Implementation of Grover's Algorithm based on Quantum Reservoir Computing

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***Abstract*—Quantum computing represents the leading edge of computational technology, leveraging the principles of quantum mechanics to execute targeted computations much faster than classical computers. In contrast to classical bits, which are limited to representing either 0 or 1, qubits, or quantum bits, exhibit the extraordinary property of superposition. This distinctive characteristic enables qubits to simultaneously occupy multiple states, empowering quantum computers to explore numerous potential solutions to a problem concurrently. This feature makes quantum computing particularly potent for specific tasks. Recent research endeavors have been sparked by the potential of advanced quantum computing technology, leading to the creation of simulations of quantum computers using classical hardware. Grover's quantum search algorithm serves as a notable illustration of quantum computing application, enabling quantum computers to conduct a database search within an unsorted array with a quadratic speedup in time efficiency compared to classical computers. This document presents the quantum Grover search algorithm and its application through 5-qubit quantum circuits, as well as a design framework to simplify the creation of an oracle for a greater number of qubits.**

***Keywords*—Quantum computation, Qubits, Oracle, Grover's algorithm, IBM Qiskit.**

I. Introduction

The exploration of quantum computing [1][2] falls within the realm of quantum information science, which revolves around the fundamental principles of storing and manipulating information. In this work, we delved into quantum computing, acquiring a comprehensive understanding of quantum bits and their properties, as well as leveraging these properties to tackle problems. We familiarized ourselves with quantum gates and their operations on qubits, simulating all the fundamental quantum gates [3][4]. Additionally, we delved into the Grover search algorithm and implemented quantum gates for Grover operations. Quantum computers exhibit significantly faster speeds compared to classical computers [5][6]. In the case of an unsorted dataset with size N, classical computers usually demand O(N) operations, whereas Grover's algorithm accomplishes this task optimally in O($\sqrt{N}$) operations.
We executed the algorithm using Qiskit, an IBM tool for computing quantum circuits, and conducted simulations for the Grover algorithm [7], presenting the results graphically with the probability of obtaining the correct output.

Within Grover's quantum search algorithm, a network with n qubits harbors 2^n = N states, with each state bearing a probability of $1/N$ for discovery. Consequently, the amplitude of each state is $1/\sqrt{N}$. Conversely, classical systems tackling the same problem necessitate a maximum of O(N) trials.

II. Background and Methodology

The Grover search algorithm, conceived by Lov Grover in 1996, stands as a quantum computing method offering a quadratic acceleration compared to classical counterparts, particularly for solving unstructured search problems [8]. It has gained renown for its proficiency in searching through unsorted databases, but its utility extends to a spectrum of tasks, encompassing cryptographic problem-solving and quantum system simulation. This can speed up a search problem quadratically. For N number of unsorted data classical computer require O (N) operations Whereas Grover is optimal and can do this in O($\sqrt{N}$) operation. The following provides a synopsis of the workings of the Grover search algorithm [9]:

A. Initialisation

Commencing the process involves establishing a superposition of all conceivable states. For instance, when seeking an item in a database housing N item, quantum parallelism is harnessed to generate a superposition of all N states as shown in Fig.1. Achieving this involves a sequence of quantum gate operations.

B. Oracle Function

Grover's algorithm hinges on the application of an oracle function, often denoted as "U_f". This oracle acts to mark

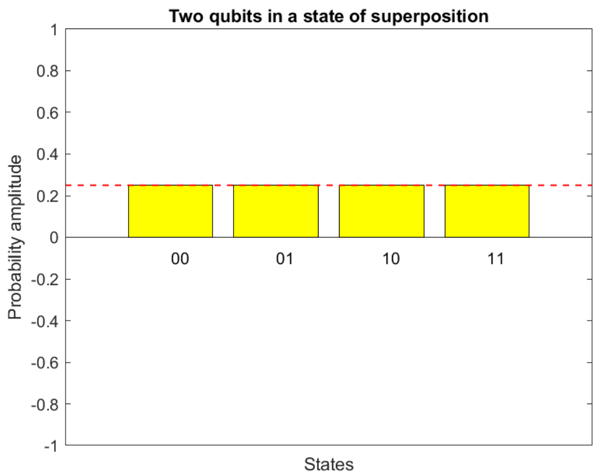

Fig. 1. Superposition of 2 qubits

the target state(s) by inverting their sign. For instance, if the objective is to locate a specific item in a database, the oracle would negate the amplitude corresponding to the target item. In Fig. 2, it is graphically shown the oracle function flipping the correct target.

