Implementation of Grover's Algorithm based on Quantum Reservoir Computing

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Abstract—**Quantum computing represents the leading edge of computational technology, leveraging the principles of quantum mechanics to execute targeted computations much faster than classical computers. In contrast to classical bits, which are limited to representing either 0 or 1, qubits, or quantum bits, exhibit the extraordinary property of superposition. This distinctive characteristic enables qubits to simultaneously occupy multiple states, empowering quantum computers to explore numerous potential solutions to a problem concurrently. This feature makes quantum computing particularly potent for specific tasks. Recent research endeavors have been sparked by the potential of advanced quantum computing technology, leading to the creation of simulations of quantum computers using classical hardware. Grover's quantum search algorithm serves as a notable illustration of quantum computing application, enabling quantum computers to conduct a database search within an unsorted array with a quadratic speedup in time efficiency compared to classical computers. This document presents the quantum Grover search algorithm and its application through 5-qubit quantum circuits, as well as a design framework to simplify the creation of an oracle for a greater number of qubits.**

Keywords—**Quantum computation, Qubits, Oracle, Grover's algorithm, IBM Qiskit.**

I. INTRODUCTION

Quantum computing is a new frontier in computer science that utilises the unique phenomena of quantum physics to solve problems that are too complex for classical computers to handle. [1]. Quantum computing algorithms are specialised procedures designed to harness the unique capabilities of quantum computers to solve problems more efficiently than classical algorithms. These algorithms leverage quantum phenomena like superposition and entanglement to explore multiple possibilities simultaneously, leading to potential speedups for certain types of tasks. One such algorithm is Grover's algorithm [2]. With Grover's algorithm searching an unsorted database is quadratically faster than classical algorithms. Grover's algorithm has been implemented on publicly available quantum computers like IBM Q 5 Tenerife (5-qubit

chip) and IBM Q 16 Rüschlikon (16-qubit chip) [3] and on programmable quantum computer consisting of a linear chain of five trapped ions [4] . While the early implementations were on small-scale devices with limited qubits, significant progress has been made in recent years.

Our goal is to investigate the readiness of current IBM quantum computers to support the execution of Grover's algorithm.

II. BACKGROUND AND METHODOLOGY

The Grover search algorithm, conceived by Lov Grover in 1996, stands as a quantum computing method offering a quadratic acceleration compared to classical counterparts, particularly for solving unstructured search problems [8]. It has gained renown for its proficiency in searching through unsorted databases, but its utility extends to a spectrum of tasks, encompassing cryptographic problem-solving and quantum system simulation. This can speed up a search problem quadratically. For N number of unsorted data classical computer require O (N) operations Whereas Grover is optimal and can do this in O($\sqrt{N}$) operation. The following provides a synopsis of the workings of the Grover search algorithm [9]:

A. Initialisation

Commencing the process involves establishing a superposition of all conceivable states. For instance, when seeking an item in a database housing N item, quantum parallelism is harnessed to generate a superposition of all N states as shown in Fig.1. Achieving this involves a sequence of quantum gate operations.

B. Oracle Function

Grover's algorithm hinges on the application of an oracle function, often denoted as "U_f". This oracle acts to mark the target state(s) by inverting their sign. For instance, if the objective is to locate a specific item in a database, the oracle
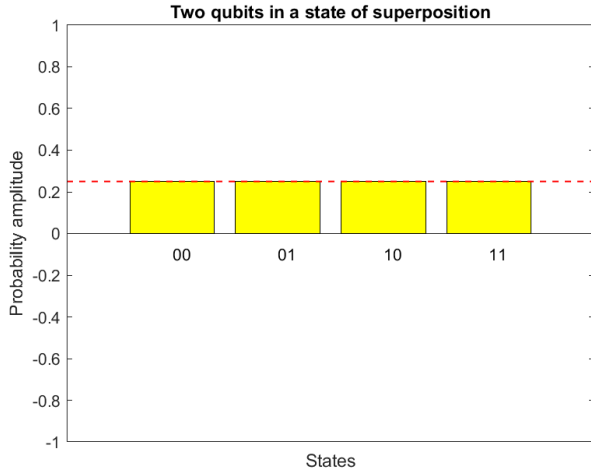
Fig. 1. Superposition of 2 qubits

would negate the amplitude corresponding to the target item. In Fig. 2, it is graphically shown the oracle function flipping the correct target.
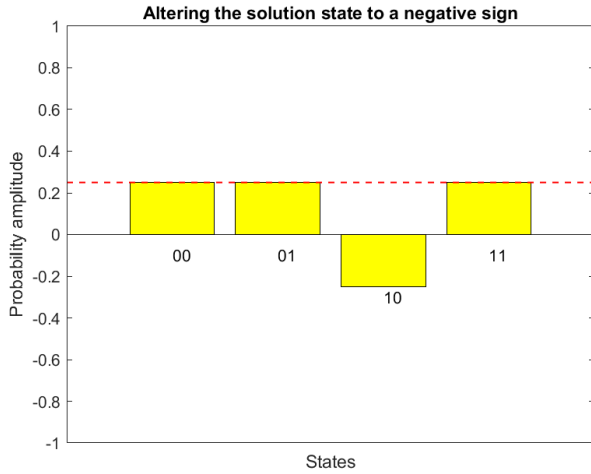

