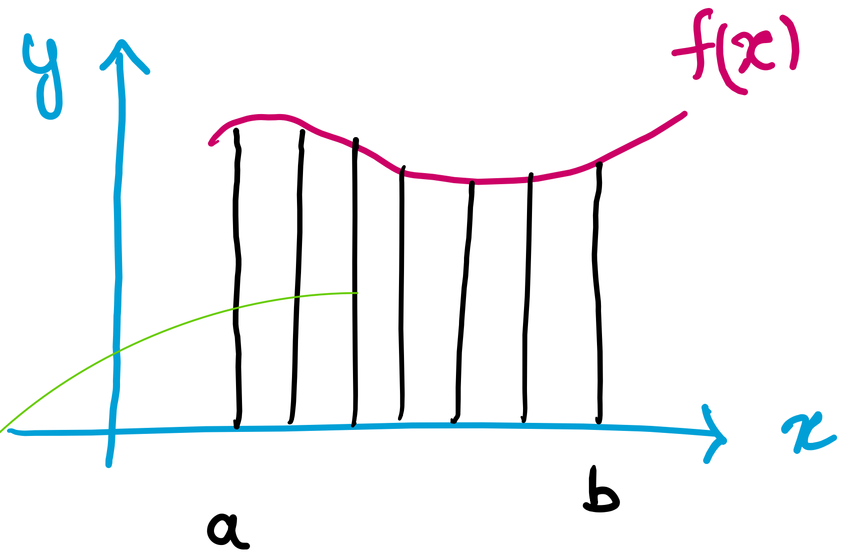
NM Lab-1

Trapezoidal Rule & Simpson's 1/3 Rule.

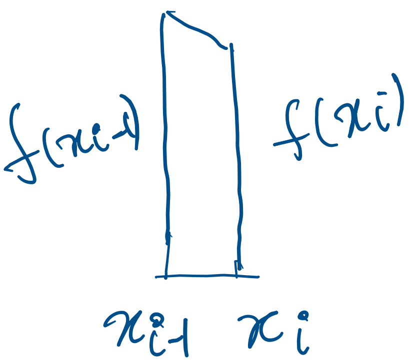
⊛ Trapezoidal Rule



divide into n parts.

$$\text{width of segment} = \frac{b-a}{n} = \Delta$$

Area of any i^{th} segment



$$\text{area}_i = \frac{1}{2} \times \Delta \times \left(f(x_{i-1}) + f(x_i)\right)$$

Linear approximation.

$$\text{Total area} = \sum_{i=1}^{n} \text{Area}_i \quad (\text{iteration})$$

$$= \frac{1}{2} \times \frac{b-a}{n} \left[f(x_0) + 2f(x_1) + 2f(x_2) \cdots\cdots 2f(x_{n-1}) + f(x_n) \right]$$

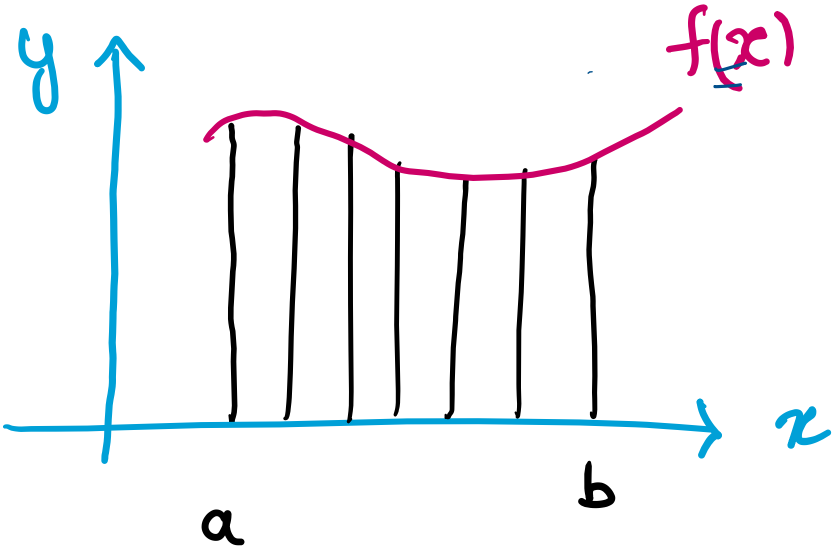
where

$$x_0 = a \qquad \Delta = \frac{b-a}{n}$$

$$x_n = b$$

$$x_i = a + \Delta(i)$$

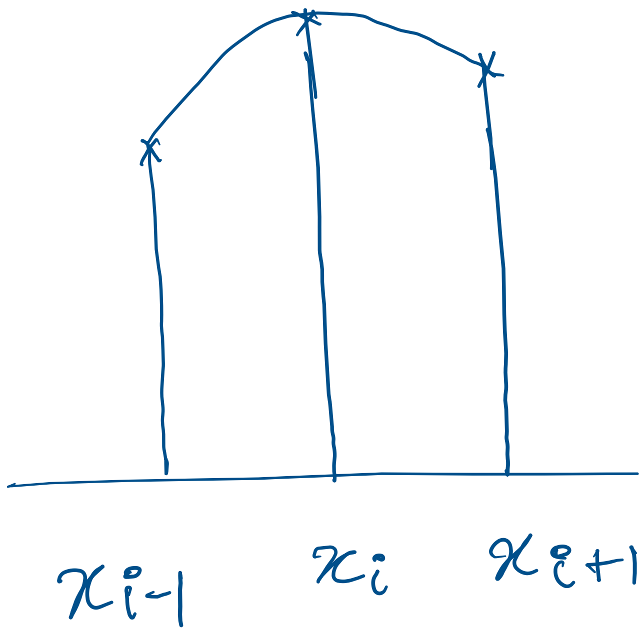
Simpson's 1/3rd Rule



divide into n (even parts)

Quadratic approximation

consider two segments
with x_i as mid point.



$i \to$ odd number

$$f(x) = ax^2 + bx + c$$

$$\begin{bmatrix} x_{i-1}^2 & x_{i-1} & 1 \\ x_i^2 & x_i & 1 \\ x_{i+1}^2 & x_{i+1} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \begin{bmatrix} f(x_{i-1}) \\ f(x_i) \\ f(x_{i+1}) \end{bmatrix}$$

after solving for unknows

a, b, c

and finding the area..

$$= \int_{x-1}^{x+1} ax^2 + bx + c$$

$$= \frac{1}{3} \Delta \left[f(x_{i-1}) + 4f(x_i) + f(x_{i+1}) \right]$$

total area

$$= \frac{1}{3} \frac{b-a}{n} \Big[f(x_0)$$

$$4f(x_1) + 4f(x_3) \cdots + 4f(x_{n-1})$$

$$2f(x_2) + 2f(x_4) \cdots + 2f(x_{n-2})$$

$$+ f(x_n) \Big]$$

where

$x_0 = a$

$\Delta = \frac{b-a}{n}$

$x_n = b$

$x_i = a + \Delta (i)$