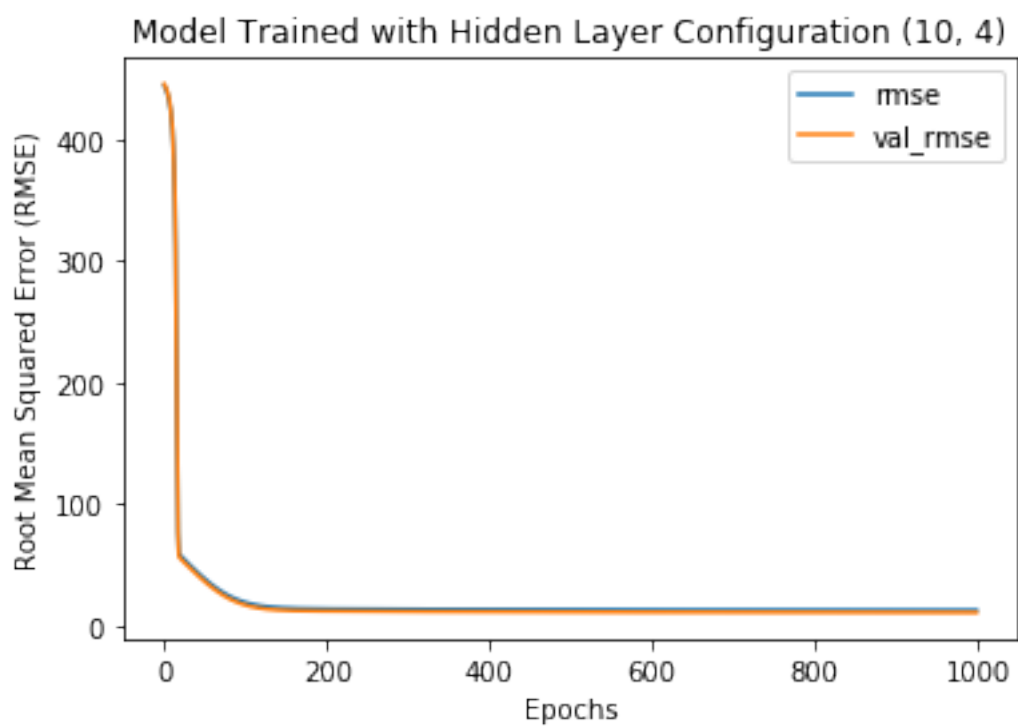
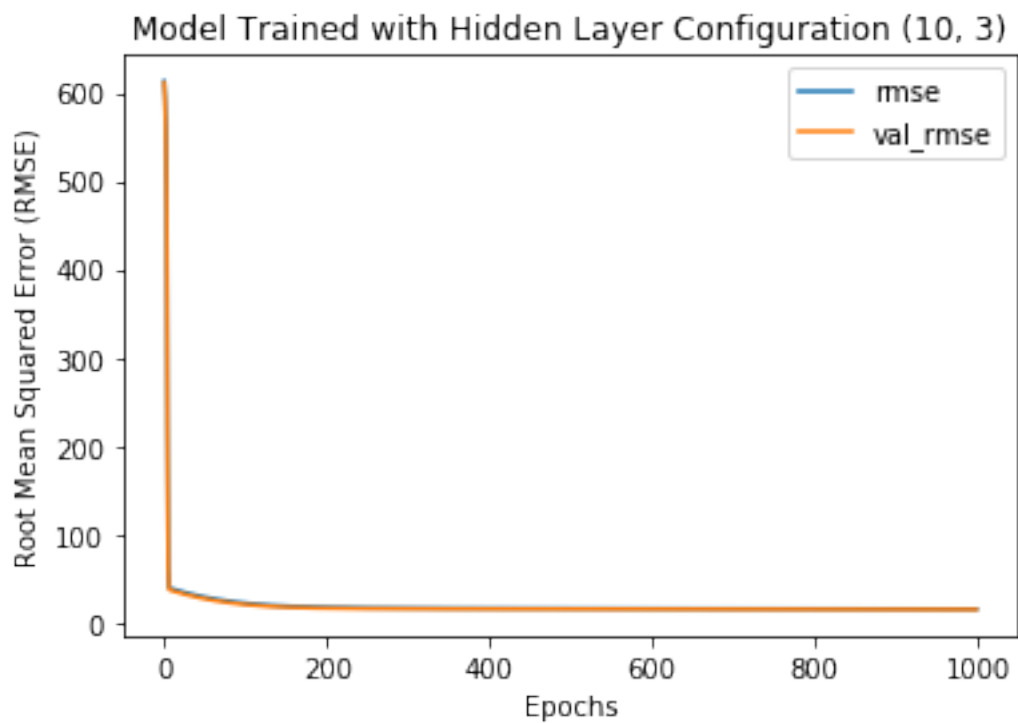
st\_r\_sqr 0.644567