Fig. 2. Altering the sign of solution state(10)

C. Amplitude Amplification

The core of the Grover algorithm is the process of amplitude amplification, which entails two central maneuvers:

- **Inversion around the Mean**: During this stage, the amplitudes are mirrored around their average value, thus boosting the amplitude of the desired state(s) while reducing the amplitudes of the non-desired states.
- **Grover diffusion operator**: In this step, the amplitudes of the target state(s) are further augmented through the application of a suite of quantum gates [10].

After acting of Grover diffusion operator, the final output will have amplified magnitude as shown in Fig. 3.

D. Reiteration

Step. 2 and step. 3 are iterated approximately $\sqrt{N}$ times to maximize the likelihood of detecting the correct state. This number of iterations ensures that the probability of identifying the correct state approaches near certainty.
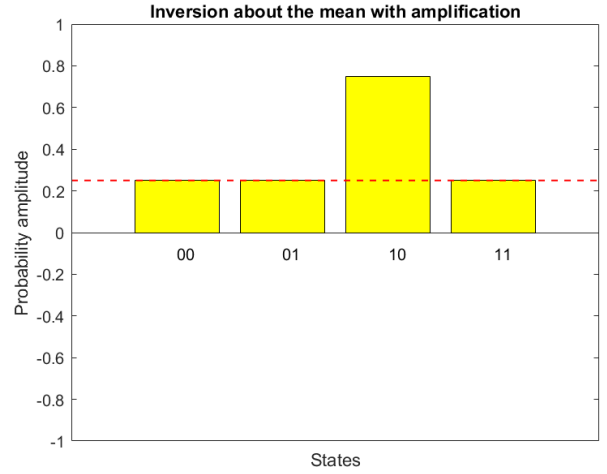

Fig. 3. Inversion about the mean with amplification.

E. Measurement

Ultimately, the quantum state is subjected to measurement. The target state is discerned with notably higher probability in comparison to the non-target state.

III. CONSTRUCTION OF ORACLE AND DIFFUSION OPERATOR

The Grover iteration, also known as the Grover operator, constitutes a crucial component of the quantum search process. It encompasses two distinct phases. At the outset, the oracle function modifies the phase of a single amplitude within a marked state. Upon completion of this phase, referred to as the diffusion layer, the indicated state amplitude is flipped. Consequently, while the amplitudes of the other states remain unchanged, the target state assumes an inverted state, resulting in a notable increase in its amplitude and a slight decrease in the amplitudes of the other states. The block diagram of Grover iteration circuit is shown below in Fig. 4.
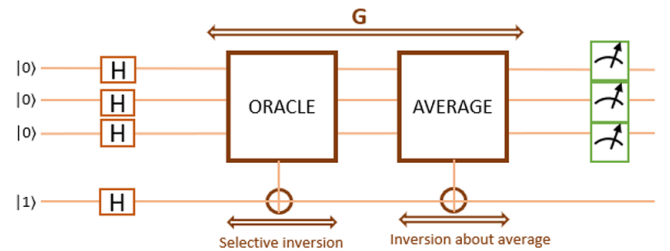

Fig. 4. Grover Iteration circuit

A. Oracle

In quantum computing, a quantum oracle [11] is a computational component that furnishes data pertinent to the particular prob lem or task undergoing processing by the quantum algorithm. The quantum oracle performs a bit-flip on the oracle qubit when the input corresponds to a valid solution. Oracle can be understood as described below.

$$Oracle(x) = (-1)^{f(x)}|x\rangle$$

$$f(t) = \begin{cases} 1, & t = x \\ 0, & otherwise \end{cases}$$

Oracles can be classified into 2 types, phase and boolean. Boolean oracles make use of ancilla qubits while phase oracles contain same number of qubits as the search problem. We have used boolean oracles depicted in Fig 5, Fig 6 and Fig 7 to implement Grover's search algorithm.
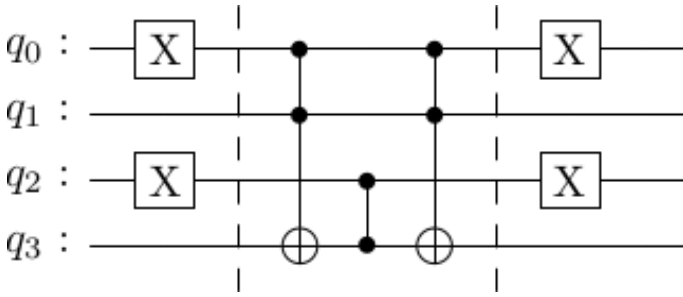