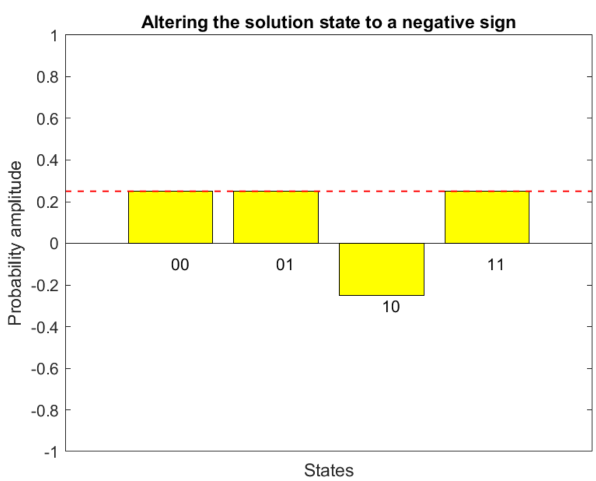

Fig. 2. Altering the sign of solution state(10)

C. Amplitude Amplification

The core of the Grover algorithm is the process of amplitude amplification, which entails two central maneuvers:

- **Inversion around the Mean**: During this stage, the amplitudes are mirrored around their average value, thus boosting the amplitude of the desired state(s) while reducing the amplitudes of the non-desired states.
- **Grover diffusion operator**: In this step, the amplitudes of the target state(s) are further augmented through the application of a suite of quantum gates [10].

After acting of Grover diffusion operator, the final output will have amplified magnitude as shown in Fig. 3.

D. Reiteration

Step. 2 and step. 3 are iterated approximately $\sqrt{N}$ times to maximize the likelihood of detecting the correct state. This number of iterations ensures that the probability of identifying the correct state approaches near certainty.

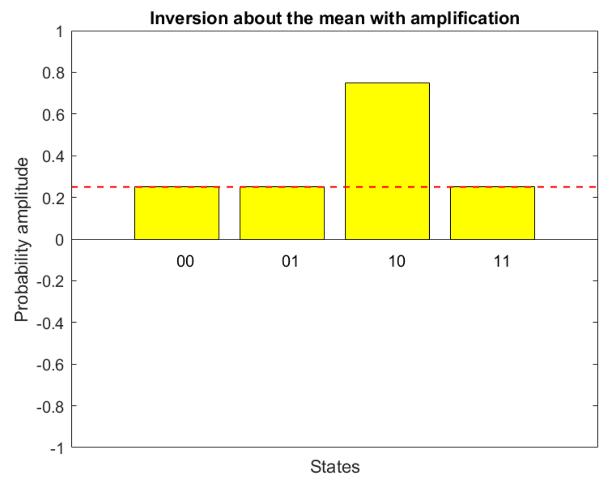

Fig. 3. Inversion about the mean with amplification.

E. Measurement

Ultimately, the quantum state is subjected to measurement. The target state is discerned with notably higher probability in comparison to the non-target state.

III. CONSTRUCTION OF ORACLE AND DIFFUSION OPERATOR

The Grover iteration, also known as the Grover operator, constitutes a crucial component of the quantum search process. It encompasses two distinct phases. At the outset, the oracle function modifies the phase of a single amplitude within a marked state. Upon completion of this phase, referred to as the diffusion layer, the indicated state amplitude is flipped. Consequently, while the amplitudes of the other states remain unchanged, the target state assumes an inverted state, resulting in a notable increase in its amplitude and a slight decrease in the amplitudes of the other states. The block diagram of Grover iteration circuit is shown below in Fig. 4.

A. Oracle

In quantum computing, a quantum oracle [11] is a computational component that furnishes data pertinent to the particular prob lem or task undergoing processing by the quantum algorithm. The quantum oracle performs a bit-flip on the oracle qubit when the input corresponds to a valid solution. Oracle can be understood as described below.

$Oracle(x) = (-1)^{f(x)}|x\rangle$

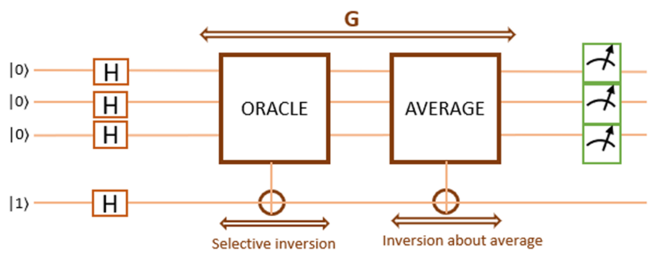

Fig. 4. Grover Iteration circuit

$$f(t) = \begin{cases} 1, & t = x \\ 0, & otherwise \end{cases}$$

Oracles can be classified into 2 types, phase and boolean. Boolean oracles make use of ancilla qubits while phase oracles contain same number of qubits as the search problem. We have used boolean oracles depicted in Fig 5, Fig 6 and Fig 7 to implement Grover's search algorithm.