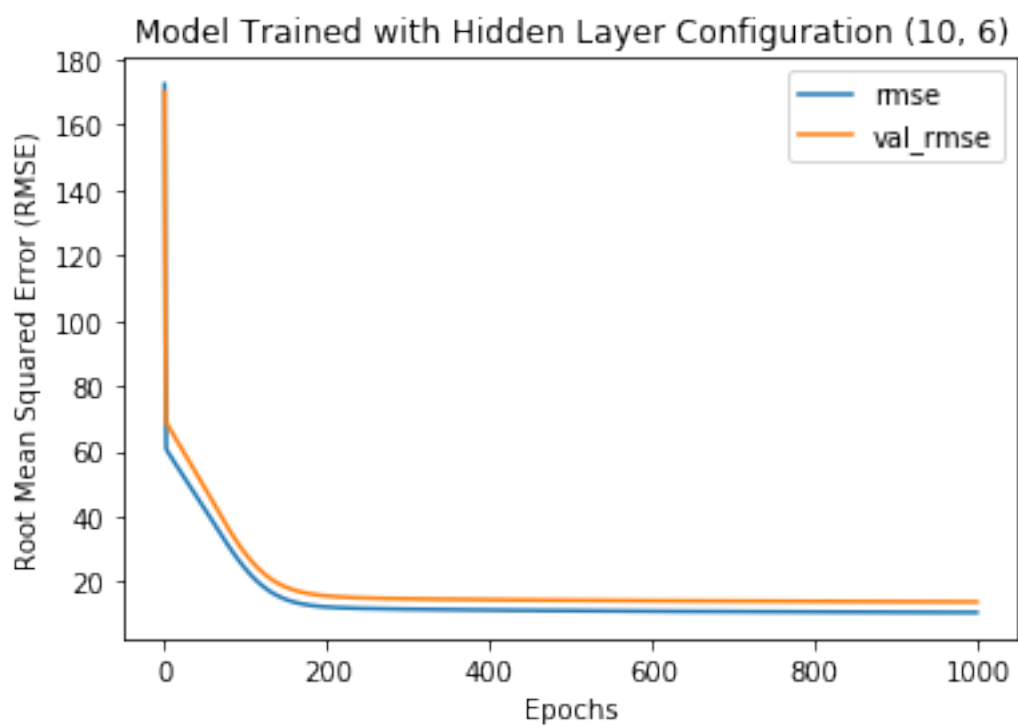
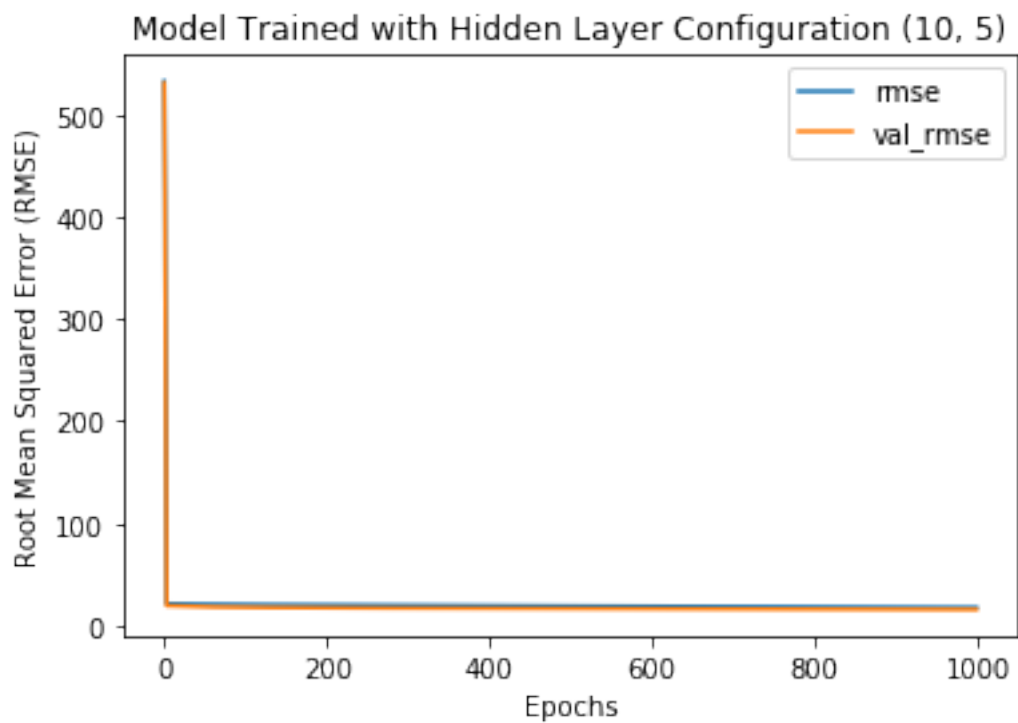
Name: 0, dtype: float64

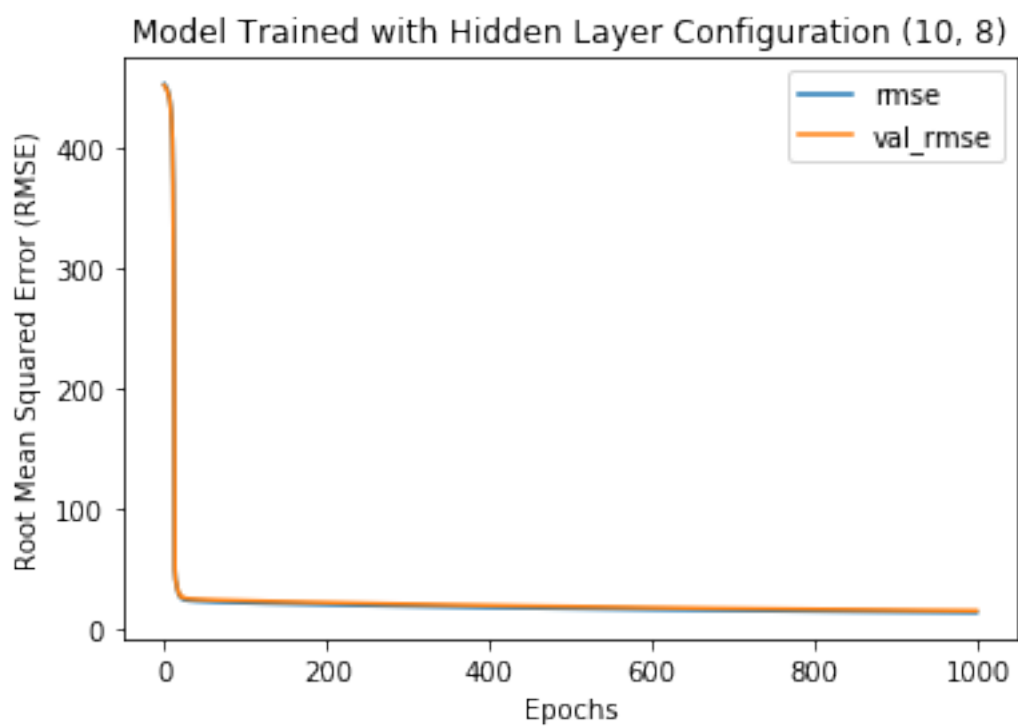
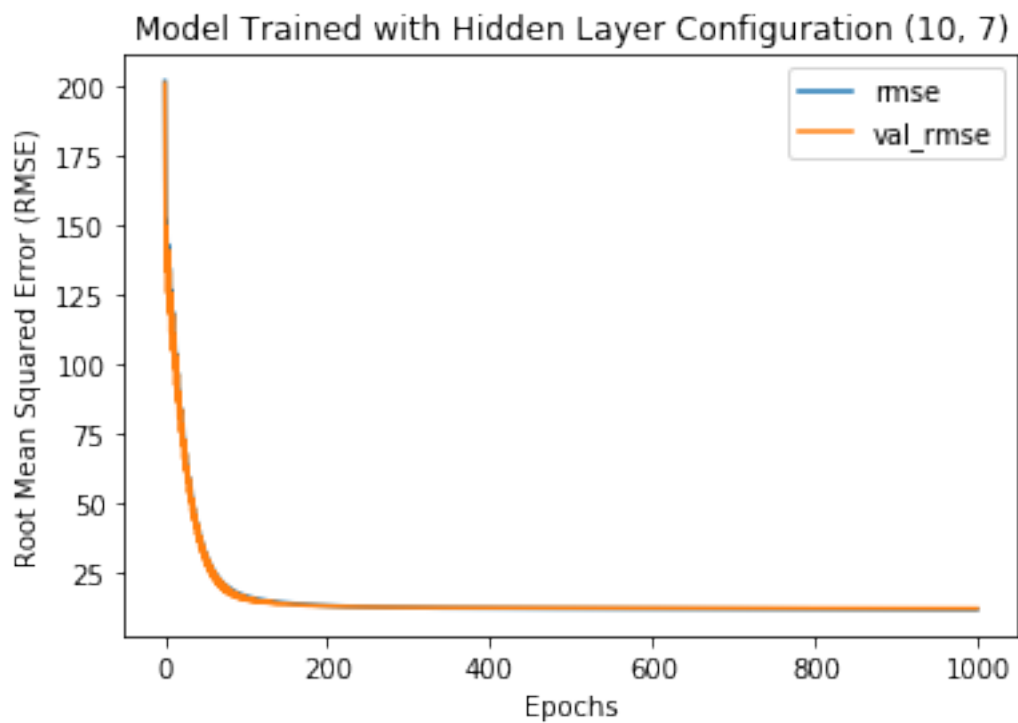
=====