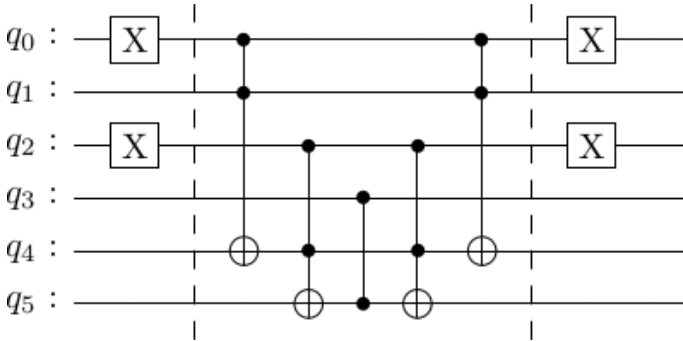Fig. 5. Oracle design for 3-qubit Grover's algorithm with 010 as solution.

Fig. 6. Oracle design for 4-qubit Grover's algorithm with01011 as solution.

B. Diffusion operator

The diffusion operation can be realised through the sequence: **HX+C^{n-1}Z+XH**. In this scenario, H symbolises the Hadamard gate, X represents the Pauli-X gate and $C^{n-1}Z$ is a controlled Z gate with n-1 control qubits. This sequence effectively flips the amplitudes around their mean value.

IV. RESULTS AND DISCUSSION

3-qubit, 4-qubit and 5-qubit Grover circuits are simulated using AerSimulator from Qiskit. In addition to that 3-qubit Grover's algorithm is implemented using phase oracles [4] on ibm_kyiv quantum computer.
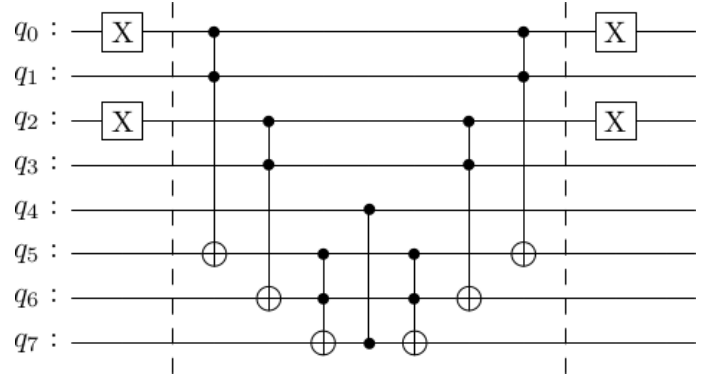

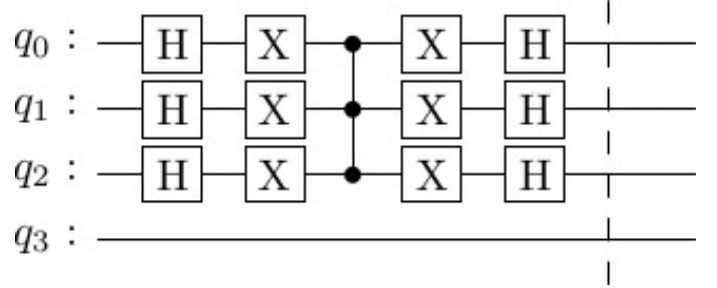Fig. 7. Oracle design for 5-qubit Grover's algorithm with 01011 as solution.

Fig. 8. Diffusion operator design for 3-qubit Grover's algorithm

A. Simulation Results

Simulation results for 3-qubit, 4-qubit and 5-qubit Grover search algorithm are shown in Fig 9, Fig 10 and Fig 11 respectively. For the 3-qubit case when searching for x=010, the algorithm yielded a success probability of 0.933. Similarly for 4 and 5 qubit cases the algorithm yielded a success probability of 0.959 and 0.999 when searching for x=0101 and x=01011 respectively. These results closely match the simulation results with **HX + ORACLE + XH** as the oracle [17]. By replacing the oracle with a C^{n-1}Z we can reduce the number of gates required to implement Grover's search algorithm.

