

1. Convert the following six statements into First Order Logic (FOL). Translate the obtained expressions to Conjunctive Normal Forms (CNFs, Steps 1-6 of Lecture 9).

1. You have a dog.
2. Amy buys cat food.
3. Anyone who owns a cat hates anything that chases any cat.
4. Every dog chases some cat.
5. Anyone who buys cat food owns a cat.
6. Someone who hates something owned by another person will not marry that person.

2. Transform the following conclusion into FOL, negate it and convert it to CNF (Steps 1-6 of Lecture 9: Knowledge Representation and Reasoning).

Amy will not marry you.

3. Based on all the previously created CNFs, finalise the conversion to CNF (Steps 7-8 of Lecture 9) and prove the statement from 2.

The FOL expression corresponding to the six statements are:

1. $\exists x \text{ Dog}(x) \wedge \text{Own}(\text{You}, x)$
2. $\text{Buy}(\text{Amy})$
3. $\forall x (\exists y (\text{Own}(x, y) \wedge \text{Cat}(y)) \Rightarrow (\forall z (\exists w (\text{Cat}(w) \wedge \text{Chase}(z, w)) \Rightarrow \text{Hate}(x, z))))$
4. $\forall x \text{ Dog}(x) \Rightarrow \exists y (\text{Cat}(y) \wedge \text{Chase}(x, y))$
5. $\forall x \text{ Buy}(x) \Rightarrow \exists y (\text{Own}(x, y) \wedge \text{Cat}(y))$
6. $\forall x \forall y (\exists z (\text{Own}(y, z) \wedge \text{Hate}(x, z)) \Rightarrow \neg \text{Marry}(x, y))$

1. Convert the following six statements into First Order Logic (FOL). Translate the obtained expressions to Conjunctive Normal Forms (CNFs, Steps 1-6 of Lecture 9).

1. Remove implications:

7. $\forall x (\neg \exists y (\text{Own}(x, y) \wedge \text{Cat}(y))) \vee (\forall z (\neg \exists w (\text{Cat}(w) \wedge \text{Chase}(z, w)) \vee \text{Hate}(x, z)))$ (from 3)
8. $\forall x \neg \text{Dog}(x) \vee \exists y (\text{Cat}(y) \wedge \text{Chase}(x, y))$ (from 4)
9. $\forall x \neg \text{Buy}(x) \vee \exists y (\text{Own}(x, y) \wedge \text{Cat}(y))$ (from 5)
10. $\forall x \forall y (\neg \exists z (\text{Own}(y, z) \wedge \text{Hate}(x, z))) \vee \neg \text{Marry}(x, y)$ (from 6)

2. Minimise negations:

11. $\forall x \forall y (\neg \text{Own}(x,y) \vee \neg \text{Cat}(y)) \vee (\forall z \forall w (\neg \text{Cat}(w) \vee \neg \text{Chase}(z,w)) \vee \text{Hate}(x,z))$
(from 7)
12. $\forall x \forall y \forall z (\neg \text{Own}(y,z) \vee \neg \text{Hate}(x,z)) \vee \neg \text{Marry}(x,y)$ (from 10)

3. Standardise variables apart:

13. $\forall x_1 \forall y_1 (\neg \text{Own}(x_1,y_1) \vee \neg \text{Cat}(y_1)) \vee (\forall z_1 \forall w_1 (\neg \text{Cat}(w_1) \vee \neg \text{Chase}(z_1,w_1)) \vee \text{Hate}(x_1,z_1))$ (from 11)
14. $\forall x_2 \neg \text{Dog}(x_2) \vee \exists y_2 (\text{Cat}(y_2) \wedge \text{Chase}(x_2,y_2))$ (from 8)
15. $\forall x_3 \neg \text{Buy}(x_3) \vee \exists y_3 (\text{Own}(x_3,y_3) \wedge \text{Cat}(y_3))$ (from 9)
16. $\forall x_4 \forall y_4 \forall z_2 (\neg \text{Own}(y_4,z_2) \vee \neg \text{Hate}(x_4,z_2)) \vee \neg \text{Marry}(x_4,y_4)$ (from 10)

4. Skolemise existentials:

17. $\text{Dog}(D) \wedge \text{Own}(\text{YOU}, D)$ (from 1)
18. $\forall x_2 \neg \text{Dog}(x_2) \vee (\text{Cat}(R(x_2)) \wedge \text{Chase}(x_2, R(x_2)))$ (from 14)
- Note:** the existential on y_2 was in the scope of the universal on x , hence the need for introduction of the function $R(x_2)$.

19. $\forall x_3 \neg \text{Buy}(x_3) \vee (\text{Own}(x_3, F(x_3)) \wedge \text{Cat}(F(x_3)))$ (from 15)

Note: the existential on y_3 was in the scope of the universal on x , hence the need for introduction of the function.