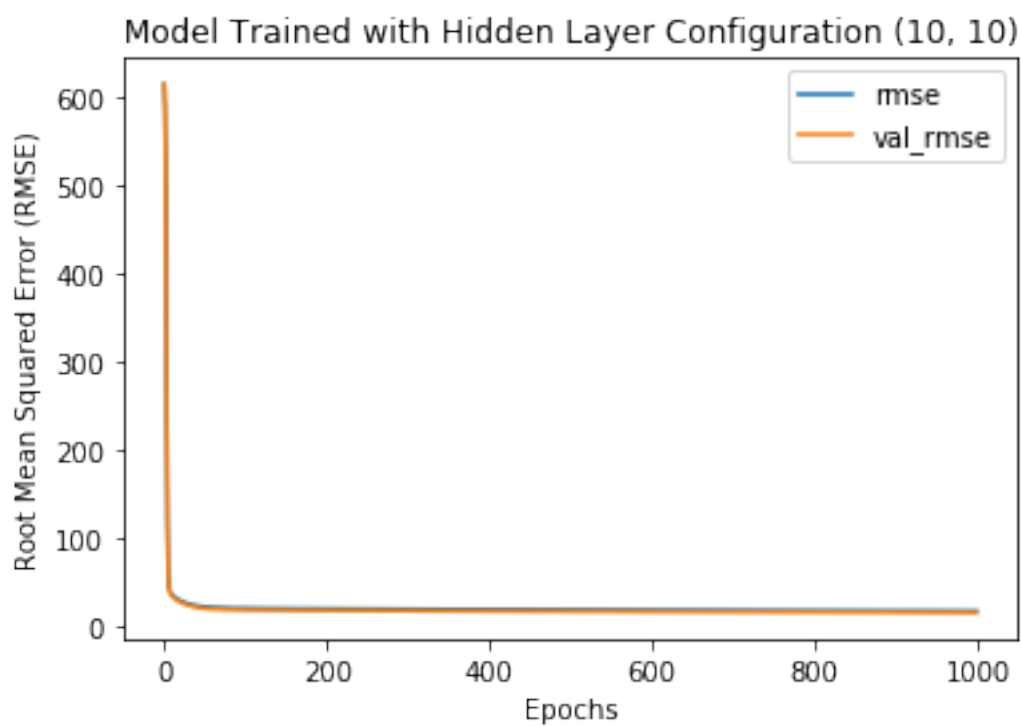
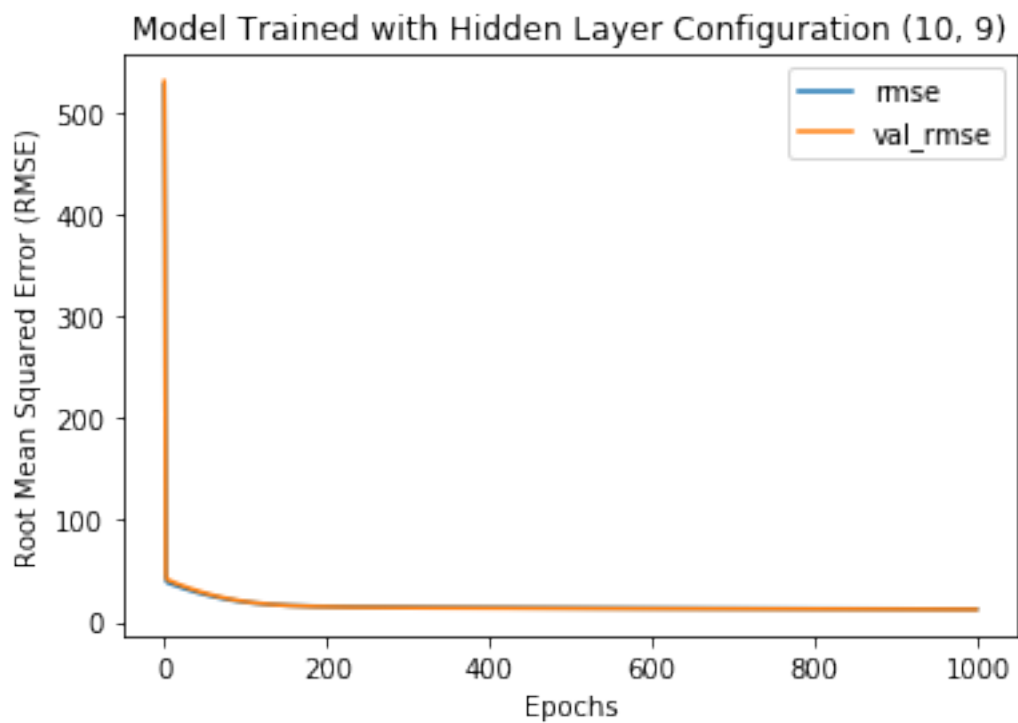












```
[19]: layer_tests_3 = dict()
      for i in range(1, 11):
          layer_tests_3[f"Test-{i}"] = build_train_test(
              fs,
              feature_cols,
              target_cols,
              layers=("auto", i, 10, 1),
              activ_func="tanh",
              epochs=1000,
              l_rate=0.3
          )
```

```
[20]: for i in range(1,11):
      print(f"Model Trained with Hidden Layer Configuration ({i}, 10)",
            end=f"\n{'-'*100}\n")

      print("Final Training Results")
      print(layer_tests_3[f"Test-{i}"]["training_results"].iloc[-1, :4],
            end=f"\n{'-'*100}\n")

      print("Final Validation Results")
      print(layer_tests_3[f"Test-{i}"]["training_results"].iloc[-1, 8:12],
            end=f"\n{'-'*100}\n")

      print("Test Set Results")
      print(layer_tests_3[f"Test-{i}"]["error_metrics"].iloc[0],
            end=f"\n{'-'*100}\n\n\n")

      ax = layer_tests_3[f"Test-{i}"]["training_results"].plot(
          y=["rmse", "val_rmse"], title=f"Model Trained with Hidden Layer
            Configuration ({i}, 10)",
          )
      ax.set_xlabel("Epochs")
      ax.set_ylabel("Root Mean Squared Error (RMSE)")
```