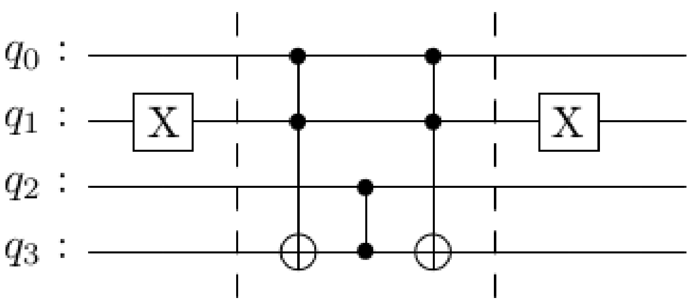

Fig. 5. Oracle design for 3-qubit Grover's algorithm with 101 as solution.

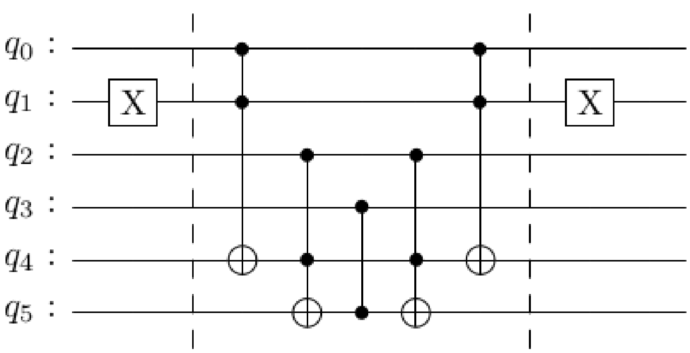

Fig. 6. Oracle design for 4-qubit Grover's algorithm with 1011 as solution.

B. Diffusion operator

The diffusion operation can be realized through the sequence: **HX+C$^{\mathbf{n-1}}$Z+XH**. In this scenario, H symbolizes the Hadamard transform,while X represents the Pauli-X gate. This sequence effectively flips the amplitudes around their mean value.

IV. Results

3-qubit, 4-qubit and 5-qubit Grover circuits are simulated using AerSimulator from Qiskit. In addition to that 3-qubit

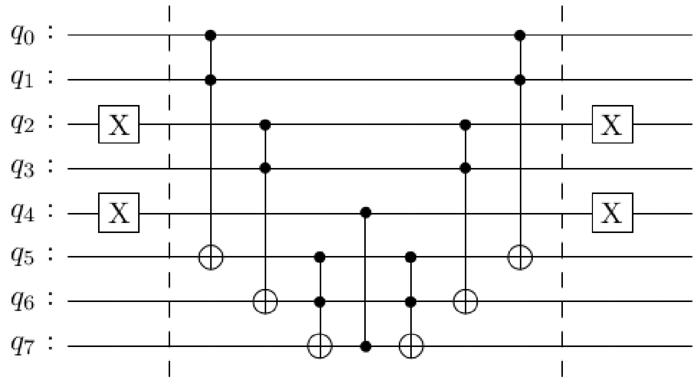

Fig. 7. Oracle design for 5-qubit Grover's algorithm with 11010 as solution.

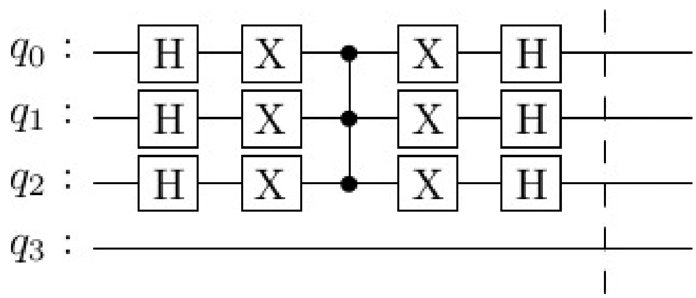

Fig. 8. Diffusion operator design for 3-qubit Grover's algorithm

Grover's algorithm is implemented using phase oracles on ibm_kyiv quantum computer.

A. Simulation Results

Simulation results for 3-qubit, 4-qubit and 5-qubit Grover search algorithm are shown in Fig 9, Fig 10 and Fig 11 respectively. For 3-qubit case when searching for x=101, the algorithm yielded a success probability of 0.944. Similarly for 4 and 5 qubit cases the algorithm yielded a success probability of 0.965 and 0.999 when searching for x=1011 and x=01011. In Qiskit qubits are indexed in reverse order compared to the standard physical convention. That is why for 4-qubit and 5-qubit cases the algorithm finds **1101** and **01011** with high probability.