5. Drop universals:

20. $\square \neg \text{Own}(x_1,y_1) \vee \neg \text{Cat}(y_1) \vee \square \neg \text{Cat}(w_1) \vee \neg \text{Chase}(z_1,w_1) \vee \text{Hate}(x_1,z_1)$ (from 13)
21. $\square \neg \text{Dog}(x_2) \vee (\text{Cat}(R(x_2)) \wedge \text{Chase}(x_2, R(x_2)))$ (from 18)
22. $\square \neg \text{Buy}(x_3) \vee (\text{Own}(x_3, F(x_3)) \wedge \text{Cat}(F(x_3)))$ (from 19)
23. $\square \neg \text{Own}(y_4,z_2) \vee \neg \text{Hate}(x_4,z_2) \vee \neg \text{Marry}(x_4,y_4)$ (from 16)

6. Convert to CNF:

24. $[\neg \text{Dog}(x_2) \vee \text{Cat}(R(x_2))] \wedge [\neg \text{Dog}(x_2) \vee \text{Chase}(x_2, R(x_2))]$ (from 21)
25. $[\neg \text{Buy}(x_3) \vee \text{Own}(x_3, F(x_3))] \wedge [\neg \text{Buy}(x_3) \vee \text{Cat}(F(x_3))]$ (from 22)

Final set:

26. $\text{Dog}(D) \wedge \text{Own}(\text{You}, D)$
27. $\text{Buy}(\text{Amy})$
28. $\neg \text{Own}(x_1,y_1) \vee \neg \text{Cat}(y_1) \vee \neg \text{Cat}(w_1) \vee \neg \text{Chase}(z_1,w_1) \vee \text{Hate}(x_1,z_1)$
29. $[\neg \text{Dog}(x_2) \vee \text{Cat}(R(x_2))] \wedge [\neg \text{Dog}(x_2) \vee \text{Chase}(x_2, R(x_2))]$
30. $[\neg \text{Buy}(x_3) \vee \text{Own}(x_3, F(x_3))] \wedge [\neg \text{Buy}(x_3) \vee \text{Cat}(F(x_3))]$
31. $\neg \text{Own}(y_4,z_2) \vee \neg \text{Hate}(x_4,z_2) \vee \neg \text{Marry}(x_4,y_4)$

2. Transform the following conclusion into FOL, negate it and convert it to CNF (Steps 1-6 of Lecture 9: Knowledge Representation and Reasoning).

Goal: $\neg \text{Marry}(\text{Amy}, \text{You})$

Negated goal: $\neg \neg \text{Marry}(\text{Amy}, \text{You})$

1. **Remove implications:** Nothing to do
2. **Minimise negations:** $\text{Marry}(\text{Amy}, \text{You})$
3. **Standardise variables apart:** Nothing to do
4. **Skolemise existentials:** Nothing to do
5. **Drop universals:** Nothing to do
6. **Convert to CNF:** Nothing to do

3. Based on all the previously created CNFs, finalise the conversion to CNF (Steps 7-8 of Lecture 9) and prove the statement from 2.

7-8: Split into disjunctive clauses & standardise variables apart:

1. $\text{Dog}(\text{D})$
2. $\text{Own}(\text{You}, \text{D})$
3. $\text{Buy}(\text{Amy})$
4. $\neg \text{Own}(\text{x1}, \text{y1}) \vee \neg \text{Cat}(\text{y1}) \vee \neg \text{Cat}(\text{w1}) \vee \neg \text{Chase}(\text{z1}, \text{w1}) \vee \text{Hate}(\text{x1}, \text{z1})$
5. $\neg \text{Dog}(\text{x2}) \vee \text{Cat}(\text{R}(\text{x2}))$
6. $\neg \text{Dog}(\text{x3}) \vee \text{Chase}(\text{x3}, \text{R}(\text{x3}))$
7. $\neg \text{Buy}(\text{x4}) \vee \text{Own}(\text{x4}, \text{F}(\text{x4}))$
8. $\neg \text{Buy}(\text{x5}) \vee \text{Cat}(\text{F}(\text{x5}))$
9. $\neg \text{Own}(\text{y2}, \text{z2}) \vee \neg \text{Hate}(\text{x6}, \text{z2}) \vee \neg \text{Marry}(\text{x6}, \text{y2})$
10. $\text{Marry}(\text{Amy}, \text{You})$

Resolution proof

11. $\neg \text{Hates}(\text{Amy}, \text{z2}) \vee \neg \text{Owns}(\text{You}, \text{z2})$ (Resolve 10, 9) Unifier: $\{\text{Amy}/\text{x6}, \text{You}/\text{y2}\}$
12. $\neg \text{Hates}(\text{Amy}, \text{D})$ (Resolve 11, 2) Unifier: $\{\text{D}/\text{z2}\}$
13. $\neg \text{Own}(\text{Amy}, \text{y1}) \vee \neg \text{Cat}(\text{y1}) \vee \neg \text{Cat}(\text{w1}) \vee \neg \text{Chase}(\text{D}, \text{w1})$ (Resolve 12, 4) Unifier: $\{\text{Amy}/\text{x1}, \text{D}/\text{z1}\}$
14. $\text{Own}(\text{Amy}, \text{F}(\text{Amy}))$ (Resolve 7, 3) Unifier: $\{\text{Amy}/\text{x4}\}$
15. $\neg \text{Cat}(\text{F}(\text{Amy})) \vee \neg \text{Cat}(\text{w1}) \vee \neg \text{Chase}(\text{D}, \text{w1})$ (Resolve 14, 13) Unifier: $\{\text{F}(\text{Amy})/\text{y1}\}$
16. $\text{Chase}(\text{D}, \text{R}(\text{D}))$ (Resolve 6, 1) Unifier: $\{\text{D}/\text{x3}\}$
17. $\neg \text{Cat}(\text{F}(\text{Amy})) \vee \neg \text{Cat}(\text{R}(\text{D}))$ (Resolve 16, 15) Unifier: $\{\text{R}(\text{D})/\text{w1}\}$

18. $\text{Cat}(\text{R}(\text{D}))$ (Resolve 5, 1) Unifier: $\{\text{D}/\text{x2}\}$
19. $\neg \text{Cat}(\text{F}(\text{Amy}))$ (Resolve 18, 17)
20. $\text{Cat}(\text{F}(\text{Amy}))$ (Resolve 8, 3) Unifier: $\{\text{Amy}/\text{x5}\}$
21. \emptyset (Resolve 20, 19)