Model Trained with Hidden Layer Configuration (1, 10)

-----

Final Training Results

mae	9.687211
mse	322.933810
r_sqr	0.891352
rmse	17.970359
Name: 999, dtype: float64	

-----

Final Validation Results

```
val_mae      9.022622
val_mse      262.254092
val_r_sqr     0.907723
val_rmse     16.194261
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          415.454993
rmse         20.382713
mae          11.214236
r_sqr        0.887585
st_mse       0.003672
st_rmse      0.060598
st_mae       0.032000
st_r_sqr     0.800421
Name: 0, dtype: float64
```

=====

Model Trained with Hidden Layer Configuration (2, 10)

-----  
-----  
Final Training Results

```
mae          7.003304
mse         160.427402
r_sqr        0.942346
rmse        12.665994
Name: 999, dtype: float64
```

-----  
-----  
Final Validation Results

```
val_mae      8.080296
val_mse      250.315750
val_r_sqr     0.935941
val_rmse     15.821370
Name: 999, dtype: float64
```

-----  
-----  
Test Set Results

```
mse          306.903190
rmse         17.518653
mae          11.458790
r_sqr        0.902450
st_mse       0.003250
st_rmse      0.057008
```

```
st_mae      0.028263
st_r_sqr     0.864117
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (3, 10)

```
-----
-----
```

Final Training Results

```
mae      9.188204
mse     257.334761
r_sqr     0.914666
rmse     16.041657
Name: 999, dtype: float64
```

```
-----
-----
```

Final Validation Results

```
val_mae     8.752097
val_mse    177.811155
val_r_sqr     0.918427
val_rmse    13.334585
Name: 999, dtype: float64
```

```
-----
-----
```

Test Set Results

```
mse      561.650211
rmse     23.699161
mae      16.234630
r_sqr     0.866639
st_mse     0.004243
st_rmse     0.065135
st_mae     0.029878
st_r_sqr     0.843230
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (4, 10)

```
-----
-----
```

Final Training Results

```
mae      7.394178
mse     171.687990
```

```
r_sqr      0.945897
rmse       13.102976
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      8.515336
val_mse     180.175509
val_r_sqr    0.937592
val_rmse     13.422947
Name: 999, dtype: float64
```

---

Test Set Results

```
mse         333.976598
rmse        18.275027
mae         11.292514
r_sqr       0.891780
st_mse      0.002213
st_rmse     0.047044
st_mae      0.022497
st_r_sqr    0.889996
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (5, 10)

---

Final Training Results

```
mae         7.882073
mse        193.996556
r_sqr       0.937152
rmse       13.928265
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      7.610485
val_mse     185.159962
val_r_sqr    0.945962
val_rmse     13.607350
Name: 999, dtype: float64
```

---

Test Set Results



```
mse          1107.365880
rmse         33.277107
mae          22.350471
r_sqr        0.606026
st_mse       0.001876
st_rmse      0.043311
st_mae       0.024061
st_r_sqr     0.895976
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (6, 10)

-----  
Final Training Results

```
mae          8.222177
mse         202.121727
r_sqr        0.926889
rmse        14.216952
Name: 999, dtype: float64
```

-----  
Final Validation Results

```
val_mae      8.057300
val_mse     165.134807
val_r_sqr    0.944909
val_rmse    12.850479
Name: 999, dtype: float64
```

-----  
Test Set Results

```
mse          237.158358
rmse         15.399947
mae          9.838986
r_sqr        0.942777
st_mse       0.002350
st_rmse      0.048479
st_mae       0.025716
st_r_sqr     0.886089
Name: 0, dtype: float64
```

```
=====
=====
```

Model Trained with Hidden Layer Configuration (7, 10)

Final Training Results

mae 7.145765  
mse 150.924877  
r\_sqr 0.953200  
rmse 12.285149  
Name: 999, dtype: float64

Final Validation Results

val\_mae 5.852051  
val\_mse 106.614364  
val\_r\_sqr 0.962687  
val\_rmse 10.325423  
Name: 999, dtype: float64

Test Set Results

mse 226.863857  
rmse 15.062000  
mae 7.475933  
r\_sqr 0.923382  
st\_mse 0.000753  
st\_rmse 0.027448  
st\_mae 0.016128  
st\_r\_sqr 0.921576  
Name: 0, dtype: float64

Model Trained with Hidden Layer Configuration (8, 10)

Final Training Results

mae 5.884373  
mse 127.769140  
r\_sqr 0.958400  
rmse 11.303501  
Name: 999, dtype: float64

Final Validation Results

val\_mae 6.984307  
val\_mse 143.126778

```
val_r_sqr      0.956033
val_rmse      11.963560
Name: 999, dtype: float64
```

---

Test Set Results

```
mse      1251.604582
rmse      35.378024
mae      24.385259
r_sqr      0.581258
st_mse      0.001336
st_rmse      0.036557
st_mae      0.019537
st_r_sqr      0.934907
Name: 0, dtype: float64
```