B. Experimental Results

For every possible value of x (with 3 classical bits), a 3-qubit Grover circuit is constructed with phase oracle and the diffusion operator discussed above. The circuits are then transpiled and run on ibm_kyiv quantum computer. The circuits are run for 1024 times and the success probabilities are plotted against the value of x we are searching in Fig 12. In all the cases we observe that the success probability is more than 0.5.

V. CONCLUSION

In summary, quantum computing presents an intriguing pathway for addressing intricate problems that surpass the capabilities of classical computers. In contrast to classical search algorithms, which require O(N) operations for N items, Grover's algorithm achieves this task in approximately O($\sqrt{N}$)
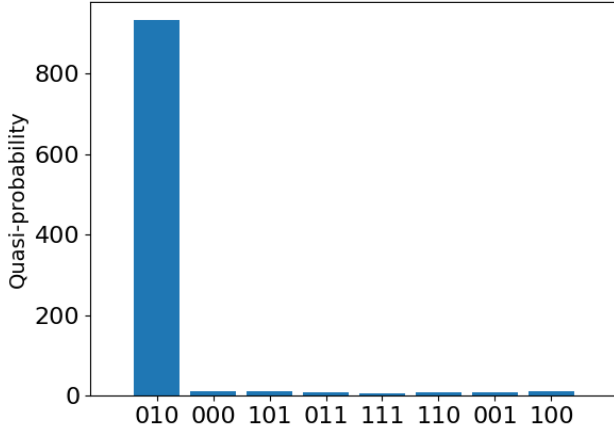
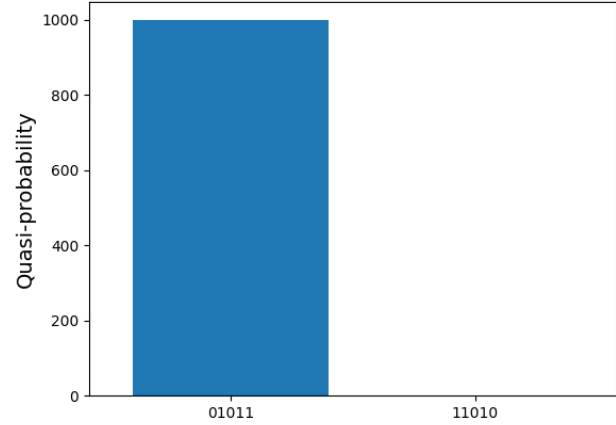Fig. 9. Results obtained when 3-qubit Grover's algorithm using oracle from Fig 5

Fig. 11. Results obtained when 5-qubit Grover's algorithm using oracle from Fig 7
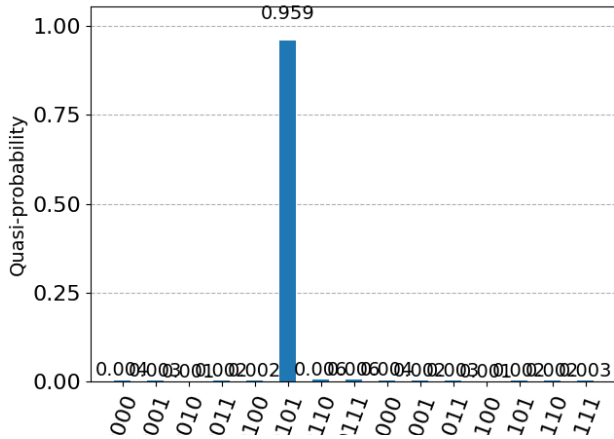

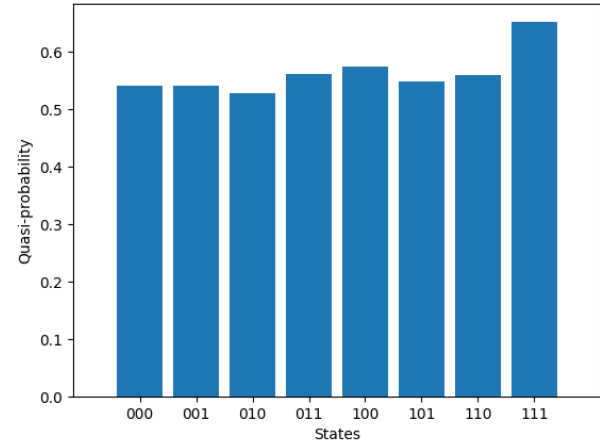Fig. 10. Results obtained when 4-qubit Grover's algorithm using oracle from Fig 6

Fig. 12. Results obtained when 3-qubit Grover's algorithm is implemented using phase oracles on ibm_kyiv

operations. Experimental results for Grover's algorithm on 3-qubit cases consistently yielded success probabilities exceeding 0.5, indicating the algorithm's effectiveness in practical applications. Subsequently, using this 5-qubit setup as a benchmark, the algorithm will be extended to n-qubits.

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