B. Experimental Results

For every possible value of x (with 3 classical bits), a 3-qubit Grover circuit is constructed with phase oracle and the diffusion operator discussed above. The circuits are then transpiled and ran on ibm_kyiv quantum computer. The circuits are ran for 1024 times and the success probabilities are plotted against the value of x we are searching in Fig 12. In all the cases we observe that the success probability is more that 0.5.

V. Conclusion

In summary, quantum computing presents an intriguing pathway for addressing intricate problems that surpass the capabilities of classical computers. In contrast to classical

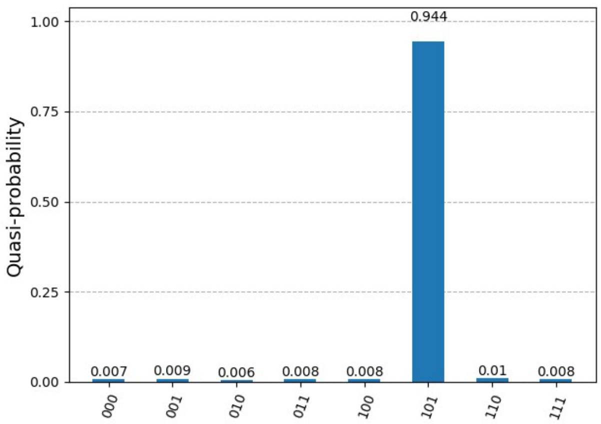

Fig. 9. Results for 101 detection

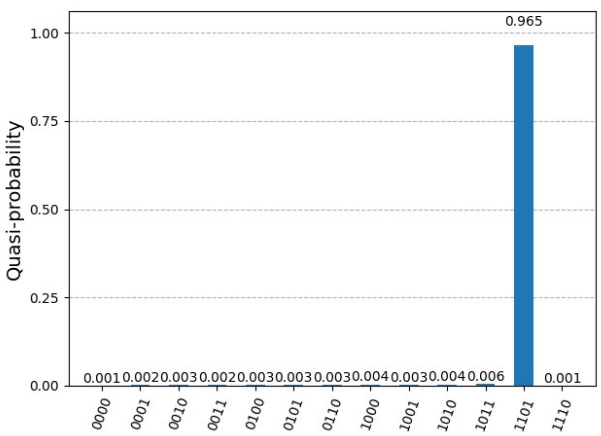

Fig. 10. Results for 1011 detection

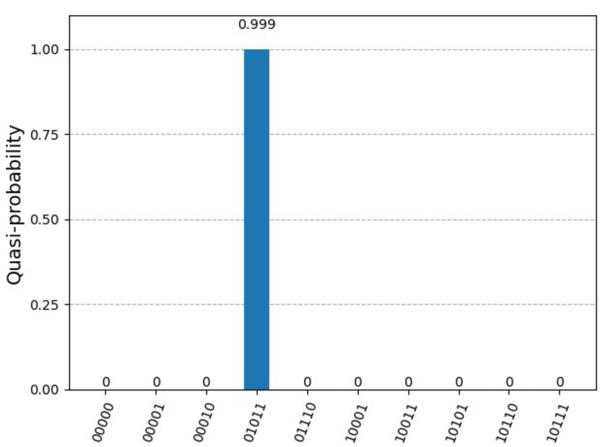

Fig. 11. Results for 11010 detection

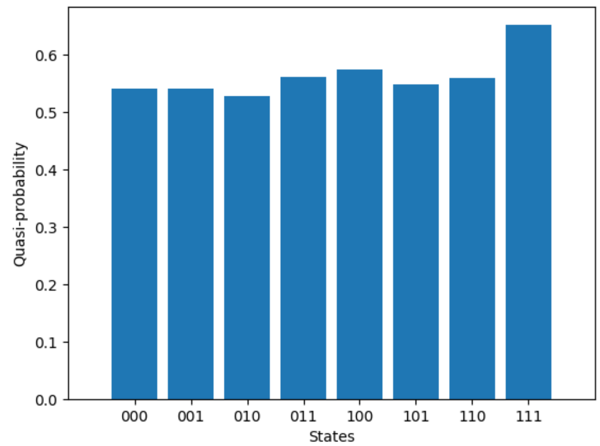

Fig. 12. Results obtained when 3-qubit Grover's algorithm is implemented using phase oracles on ibm_kyiv

search algorithms, which require O(N) operations for N items, Grover's algorithm achieves this task in approximately O($\sqrt{N}$) operations. Experimental results for Grover's algorithm on 3-qubit cases consistently yielded success probabilities exceeding 0.5, indicating the algorithm's effectiveness in practical applications. Subsequently, using this 5-qubit setup as a benchmark, the algorithm will be extended to n-qubits.

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