---

Model Trained with Hidden Layer Configuration (9, 10)

---

Final Training Results

```
mae      8.540181
mse      213.223943
r_sqr      0.928560
rmse      14.602190
Name: 999, dtype: float64
```

---

Final Validation Results

```
val_mae      10.470772
val_mse      331.918736
val_r_sqr      0.887989
val_rmse      18.218637
Name: 999, dtype: float64
```

---

Test Set Results

```
mse      484.287486
rmse      22.006533
mae      11.519720
r_sqr      0.864286
st_mse      0.000800
st_rmse      0.028284
st_mae      0.017758
st_r_sqr      0.930837
```

Name: 0, dtype: float64

=====

Model Trained with Hidden Layer Configuration (10, 10)

-----

Final Training Results

mae 6.367745

mse 139.122132

r\_sqr 0.956971

rmse 11.795005

Name: 999, dtype: float64

-----

Final Validation Results

val\_mae 6.180989

val\_mse 119.222941

val\_r\_sqr 0.960460

val\_rmse 10.918926

Name: 999, dtype: float64

-----

Test Set Results

mse 1062.005232

rmse 32.588422

mae 21.931587

r\_sqr 0.617991

st\_mse 0.001747

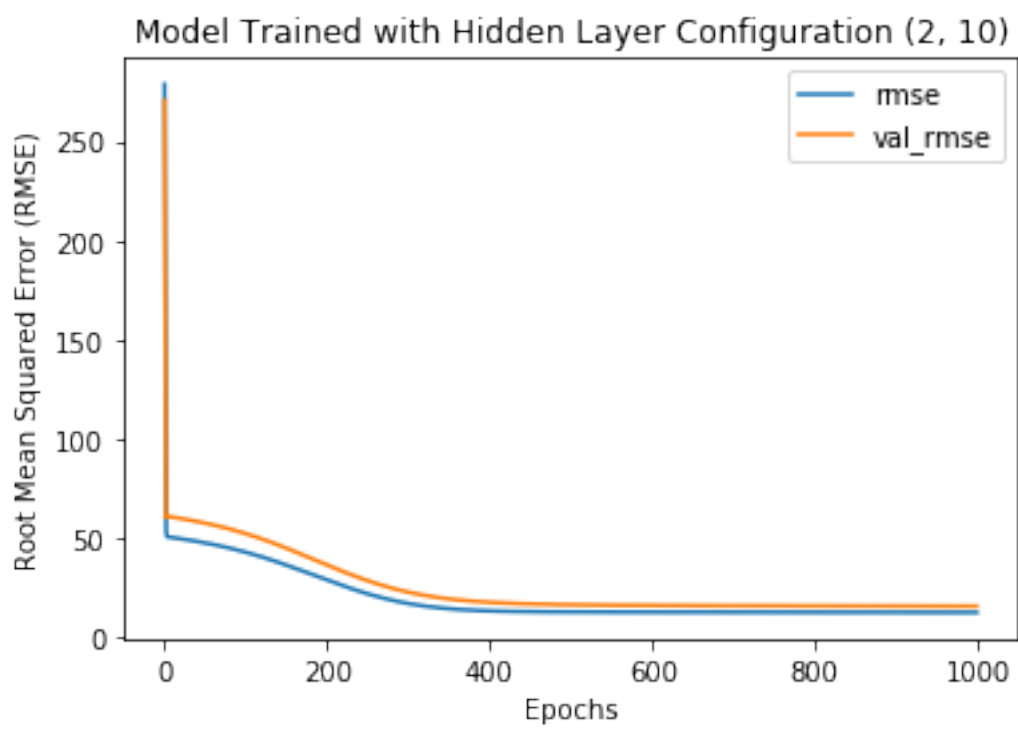
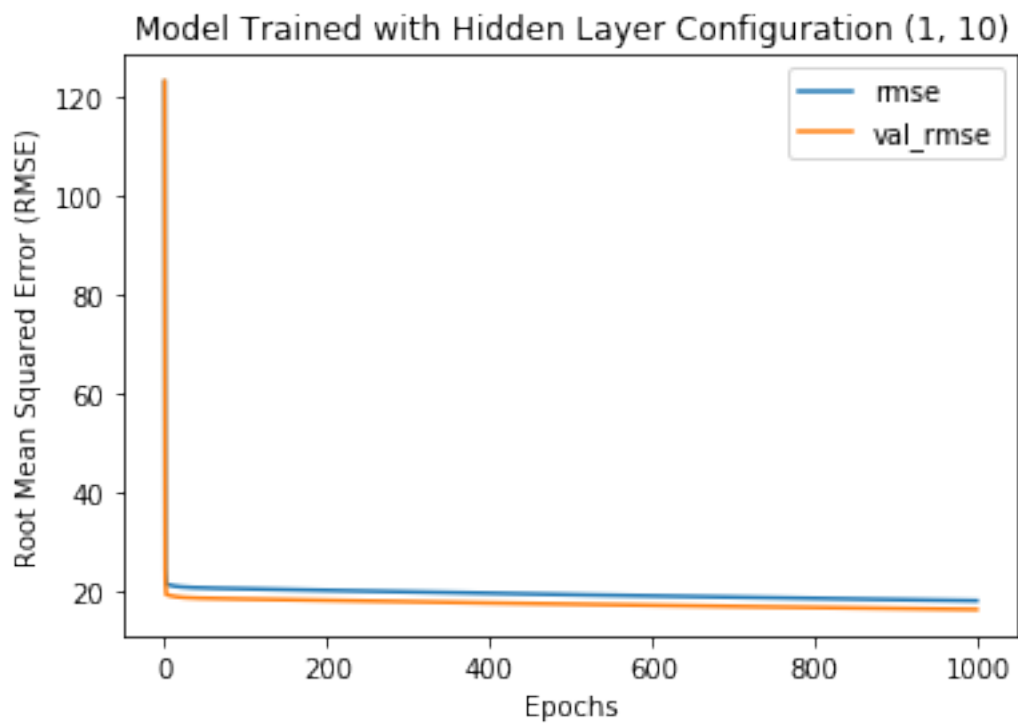
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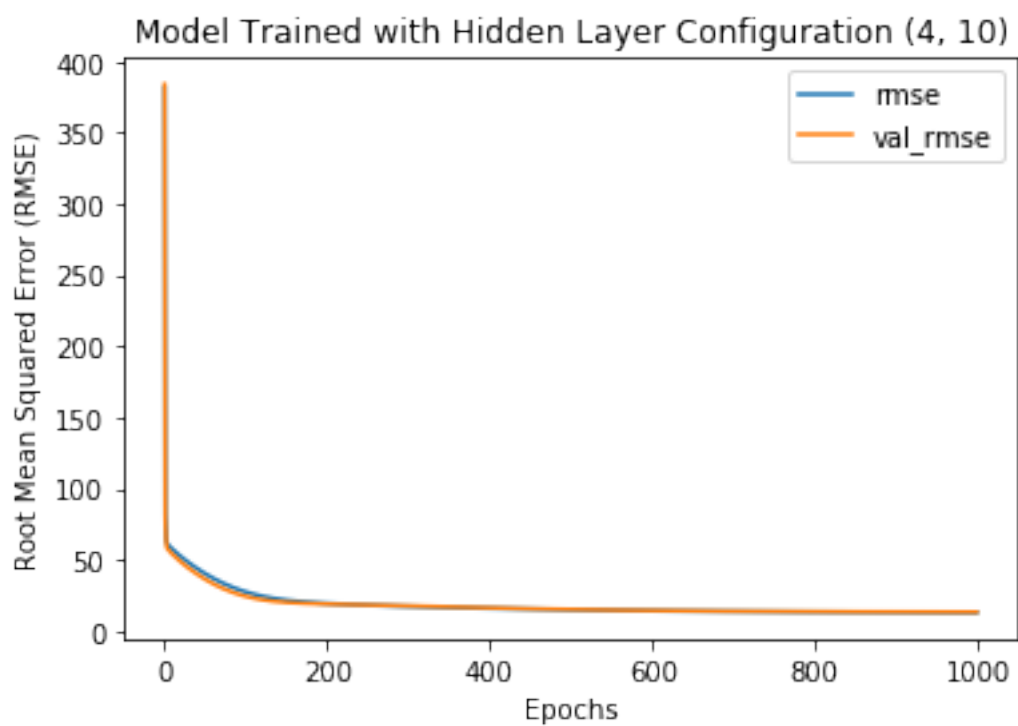
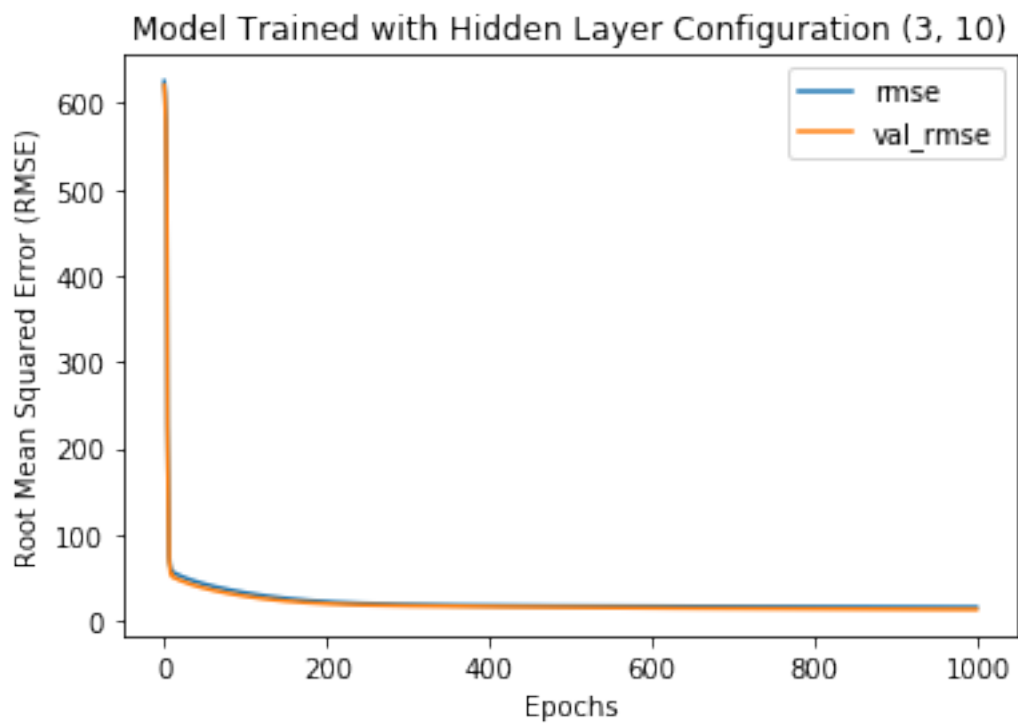
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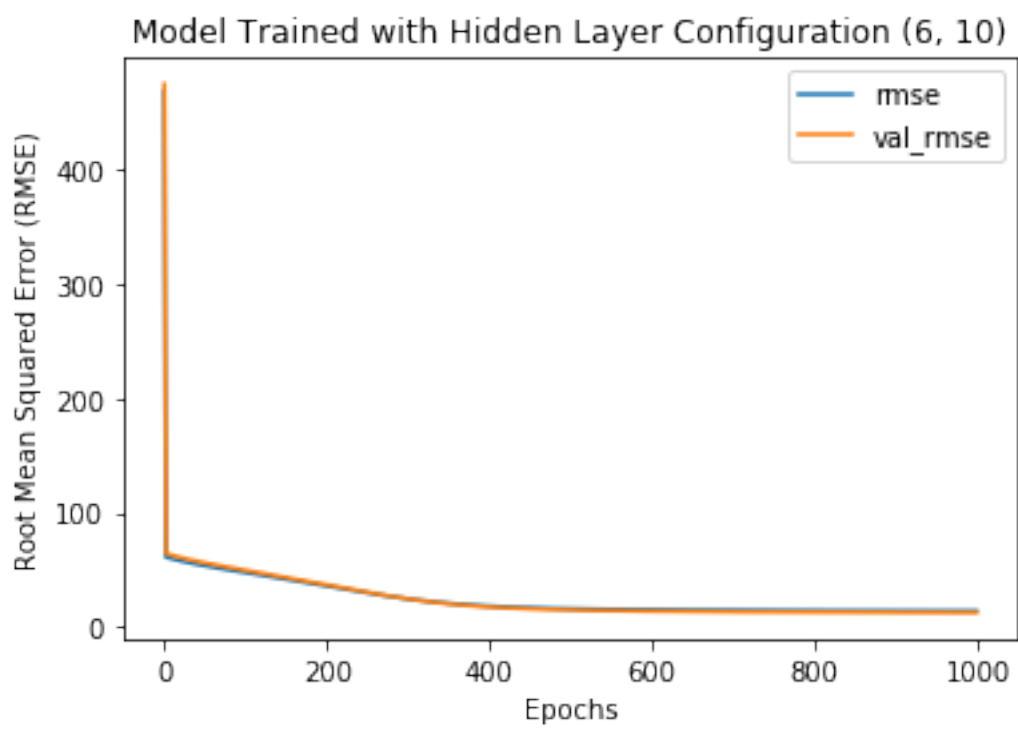
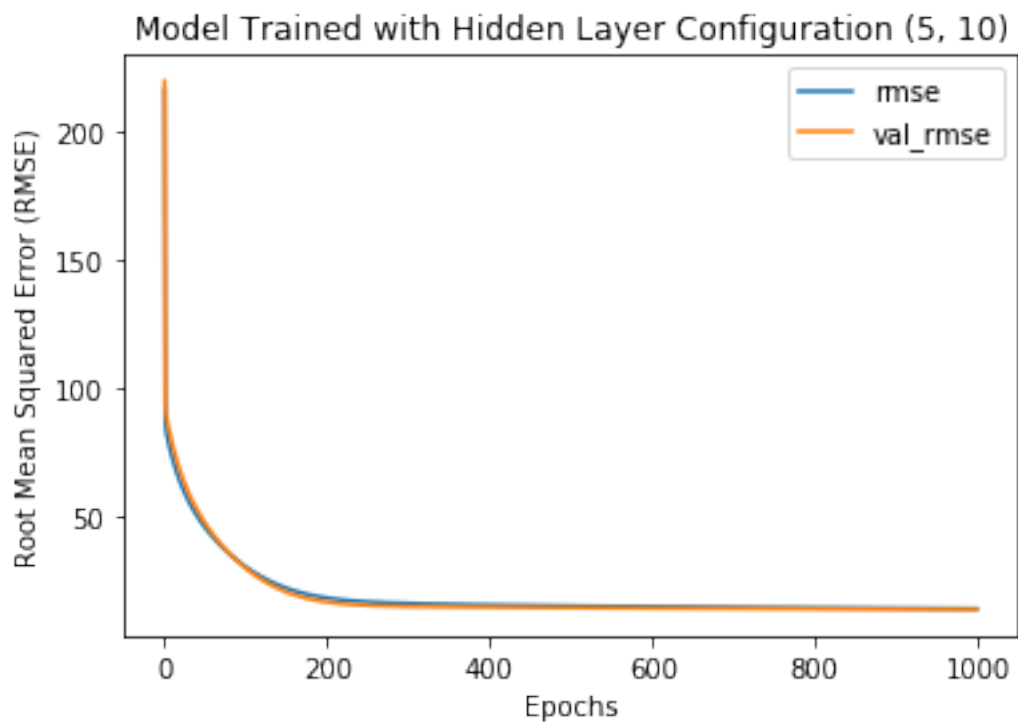
st\_r\_sqr 0.926734

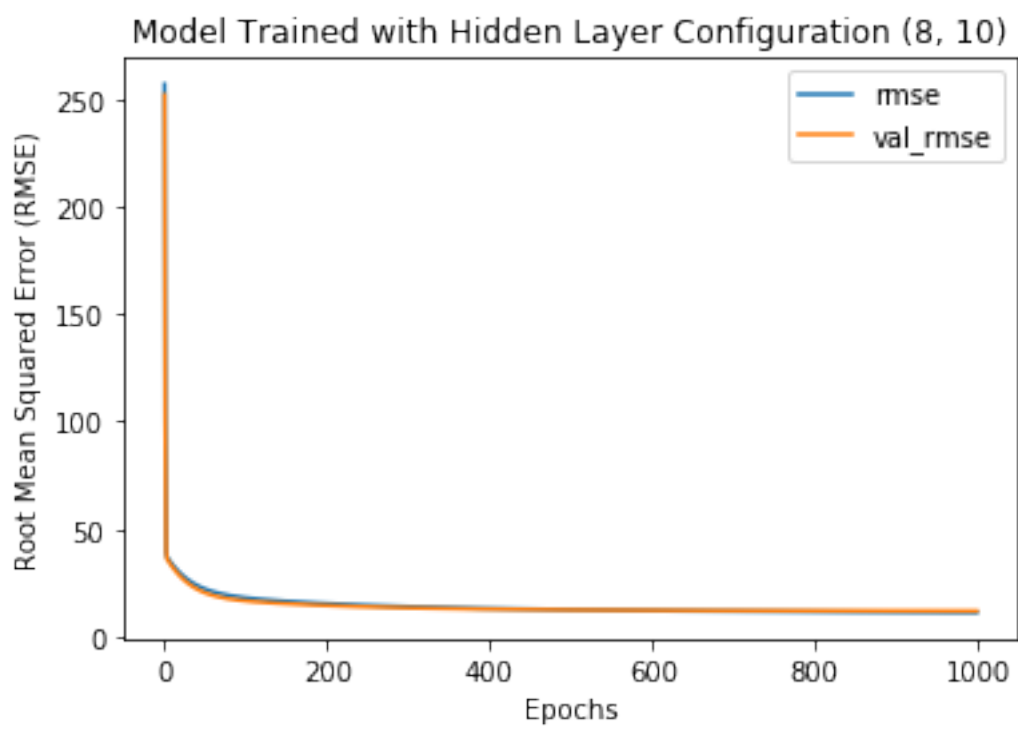
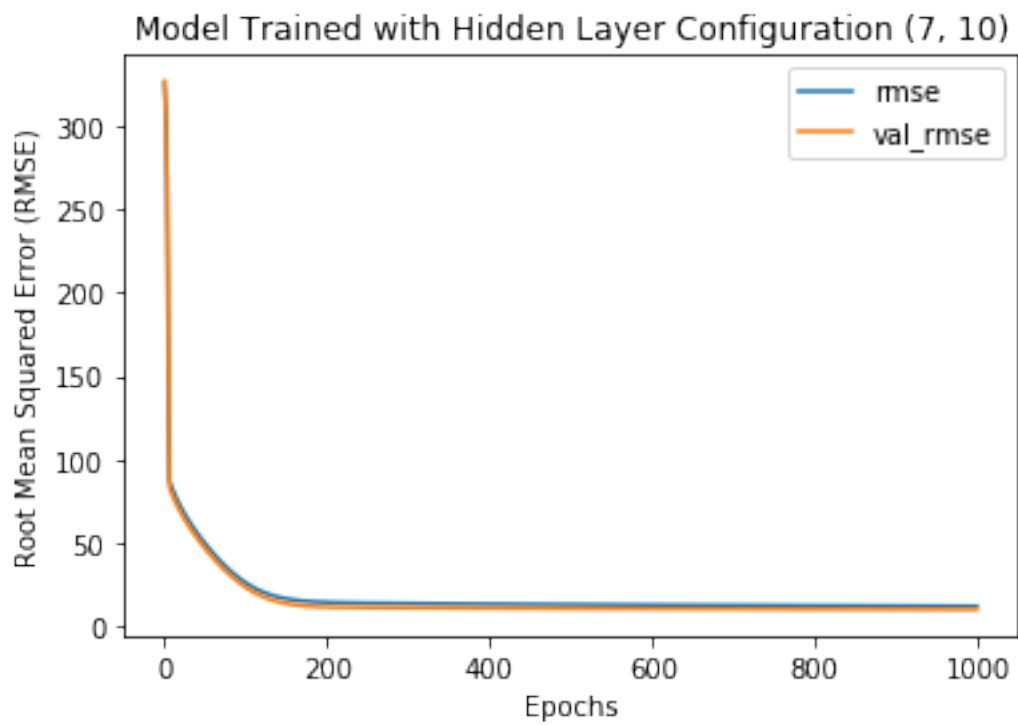
Name: 0, dtype: float64

=====

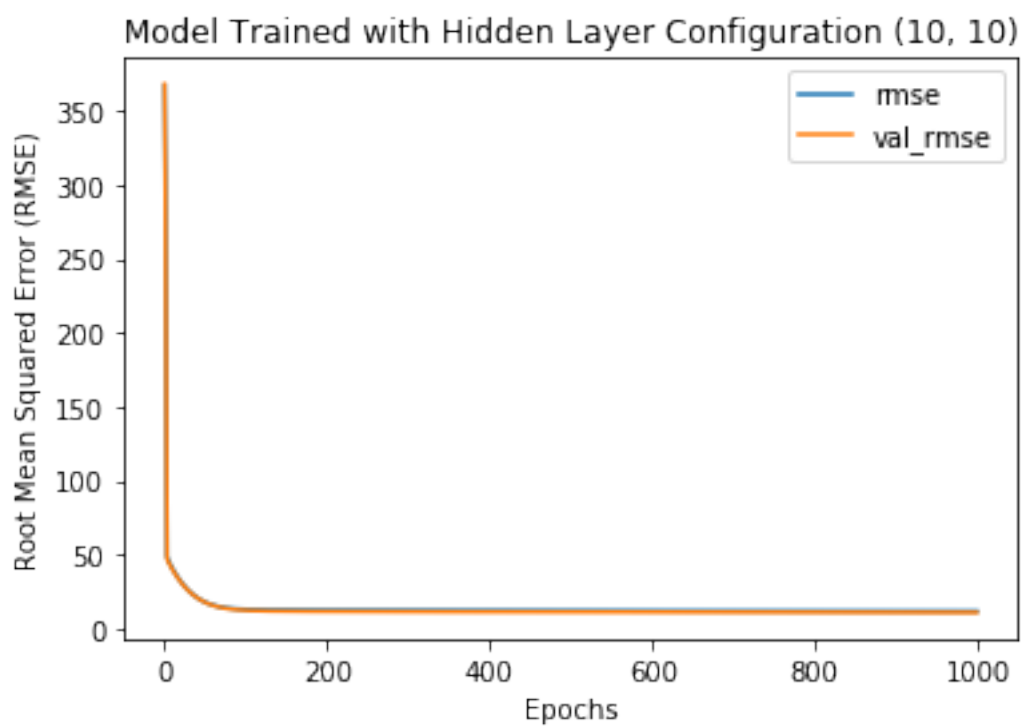
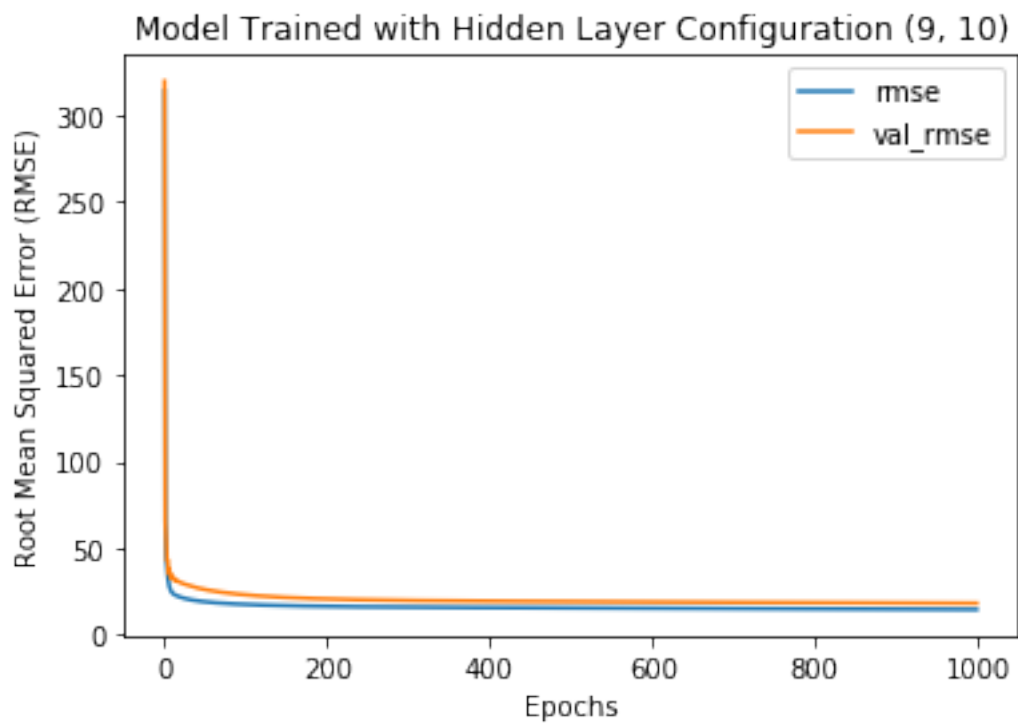












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